MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 21/1975

Gruppentheorie
18. 5. - 24. 5. 1975

Die diesjährige Gruppentheorietagung wurde von den Herren Professoren W. Gaschütz (Kiel), K.W. Gruenberg (London) und B. Huppert (Mainz) geleitet. Ein breites Vortragsangebot aus zahlreichen Gebieten der Theorie der endlichen und unendlichen Gruppen vermittelte eine gute Übersicht über den derzeitigen Stand verschiedener Forschungen. Die vortragsfreie Zeit wurde zu intensivem wissenschaftlichem Gedankenaustausch genützt.

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J.S.Wilson, Cambridge
G.Zappa, Florenz

Vortragsauszüge

J.L.ALPERIN: An Experiment in Group Representations

Motivated by problems in block theory and in the representations and cohomology of groups of Lie type we examine closely the representations of $SL(2,2^n)$ in characteristic two. We discuss the structure of projective modules and calculate some one-dimensional cohomology. The ring of irreducibly generated indecomposables is studied and found to be semi-simple. These are the indecomposable summands of tensor products of irreducible modules.

G.BAUMSLAG: A survey of 1-relator groups

A survey of groups with a single defining relation was given.

R.BIERI: Normal subgroups in groups of cohomology dimension 2

The Lyndon spectral sequence together with Stallings' Theorem on f.g. groups G with $H^1(G;\mathbb{Z}G) \neq 0$ is used to get the following results (which are new even in the 1-relator group case):

Theorem A. Let G be a f.g. group of cohomology dimension $cdG \leq 2$, and let N be a f.p. normal subgroup in G. Then either N is free or of finite index in G.

Theorem B. If, in addition, G is f.p., then G/N is always free-by-finite.

As an application on can classify all f.p. groups G with $cdG \leq 2$ and non-trivial centre. The same homological methods can also be used to prove

Theorem C. Let $G = G_1 *_S G_2$ be an amalgamated product with S f.p. and $1 < |G_i:S| < \infty$, for $i = 1,2$. If $cdG \leq 2$ then G_1, G_2 and S are free.

M.J.COLLINS: 3-structure in finite simple groups

In studying finite simple groups in which all 2-local subgroups are 2-constrained, one is led to examine the 3-local structure. If 3-local subgroups are not 3-constrained, one poses two questions:

1) If N is a 3-local subgroup of "large enough" 3-rank, is $O_2(N) = 1$?
2) Does an analogue of Aschbacher's component theorem exist ?

These lead to the following condition in an attempt to differentiate between groups of Lie type of even characteristic and sporadic groups -

(*) G contains a subgroup A isomorphic to SL(2,q), q even, and if t is an element of order 3 in A, $C_G(t) \leq N(A)$.

Some consequences and a general strategy are discussed.

U.DEMPWOLFF: On primitive permutation groups

Let G be a primitive permutation group on the finite set Ω and Δ be a sub-orbit of G_α ($\alpha \in \Omega$). The following results are discussed:

a) If $G_\alpha^\Delta \cong L_2(q)$ (natural permut. representation) then $G_\alpha \cong L_2(q)$, or $p > 2$, $q = p^n$ and $G_\alpha \cong L_2(q) \ast Y$, $Y \cong S_p$-normalizer of $L_2(q)$.

b) If $G_\alpha^\Delta \cong G_\alpha^{\Delta'} \cong U_3(q)$ and $|\Delta| \not\equiv 0 \bmod p$ then $G_\alpha \cong U_3(q) \ast Y$, Y suitable subgroup of a S_p-normalizer of $U_3(q)$.

R.FOOTE: Finite groups with a standard subgroup of 2-rank 1

A finite group G has a standard subgroup L if L is quasisimple, $\forall g \in G[L,L^g] \neq 1$, and $K = C_G(L)$ is tightly embedded with $N_G(K) = N_G(L)$. We extend M.Aschbachers's Standard Form Theorem (Theorem 1 of "On finite groups of component type") with the following result:

Theorem. Assume G is of component type, $F^*(G)$ simple and for every involution $t \in G$ the components of $C_G(t)$ are quasisimple. Assume further that G has an involution t and component A of $C_G(t)$ with A of 2-rank 1 and A maximal in the component ordering; moreover, if t_1 is an involution in G and A_1 is a component of $C_G(t_1)$ with A a homomorphic image of A_1, assume A_1 is also maximal. Then either A is a standard subgroup, in which case for some odd $q > 3$ $F^*(G) \cong L_3(q)$, $U_3(q)$, $G_2(q)$, ${}^3D_4(q)$ or ${}^3D_4(3)$, or A is not standard and $F^*(G) \cong S_4(q)$, $L_4(q)$ $q \not\equiv 1 \pmod 8$, $U_4(q)$ $q \not\equiv 7 \pmod 8$, q odd > 3 or $G_2(3^{2n+1})$ $n \geq 1$.

E.FORMANEK: Commutators in free groups

J.R. Stallings has made a group-theoretic conjecture equivalent to the Poincaré conjecture. Some aspects of Stallings' conjecture in the simplest case will be discussed.

W.GASCHÜTZ: Untergruppenkriterien

$f(x_1,\dots,x_n) = \sum_1^n a_i x_i$, $a_i \in \mathbb{Z}$, wird Untergruppenkriterium für die Klasse $\mathfrak{A}$ der abelschen Gruppen genannt, wenn für jeden unter f abgeschlossenen Komplex $K \neq \emptyset$ einer abelschen Gruppe G gilt: $K \leq G$.

Beispiele: $f = x_1 - x_2$, $f = x_1 + 2x_2 - x_3$. Ist f Untergruppenkriterium für $\mathfrak{A}$, so ist $\sum_1^n a_i = 0$ oder 2 und

(*) $(a_1,\dots,a_{i-1},a_{i+1},\dots,a_n) = 1$ für alle i .

Ist $< 1 >_f$ die kleinste unter f abgeschlossene Teilmenge von $\mathbb{Z}$, die 1 enthält, so ist bei (*) $< 1 >_f = \mathbb{Z}$ notwendig und hinreichend, daß f Untergruppenkriterium für $\mathfrak{A}$ ist. Hierfür wiederum ist $0 \in < 1 >_f$ notwendig und hinreichend. Damit ist für $\sum a_i = 0$ alles geklärt. Ist $\sum a_i = 2$, so konnten bisher keine genaueren Resultate erzielt werden.

H.HEINEKEN: Untere Schranken für die Nilpotenzklasse

Für großes n weiß man sehr wenig über die mögliche Nilpotenzklasse von n-Engelgruppen. Um eine Vorstellung davon zu bekommen, welche Klasse gewiß noch vorkommt, werden von 3 Elementen erzeugte Gruppen betrachtet, die zyklische Erweiterungen geeigneter Gruppen der Klasse zwei sind. Dabei sind die auftretenden Klassen abhängig von den auftretenden Primzahlen.
(Untersuchung gemeinsam mit N.D.GUPTA)

Z.JANKO: Über die endlichen einfachen Gruppen mit den großen extra-speziellen 2-Untergruppen

Sei G eine endliche einfache Gruppe, welche eine Involution z besitzt, so daß $H = C_G(z)$ die folgenden Eigenschaften hat:

(i) $E = O_2(H)$ ist eine extra-spezielle 2-Gruppe.

(ii) $C_H(E) \leq E$.

Wir sagen, daß E eine "große" extra-spezielle 2-Untergruppe von G ist, weil:

$$E = O_2(C_G(E')) \text{ und } C_G(E) \leq E \text{ .}$$

Das Ziel ist es, die Struktur von H und G zu bestimmen. Dies ist sehr schwierig, weil die Mehrheit der 25 sporadischen einfachen Gruppen so eine große extra-spezielle 2-Untergruppe besitzt.

A.KERBER: Characters of wreath products and applications

If V denotes a vector space, if $n \in \mathbb{N}$, and $\overset{n}{\otimes} V := V \otimes \dots \otimes V$, then the wreath product $GL(V) \wr S_n$ acts on $\overset{n}{\otimes} V$ by

$$(f;\pi)(v_1 \otimes \dots \otimes v_n) := f(1)v_{\pi^{-1}(1)} \otimes \dots \otimes f(n)v_{\pi^{-1}(n)}$$

If $\pi = \prod_{=1}^{c(\pi)} (j_\nu \dots \pi^{k_\nu - 1}(j_\nu))$, where $j_\nu \leq \pi^r(j_\nu) \leq j_{\nu+1}$, $\forall\, r,\nu$, and $g_\nu(f;\pi) := f(j_\nu)f(\pi^{-1}(j_\nu))\dots f(\pi^{-k_\nu+1}(j_\nu))$, $1 \leq \nu \leq c(\pi)$, then it is easy to see that we have for the above operation of $(f;\pi)$ on $\overset{n}{\otimes} V$:

$$\operatorname{trace}\,(f;\pi) = \prod_\nu \operatorname{trace}\, g_\nu(f;\pi).$$

It is shown, how this can be applied to

1. characters of wreath products,
2. the enumeration theory of combinatorics,
3. symmetrizqtion of representations.

An example for 3. is provided by Kostant's proof of the theorem of Amitsur and Levitzki that the m m-matrix ring over C satisfies the standard polynomial identity.

R.MAIER: Faktorisierbare p-auflösbare Gruppen

Die Beweisidee zu folgendem Satz wurde vorgetragen:

Satz: (a) Sei s eine natürliche Zahl und p eine Primzahl mit $p \geq 2s$. Sei $G = G_1 \cdot G_2 \dots \cdot G_r$ eine p-auflösbare endliche Gruppe, die das Produkt von paarweise vertauschbaren Untergruppen G_i ist. Sind die p-Sylowgruppen P_i von G_i polyzyklisch der Länge $\leq s$ und gilt $P_i \trianglelefteq G_i$ für $p = 2$, so hat G die p-Länge höchstens 1.

(b) Zu jeder natürlichen Zahl $s \geq 2$ und zu jeder Primzahl p mit $2 < p \leq 2s-1$ gibt es eine auflösbare Gruppe $G = G_1 \cdot G_2$ von der p-Länge 2, die das Produkt von zwei Untergruppen G_1 und G_2 ist, welche beide polyzyklische p-Sylowgruppen der Länge $\leq s$ besitzen.

M.L.NEWELL: Splitting theorems for abelian-by-supersoluble groups

We proved the following result: Let A be an abelian minimal normal subgroup of a group G. If G/A is hypercyclic and A is non-cyclic then A has a unique conjugacy class of complements in G.
Further we showed that if the rank of A considered as a vector space over $\mathbb{Z}_p$ or $\mathbb{Q}$ is finite, then "hypercyclic" can be replaced by "locally supersoluble" in this result.

P.M.NEUMANN: Infinite groups of finitary permutations

I talked about theorems describing the structure of a transitive group G of finitary permutations:

Theorem 1. If N is a normal subgroup of G then either

(a) all N-orbits are finite and N is a restricted subdirect power of some finite group; otherwise

(b) the degree is infinite, N is transitive and $N \geq G'$.

Theorem 2. Either

(a) there is an ascending chain $\{N_i\}_{i:=1,2,\ldots}$ of intransitive normal subgroups whose union is G (in which case the degreee must be $\aleph$), or

(b) there is a finite transitive group H and a cardinal number $\mathfrak{m}$ such that

$$W' \leq G \leq W$$

where W:= Hwr $SF_{\mathfrak{m}}$ (and $SF_{\mathfrak{m}}$ denotes the finitary symmetric group of degree $\mathfrak{m}$).

These theorems, and some more detail, are to appear in a paper in Archiv der Mathematik.

M.F.NEWMAN: A practical method for calculating nilpotent quotient groups

A description will be given of the calculation of a presentation for the free group B(4,4) of rank four in the variety of groups of exponent four from which can be read off such information as: the order of B(4,4) is 2^{422}, the nilpotency class of B(4,4) is 10.

J.NEUBÜSER: Gruppentheorie auf Computern

Der Vortrag gibt einen Überblick über gegenwärtig verfügbare Rechenverfahren.

1. Präsentationsalgorithmen, insbesondere

1.1 Todd-Coxeter-Verfahren

1.2 J.Cannon's Verfahren zur Bestimmung einer Präsentation

1.3 Sims' Verfahren zur Bestimmung von Untergruppen von gegebenem Index

1.4 Reidemeister-Schreier Algorithmus

1.5 MacDonald-Wamsley Algorithmus für nilpotente Faktorgruppen

1.6 Schur-Multiplikator

2. Permutationsalgorithmen von Sims (base, Stabilizer-chain, strong generating system)

3. Strukturalgorithmen

3.1 Bestimmung partieller Strukturen, z.B. Klassen, Zentralisatoren, Normalisatoren, Sylowgruppen, gewisse charakteristische Untergruppen

3.2 Untergruppenverband, Normalteilerverband

3.3 Automorphismengruppen

4. Darstellungstheoretische Algorithmen

4.1 J. Dixon's Algorithmus zur Bestimmung der Charakterentafel aus den Klassenmultiplikationskoeffizienten

4.2 Interaktive Methoden (Livingstone)

Seit ca. 3 Jahren wird gemeinsam von einem Team in Legdney (J.Cannon) und einem Team in Aachen ein transportables Programmsystem entwickelt, das insbesondere diese Verfahren enthalten soll. Dieses beruht auf einem maschinenabhängigen Speicherverwaltungsprogramm; eine benutzerorientierte Sprache für das System wird zur Zeit geschrieben.

J.MENNICKE: Some groups of exponent 8

Let $B(2,8) = < a,b \mid \omega^8 = 1 >$ denote the Burnside group of exponent 8 with two generators. We study some finite subgroups of $B(2,8)$.

Consider $\overline{B}_1 := < y_1 := a^2, z_1 := b^4a^4b^4a^2 > \leq B(2,8)$.

One can easily see that $\overline{B}_1$ is a homomorphic image of

$B_1 := < y,z \mid y^4 = (yz)^4 = (yz^{-1})^2 = 1, \omega^8 = 1 >$.

Theorem: (i) B_1 is a finite group of order

$$|B_1| \leq 2^{56}$$

(ii) $(((B_1^4)^2)^2)^2 = 1$

The proof consists in computation with generators and relations.

The work reported here was carried out jointly by F.GRUNEWALD and the speaker.

G.MICHLER: On indecomposable modules of finite groups

Es wurde ein neuer Beweis für den Satz von E.C. DADE über Blöcke mit zyklischer Defektgruppe gegeben.

H.PAHLINGS: Characterizing groups by their character tables

A finite group G is said to be determined by its character table, if any group with the same table of complex irreducible characters as G is isomorphic to G. The following results are discussed:

Theorem 1. The groups $Sp(2n,2^k)$, $O^{\pm}(2n,2^k)$, $PSU(n,2^{2k})$, ${}^{\pm}O^{\pm}(n,3)$ (except for ${}^{\pm}O^{+}(4,3)$), ${}^{\pm}O^{\pm}(n,5)$ and the Fischer groups F_{22}, F_{23}, F_{24} are determined by their character tables.

Theorem 2. The Weyl groups of the simple complex Lie algebras are determined by their character tables except for $W(B_2)$ and $W(D_{4k})$ $(k \geqq 1)$.

K.W.ROGGENKAMP: An extension of central extension theory

For K a commutative ring and G a group the equivalence between group extensions $1 \longrightarrow C \longrightarrow N \longrightarrow G \longrightarrow 1$, C a KG-module via conjugation and exact sequences $0 \longrightarrow C \longrightarrow M \longrightarrow \mathfrak{g}_K \longrightarrow 0$ of KG-modules, where $\mathfrak{g}_K$ is the augmentation ideal of KG is discussed. If $\vartheta : G \longrightarrow K^*$, where K^* is the group of units of K, is a homomorphism, one can consider "generalized central extension" i.e. exact sequences, where the kernels are K-modules and G operates via ϑ . The corresponding "Darstellungsgruppen" are constructed and results on the uniqueness presented.

J.ROSEBLADE: Prime Ideals in group algebras of free Abelian groups

I discussed the prime ideals P of a group algebra KA = S with A free Abelian of finite rank which were stabilized by some subgroup of finite index in some subgroup G of Aut A. If $d_G(S)$ denotes the length r of a maximal chain $P_o < P_1 < \ldots < P_r$ of primes stabilized by some subgroup of finite index in G and if $d_G(A)$ denotes the length s of a maximal chain $1 = B_o < B_1 < \ldots < B_s = A$ of pure subgroups of A all stabilized by some subgroup of finite index of G then the theorem proved maybe stated "$d_G(A) = d_G(S)$".

This generalizes to a wider class of A and can be used to give formulae for dim(R) where R is the group algebra of a polycyclic group. Douglas Brewster and I proved the result.

R.SCHMIDT: Maximal subgroups and lattice isomorphisms of finite groups

We say that a group G is determined by its subgroup lattice L(G) if every group H with L(H) $\simeq$ L(G) is isomorphic to G. To find such groups G we studied the question under which conditions conjugacy classes of maximal subgroups of G are mapped onto conjugacy classes under lattice isomorphisms. We got the result that the following groups are determined by their subgroup lattice :

1) triply transitive permutation groups of degree ≥ 4 generated by involutions ,

2) all known simple doubly transitive permutation groups ,

3) certain rank-3-groups, especially all projective symplectic , unitary and orthogonal (Char $\neq 2$) groups of sufficiently high dimension.

We finally mentioned that for $n = 3^r$, r odd, $r \geq 3$ not every lattice isomorphism of the alternating group A_n is induced by a group isomorphism although every A_n is determined by its subgroup lattice ($n \geq 4$).

G.E.WALL: The Lie ring of a group of prime exponent

The Lie ring L(G) of a group G of prime exponent p is an algebra over the field of p elements which satisfies the (p-1)st Engel condition. When G has nilpotency class $\leq 2p-2$, these are the only conditions imposed on L(G). But when G has larger class, further conditions are imposed. The talk is concerned with these additional conditions when G has class $\leq 3p-3$.

H.WIELANDT: Two lattices of generalized subnormal subgroups

For any class p of pairs (A,G) where G is a group and A a subgroup of G, there are two localizations:

(i) $A \ {}_L p \ G$ iff for any finitely generated group $A_o \leqq A$ there exists A^* such that $A^* \ p \ G$ and $A_o \leqq A^* \leqq A$;

(ii) $A \ {}_L p_L \ G$ iff for any two finitely generated groups $A_o \leqq A$ and $G_o \leqq G$ such that $A_o \leqq G_o$, there exist A^* and G^* such that $A^* \ p \ G^*$, $A_o \leqq A^* \leqq A$ and $G_o \leqq G^* \leqq G$.

In this notation the theorem of Roseblade and Stonehewer on subnormal subgroups [J. Algebra 8] says that A sn G and B sn G imply $\langle A,B \rangle \ {}_L sn \ G$. An immediate consequence of this basic result is the new

Theorem: For any group G, both ${}_L sn$ G and ${}_L sn_L$ G are sublattices of the subgroup lattice of G.

These lattices enjoy additional properties with respect to limit processes. After the lecture, J.S. Wilson pointed out that ${}_L sn_L$ is, in a sense, the best possible solution of the old problem of finding the "right" generalization of subnormality from finite to infinite groups.

J.S.WILSON: Finitely generated simple and characteristically simple groups

Examples of infinite finitely generated simple and characteristically simple groups were discussed briefly, and the following result was stated:

<u>Theorem A.</u> Let r and s be integers with $r \geq 2$ and $s \geq 7$. There are $2^{\aleph_0}$ non-isomorphic characteristically simple groups which

(a) are not direct powers of simple groups,

(b) are homomorphic images of $C_r * C_s$, and

(c) have (at least) $\aleph_0$ non-isomorphic simple images.

It was remarked that the simple images S of the groups of Theorem A have the property that all of their finite direct powers are 2-generator groups; but that this is not altogether surprising because of the very elementary

<u>Lemma.</u> Let S be any infinite simple d-generator group. Then every finite direct power S^n of S is a (d+1)-generator group.

G.ZAPPA: <u>On normal Fitting classes</u>

Let G be a soluble finite group, p a prime, M a p-principal fyctor of G. Let D_M be the representation of G in GL(M) obtained by M; $\forall\, g \in G$, let $d_M(g)$ be the determinant of the matr-ix $D_M(g)$ $(d_M(g) \in GF(p))$.

<u>Theorem.</u> Let $\mathcal{X}$ be a Fitting class, R the maximal normal $\mathcal{X}$-subgroup of G, S a principal G-series of R, and $M_1, \dots, M_r$ the principal p-factors $\in$ S. For every $g \in G$, let $d_{G,R,P}(g) = \prod_{i=1}^{r} d_{M_i}(g)$. Then the class $F = \{G \mid G$ soluble, $d_{G,R,P}(g) = 1 \;\; \forall\, g \in G\}$ is a normal non-identical Fitting class $\mathcal{A}(p,\mathcal{X})$.

If $\mathcal{H}$ is the class of finite soluble groups, the theorem becomes a known result of Blessenohl and Gaschütz (MZ 118, 1-8).

Also the normal class $\mathfrak{A} = \bigcap_{p,\mathcal{H}} \mathfrak{A}(p,\mathcal{H})$ is characterized.

Karsten Johnsen (Kiel)

TAGUNGSBERICHT NR: 22 fehlt

lt. Brief von Frau Mayenburg aus Karlsruhe vom 17. Dez. 1979 ist dieser Bericht nicht mehr zu erwarten.

Fritsch