MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 25/1975

Maßtheorie

15. 6. bis 21. 6. 1975

Die Tagung stand unter der Leitung von A. Ionescu-Tulcea (Evanston, U.S.A.) und D. Kölzow (Erlangen).
Vor einem Auditorium von 52 Wissenschaftlern aus 13 Ländern wurden 38 Vorträge gehalten.
Seit der Tagung über "Maßtheorie und Boole'sche Algebren" im Jahre 1961 war dies - von einigen Tagungen mit spezielleren Themen abgesehen - die erste Konferenz in Oberwolfach, welche das Gesamtgebiet der Maßtheorie zum Gegenstand hatte.

Teilnehmer

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S.D. Chatterji, Lausanne (Schweiz)
G.H.Y. Chi, Gainesville (U.S.A.) z.Zt. Bukarest (Rumänien)
J.R. Choksi, Montreal (Kanada)
J.P.R. Christensen, Kopenhagen (Dänemark)
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J.D.M. Wright, Reading (Großbritannien)
A.C. Zaanen, Leiden (Niederlande)

Vortragsauszüge

Allgemeine Maßtheorie

J.R. CHOKSI : Set and point transformations of measure spaces

For an arbitrary σ-finite Baire measure μ on an arbitrary locally compact, σ-compact homogeneous space M, under the action of a locally compact, σ-compact group G, it is shown (jointly with R.R. Simha), that every automorphism of the measure algebra of μ can be induced by an invertible, completion Baire measurable point transformation of M. If M is itself of the form G/L and μ is taken to be the essential Baire measure, then the result holds without any assumptions of σ-compactness. This generalizes results of the author (compact groups and products of Polish spaces), Maharam (direct product measure on a product of Polish spaces) and the classical result of von Neumann (Polish spaces). The correspon-ding results for Borel measures are known to be in general false.

D. FREMLIN : Pointwise compact sets of measurable functions

Let (X, Σ, μ) be a probability space, M(X) the space of real-valued measurable functions on X. On M(X) let τ_p be the topology of pointwise convergence, and τ_m the (non-Hausdorff) topology of convergence in measure.

Theorem 1 : If (X, Σ, μ) is perfect and $A \subseteq M(X)$ is τ_p-compact, then it is τ_m-compact.

Corollary: If τ_m separates the points of A, then τ_m and τ_p agree on A.

Theorem 2 : If $X_0, \dots, X_n$ are Radon measure spaces and $X = X_0 \times \cdots \times X_n$ is given the product Radon measure, then any separately continuous function $f: X \to R$ is jointly measurable.

Problems: 1. Without assuming the existence of a real-valued measurable cardinal, find a probability space X and $A \subseteq M(X)$ τ_p-compact but not τ_m-compact. 2. Using special axioms if convenient, find a probability space X and a τ_p-compact $A \subseteq M(X)$, which is separated by τ_m, but not τ_m-compact.

D. KAHNERT : Haar-Maß und Hausdorff-Maß

Ist G eine separable, lokalkompakte Gruppe von endlicher topologischer Dimension und U eine kompakte Umgebung der Null, dann gibt es eine linksinvariante Metrik d auf G, die die Gruppentopologie erzeugt, und eine Hausdorff-Funktion h, sodaß das Hausdorff-Maß L_h bezüglich (G,d) ein Haar-Maß auf G ist und

$\limsup_{q\to 0} \log N_q(U)/\log(1/q) = \dim G$ gilt.
Dabei ist $N_q(U) = \min\{k\in\mathbb{N}, \text{es ex.} (X_i): \bigcup_{i=1}^{k} X_i = U; d(X_i)\leq q\}$ für $q>0$.
Lediglich für spezielle metrisierbare, lokalkompakte Gruppen unendlicher Dimension wird gezeigt, daß jedes Haar-Maß ein Hausdorff-Maß ist.

K. MUSIAŁ : <u>Inheritness of compactness and perfectness of measures by thick subsets</u>

Let $(X, \mathfrak{B}, \mu)$ be a compact measure space and $Z \subseteq X$ be a thick subset. (We assume $\mu(X)=1$ and $\mathfrak{B}$ to be generated by $\aleph_1$ sets) Then the restriction of μ to Z is a compact measure iff there exists a net $(\mathfrak{B}_\lambda)_{\lambda<\omega_1}$ of coutably generated sub-σ-algebras of $\mathfrak{B}$ and a net $(N_\lambda)_{\lambda<\omega_1}$ of μ-null sets with the following properties: $\mathfrak{B}_0 \subseteq \mathfrak{B}_1 \subseteq \dots$; $\mathfrak{B} = \bigcup_{\lambda<\omega_1} \mathfrak{B}_\lambda$; $N_0 \subseteq N_1 \dots$; for every limit λ: $N_\lambda = \bigcup_{\gamma<\lambda} N_\gamma$, $\mathfrak{B}_\lambda = \sigma(\bigcup_{\gamma<\lambda} \mathfrak{B}_\gamma)$; $A \cap Z \neq \emptyset$ for every $\mathfrak{B}_\lambda$-atom $A \subseteq X \setminus N_\lambda$.
Using this theorem it is easy to construct examples of perfect but non-compact measures. Question: If $(X, \mathfrak{A}, \mu)$ is a compact measure space and $\mathfrak{B}$ is a sub-σ-algebra of $\mathfrak{A}$ is $\mu|_{\mathfrak{B}}$ compact ?

T.P. SRINIVASAN : <u>Remark on the Fubini theorem</u>

The following question was studied: Consider a class of functions for which the Fubini theorem holds. Under which conditions does it hold for the class of functions which are integrable after Daniell extension?

A.H. STONE : <u>Topology and measure theory</u>

If f and g are measurable maps from a measure space $(X, \mathfrak{B}, \mu)$ to a normed linear space Y, need f+g be measurable? The answer "yes" follows if μ is complete and there are no cardinals of measure zero, and "yes" follows without settheoretic assumptions if X is an absolutely analytic space and $\mathfrak{B}$ it's Borel field.
A second question: if B is a G_δ-subset of $P\times P$ (P the irrationals) meeting each vertical in an uncountable dense subset, and T is a Borel isomorphism of P onto P, it can be shown, that there exists an absolutely measurable T^* of B onto B, such that $T\circ\pi = \pi\circ T^*$. (π projection) But it would be desirable to be able to drop "dense" and especially to replace "G_δ" by "Borel" (which would answer a question of J. Choksi). Finally if X is a compact metric space with a regular non-atomic Borel measure μ and $\mu(X) < 1$, does there exist a measure preserving homeomorphism

of X into the Hilbert cube (with product Lebesgue measure)? The answer is "yes" at least in some low dimensional cases. In this connection a measure preserving form of Urysohn's lemma is valid.

Endlich additive Maße

J.P.R. CHRISTENSEN : Some results and problems with relations to a problem by D. Maharam

Let $(X,\mathfrak{A})$ be a set with a Boolean algebra $\mathfrak{A}$ of subsets. $\varphi:\mathfrak{A}\to\mathbb{R}$ is a submeasure if (1) $\varphi(\emptyset) = 0$, $A\subseteq B \Rightarrow \varphi(A)\leq\varphi(B)$ (2) $\varphi(A\cup B)\leq \varphi(A)+\varphi(B)$. φ is pathological, if it does not dominate a non negative finitely additive measure. D. MAHARAM's problem (essentially): Can φ be pathological if φ is sequentially point continuous? Some results and problems related to this question were discussed.

L.E. DUBINS : Countably additive measures generate finitely additive ones

Let γ be any proper, Lebesgue measurable disintegration for the usual Jordan measure λ on the unit circle Z with respect to the Borel partition Π consisting of all cosets of the subgroup Q of Z, consisting of all $z\in Z$ having finite period. Then for λ-almost all $h\in\Pi$, the probability measure $\gamma(h)$ on h, being invariant under the action of Q on h, is necessarily purely finitely additive.

D. MAHARAM : Finitely additive measures on the integers

Let μ be a finitely additive probability measure on $\mathcal{P}(\mathbb{N})$, the power set of the positive integers. Denote the ideal of μ-null-sets by $\mathfrak{N}_\mu$. The principal topics are: (1) the range of μ (2) the structure of the quotient algebra $E_\mu = \mathcal{P}(\mathbb{N})/\mathfrak{N}_\mu$ (3) the relations between μ and μ', where μ' is equivalent to μ in the sense that $\mathfrak{N}_\mu = \mathfrak{N}_{\mu'}$ (4) the existence of liftings of E_μ (5) applications to the topology of $\beta\mathbb{N}$ and to the structure of $\{0,1\}^{\mathbb{N}}$ as a compact topological group.

Liftings

W. STRAUSS : Beschränkte lineare Liftings

Wenn $(T,\mathfrak{m},\mu)$ ein Maßraum ist, so wird unter einem beschränkten linearen Lifting von $\mathcal{L}^\infty = \mathcal{L}^\infty(T,\mathfrak{m},\mu)$ eine beschränkte lineare Abbildung $l:\mathcal{L}^\infty \to \mathcal{L}^\infty$ verstanden mit den Eigenschaften: $f=l(f)$ μ-fast überall, $l(f) = l(g)$ für μ-fast gleiche f, g. Es gilt

Satz: 1) Wenn für $\mathcal{L}^\infty$ ein beschränktes lineares Lifting existiert mit Norm $\|l\| < 2$, so ist $(T,\mathfrak{m},\mu)$ zerlegbar.

2) Es ex. ein unzerlegbarer Maßraum, so daß der zugehörige $\mathcal{L}^\infty$ ein beschränktes lineares Lifting l der Norm $\|l\| = 2$ besitzt.

T. TRAYNOR : An elementary proof of the lifting theorem

The lifting theorem for a complete totally finite measure space is proved in an elementary way. The essence of the proof is the following replacement for the IONESCU-TULCEA martingale argument. Let $\mathfrak{A}_1 \subseteq \mathfrak{A}_2 \subseteq \dots$ be a sequence of σ-fields of measurable sets containing the null-sets, each endowed with a lifting l_k (density would be enough [remark of S. GRAF]) such that l_{k+1} extends l_k. Let $d(A,k,r)$ be the largest $E \in l_k(\mathfrak{A}_k)$ such that $\mu(A \cap F) \geq r\mu(F)$ for all $F \in \mathfrak{A}_k, F \subseteq E$. Then: $d(A) = \bigcap_{r<1} \bigcup_{n\in\mathbb{N}} \bigcap_{k\geq n} d(A,k,r)$ defines a density on the σ-field generated by $\bigcup_{n\in\mathbb{N}} \mathfrak{A}_n$ which extends each l_k. The remainder of the proof is like that of the IONESCU-TULCEA's.(For details see Pacific J. Math. 53(1974)) S. GRAF (Schnitte Boolescher Korrespondenzen und ihre Dualisierung, Dissertation, Erlangen, 1973) has essentially the same proof, but defines $d(A,k,r)$ in terms of Radon-Nikodým-derivatives.

H. VON WEIZSÄCKER : Some negative results in the theory of lifting

It is shown that two classical theorems in the theory of lifting are sharp in a certain sense: (1) An example of an ideal in a Boolean algebra is given showing that the completeness assumption in a theorem of von Neumann - Stone on the construction of a lifting from a lower density can not be weakened in general. The construction illustrates the fact that the metrizability assumptions in E. MICHAEL's selection theorems are essential.
(2) Let Ω be a locally compact group with Haar measure μ. Then there is a lifting commuting with all left translations in Ω. (A. and C. IONESCU-TULCEA) We proof that such a lifting can not commute with any other continuous bijection on Ω, which transforms

μ-nullsets into μ-nullsets whenever Ω is connected and μ is σ-finite. The proof is based on a lemma on automorphisms of complete Boolean algebras.

Differentiation von Maßen

S.D. CHATTERJI : Differentiation of measures

Let $\mathfrak{A}_n \subseteq \mathfrak{A}_{n+1}$ be a sequence of algebras on a space Ω ; let Θ be a finitely additive real set-function of bounded variation on $\mathfrak{A}_\infty = \bigcup_n \mathfrak{A}_n$ and let μ be a fixed positive, finite, σ-additive measure defined on a σ-algebra $\Sigma \supseteq \mathfrak{A}_\infty$. Under these conditions it is known (see CHATTERJI, Manuscripta Math.,4(1973)) that lim $D\pi_n\Theta = D\Theta$ a.e.(μ) where $\pi_n\Theta$ = restriction of Θ to $\mathfrak{A}_n$ and $D\pi_n\Theta$ is the Radon-Nikodym-derivative w.r.t. μ of the absolutely continuous part of $\pi_n\Theta$ calculated w.r.t. $\mathfrak{A}_n$ (the same for $D\Theta$ on $\mathfrak{A}_\infty$). This theorem contains the classical martingale theorem as well as the Lebesgue differentiation theorem in $\mathbb{R}^n$. Various generalizations of this to the case of general Θ_n replacing $\pi_n\Theta$ and taking values in a Banach space X satisfying Radon-Nikodym theorem are given where the basic measure μ need not be finite or is only finitely additive. Relationships with the classical Vitali-Besikovitch-theorems are mentioned.

M. DE GUZMAN : Differentiation of integrals in $\mathbb{R}^n$

The problems treated in this communication referred to the local theory of differentiation, i. e. to the extensions of the classical theorem of Lebesgue on differentiation. The development of the theory has lead to intersting connections with covering lemmas of different types and with weak-type properties for the Hardy-Littlewood maximal operator and their generalizations. Several recent results concerning these relationships were discussed and some open problems in the field were stated. (Reference: M. DE GUZMAN, Differentiation of integrals in $\mathbb{R}^n$, Lecture Notes Universidad complutense de Madrid, 1974)

F. TOPSOE : Packings and coverings with balls in Euclidean space

The following theorem was proved and discussed:

Theorem: Let $X = \mathbb{R}^d$ with some norm. Let $A \subseteq X$, and $\mathcal{S}$ be a class of closed balls such that $\forall_{x\in A} \exists_{0\leq\alpha<1} \forall_{\varepsilon>0} \exists_{[y,r]\in\mathcal{S}} \|x-y\| \leq r\alpha,\ r\leq\varepsilon$

Let μ be any Radon measure on X

(i) There exists $(B_n)_n$ disjoint in $\mathcal{S}$, such that $\mu^*(A \setminus \bigcup B_n) = 0$
(ii) To $\varepsilon > 0$ there exists $(B_n)_n$ in $\mathcal{S}$, such that $\bigcup\limits_n B_n \supseteq A$ and $\sum\limits_n \mu(B_n) \leq \mu^*(A) + \varepsilon$

Both in (i) and (ii) the sets B_n can be chosen inside any prescribed open set containing A.

Radon-Nikodým'scher Satz für vektorwertige Maße

G.Y.H. CHI : On Radon-Nikodým theorem for a class of locally convex spaces

A locally convex space (l.c.s.) E has property (BM) iff for all bounded subsets B of the space $l_N\{E\}$ of absolutely summably sequences in E there exists an absolutely convex bounded closed metrizable $M \subseteq E$ such that $\sum\limits_{i=1}^{\infty} p_M(x_i) \leq 1$ for all $(x_i)_i \in B$ (p_M denotes the Minkowski-functional of M). The following analogue of RIEFFEL's Radon-Nikodým theorem can be established:

Theorem: Let E be a quasicomplete l.c.s. with property (BM) and let (Ω, Σ, μ) be a complete probability space. Let $m: \Sigma \to E$ be a (countably additive) vector measure. Then m has a strongly (Pettis) integrable derivative w.r.t. μ iff: (i) m is absolutely continuous w.r.t. μ (ii) m has bounded variation (iii) m has locally relatively compact average range.

Examples of l.c.s. having property (BM) are: Frechet spaces, (LF)-spaces, Montel (DF)-spaces, strong duals of metrizable Montel-spaces, strong duals of metrizable Schwartz-spaces, precompact duals of separable metrizable spaces.

J. DIESTEL : The Radon-Nikodým theorem and spaces of operators

The relationships of the RNP with the coincidence of integral and nuclear operators were discussed; in particular it is noted that if X^* has RNP and the approximation property then X^* has the metric approximation property. The classical Dixmier-v. Neumann--Schatten duality theory, i. e. K(X) (the compact operators) has $N(X^*)$ (nuclear operators) as a dual and $N(X^*)$ has $L(X^{**})$ (linear operators) as a dual, is generalized by the corollary fact that if X^* has RNP and the approximation property, then this duality is valid too. The problem of which spaces of operators have RNP was then discussed noting that X^* has RNP and the approximation property implies $N(X^*)$ has RNP and there is a "good" sufficient condition that L(X,Y) has RNP.

R.E. HUFF : Geometric characterizations of the Radon-Nikodým property

A Banach space is said to have the Radon-Nikodým property (RNP) provided every absolutely continuous X-valued measure of bounded variation on $[0,1]$ has a Bochner-integrable Radon-Nikodým derivative. The talk was concerned with recently obtained internal, geometric characterizations of the RNP in terms of the existence of extreme points, exposed points, and supporting hyperplanes for closed bounded sets in X. (The results discussed are due to several people; particularly to R.R. PHELPS, and to P.D. MORRIS and the speaker jointly)

Z. LIPECKI and K. MUSIAŁ : On the Radon-Nikodým derivative of a measure taking values in a Banach space with basis

(communicated by Z. Musiał)

Let (Ω, Σ, μ) be a positive finite measure space and let X be a Banach space with basis $\{x_n\}$ and the associated coefficient functionals $\{x_n^*\}$.

Theorem: Let $\{f_n\} \subseteq L_1(\mu)$ and a measure $\nu : \Sigma \to X$ be such that $\forall\, E \in \Sigma\ \forall\, n \in \mathbb{N} :\ x_n^* \nu(E) = \int_E f_n \, d\mu$. Then the following are equivalent: (i) ν has a Pettis-μ-integrable RN-derivative

(ii) $\sum_i f_i(\cdot) x_i$ converges strongly μ-almost everywhere

(iii) $\sum_i f_i(\cdot) x_i$ converges weakly in μ

Either of them implies that: $\nu(E) = (P)\text{-}\int_E \sum_n f_n(\cdot) x_n \, d\mu \quad \forall E \in \Sigma$

Darstellungssätze vom Riesz'schen Typ

J. BLIEDTNER : Representation of positive linear functionals by measures

Let X be a locally compact space, $P \subseteq C(X)$ be a convex cone such that the space $K(X) \subseteq C(X)$ of functions with compact support is contained in $C_P(X) := \{ f \in C(X) \mid \exists\, p \in P : |f| \leq p \}$. By a theorem of Choquet every positive linear functional τ on P is represented by a positive Radon measure μ on X provided P is an adapted cone, i.e. $\forall p \in P\ \exists q \in P : \forall \varepsilon > 0\ \exists K \subseteq X$ compact such that $p \leq \varepsilon q$ on $X \setminus K$. This implies especially that $\tau : P \to \mathbb{R}_+$ positive linear satisfies the following condition : $(*)$ $\forall\, p \in P\ \forall \varepsilon > 0\ \exists q \in P\ \exists K \subseteq X$ compact such that $p \leq q$ on $X \setminus K$ and $\tau q \leq \varepsilon$. We have the following:

Proposition: Suppose $\tau : P \to \mathbb{R}_+$ satisfies (*). Then τ can be represented by a measure μ.

This proposition has applications to potential theory (existence of balayaged measures). A connection of condition (*) to the adaptedness of P is given by the following

Proposition: Consider the statements: (1) P is adapted (2) Every $\tau : P \to \mathbb{R}_+$ positive linear satisfies (*) (3) The set $P_\sigma := \{\sum_n p_n \in C(X) \mid (p_n)_n \in P\}$ is adapted. Then (1)$\Rightarrow$(2)$\not\Rightarrow$(3)

B. FUCHSSTEINER : A generalization of the Riesz-Choquet-representation theorem

Let X be an arbitrary set and F(X) be a cone of upper bounded functions $f: X \to [-\infty,\infty[$ containing the constants. F(X) is called a Dini-cone if for every decreasing sequence $(f_n)_n$ in F(X) we have: $\sup_{x\in X} \inf_{n\in\mathbb{N}} f_n(x) = \inf_{n\in\mathbb{N}} \sup_{x\in X} f_n(x)$. A positive homogeneous and additive functional $\mu: F(X) \to [-\infty,\infty[$ is called a state if for all $f \in F(X)$ $\mu(f) \leq \sup_{x\in X} f(x)$.

Theorem: For every state μ there is a probability measure ν (defined on the σ-algebra over X generated by F(X)) such that $\mu(f) \leq \int_X f\, d\nu$ $\forall f \in F(X)$ if and only if F(X) is a Dini-cone. In case that X is a topological space and F(X) consists of continuous functions then ν can be extended to a σ-algebra containing all compact sets.

This theorem is proved via continuous decomposition properties in the state space. Choquet's theorem is a special case of this result since the restrictions to the extreme boundary of upper semicontinuous functions on a compact convex set form a Dini-cone (maximum principle).

J.D.M. WRIGHT : Measures with values in partially ordered linear spaces

Let V be a monotone σ-complete partially ordered linear space. A V-valued measure on a measurable space $(X,\mathcal{S})$ is a map m from $\mathcal{S}$ into V^+ sucht that: (i) $\forall E,F \in \mathcal{S}: m(E)+m(F)=m(E\cup F)+m(E\cap F)$ (ii) if $(E_n)_n$ is an increasing sequence in $\mathcal{S}$, then $m(\bigcup_n E_n) = \bigvee_n m(E_n)$

Theorem: Let X be a compact Hausdorff space and $\varphi: C(X) \to V$ a positive linear operator. Then there exists an unique V-valued Baire measure m on X such that $\varphi(f) = \int_X f\,dm$ $\forall f \in C(X)$. Further if V is monotone complete then there exists an unique Borel measure μ such that μ extends m.

Problem: Give a constructive proof of this theorem.

If the analogue of the Hopf-Kolmogorov extension theorem holds for V we say that V has the measure extension property.

Theorem: V has the measure extension property if and only if each V-valued Baire measure on each compact space is regular.

Specialize by requiring V to be a lattice. By a result of MATTHIS if V is weakly σ-distributive then V has the measure extension property. The converse of this result follows from a result of the speaker who proved that V is weakly σ-distributive if and only if each V-valued Baire measure on each compact Hausdorff space is regular.

Vektorwertige Maße - Integration, Liapunov - Eigenschaft

G. KNOWLES : On Liapunov vector measures

A vector measure on a σ-algebra $\mathfrak{A}$ taking values in a quasi-complete locally convex space X is called Liapunov iff the range of its restrictions to $\mathfrak{A}_F = \{E \in \mathfrak{A} | E \subseteq F\}$ is convex and weakly compact for every $F \in \mathfrak{A}$. It is called closed, iff the associated metric space of equivalence classes is complete. A necessary and sufficient condition for a closed vector measure to be Liapunov is: For every non-m-null set $E \in \mathfrak{A}$ there exists a bounded measurable function f, non zero on E, such that $\int_E f\, dm = 0$.

From this theorem it can be shown that for every vector measure there is a Liapunov measure having the same (closed convex hull of) range and that every vector measure can be decomposed into a Liapunov part and another with the property that integration is essentially a 1-1 map from the bounded mesurable functions into X.

P. MASANI : Pettis integration in Hilbert space

Let H be a Hilbert space, not necessarily separable, and $\mathfrak{B}_{\mathcal{N}}$ be the σ-algebra generated by the class of open balls in H. Let $\mathfrak{A}$ be a σ-algebra over a set Ω. A function $f \in H^{\Omega}$ is $\mathfrak{A}, \mathfrak{B}_{\mathcal{N}}$ measurable iff $\forall x \in H$, $|f(.)-x|$ is $\mathfrak{A}$ measurable. For a positive measure μ on $(\Omega, \mathfrak{A})$ and $0 < p \leq \infty$ define $L_{p,\mu} := \{f \in H^{\Omega}: f \text{ is } \mathfrak{A}, \mathfrak{B}_{\mathcal{N}} \text{ measurable and } \int_{\Omega} |f|^p\, d\mu < \infty\}$. $L_{p,\mu}$ is not a vector space in general for any p, but it is "complete" in a slightly extended sense under the usual L_p norms.

Let $\mathfrak{B}_{\mathcal{N}_\omega}$ be the σ-algebra generated by the base $\mathcal{N}_\omega$ of weak neighborhoods of H. Then $\mathcal{B}_{\mathcal{N}_\omega} \subseteq \mathcal{B}_{\mathcal{N}}$. Hence every $f \in L_{1,\mu}$ is scalarly measurable, and in fact Pettis integrable. There exists $f \in L_{1,\mu}$ such that range f is not μ-essentially separable. Hence $L_{1,\mu}$ is a proper extension of the Bochner class $\mathcal{L}_{1,\mu}$. Functions $f \in L_{1,\mu} \setminus \mathcal{L}_{1,u}$ are encountered in the Lévy inversion for H-valued c.a.o.s. measures over non-second countable l.c.a. groups.

G.E.F. THOMAS : Vector integration

The purpose of this talk is to discuss a property of the Pettis integral which is not shared by the Bochner integral.

Theorem: Let X and Y be Banach spaces; X separable with $X \hookrightarrow Y$. Let $f: \mathbb{R} \to Y$ be a Pettis integrable function such that (i) $f(t) \in X$ $\forall t$, (ii) $\int_A f\, d\nu \in X$ $\forall A \in \mathcal{B}$. Then $f: \mathbb{R} \to X$ is Pettis integrable.

Corollary: X separable, $T: D_T \to X$ a closed operator, $f: \mathbb{R} \to D_T$ a function which is integrable as X-valued function such that $\int_A f\, d\nu \in D_T$. Then Tf is integrable and $\int_A Tf\, d\nu = T\int_A f d\nu$.

This theorem can be proved by using the theorem of Banach-Dieudonné. It can be generalized to a Suslin locally convex space by using a very intersting theorem of CHRISTENSEN : Let $m: \mathcal{P}(\mathbb{N}) \to G$ be an additive set function to an abelian topological group. Assume m is a Borel map (identifying $\mathcal{P}(\mathbb{N})$ with $\{0,1\}^{\mathbb{N}}$) Then m is countably additive.

Funktionale auf Vektorverbänden

K. BICHTELER : Function metrics with dominated convergence theorems

A countably subadditive function metric G on a Stone-lattice R of elementary functions is called weak upper gauge (WUG) if $G(\varphi_n) \to 0$ for disjoint dominated sequences $(\varphi_n)_n$ in R. The dominated and monotone convergence theorems hold in the closure $L^1(G)$ of R. Measurability is defined via the Lusin property and has all desired properties.

Applications: (1) All topological vector lattices L in which disjoint sequences of positive elements converge to zero have order continuous topology and so have their topological completions which are space $L^1(\mathfrak{g})$, $\mathfrak{g}$ a hyperstonean collection of WUG's. One gets: representations of the dual L' if L is locally convex, order characterizations of Σ -completeness, weak sequential completeness, reflexivity etc. (2) Integration of vector measures.

W.A.J. LUXEMBURG : <u>A new proof of Pallu de la Barriere's representation theorem for normal states of von Neumann algebras</u>

Let $\mathfrak{M}$ be a commutative von Neumann algebra of operators on a Hilbert space H and φ be a normal state on $\mathfrak{M}$. Then, by Pallu de la Barriere's theorem, there exists a vector $x \in H$ such that $\varphi(A) = \langle Ax,x \rangle$ for all $A \in \mathfrak{M}$. This result can be easily derived from the following Radon-Nikodym type theorem for normal functionals on Riesz spaces: Let L be a Dedekind complete Riesz space and φ, ψ a pair of positive normal linear functionals on L. Then ψ is absolutely continuous w.r.t. φ if and only if there exists an orthomorphism T on L (i.e. a positive linear transformation of L into L commuting with all projections of L) such that $\psi(f) = \varphi(Tf)$ for all $f \in L$.

A.C. ZAANEN : <u>Normed spaces for which the space of all singular bounded linear functionals is an abstract L-space</u>

If L is a normed vector lattice, L^* is the space of all bounded linear functionals on L and L_c^* the subspace of L^* consisting of all sequentially order continuous elements in L^*, then L_c^* is a band in the vector lattice L^*. The elements in the disjoint complement L_s^* of L_c^* are called the singular bounded linear functionals on L. For L an Orlicz space it was proved by T. ANDO (1960) that L_s^* is an L-space; this was generalized to some "boundary" cases (discontinuous Orlicz function) by M.M. RAO (1968).

<u>Definition:</u> The normed vector lattice L is called a semi-M space if, for any positive u_1, u_2 in L of norm one and for any sequence $u_1 \vee u_2 \geq v_n \downarrow 0$ in L we have $\lim \|v_n\| \leq 1$.

<u>Theorem</u> (E. DE JONGE): If L is semi-M, then L_s^* is an L-space.
(1974) If L has the "principal projection property", the converse also holds.

Since all Orlicz spaces are semi-M, this takes care of the elder results.

Probleme in Verbindung mit Quantentheorie

L. GROSS : Logarithmic Sobolev inequalities

Inequalities of Sobolev type are proven in infinite dimensions. Specifically, if ν denotes Gauss measure on $\mathbb{R}^m$, $m=1,2,\ldots,\infty$, then we proof $\int |f|^2 \ln |f| \, d\nu \leq \int |\text{grad } f|^2 \, d\nu + \|f\|_2^2 \cdot \ln \|f\|_2$

Applications to quantum field theory are described.
(Reference: L. GROSS: Logarithmic Sobolev inequalities, to appear in Amer. J. of Math.)

W. HACKENBROCH : Vector measures and spectral theory

The aim of the talk was to show how vector measures are useful in function space representation of ordered vector spaces both to describe the size of the representing function system and to give (spectral) integral representations. In the "commutative" case of (σ-complete) vector lattices E there is a vector lattice isomorphism $E \cong L^1(\pi, \mathbb{R})$ which generalizes Freudenthal's spectral theorem and also gives Kakutani's representation of AL-spaces. In "non-commutative" situations arising in C^*-algebra theory and also in axiomatic quantum mechanics one has a vector space A of bounded real functions on some set Ω such that (1) every face $F := \{t \in \Omega : a(t) = 0\}$ (for some $0 \leq a \in A$) has a complementary face $F^\perp \subseteq \Omega \setminus F$ and (2) every F gives rise to a projection P_F from A into A defined by $P_F a = a$ on F and $P_F = 0$ on $F^\perp$ for $0 \leq a \in A$. Then if A is pointwise monotone σ-complete we have a spectral theorem: $a = \int \lambda \pi(d\lambda)$ for some spectral measure π on $\mathbb{R}$ iff a is measurable w.r.t. the logic of all faces in an appropriate sense.

D.A. KAPPOS : Measure theory on orthomodular lattices

The concept of an orthomodular σ-poset, i.e. an orthocomplemented partially ordered set L with addition such that for every sequence $(a_n)_n$ of pairwise orthogonal elements of L $\bigvee_n a_n$ exists and equals $\sum_n a_n$. In case that L is an orthomodular lattice it was shown that L is the union of a family of maximal Boolean sublattices. With the help of this result, the concepts of probability measures, measurable functions, random variables, experiments and of observables can be defined.

Wahrscheinlichkeitstheorie und Stochastische Prozesse

W.M. BOGDANOWICZ : A new approach to the theory of probability via algebraic categories

In this talk were presented statistical considerations leading to the definition of the category EXP of expectation spaces and the category ARRP of all representations of random processes. Properties of morphisms of such categories were established and some isomorphic categories were derived. This permits one to define two functors: STAT and PHYS from the category ARRP into itself which are idempotent. The first functor does not distinguish statistically equivalent random processes and the second one does not distinguish processes with the same set of trajectories. Image of ARRP by PHYS represents a category which,generated by means of Baire functions,is identifiable with the category of all random processes. Then the isomorphic categories of (1) Baire probability spaces, (2) Baire measure spaces, (3) Kolmogorov's distribution spaces, (4) Bochner's positive definite functions, are derived giving a better insight into the classical theory of random processes.

V. GOODMAN : Transition probabilities for a Banach space-valued Brownian motion

A mean zero Gaussian measure, μ, defined on the Borel algebra of a real separable Banach space, B, determines an independent increment stochastic process $\{W(t)\}$, $t \geq 0$, with transition probabilities given by $P(\{x + W(t) \in E\}) = \mu(t^{-\frac{1}{2}}E - x)$ where $t > 0$, x B, $E \subseteq B$. The process $\{x + W(t)\}$ is called B-valued Brownian motion. A new process is obtained by truncating a sample path at the point of first exit from a fixed open subset U of B. Transition probabilities for this process are studied through characterization of their Radon-Nikodým derivatives relative to the Brownian motion transition probabilities. In addition, the result that the Yeh-Wiener process in two variables is a $C([0, 1])$ valued Brownian motion with μ = classical Wiener measure is discussed. It is conjectured that the Radon-Nikodým derivative of transition probabilities for $U = \{x \mid \sup_{0 \leq s \leq 1} x(s) \leq 1\}$ is

$$1 - \exp\left[- \frac{2}{t} \cdot \min_{0 \leq s \leq 1} \frac{y(s) - 1}{s} \, (x(s) - 1) \right]$$

S. KAKUTANI : A problem in equidistribution

Let $\{P_n: n=1,2,\dots\}$ be a sequence of partitions of the unit interval $[0,1]$, where $P_n = \{0=x_1^n < x_2^n < \dots < x_{m_n}^n = 1\}$ $n=1,2,\dots$

We say that $\{P_n: n=1,2,\dots\}$ is equidistributed on $[0,1]$ if

$$\lim_{n\to\infty} \frac{1}{m_n+1} \sum_{k=0}^{m_n} f(x_k^n) = \int_0^1 f(t)\,dt$$ for any real valued continuous function defined on $[0,1]$.

Let $0<\alpha<1$. Consider the sequence $\{P_n(\alpha): n=1,2,\dots\}$ of partitions of $[0,1]$ defined, by induction, as follows: (i) $P_1(\alpha) = \{0,1\}$ (ii) $P_{n+1}(\alpha)$ is obtained from $P_n(\alpha)$ by decomposing all intervals of $P_n(\alpha)$ which are of maximal length into two parts with the ratio $\alpha : 1-\alpha$. It is shown that $\{P_n(\alpha): n=1,2,\dots\}$ thus obtained is equidistributed on $[0,1]$. The proof is based on a result in ergodic theory obtained in a joint paper of the speaker with HAJIAN and ITO which appeared in the Advances in Math. in 1972. (vol. 9, pp. 52-65; "S is ergodic" on page 63)

Z.R. POP-STOJANOVIC : Absolute continuity of measures generated by Ito-McShane stochastic differential equations

The setting: Let $(\Omega, \mathfrak{F}, P)$ be a probability space and let $(\mathfrak{F}_t, t\in[0,1])$ be a family of σ-subalgebras of $\mathfrak{F}$, such that $\forall s,t\in[0,1]: s\le t \Rightarrow \mathfrak{F}_s \subseteq \mathfrak{F}_t$. Denote by $(\underline{C}, \mathfrak{B})$ a measurable space of continuous functions $X = (X_t: t\in[0,1])$ with $\mathfrak{B}$ the σ-algebra generated by $\{X_s: s\le 1\}$ (X_s are $\mathfrak{F}$ measurable). Finally let $Z = (Z_t, \mathfrak{F}_t: t\in[0,1])$ be a quasi-martingale with $Z_0 = 0$.

Definition: A continuous random process $\xi = (\xi_t, \mathfrak{F}_t: t\in[0,1])$ is called of Ito-McShane type (w.r.t. Z) if there exists a random process $\beta = (\beta_t, \mathfrak{F}_t: t\in[0,1])$ such that (i) $\int_0^1 |\beta_t(\omega)|\,dt < \infty$ a.e.(P) and (ii) $\forall t\in[0,1]: d\xi_t = \beta_t(\omega)dt + dZ_t$, $\xi_0 = 0$, where the last equation is Ito-McShane stochastic differential equation.

Let μ_ξ, μ_z be measures on $(\underline{C}, \mathfrak{B})$ corresponding to ξ and Z, i.e. $\forall B\in\mathfrak{B}: \mu_\xi(B) = P(\{\omega: \xi(\omega)\in B\})$, $\mu_z(B) = P(\{\omega: Z(\omega)\in B\})$.

Then the following holds:

Theorem: Let ξ be of Ito-McShane type w.r.t. Z.

If $\int_0^1 \beta_t^2(\omega)\,dt < \infty$ a.e. (P) then $\mu_\xi \ll \mu_z$

Maße auf Mannigfaltigkeiten und uniformen Räumen

G. FICHERA : Homology spaces of k-measures and related inequalities for differential forms

Homology spaces of k-measures on a compact differentiable manifold V were defined. The related isomorphism theorems were shown to be connected with some "a priori" inequalities for smooth differential forms defined on V.
(Reference: G. FICHERA : Spazi lineari di k-misere e di forme differenziali, Proc. Int. Symp. on linear spaces, Jerusalem 1960, pp. 175-226 (1961)

Z. FROLIK : Measure theory on uniform spaces

In this talk the definitions of uniform and free uniform measures on uniform spaces were given. Then topological properties of the spaces of uniform and free uniform measures were derived (e.g. compact sets coincide in the strong and the weak topology). Further the problem of reducing the theory to "simple spaces" was discussed. Finally problems and conjectures were stated and examples given.

Maßtheoretische Probleme in der Mathematischen Ökonomie

D. SONDERMANN : Measure theory and economic equilibrium analysis

A large economy can be described as an atomless distribution on the space T of agent's characteristics. Let $\mathcal{E}$ denote the space of all atomless distributions on T and S the price simplex. Then equilibrium analysis is concerned with the study of the Walras-correspondence W from the parameter space $\mathcal{E}$ into the state space S which associates to each economy E the set W(E) of price equilibria for this economy. Four aspects have been studied, so far, in equilibrium analysis: (1) Existence (i.e. $W(E) \neq \emptyset$) by means of algebraic topology and measure theory (2) Computation by means of combinatorics (3) Uniqueness and (4) Continuity by means of global analysis.

Measure theoretical problems in equilibrium analysis:
(1) Is there a "canonical" measure on T ? (2) What measures on T give rise to a C^1 demand function ? (3) What measures on T give rise to an excess demand which is a gradient field on S ? (4) Will a "smooth" distribution on T yield a "smooth" excess demand ?

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