MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 14 /1975

Kommutative Algebra und Algebraische Geometrie

30.3. bis 5.4.1975

Zum ersten Mal fand in diesem Jahr in Oberwolfach eine Tagung über Kommutative Algebra und Algebraische Geometrie statt. Sie stand unter der Leitung von E. Kunz (Regensburg), H.-J. Nastold (Münster) und L. Szpiro (Paris).

Trotz mäßigem Wetter herrschte eine gelockerte und anregende Atmosphäre mit vielen Gesprächen und Diskussionen unter den Teilnehmern.

Die 17 Vorträge befaßten sich mit den verschiedensten Spezialfragen. Auf dem Gebiet der Kommutativen Algebra Untersuchungen über Fixringe von Gruppenoperationen (1,2), über Erzeugendensysteme von Idealen, Moduln und freien Auflösungen (3-5), analytische Lösungen von Funktionalgleichungen (im Anschluß an M.Artin) (6), über analytische Verzweigung (7), Derivationen (8), seminormale Ringe (9), Ringe holomorpher Funktionen (10). Auf dem Gebiet der Algebraischen Geometrie Verallgemeinerung der Ergebnisse von W. Barth und A. van de Ven über Varietäten von kleinem Grad auf Grundkörper beliebiger Charakteristik (11), eine neue Definition von Chernschen Klassen mit Ansätzen zur Verallgemeinerung des Riemann-Roch'schen Satzes (12,13), Cohomologie von Schemata mit Gruppenoperation (14,15), Modulräume (16), lokale Picard-Schemata (17).

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Vortragsauszüge

R.M. FOSSUM: Complete non Cohen-Macaulay local factorial rings in characteristic p

Let the cyclic group of order p^n act on the ring of formal power series $k[[X_0,\ldots,X_{p^n-1}]]$ by cyclically permuting the indeterminates. The ring of invariants is shown to be a noetherian complete local factorial ring of depth at most $p^{n-1} + 2$, and hence not Cohen-Macaulay when $p^n > 4$. When $p^n = 4$, the ring of invariants is the completion of Bertin's example and hence has depth 3, so is not Cohen-Macaulay.

The calculation of the cohomology groups $H^i(\mathbb{Z}/p^n\mathbb{Z}, G_m k[[X_0,\ldots,X_{p^n-1}]])$ used in establishing that the ring of invariants is factorial also has applications to the theory of Azumaya algebras.

M.P. MALLIAVIN: Some rings of invariants of nilpotent groups

Let k be an algebraically closed field of characteristik O. All algebraic affine k-groups are supposed to be linear. Let R be a noetherian and regular k-algebra. All the actions of algebraic affine k-groups V on R are supposed to be rational and by the way of k-automorphisms of k-algebras. We note R^V the k-algebra of invariants.

Proposition 1 - Let G be a connected and simply connected semi-simple algebraic affine k-group. Let H be a connected nilpotent maximal sub-group of G. Then R^H is a noetherian Cohen-Macaulay k-algebra, if G acts on R.

Proposition 2 - Let G be Sl(2,k) and N be a maximal unipotent subgroup of G. Then R^N is a Cohen-Macaulay noetherian k-algebra.

J. HERZOG: On the Hilbert-function of 1-dim. CM-rings

Examples of 1-dim. CM-rings (R,m) are given with $\mu(m) - \mu(m^2) > 0$ (μ denotes the minimal number of generators of an ideal). It is shown that $\mu(m^n) > n$ if n is less than the multiplicity of R.

W. BRUNS: On systems of generators of a module

We generalize the theorem of D. Eisenbud and E.G. Evans on basic elements in finitely generated modules M over a noetherian ring R (J. Algebra 27, 278-3o5) to an assertion about the existence of elements in M, which are basic (at certain prime ideals p) in factor modules $M/M_1,\dots,M/M_s$. This result enables us to construct systems of generators of M, whose subsets $\{x_{i_1},\dots,x_{i_k}\}$ are parts of minimal systems of generators of M_p, if ht p (in a second version: codh R_p) does not exceed a certain number depending on k. As a corollary one gets a positive answer to a question of D. Buchsbaum and D. Eisenbud concerning determinantal ideals (Advances in Math. 12, 84-139, p.125, Remark).

D. EISENBUD: Resolutions of modules over complete intersections

Theorem: Let S be a regular local ring, and let $R = S/_{(f)}$. If M is a finitely generated R-module, then the minimal free (over R) resolution of M is eventually periodic, of period ≤ 2.

This theorem and its generalization to the case of complete intersections ($R = S/_{(f_1,\dots,f_n)}$, $f_1,\dots,f_n$ an S-sequence) will be discussed.

M. ANDRÉ: Analytic solutions in positive characteristic

By means of the excellence of the rings of convergent power series and of the cotangent complex theory, Artin's result on analytic solutions is proved in characteristic p too (see Inventiones mathematicae 5 (1968) 277-291 for the case of characteristic O). If a set of analytic equations $f_1(x,y)=\ldots=f_2(x,y)=0$ with m variables x and n variables y has a formal solution $\bar{y}(x)$, then it has an analytic solution $y(x)$ agreeing with $\bar{y}(x)$ up to degree c fixed in advance.

C. LECH: Flat couples associated with analytic ramifications

A couple (R_o,R_1) of (noetherian) local rings is called flat if R_1 contains R_o as a subring and if R_1 is faithfully flat as an R_o-module. It is called equi-dimensional if R_o and R_1 have the same Krull dimension. Our result, obtained together with Tomas Larfeldt, can be described as follow:

Any equidimensional flat couple of local rings is in a certain sense equivalent to one of the form $(R_p,R^*_{p^*})$, where R is a local ring, R^* its completion, p a prime ideal of depth one in R, and p^* a minimal prime ideal of pR^*. In a somewhat weaker sense it is equivalent to a (non-flat) couple of the form (R_p,R).

The equivalence relations preserve or almost preserve, respectively, the Hilbertseries and Poincaréseries of the local rings involved.

H. MATSUMURA: Integrable derivations in rings of char. p.

Let A be a noetherian ring. A (Hasse - F.K. Schmidt) differentiation of A is a sequence $\underline{D} = (D_o,D_1,D_2,\ldots)$ of additive maps D_i: $A \longrightarrow A$ such that D_o = identity and $D_n(ab) = \sum D_i(a)D_{n-i}(b)$.

A derivation D of A into itself is called <u>integrable</u> if there exists a differentiation $\underline{D} = (1, D_1, D_2, \ldots)$ with $D_1 = D$.

Examples: 1) If A is a field, any derivation is integrable.

2) If A is a regular local ring of char. p, $D \in \mathrm{Der}(A)$, $x \in m_A$, $Dx \notin m_A$, $D^p = aD$ for some $a \in A$, then D is integrable in the completion $\hat{A}$ of A (i.e. the natural extension of D to $\hat{A}$ is integrable).

Let (A,m,K) be a local domain of char. p, and k be a subfield of A containing a p-basis of K. Let $\mathrm{Der}_k^*(A)$ denote the set of the derivations of A over k which are integrable in $\hat{A}$ over k. Then it is a submodule of $\mathrm{Der}_k(A)$ and we have

$$\mathrm{rank}_A \, \mathrm{Der}_k^*(A) \leq \dim A.$$

S. GRECO: <u>Seminormality of Rings and Algebras</u>.

A noetherian reduced ring A with finite integral closure $\bar{A}$ is <u>seminormal</u> (SN) if whenever $A \subset B \subset \bar{A}$ and $\mathrm{spec}(B) \longrightarrow \mathrm{spec}(A)$ is a homeomorphism with trivial residue field extensions, then $B = A$. This is equivalent to: $\mathrm{Pic}(A) = \mathrm{Pic}(A[T])$. Results:

1. If $f: A \longrightarrow A'$ is normal and A is SN, A' is SN. In particular: (a) $S^{-1}A$ is SN if A is; (b) the completion of an excellent SN local ring is SN.
2. A is SN iff A_p is SN whenever $\mathrm{depth}(A_p) = 1$.
3. A Gorenstein ring A is SN iff for any 1-dimensional localization (B,m,k) $m = m\bar{B}$ and either: (a) $\bar{B}$ is local, $\bar{B}/m\bar{B} = K$ is a field and $[K:k] \leq 2$; or (b) $B/mB = k \times k$.
4. A surface in 3-space is seminormal iff it has at most double biplanar curves; a surface with ordinary singularities only is SN.
5. Let G be a finite abelian group of order m. Then group ring AG is SN iff A is SN, $m1_A$ is regular and A/mA is reduced.

K. LANGMANN: Noethereigenschaften für den Ring aller holomorphen Funktionen

Ist G eine kompakte Teilmenge eines komplexen Raumes, so ist der Ring $\Gamma(G,\mathcal{O})$ aller in einer Umgebung von G holomorphen Funktionen im allgemeinen nicht noethersch. Wir zeigen, daß dies genau dann der Fall ist, wenn alle von n = dim G Elementen erzeugten Ideale endlich viele minimale Primoberideale haben. Dies ist weiter äquivalent damit, daß diese Ideale nur endlich erzeugte minimale Primoberideale haben. Schließlich geben wir noch ein geometrisches Kriterium, wann $\Gamma(G,\mathcal{O})$ noethersch ist.

R. HARTSHORNE: Projective varieties of small degree

I will discuss the result that "a non-singular projective variety whose degree is small sompared to the dimension of the least projective space containing it, is necessarily a complete intersection", following work of W.BARTH, A.van de VEN, and myself.

W. FULTON: Riemann-Roch and Canonical Classes for Schemes (2 Vorträge)

A locally noetherian scheme X has a Chow homology group A.X of cycles modulo rational equivalence, which is covariant for proper morphisms. It is possible to define Chern classes of bundles as operators on these Chow groups, with all the expected properties. This enables one to formulate a Riemann-Roch conjecture for quasi-projective schemes over a regular base scheme which would generalize the theorem of Baum, MacPherson and the speaker when the base is a field [I.H.E.S., to appear]. A corollary would be that the Grothendieck group of sheaves and the Chow group agree up to torsion. In codimension one this is related to a proof that regular local rings are factorial.

Any smoothable morphism from a scheme X to a scheme Y has a canonical class in the Chow group A.X; when the map is a complete intersection the canonical class is dual to the Chern class of the virtual tangent bundle T_f. When Y is a point, we get canonical classes on singular varieties. The construction depends on a notion of inverse Chern classes for sheaves.

G. HORROCKS: Properties of schemes inherited from orbits

Certain cohomological properties of schemes with a group action can be deduced from the corresponding properties for the orbits. Sufficient conditions for such properties are obtained and these are used to relate the low dimensional cohomology of schemes with solvable group actions to that of suitably chosen low dimensional invariant subschemes.

B. IVERSEN: Equivariant cohomology for torus action

An equivariant cohomology theory for torus actions are constructed on the basis of etale cohomology, and a localization theoren for this theory is proved. This is applied to analyse ordinary cohomology of a smooth projective variety X on which a 1-dim. torus acts. The result is a computation of $H^{\cdot}(X)$ in terms of cohomology of the fixed points and the sign of the weights in the tangent spaces of the fixed points. In case k = C this is refined to calculate Hodge cohomology, $H^{\cdot}(X,\Omega^{\cdot}_X)$.

H. POPP: Moduli of algebraic varieties

Für kanonisch polarisierte algebraische Mannigfaltigkeiten, Flächen allgemeinen Typs, polarisierte K-3 Flächen, stabile Kurven wird gezeigt, daß über dem komplexen Zahlkörper grobe Modulräume existieren, welche algebraische Räume sind.

Weiter wird die Einordnung der Modultheorie in die Klassifikationstheorie algebraischer Mannigfaltigkeiten beschrieben und angegeben, wie die universellen Familien für stabile Kurven vom Geschlecht g mit n-Teilungspunktstruktur (die Existenz dieser Familien wird nachgewiesen) benutzt werden können, um Mannigfaltigkeiten, die eine Faserung von Kurven vom Geschlecht g tragen, im birationalen Sinne zu klassifizieren.

J.F. BOUTOT: Local Picard Scheme

Let k be a field, R a noetherian local k-algebra with maximal ideal $\underline{m}$ and residue field k. For every noetherian k-algebra A, let $R\hat{\otimes}_k A = \varprojlim (R/\underline{m}^n \otimes_k A)$. Let $V = \mathrm{Spec}(R) - V(\underline{m})$ be the punctured spectrum of R and $\hat{V}_A$ the inverse image of V in $\mathrm{Spec}(R\hat{\otimes}_k A)$. We define the local Picard functor $P = \underline{\mathrm{Pic\ loc}}_{R/k}$ as the sheaf associated for the etale topology to $A \longmapsto \mathrm{Pic}(\hat{V}_A)$ and prove that, if $H^1_{\underline{m}}(R)$ and $H^2_{\underline{m}}(R)$ are of finite length [e.g. if $\hat{R}$ is normal of dimension ≥ 3], P is representable by a locally algebraic group over k whose tangent space at the origin is isomorphic to $H^2_{\underline{m}}(R)$.

We consider in particular the case where R is the ring of germs of holomorphic functions at a point of a complex analytic space or the local ring at the vertex of a cone over a projective variety.

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