MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 20/1975

Ringe, Moduln und homologische Methoden

11.5. bis 17.5.1976

Die traditionsreiche Tagung fand auch in diesem Jahr ein sehr großes internationales Interesse.

Wegen der großen Zahl der Vortragsanmeldungen konnte leider nicht allen Vortragswünschen entsprochen werden. Die kooperative Einstellung der Teilnehmer ermöglichte es den Veranstaltern A. Rosenberg, Ithaca, und F. Kasch, München, ohne Schwierigkeit einen Ausgleich zu finden und ein Vortragsprogramm aufzustellen, das die verschiedenen Entwicklungsrichtungen zur Geltung brachte. Die Veranstalter sehen den Wert einer nicht ganz eng spezialisierten Tagung mit darin, daß die Teilnehmer auch unmittelbare Informationen aus Nachbargebieten zu denen ihrer eigenen Forschung erhalten.

Zahlreiche vorhandene wissenschaftliche Kontakte wurden fortgeführt und neue angeknüpft.

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Teilnehmer

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Long, F.W., Aberystwyth
Malliavin, M.P., Paris
Martindale, W.S., Amherst
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DeMeyer, F., Zürich
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Mueller, B.J., Hamilton
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Procesi, C., Pisa
Ringel, C.M., Bonn
Robson, J.C., Leeds
Roggenkamp, K.W., Stuttgart
Roos, C., Delft
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Schneider, H., München
Schneider, H.-J., München
Simson, D., Torun
Small, L.W., La Jolla
Strooker, J.R., Utrecht
Sweedler, M., Ithaca
Vamos, P., Sheffield
Wiegandt, R., Budapest
Zelinsky, D., Evanston
Zelmanowitz, J., Santa Barbara
Zimmermann, B., München
Zimmermann, W., München

Vortragsauszüge

H. Bass: Ranks of projective $\mathbb{Z}G$-modules

M. Brodmann: A macaulayfication of local domains

Sei R ein noetherscher, halblokaler Integritätsbereich. Sei $\hat{\delta}(R) = \min\{\dim(\hat{\mathfrak{p}}) \mid \hat{\mathfrak{p}} \in \mathrm{Ass}(\hat{R})\}$, depth (R) die maximale Länge aller R-regulären Folgen aus dem Jacobson-Radikal J(R). Dann gilt allgemein $\hat{\delta}(R) \geq \mathrm{depth}(R)$; in sehr vielen Fällen ist die Ungleichung echt. Es wird ein Modell R' von R konstruiert, für welches gilt:

$$\mathrm{depth}\,(R') = \hat{\delta}(R').$$

Dabei ist R' eine Lokalisierung einer R-Algebra von endlichem Typ im Quotientenkörper von R. Weiter ist R ein Unterraum von R', $R \leq R'$, $\hat{\delta}(R') = \hat{\delta}(R)$ und R und R' haben dieselbe Krull'sche Dimension.

R.T. Bumby: Amitsur Cohomology and Group Cohomology

Analogs of reduction theorems of the cohomology of groups are obtained for certain complexes of Amitsur cohomology. Application is made to the units of Galois extensions of number fields. For cyclic extensions, this leads to stable periodicity. Quadratic extensions of $\mathbb{Z}$ already show that this stability may be deferred arbitrarily long. (This represents joint work with David E. Dobbs).

W.D. Burgess: Pierce Sheaves and Rings generated by their Units

The purpose of this work, done jointly with W. Stephenson, is to use Pierce's representation of a ring, as a ring of sections of a sheaf, to improve some results of Henriksen on rings generated by their units.
In particular, if R is a regular ring of bounded index of nilpotency then: (i) If $\mathbb{Z}/2\mathbb{Z}$ is not a homomorphic image of R then every element of R is a sum of two units. (ii) If $\mathbb{Z}/2\mathbb{Z}$ is a homomorphic image modulo exactly one ideal then every element of R is a sum of ≤ 3 units, 3 is the best possible. (iii) Otherwise R is not generated by its units.

The same theorem can be shown to apply to a much wider class of regular rings by iterating the Pierce sheaf process and using transfinite induction. The same techniques apply to other types of problems.

P. M. Cohn: <u>Gleichungen über Schiefkörpern</u>

Sei K ein Schiefkörper (über einem kommutativen Grundkörper k), R ein K-Ring, d.h. ein Ring mit kanonischem Homomorphismus $K \longrightarrow R$. Eine Matrix A über R heißt <u>eigentlich</u>, wenn es einen K-Ring-Homomorphismus von R in einen Schiefkörper gibt, der A singulär macht. Die allgemeinste Gleichung in $x_1,\dots,x_n$ über K hat die Form $p = 0$, wo $p \in F = K *_k k\langle x_1,\dots,x_n\rangle$, und die Frage, ob $p = 0$ lösbar ist, läßt sich so formulieren: Ist p eigentlich (als Element von F)? Eigentlichkeit bleibt erhalten bei elementarer Transformation und Rändern mit I, daher läßt sich das Problem "linearisieren": p läßt sich auf eine Matrix der Form $B - \sum A_i x_i$ $(B, A_i \in M_r(K))$ reduzieren, welche genau dann eigentlich ist, wenn $p = 0$ lösbar ist. Im Falle einer Veränderlichen führt dies auf ein Eigenwertproblem über K, das sich bis jetzt nur in einigen speziellen Fällen lösen läßt.

V. Dlab: <u>Classification of linear transformations</u>

A report on a joint work with Dr. C.M. Ringel establishing a complete classification of real linear transformations between two complex vector spaces in terms of matrices.

F. Eckstein: <u>Perfect topologies</u>

All rings are commutative. Let $\mathcal{F}$ be an idem-potent ideal filter, $Q_{\mathcal{F}}(R)$ the ring of quotients w.r.t. $\mathcal{F}$ and $\varphi : R \longrightarrow Q_{\mathcal{F}}(R)$ the canonical map. $\mathcal{F}$ is perfect iff $\varphi(I)Q_{\mathcal{F}} = Q_{\mathcal{F}}$ for all $I \in \mathcal{F}$. $Q_{\mathcal{F}}(R)$ is a topological ring without open ideal w.r.t. the topology it inherits from R. Conversely if A is a topological ring without open ideals containing a linearly topologized open subring, ten $A \cong Q_{\mathcal{F}}(R)$ and $\mathcal{F}$ is a perfect topology.
If R is Noetherian, then any idem-potent filter is generated by prime ideals.

<u>Theorem:</u> Let R be a commutative Noetherian ring, and $\mathcal{F}$ a perfect topology on R such that R is $\mathcal{F}$-torsion free, then any minimal prime ideal in $\mathcal{F}$ has rank 1.

<u>Corollary</u> (Rubin): Let R be a commutative Noetherian ring such that any idempotent filter is a perfect topology, then Krull-dimension of R $\leq$ 1.

<u>Prop.:</u> Let R be a commutative Noetherian ring, $\mathcal{P} \subseteq$ Spec R such that no $P \in \mathcal{P}$ belongs to $\{o\}$ and such that all $P \in \mathcal{P}$ have rank one and for all maximal ideals M of R P_M contains a principal ideal of $\mathcal{F}_M$, then $\mathcal{F}$, the filter generated by $\mathcal{P}$ is perfect and R is $\mathcal{F}$-torsion free.
In particular this is true for Krull-dim. of R $\leq$ 1 or R regular.

J.W. Fisher: <u>Rings generated by their units</u> (joint work with Robert L. Snider)

In 1958 Skornyakov conjectured that all von Neumann regular rings in which 2 is a unit are generated by their units. The conjecture was finally settled in the negative by Bergman in 1974. At about the same time, we proved the conjecture in the affirmative for the class of regular rings which have primitive factor rings Artinian. In fact we proved that such rings are unit regular.
Our most general theorem is as follows: Let R be a strongly π-regular ring, i. e., for each a in R, there exists a positive integer n and x,y in R such that $a^n=a^{n+1}x$ and $a^n=ya^{n+1}$.
If 2 is a unit in R, then each element of R can be expressed as a sum of 2 units. Included in the class of strongly π-regular rings are algebraic algebras, locally Artinian rings, and rings with prime factor rings Artinian.

O. Gerstner: <u>Zwei Begriffe von Reflexivität für Integritätsbereiche</u>

<u>Definition:</u> Der Integritätsbereich R, welcher kein Körper ist, heiße <u>p-reflexiv</u>, wenn A = 0 der einzige endlich erzeugte R-Modul mit $\mathrm{Hom}_R(A,R) = 0 = \mathrm{Ext}^1_R(A,R)$ ist.

<u>Satz:</u> (1) Jeder noethersche, reflexive (s.E.Matlis,J. of Alg.<u>8</u>) Integritätsbereich ist p-reflexiv.

(2) Jeder noethersche, p-reflexive Integritätsbereich endlicher injektiver Dimension ist reflexiv.

Beispiele: (1) $\mathbb{R}[[X]]$ ist p-reflexiv, aber nicht vollreflexiv (s. O.G.-L.Kaup-H.G.Weidner, Arch.Math. 20).

(2) $\mathbb{R}[X,Y]$ ist vollreflexiv, aber nicht p-reflexiv.

Theorem: Es sei R ein noetherscher, p-reflexiver und vollreflexiver Integritätsbereich. Dann verschwindet jeder R-Modul A mit $Hom_R(A,R) = 0 = Ext_R^1(A,R)$ (sofern A mit einem Erzeugendensystem nicht meßbarer Mächtigkeit auskommt).

Im Falle eines Hauptidealrings R folgt das Theorem aus der erwähnten gemeinsamen Arbeit; im allgemeineren Fall verwendet der Beweis Fortsetzungssätze für auf $R^I \subset R^J$ definierte Linearformen und soll demnächst erscheinen.

A. Goldie: On a theorem of M. Artin

A proof is presented of the theorem of M. Artin on the relationship between rings with polynomial identities and Azumaya algebras. The proof uses the Formanek polynomial as a trace on one variable.

M. Hochster: Homological conjectures on local rings and Cohen-Macaulay modules

The talk would be a survey of the status of various homological questions about local (commutative, Noetherian) rings (e. g. Serre's conjecture on multiplicities, Bass' conjecture, M. Auslander's zerodivisor conjecture, the intersection conjecture of Peskine-Szpiro, the question of whether a commutative module-finite extension algebra R of a regular Noetherian ring A has A as a direct summand as A-modules, etc.) and their relationship to various results, some known and some conjectured, concerning the existence of big and small Cohen-Macaulay modules.

G. Krause: Krull-dimension of idealizers

Let S be a ring with identity, L a semi-maximal left ideal of S (=intersection of finitely many maximal left ideals), and let $R = \mathbb{I}_S(L)$ denote the idealizer of L in S (=largest subring of S which contains L as a two-sided ideal). If $K\text{-}dim_S(M)$ denotes the Krull-dimension of the left S-module M and $1.K\text{-}dim(S) = K\text{-}dim_S({}_SS)$ denotes the left Krull-dimension of S, then the following is true.

<u>Theorem:</u> (i) Let M be a left S-module. Then $K\text{-}dim_S(M)=K\text{-}dim_R(M)$ if either side exists.

(ii) If M is a left R-module with Krull-dimension then $K\text{-}dim_S(S \otimes_R M) = K\text{-}dim_R(M)$ if and only if $S \otimes_R M \neq 0$ or M=0.

(iii) l.K-dim(R) = l.K-dim(S) if either side exists.

Precisely the same statements are true for the Gabriel-dimension.

J. Lambek: <u>Localization and Duality</u>

This is a report of joint work with B.A. Rattray. Any pair of adjoint functors between two categories $\mathfrak{A}$ and $\mathfrak{B}$ induces an equivalence between certain fixed subcategories. Of special interest is the case when one gets a reflective subcategory of $\mathfrak{A}$, hence a coreflective subcategory of $\mathfrak{B}$. Many, if not all equivalence and duality theorems in mathematics can be obtained in this way, for example, those of Morita, Pontrjagin, Stone, Gelfand, Kaplansky and Matlis. The last two are special cases of the following theorem: Let I be a quasi-injective right R-module with the discrete topology. Then there is a duality between the limit closure of I in the category of continuous right R-modules and the full subcategory of abstract left E-modules cogenerated by $_E I$, where $E = End_R(I)$.

R.G. Larson: <u>Hopf algebra orders in group algebras</u>

The existence of non trivial Hopf algebra orders in the group algebra of a finite nilpotent group gives information on the structure and representation theory of the group. We describe how to construct such orders using p-adic group valuations, and discuss some applications. Possible extensions and open questions are discussed.

F. W. Long: <u>Dimodule algebras and Generalized Clifford Algebras</u>

For a finite Abelian group Γ, a Γ-dimodule algebra is an algebra (over some ground ring R) which is graded by Γ and acted on by Γ in such a way that the grading and action commute. We then define a twisted tensor product, or "smash product" of two such Γ-dimodule algebras.

The generalized Clifford Algebras (as defined by A.O. Morris and others) can be obtained as a smash product of suitable Γ-dimodule algebras. The structure of these smash products is analysed and used to determine the algebra structure of the generalised Clifford Algebras.

F. DeMeyer: The Brauer group of a polynomial ring

Let R be a local ring with maximal ideal I. The kernal of the induced homomorphism $B(R) \to B(R/I)$ on Brauer groups is described. This homomorphism is onto if R/I is a field from algebraic number theory. In general the question is open. Let R be a normal Noetherian domain and let Ref(R) be the isomorphism classes of reflexive R-modules M with $End_R(M)$ projective over R. Under the multiplication $M \circ N = (M \otimes N)^{**}$ Ref(R) is a monoid and if Pro(R) is the submonoid of projective modules, then Ref(R) / Pro(R) is a group. There is an induced homomorphism $Ref(R[x]) / Pro(R[x]) \to Ref(R) / Pro(R)$ whose kernal is denoted $Ref'(R[x])$. If $B'(R[x])$ is the kernal of $B(R[x]) \to B(R)$ then the sequence

$$0 \to Ref'(R[x]) \to B'(R[x]) \to B'(K[x])$$

is exact. No examples where $Ref'(R[x]) \neq 0$ are known. Let I be an ideal of depth = 1 in $\mathbb{R}[x_1,...,x_n]$ where $\mathbb{R}$ is the real numbers and let $R = \mathbb{R}[x_1,...,x_n]/I$. Then R is the affine ring of a real affine algebraic curve and B(R) is a direct sum of r-copies of the cyclic group of order 2 where r is the number of real components of the curve.

G. O. Michler: Unzerlegbare Moduln endlicher Gruppen

Die Theorie der unzerlegbaren Moduln eines Blocks mit zyklischer Defektgruppe wurde über beliebigen vollständigen, diskreten Bewertungsringen ungleicher Charakteristik entwickelt. Als Spezialfälle wurden Sätze von J.L. Alperin-G.Janusz, E.C. Dade, J.A. Green, G. Janusz, H. Kupisch und B.Röthschild-W. Feit erhalten.

B. J. Mueller: Localization of noetherian rings at cycles

A semiprime ideal S of a noetherian ring R is uniquely the finite irredundant intersection of prime ideals P_i. Call such S classical, if the multiplicative set $\mathcal{C}(S)$ of modulo S regular elements satisfies the Ore condition and if the corresponding quotient ring R_S has the Artin-Rees property, then R_S is a semilocal noetherian ring with the maximal ideals P_iR_S. Call the set $\{P_1,\ldots,P_n\}$ a cycle, if $S = \bigcap_{i=1}^{n} P_i$ is classical but no proper subset has classical intersection. Then a prime ideal belongs to at most one cycle, and for any classical semiprime ideal, the associated finite set of prime ideals is uniquely the (disjoint) union of cycles. These results are obtained by studying the completion $\hat{R}_S$; they suggest to regard the quotient rings R_S corresponding to the cycles as the natural localizations of R. The cycles can be explicitely determined for rings which are finite modules over their center, for HNP rings, and for certain enveloping algebras of Lie algebras; preliminary results are also known for FBN rings, in particular for rings with polynomial identity. There is some evidence that all FBN rings might have enough cycles, ie. that every prime ideal belongs to a cycle.

N. Nöbauer: Kompatible Funktionen auf Moduln und Ringen

Sei A eine (universale) Algebra, $F_k(A)$ die von allen Funktionen $f: A^k \longrightarrow A$ bezüglich punktweiser Ausführung der Operationen von A gebildete Algebra. Die Funktion $f \in F_k(A)$ heißt kompatibel, wenn für jede Kongruenz Θ auf A aus $a_i \Theta b_i$, $i=1,2\ldots k$, stets folgt

$$f(a_1,a_2 \ldots a_k) \Theta f(b_1,b_2 \ldots b_k)$$

Es sei $C_k(A)$ die Menge aller kompatiblen Funktionen von $F_k(A)$. Ferner sei $P_k(A)$ die durch die konstanten Funktionen und die Projektionen erzeugte Teilalgebra von $F_k(A)$ (ihre Elemente sind die "Polynomfunktionen").
Klarerweise gilt $P_k(A) \subseteq C_k(A)$. Die Algebra A heißt k-affin vollständig, wenn gilt $P_k(A)=C_k(A)$.
Problem Nr. 6 in Grätzer, Universal Algebra, lautet: Man bestimme alle k-affin vollständigen Algebren. Dieses Problem wird hier gelöst für die endlichen Moduln und die endlichen Ringe mit zyklischer additiver Gruppe.

C. Procesi: The invariant theory of matrices

In the spirit of classical Invariant theory we prove the first and second fundamental theorem for invariants of i $n \times n$ matrices $X_1, \ldots, X_i$ under conjugation by one of the classical groups. Similarly for matrix valued concomitants. For $Gl(n,K)$, char $K=0$, the theorem is:

The ring of invariants T is generated by $Tr(X_{i_1} \ldots X_{i_s})$ $s \leq 2^n-1$. The algebra S of matrix valued concomitants is spanned, over T, by the monomials $X_{i_1} \ldots X_{i_t}$, $t \leq 2^n-2$. (First fundamental theorem).

Every relation between invariants and concomitants is a "consequence" of the Hamilton Cayley relation (Second fundamental theorem).

Similar results for $O(n)$, $Sp(n)$ and $U(n)$.

C. M. Ringel: Indecomposable modules over finite dimensional algebras

Let A be a finite dimensional k-algebra. An A-module X is said to be locally indecomposable, provided every finite dimensional subspace of X is contained in a finite dimensional indecomposable submodule. Locally indecomposable modules do not have to be indecomposable; in fact, for all wild algebras, it is easy to construct locally indecomposable modules which are direct sums of infinitely many indecomposable modules. The remaining tame case seems to be of interest. For a tame k-species (and for the corresponding k-algebra), every locally indecomposable module is the direct sum of at most 6 indecompsable (and locally indecomposable) modules, and all indecomposable and locally indecomposable modules can be described. There are two different types: either all non-zero endomorphisms are surjective, or all are injective.

J. C. Robson: Unity in simple noetherian rings

I will prove a result on rings and factor rings which lead both to a proof that certain simple noetherian rings have unity element and to a counterexample to the general claim.

I. C. Roos: On regularities of rings

In ring theory many so-called regularities appear, such as the von Neumann regularity, the Perlis-Jacobson regularity, the Brown-Mc Coy regularity and many other ones, for each of which it has been shown that every ring R contains a greatest "regular" ideal. It is possible to define a general notion of regularity for rings, including all these well-known regularities, such that any regularity $\mathfrak{F}$, in this general sense, determines a subradical $\bar{F}$ and a radical F *). For any ring R, F(R) is the greatest "regular" ideal of R, and $\bar{F}(R)$ is the greatest ideal of R which is, in a certain sense, regular relative to R.
With the help of integral polynomials one may produce regularities. The so arising regularities are called polynomial regularities. All well-known regularities are polynomial regularities.

*) By a subradical a function is meant which assigns to any ring R an ideal $\varrho(R)$ of R, such that (1) if $\mu: R \to S$ is a ring epimorphism, then $\mu(\varrho(R)) \subset \varrho(S)$ (2) if A is an ideal of R then $A \subset \varrho(R)$ if $\varrho(A)=A$ (3) $\varrho(R/\varrho(R)) = 0$ for any ring R.
A radical is nothing else as an idempotent subradical.

M. Schacher: Algebraic-valued Functions on Non-commutative Rings

Sample Theorem: A division ring satisfying $(xy-yx)^{n(x,y)} \in Z$; Z = center, n(x,y) depending on x and y, is at most 4-dimensional over Z.

H.-J. Schneider: On finite Hopf algebras

Let R be a commutative ring. A Hopf algebra H over R is called finite, if H is finitely generated and projective as R-module. (Spec H is a finite group scheme for H commutative). After describing explicitly extensions of finite Hopf algebras by 2-co cycles (in some cases one gets all extensions by this method), applications were given in case $R=\mathbb{Z}$. Using this description one can classify finite group schemes of rank p^2, p=2,3 over $\mathbb{Z}$.
Finally I sketched a direct proof of Mazur's result.
$Ext_{\mathbb{Z}}(\mu_p, \mathbb{Z}/(p)) = 0$, p a regular prime (this proof gives also information for irregular primes).

D. Simson: Pure-perfect categories and algebras of finite representation type

Let A be a locally finitely presented Grothendieck category and let fp(A) be its full subcategory consisting of all finitely presented objects. A is pure-perfect if each its object has a pure-projective cover. The following statements are equivalent: (1) A is pure-perfect, (2) A is locally noetherian and every its object is a coproduct of noetherian objects, (3) The functor category (fp(A),Ab) is locally artinian, (4) A coproduct of pure-injective objects in A is pure-injective, (5) Each object of A is a Mittag-Leffler object, (6) Each pure-projective object of A is pure-injective. If C is a small abelian category with a finite number of non-isomorphic simple objects then the categories Lex C and Lex C^{op} are pure-perfect iff C is both noetherian and artinian and has only a finite number of non-isomorphic indecomposable objects. If R is an artin algebra then the category R-Mod is pure-perfect iff R is of finite representation type.

L. W. Small: Centers of PI-Rings

The following theorems were proved:
Theorem 1: If $R = A[x_1,...,x_n]$ is PI, right Artin and A commutative and semi-primary, then R is a finite module over A.
Theorem 2: If $R = A[x_1,...,x_n]$ is PI, prime, Noetherian and of Krull dimension one, then R is a finite module over its center (A is commutative). Counterexamples were presented to various possible generalizations.

J. R. Strooker: Die Fundamentalgruppe der allgemeinen linearen Gruppe

Für eine vollkommene Gruppe G tritt der Schursche Multiplikator $H_2(G,\underline{Z})$ als Kern einer universellen zentralen Erweiterung von G auf. Sei andererseits F ein gruppenwertiger Funktor auf Ringen. In einer gemeinsamen Arbeit mit Herrn O. E. Villamayor (Advances in Math. 15, Febr. 1975) hat der Vortragende auf "topologischer" Art eine "universelle Überlagerung" $\hat{F} \to F$ von F eingeführt und "Homotopiegruppen" π_0, π_1 durch die exakte Folge

$$1 \to \pi_1 F \to \hat{F} \to F \to \pi_0 F \to 1$$

von Funktoren definiert. Wir zeigten dort, daß $\pi_1 GL = H_2(EL,\underline{Z}) = K_2$, also daß die Fundamentalgruppe der allgemeinen linearen Gruppe der Multiplikator seiner elementaren Untergruppe ist, das heißt der Milnorsche K_2.
Satz. Sei A eine semilokale Algebra über einem Körper. Im allgemeinen gilt: Die "Zusammenhangskomponente" von $GL_n(A)$ ist die elementare Untergruppe $EL_n(A)$. Für $n \geq 3$ gilt $\pi_1 GL_n(A) = H_2(EL_n(A),\underline{Z})$ für n=2 ist die Fundamentalgruppe eine Erweiterung vom Schurschen Multiplikator. Ausnahmen für die kleinsten Körper. Beim Beweis werden Methoden und Ergebnisse benützt von Steinberg, Cohn, Silvester, Dennis, Bak und Stein.

M. E. Sweedler: Corings, birings and Hopf rings

A K-coring is a K-bimodule M with suitable maps $\Delta : M \longrightarrow M \otimes_K M$ and $\varepsilon : M \rightarrow K$. If M is a K-coring then the left (or right) K-dual to M is a ring (with unit ε) to which K maps. The image of K in the dual need not be central; thus, the dual need not be a K-algebra. Birings and Hopf rings are defined in analogywith the development coalgebra, bialgebra, Hopf algebra. Example: Suppose K is a field extension of k then $M = K \otimes_k K$ is a K Hopf ring. The K-dual to M is naturally isomorphic to $End_k K$ and the $X_{K/k}$ bialgebra structure on $End_k K$ - independently studied by Winter and Sweedler - naturally arises from the biring structure of M. Galois Hopf rings are analogous to the duals of Galois Hopf algebras $K \otimes_k K$ is a Galois Hopf ring for extension K/k. There is a one-one correspondence between the fields intermediate - between-k-and-K and the quotient Hopf rings of $K \otimes_k K$. This result does not depend upon K being a finite extension of k and may even be generalized to the case where K and k are division rings. In the latter case $K \otimes_k K$ is merely a coring, and the one-one correspondence is between the fields intermediate-between-k-and-K and the quotient corings of $K \otimes_k K$. The Jacobson-Bourbaki correspondence is dual to this quotient coring correspondence.

P. Vâmos: Rings with Morita duality

Let $R \subseteq T$ be rings. Then the following hold:
(1) If ${}_R T$ is finitely generated by elements that centralize R,

then R has duality $\Rightarrow$ T has duality.

(2) If R is commutative and T is a f. g. R-algebra, then T has duality $\Rightarrow$ R has (self) duality.

(3) If R is commutative, T is a linearly compact (l.c.) R-module (discrete topology) and T is an R-algebra, then R has duality $\Rightarrow$ T has (self duality).

(4) If R and T are commutative, ${}_RT$ is l.c., then T has duality $\Rightarrow$ R has duality.

(5) If R is commutative and $P \neq 0$ is a prime ideal comparable to every other ideal of R, then R has duality if and only if R_P has duality and the quotient field of R/P is l.c.

Using these results some progress can be made on a question of Zelinsky's: Which commutative rings are l.c. in the discrete topology? In particular, (5) guarantees the existence of a local integrally closed domain with duality, which is neither noetherian nor a valuation ring. One also notices that (4) implies that a commutative ring with l.c. quotient field has duality.

R. Wiegandt: Radicals and the existence of structure theorems

The purpose of introducing radicals was to obtain structure theorems for semisimple rings. Concerning the existence of structure theorems the following are proved. i) A necessary and sufficient condition is given for a class M of rings such that each UM-semisimple ring is a subdirect sum of M-rings. ii) There is no radical class R (different from the class of all rings) such that every R-semisimple ring is a complete or discrete direct sum of subdirectly irreducible rings; iii) Let R be a radical class. Each R-ring is a discrete or complete direct sum of subdirectly irreducible R-rings iff R is the lower radical of a set $\{Z(p_i^\infty) : p_i$ ranges over a set of primes$\}$. iv) Considering semigroups with 0, there is no radical class such that every semisimple semigroup (radical semigroup) is a disjoint union or a complete or discrete direct product of simple or of subdirectly irreducible semigroups. The results are joint with M. A. Rashid, L. Márki and P. N. Stewart.

J. Zelmanowitz: An extension of the Jacobson Density Theorem

Loosely stated it can be demonstrated that the following three conditions are equivalent for a ring R: (1) R is, "up to a scalar", a (Jacobson) dense ring of linear transformations; (2) R is "locally" an order in a ring of linear transformations; (3) R has a faithful representation of a "special type". This result unifies and extends much previous work on special cases, including particularly the Jacobson Density Theorem and the work of R.E. Johnson/ S.A. Amitsur on prime nonsingular rings with uniform 1-sided ideals. Associated with this theorem there is an allied theory of "weakly primitive" rings to be explored, including the existence of a potentially interesting radical.

B. Zimmermann: Injectivity, chain conditions and the socle of endomorphism rings

The group of R-homomorphisms $Hom_R(M,A)$, where M,A are modules over an associative ring R with 1, is looked at as a module over the endomorphism ring S of M. Under certain weak assumptions on M, the following is true: $Hom_R(M,-)$ carries injective envelopes of R-modules into injective envelopes of S-modules iff M generates all its submodules. Moreover, for M a module of the latter type, $Hom_R(M,-)$ has nice properties concerning the preservation of chain conditions and the socle. Many results in this area, in particular those for M projective, are special cases.

W. S. Martindale: Lie Isomorphisms of the Skew Elements of a Prime Ring with Involution

Let A and B be closed prime algebras over a field C (i.e., C is the extended centroid of A and of B). Next suppose that A and B have involutions of the first kind. We denote the skew elements of A and B by K and L respectively. We furthermore assume that A has two nonzero orthogonal symmetric idempotents e_1 and e_2 such that $e_1 + e_2 \neq 1$ and such that for i = 1,2 e_i lies in the associative subring generated by $e_iAe_i \cap [K,K]$. Finally we assume that $(A:C) > 25$ and that the characteristic of A and B is unequal to 2 or 3.
Theorem: Suppose ϕ is a Lie isomorphism of $[K,K]$ onto $[L,L]$. Then there exists a subring T of A such that:
(1) T contains $[K,K]$ and T contains a nonzero ideal of A
(2) There is a unique (associative) isomorphism of T into B which agrees with ϕ on $[K,K]$.

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Gefördert durch
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