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Inhaltsverzeichnis zum Vortragsbuch Nr. 70

Synthetische Methoden in der algebraischen Geometrie

13. bis 19. April 1986

(Algebraische Geometrie vom synthetischen Standpunkt - Gästebuchtitel!)

Anwendung der Graßmannmannigfaltigkeiten

Sind P ein n-dimensionaler projektiver Raum und T ein Teilraum von T, so ist T selbst ein projektiver Raum und führt seinerseits zu einem projektiven Faktorraum P/T. Vom synthetischen Standpunkt her ist von besonderem Interesse, daß nach einem Satz von Burau für jedes $k<n$ die Graßmannmannigfaltigkeiten von T und P/T sich so in die Graßmann $G_{n,k}$ einbetten lassen, daß sie der Schnitt von $G_{n,k}$ mit dem von ihnen im Graßmannraum aufgespannten Teilraum sind. Es wird u.a. erläutert, wie sich aus diesem Satz die maximalen Teilräume von $G_{n,k}$ und ihre Lage zueinander herleiten lassen und wie sich die $G_{n,k}$ aus Teilräumen und Treffgeraden induktiv aufbauen läßt.

15. 4. 86 H. Hotje (Hannover)

Flächen 4. Ordnung

Mit Hilfe der im Rahmen dieser Tagung entwickelten und untersuchten synthetischen Methoden ist es möglich, die Flächen 4. Ordnung geeignet zu klassifizieren. Insbesondere die „klassischen" Eigenschaften der Kummerschen- und der Weddle-Flächen werden mit diesen Verfahren deutlich.

17. April 1986 D. Windelberg (Hannover)

Graßmannsche Mannigfaltigkeiten

Ausgehend von der Tensoralgebra und der Graßmann-Algebra über einem Vektorraum (V,K) beliebiger Dimension und beliebiger Charakteristik wurden Segresche und Graßmannsche Mannigfaltigkeiten betrachtet. Es wurden einfache Beweise zur Gewinnung der für die Graßmannschen Mannigfaltigkeiten grundlegenden Büschelabbildungen und ihrer wesentlichen Eigenschaften angegeben. Ferner wurde der Fall der $G_{n,1}$ genauer diskutiert und auf die Möglichkeit einer Projektion von der $S_{n,n}$ auf die $G_{n,1}$ bei beliebiger Charakteristik eingegangen.

E. M. Schröder (Hamburg)

Synthetische Einführung der Veroneseschen Mannigfaltigkeiten.

Geht man in einem s-fachen Tensorprodukt $\otimes: W_1 \times \ldots \times W_s \to W$ von den beteiligten Vektorräumen W_i, W über zu den abgeleiteten projektiven Räumen $X_i = W_i^*/K^*$, $P := W^*/K^*$, so gelangt man zum Segre-Produkt $\cdot : X_1 \times \ldots \times X_s \to P$, und die von den reinen Tensoren $W_1^* \otimes \ldots \otimes W_s^*$ abstammende Bildmenge $S := X_1 \cdot \ldots \cdot X_s$ ist eine Segre-Mannigfaltigkeit. Die inzidenzgeometrischen Eigenschaften der s Scharen der maximalen ganz auf der Segre S liegenden projektiven Unterräume gestatten die folgende synthetische Definition von $X_1 \cdot \ldots \cdot X_s$: Es seien $X_1, \ldots, X_s$ projektive Räume endlicher Dimension über demselben kommutativen Koordinatenkörper K. Für $s=1$ setzen wir $S := X_1$ und $\langle S \rangle := X_1$. Für $s-1 \geq 1$ seien die Segre $S = X_1 \cdot \ldots \cdot X_{s-1}$ und der von ihr aufgespannte projektive Raum $\langle S \rangle$ bereits erklärt, und es gelte $\dim \langle S \rangle = m$, $\dim X_s = n$. Wir betrachten einen projektiven Oberraum P von $\langle S \rangle$ der Dimension $(m+1)(n+1)-1$ und in P ein System $\mathfrak{B}$ von $n+2$ projektiven Unterräumen $B_0, B_1, \ldots, B_{n+1}$, die zu je $n+1$ in P unabhängig liegen, mit $B_0 = \langle S \rangle$. Die Vereinigungsmenge $\bigcup_{b \in S} \mathrm{Treff}(b, \mathfrak{B})$ über alle Treffräume des Bezugssystems $\mathfrak{B}$ durchstreift dann eine Segre $X_1 \cdot \ldots \cdot X_s$

und für den umgebenden Raum P gilt $\dim P+1=\prod_{i=1}^{s}(\dim X_i+1)$. Jede solche Segre $X_1\cdot\ldots\cdot X_s$ läßt sich als Teilstruktur einer speziellen Segre $X^s:=\underbrace{X\cdot\ldots\cdot X}_{s}$ auffassen, wenn X ein projektiver Raum über demselben Koordinatenkörper mit $\dim X\geq\max\{\dim X_i\mid i\in\{1,\ldots,s\}\}$ ist. Als wichtige Unterstrukturen von X^s erhält man jetzt die Veroneseschen Mannigfaltigkeiten $V^{\pi_1\ldots\pi_s}:=\{\pi_1(x)\cdot\ldots\cdot\pi_s(x)\mid x\in X\}$, $\pi_i\in PGL(X_i)$. Die Schmiegräume der Segre's und ihrer Veronesen werden mittels des durch $d(x,y):=|\{i\in\{1,\ldots,s\}\mid x_i\neq y_i\}|$, $x=x_1\cdot\ldots\cdot x_s$, $y=y_1\cdot\ldots\cdot y_s$ auf X^s definierten Knickabstandes erklärt. Außerdem werden die Automorphismengruppen sowie die dualen Strukturen dieser Grundmannigfaltigkeiten beschrieben.

G. Kist (München)

Oskulanten

Es sei $S=X^s$ eine Segre-Mannigfaltigkeit, so daß für alle Veronese-Mannigfaltigkeiten $V^r\subset S$ der A-Raum A^r und der Raum $\langle V^r\rangle$ der Veronese V^r komplementär liegen. Es seien $V^{r_1}, V^{r_2}\subset S$ zwei Veronesen mit $r_1\leq r_2$. Die Veronese V^{r_1} heißt mit V^{r_2} (im Punkt p) verbunden von der Stufe r_2-r_1, wenn es eine innere Projektion φ der Stufe r_2-r_1 gibt mit $\varphi(V^{r_2})=V^{r_1}$ (und $V^{r_1}\cap V^{r_2}=p$). Es sei jetzt $V^s\subset S$ eine Veronese und $p\in V^s$. Alle mit V^s in p verbundenen Veronesen V^{s-t} haben bei der Projektion ϱ aus dem A-Raum A^s auf $\langle V^s\rangle$ das gleiche Bild $\varrho(V^{s-t})$. Das Bild $\varrho(V^{s-t})$ ist eine Veronese und wird Oskulante von V^s im Punkt p genannt. Es gilt: $\langle\varrho(V^{s-t})\rangle$ ist der Schmiegraum $T(V^s,s-t,p)$ der Stufe $s-t$ von V^s in p;

$$\varrho(V^{s-t})=\{T(V^s,s-t,p)\cap T(V^s,t,x)\mid x\in V^s\setminus\{p\}\}\cup\{p\}.$$

17.4.86 Hans-Joachim Kroll (TU München)

Verlagerungen und Projektionen

Sei X ein n-dimensionaler projektiver, pappuscher Raum, ~~s~~ und V^s eine Veronese. Zu einer direkten Zerlegung $X = A \oplus B$ wird in Abhängigkeit von $t \in \{0,1,\ldots,s-1\}$ eine direkte Zerlegung des von V^s aufgespannten Raumes $\{V^s\rangle = E_t \oplus \overline{E}_t$ gesucht. Seien $V_a^s := \{\overbrace{a\cdots a}^{s\text{-mal}} : a \in A\}$ und $V_b^s := \{b\cdots b : b \in B\}$ die von A bzw. B bestimmten Unterveronesen von V^s und $T(V^s, t, V_a^s)$ bzw. $T(V^s, s-t-1, V_b^s)$ Schmiegräume von V^s, so gilt für $E_t := T(V^s, t, V_a^s)$ bzw. $\overline{E}_t := T(V^s, s-t-1, V_b^s)$ die Behauptung $\langle V^s\rangle = E_t \oplus \overline{E}_t$.

Die Projektion $\pi : \begin{cases} \langle V^s\rangle \setminus E_t \to \overline{E}_t \\ \quad x \quad \to (x+E_t) \cap \overline{E}_t \end{cases}$ heißt dann

<u>(s,t)-Verlagerung</u>. Wir betrachten nun die Restriktion $\pi|V_s$. Dabei wird $V_a^s = \langle V^s\rangle \cap E_t$ kein Bild zugeordnet, während die Punkte von $I_{t+1} := T(V^s, t+1, V_a^s) \cap T(V^s, s-t-1, V_b^s)$ keine Urbilder in V^s besitzen. Durch die <u>Dilatation</u> oder Aufblähung genannte Abb.

$$\tilde{\pi} \begin{cases} V_a^s \longrightarrow \mathfrak{P}(I_{t+1}) \\ (a\cdots a) \longrightarrow \rho(\underbrace{a\cdots a}_{(s-t-1)\text{-mal}}\underbrace{b\cdots b}_{(t+1)\text{-mal}}) \end{cases}$$

(Def von ρ siehe Kooll) wobei b ganz B durchläuft, wird die (s-t)-Verlagerung ergänzt

17.4.86. A. Kreuzer

Synthetische Geometrie einiger Typen von Flächen 3.4. Ordnung der X_3.

Die ~~Flächen 3~~ Kurven 3. und 4. Ordnung der Ebene X_2 werden je durch hyperebene Schnitte der Veronese'schen $V_2^3 \subset P_9$ und $V_2^4 \subset P_{14}$ definiert. Zu Beginn wird kurz erklärt, wie man hiermit die einfachsten singulärer Punkte, d.h. Doppelpunkte und Spitzen, bei ebenen Punkte ebener Kubiken und Quartiken genannt f_1^3 und f_1^4, erklären kann.

Analog ~~k~~ werden die einfachsten singulären bei kubischen Flächen $f_2^3 \subset X_3$, die durch P_{18} –

Schnitte einer $V_3^3 \subset P_{19}$ erklärt sind, definiert.

Ein wesentliches Kapitel der Theorie der Flächen f_2^3 sind die 27 Geraden. Es wird vorgetragen, wie man diese und damit die ganze f_2^3 auf folgende Weise erhält:

Man projiziere eine V_2^3 aus dem Raum

$$Z_5 = \sum_{i=1}^{6} {}^{i}A_0^3 ,$$

wo ${}^{i}A_0^3$ $(i=1,\dots,6)$ die V^3-Bilder der von 6 Punkten allgemeiner Lage der Ebene sind. Dann entsteht aus der V_2^3 eine kubische Fläche f_2^3, und die 27 Geraden darauf ergeben sich auch auf diese Weise einfach, basierend auf einer ausgezeichneten Doppelsechs. Diese Behandlung stammt, natürlich nicht auf diese mehrdimensionale Weise, von A. Clebsch (1833–72).

Es wird im Vortrag noch folgendes kurz erwähnt: Regelflächen f_2^3, ferner der Beweis dafür, dass ein f_2^3 nicht mehr als 4 isolierte singuläre Punkte besitzen kann. Eine f_2^3 mit 4 verschiedenen singulären Punkten ist dann dual zur Steinerschen Römerfläche f_2^4, die ihrerseits wiederum allgemeine Projektion der $V_2^2 \subset P_5$ ist.

17.4.86 W. Burau.

Kegelspitzenmannigfaltigkeiten

Sei eine V_n^2 gegeben und ein projektiver Raum $P = P_{\binom{n+2}{2}-n-2} \subset \langle V_n^2 \rangle$. Sei ferner $v^2: P_n \to V_n^2$ und M die Menge aller Punkte $Q_0 \in P_n$ mit $T(V_n^2, 1, v^2(Q_0)) \cap P \neq \emptyset$. Die Menge M heißt Kegelspitzenmannigfaltigkeit. Ist $M = P_n$, so gibt es ein $R_0 \in P_n$ mit $T(V_n^2, 1, v^2(R_0)) \subset P$. Schließt man diesen Trivialfall aus und ist $g \subset P_n$ eine Gerade und ferner $F \subset v^2(g) + g \cdot P_{n-2}$

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die durch die Gerade g und ihre Schmiegräume bestimmte Regelmannigfaltigkeit, so ist $F \cap P \neq \emptyset$ (Bezout) und somit M eine Hyperfläche der Ordnung $n+1$.
Im weiteren werde P durch Punkte aus V_n^2 aufgespannt. Ist $n=3$, so heißt M Weddlefläche. Sind $P_0^7 \subset V^4(M)$ und ist $V^4(Q_0) \in \sum P_0^7$, so ist $Q_0 \in M$. Es ist daher $V^4(M) = H \cap V_3^4$.
M enthält 6 Doppelpunkte und 25 Geraden sowie eine V_1^3 durch die 6 Doppelpunkte. Projiziert man $V^4(M)$ aus dem V^4(Bild) der 6 Doppelpunkte, so ergibt sich als Bild eine Kummerfläche. Die 15 Verbindungsgeraden der Doppelpunkte werden in 15 Doppelpunkte der Kummerfläche abgebildet. Der 16te ist Bild der V_1^3.
Die Gemonainvolution $(x - \frac{1}{x})$ mit den Fundamentalpunkten in ~~4~~ der den Raum P aufspannenden Punkte, bildet die Weddlefläche in eine weitere ab. Die V_1^3 wird dabei in eine Gerade durch zwei Doppelpunkte abgebildet und die Gerade in die V_1^3 durch die Doppelpunkte. Kein Doppelpunkt der Kummerfläche ist somit ausgezeichnet.

17.4.86 H. Timmermann

Rang R- Mannigfaltigkeiten.

Ist $S_{m,n} = X \cdot Y$ eine Segre, so wird die Rang R-Mannigfaltigkeit $S_{m,n}^{(k)}$ definiert als

$$S_{m,n}^{(k)} = \bigcup\{S_{m,k}\}$$

wobei $S_{m,k}$ alle entspr. Untersegres der $S_{m,n}$ durchläuft.
Es ist $S_{m,n}^{(0)} = S_{m,n}$ und $S_{m,n}^{(n-1)}$ ist die Determinanten-Mannigfaltigkeit. Es gilt folgendes:
Die $S_{m,n}^{(k)}$ besitzt $k+1$ Scharen $\mathcal{S}_1 \dots \mathcal{S}_{k+1}$ definiert durch

~~$\mathcal{S}_i = \{\langle S_{m-k+1,k-1} \cup$~~

$$\mathcal{S}_i = \{\langle S_{m,k-i} \cup S_{i-1,n}\rangle\}:$$

(i) Jede Schar überdeckt die $S_{m,n}^{(k)}$
(ii) Je zwei Scharen liegen disjunkt
(iii) Jedes $\mathcal{S}_i$ besteht aus max. proj. Teilräumen

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(iv) Jeder Punkt aus $S_{m,n}^{(k)} \setminus S_{m,n}^{(k-1)}$ liegt auf eind. best. Raum aus $\mathcal{S}_1$ und $\mathcal{S}_{k+1}$ und ist Durchschnitt aller ihn enthaltenden Räume aus $\mathcal{S}_i$, $1 < i \leq k$.

(v) Jede Gerade der $S_{m,n}^{(k)}$ ist in einem $X \in \mathcal{S}_i$, i geeignet, enthalten.

Synth. Beweise für (i), (ii), (iv) sind einfach; diese für (iii) und bes. für (v) ist noch Problem.

I. Herzer

Cremona-Transformationen

Cremona Transformationen kann man mit Hilfe homaloider Netze $\{f_{n-1}^{s}\}$ in synthet. Weise erklären als Zusammensetzung einer Veronese-Abbildung V^s mit einer Projektion

$$pr: \langle V_n^s \rangle \xrightarrow[Z]{} \bar{P}_n$$

wobei das Zentrum $Z = Z_{n_s-n-1}$ von dem homaloiden Netz abhängt. Diese synth. Definition gestattet es auch, in einfacher Weise Jonquière-Transformationen für jedes Pn zu erklären. Timmermann konnte zeigen, daß die Jonquière Transform. als Produkte von quadrat. Cremona Transformationen dargestellt werden kann. Da man in der Ebene zeigen kann, daß die Jonquière Transformationen die Gruppe der ebenen Cremona Transformationen erzeugt, erhält man so einen synthet. Beweis des Satzes von Max Noether, daß die ebenen Cremona Transformationen von den quadrat. erzeugt werden.

Heinrich Wefelscheid

Rationale Normregelgebilde

Ist Y ein n-dimensionaler projektiver Pappos-Raum $(n \geq 1)$ und $S = \overset{r}{\underset{i=1}{\bigoplus}} S_i$ $(r \geq 1)$ ein weiterer projektiver Raum (der mit Y in einem Universalraum eingebettet liegt) derart, daß es Veronese Abbildungen

$$\pi_i : Y \longrightarrow \mathbb{V}^{(i)} \subset S_i$$

mit $\langle \mathbb{V}^{(i)} \rangle = S^i$ gibt, so heißt nach W. BURAU die Punktmenge

$$\mathbb{F} := \bigcup_{x \in Y} (\pi_1(x) + \cdots + \pi_r(x))$$

eine rationales Normregelgebilde mit Leitveronesen $\mathbb{V}^{(i)}$.

Die Segreschen Mannigfaltigkeiten $\mathbb{S}_{n,r-1}$ ordnen sich hier unter.

Es wird auf das "Zerschneiden" und "Verkleben" von Normregelgebilden eingegangen. Ferner ist $\mathbb{F}$ in kanonischer Weise ein Normregelgebilde $\hat{\mathbb{F}}$ des dualen projektiven Raumes $\hat{S}$ von S zugeordnet (Zusatzvoraussetzung über die Charakteristik von Y notwendig!)

Insbesondere der Fall $n=1$, $s=2$ (Normregelflächen) wird ausführlicher behandelt.

17.4.1986

Hans Havlicek, Wien

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Variationsrechnung

20. – 26. April 1986

Minimal Surfaces With Free Boundaries

We consider the problem to minimize n-dimensional area among currents T whose boundary (or part of it) is supposed to lie in a given hypersurface of $\mathbb{R}^{n+k}$. In the case of codimension one ($k=1$) we prove that the singular set of T, i.e. the set of points where spt T is not an embedded submanifold (with boundary), has codimension at least seven. An example shows that this result is optimal. For $k>1$ we show the finiteness of the mass of the free boundary and estimate the upper $(n-1)$-dimensional density of the free boundary. An important ingredient is a regularity result for stationary varifolds with a free boundary by Jost and myself. Applications include regularity results for minimal hypersurfaces with prescribed homology class of the boundary, boundary regularity for solutions of a partitioning problem, and the the existence of a minimal embedded disk inside a given convex body in $\mathbb{R}^3$ (joint work with Jost).

Michael Grüter, Düsseldorf

Free boundary problems for surfaces of constant mean curvature

Let S be a surface contained in a ball $B_R(0)$ in $\mathbb{R}^3$ and C^4-diffeomorphic to the standard sphere S^2. A disc-surface of constant mean curvature H supported by S and meeting S orthogonally along its boundary induces a non-constant solution $X \in C^2(B, \mathbb{R}^3) \cap C^1(\bar{B}, \mathbb{R}^3)$ of the system

(1) $\Delta X = 2H X_u \wedge X_v$ in B

(2) $|X_u|^2 - |X_v|^2 = 0 = X_u \cdot X_v$ in B

(3) $X(\partial B) \subset S$

(4) $\frac{\partial}{\partial n} X(w) \perp T_{X(w)} S$, $\forall w \in \partial B$.

Here, $B = \{w = (u,v) \in \mathbb{R}^2 \mid u^2 + v^2 < 1\}$, $X_u = \frac{\partial}{\partial u} X$, "$\wedge$" denotes exterior product in $\mathbb{R}^3$, n is the unit normal on ∂B, "$\perp$" means orthogonal, and $T_p S$ denotes the tangent space to S at p.

__Theorem__: For almost any H (in the sense of Lebesgue measure) with $|H| R < 1$ there exists a non-constant solution to (1)-(4).

The result extends an earlier result by the author for minimal surfaces ($H = 0$), cp. Inv. Math. (1984). For the proof one analyzes the evolution problem associated with (1)-(4) using methods developed for harmonic mappings of surfaces, cp. Struwe, Comm. Math. Helv. (1985).

Michael Struwe, Zürich

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On the Morse Index of Minimal Surfaces in $\mathbb{R}^p$ with Polygonal Boundaries

Let $\Gamma \subset \mathbb{R}^p$ $(p \geq 3)$ denote a Jordan polygon with $N+3$ $(N \geq 1)$ vertices. Then the minimal surfaces x spanning Γ correspond to the critical points $\mathring{\tau}$ of an analytic function $\Theta : T \to \mathbb{R}$ in N variables. This function was introduced by M. Shiffman and the regularity of Θ has been investigated by E. Heinz.

Now we are interested in the correspondence of the second order: The second derivative of Θ is given by its Hessian $H(\mathring{\tau})$, the second variation of the (eigenvalue problem of this) minimal surface x is described by the Schwarz operator S_x. We first study the eigenvalue problem of this singular differential operator by variational methods. Counting the negative eigenvalues of S_x we obtain the Morse index m_x of x.

As central result we prove: The Morse index m_x of the minimal surface x coincides with the Morse index $m(\mathring{\tau})$ of Θ (the number of neg. eigenvalues of $H(\mathring{\tau})$) at the corresponding critical point $\mathring{\tau} \in T$. This is achieved by comparison of the finite and infinite dimensional eigenvalue problems of $H(\mathring{\tau})$ and S_x, using certain simultaneous variations of the minimal surface.

Literature: F. Sauvigny; Math. Zeit 1985, manuscripta math. 1985

Friedrich Sauvigny, Clausthal-Zellerfeld

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Zur Regularität von Variationsproblemen mit nichtkonvexem Hindernis

Es werde das Dirichletintegral $\int_\Omega |\nabla u|^2 dx$, $\Omega^{bd} \subset \mathbb{R}^n$, in der Klasse

$$\mathcal{M} = \{ u \in u_0 + \mathring{H}_2^1(\Omega)^N \mid u^N(x) \geq g(x, u^1, \dots, u^{N-1}(x)), \text{ a.e.} \}$$

minimiert; u_0 und $\partial\Omega$ seien o.E. glatt. Hierbei seien $g(x,y)$ und $\frac{\partial g}{\partial y}(x,y)$: $\Omega \times \mathbb{R}^{N-1} \to \mathbb{R}$ von der Klasse C_1, aber nicht notwendig konvex.

Unter gewissen Voraussetzungen (z.B. $g = g(x,|y|^2)$ mit $\frac{\partial g}{\partial t}(x,t) \geq 0$ or $N=2$, $\frac{\partial g}{\partial y}(x,y) < 0$) wird gezeigt, daß ein Minimum $u \in H_2^1(\Omega)^N$ beschränkt ist und zur Klasse $C_{1,\alpha}(\bar{\Omega})^N$ für alle $\alpha < 1$ gehört, wobei die Norm a-priori abgeschätzt werden kann.

Michael Wiegner (Bayreuth)

The obstacle problem for energy minimizing maps

We generalize the known partial regularity theorems for harmonic maps to the case of vector valued obstacle problems. To be precise let us consider Riemannian manifolds X^n, Y^k (embedded in Eucl. space $\mathbb{R}^N$) and let $M \subset Y$ be a bounded smooth domain not touching the boundary of Y. For $\Omega := \text{Int}(X)$ we define the space of comparison functions $H^1(\Omega, \overline{M}) := \{u \in H^1(\Omega, \mathbb{R}^N) : u(x) \in \overline{M} \text{ a.e.}\}$ and introduce the class $\mathcal{C} := \{u \in H^1(\Omega, \overline{M}) : E(u) < E(v)$ for all $v \in H^1(\Omega, \overline{M})$, $\text{spt}(u-v) \subset\subset \Omega\}$ of local minimizers under the side condition $u(\Omega) \subset \overline{M}$. The following regularity results were obtained in collaboration with F. Duzaar.

A. (1st interior partial regularity) $u \in \mathcal{C} \Longrightarrow \mathbb{H}^{n-2}(\Omega \cap \text{Sing}\, u) = 0$.
B. (optimal interior partial regularity) $u \in \mathcal{C} \Longrightarrow \mathbb{H}\text{-dim}(\Omega \cap \text{Sing}\, u) \leq n-3$;
for $n = 3$: the interior singularities are isolated.
C. (boundary regularity) if $u \in H^1(\Omega, \overline{M})$ minimizes for smooth boundary values $\partial\Omega \to \overline{M} \Longrightarrow \text{Sing}\, u \subset\subset \Omega$.
D. (removable singularities) if $M \subset \mathbb{B}(p)$ for a regular ball $\mathbb{B}(p)$ and if M is star shaped w.r.t. $p \Longrightarrow \text{Sing}\, u = \emptyset$.

In the Euclidean case $\Omega \subset \mathbb{R}^n$, $M \subset \mathbb{R}^N$, A. holds for local minima of splitting functionals $Fu = \int_\Omega a_{\alpha\beta}(x,u)\, B^{ij}(x,u)\, \partial_\alpha u^i \partial_\beta u^j \, dx$, and B, C are true if $\partial_u a \equiv 0$. Moreover, in all cases we can impose an additional integral constraint of the form $\int_\Omega g(x,u)\, dx \equiv \text{const}$ on the comparison maps. The proofs use methods of Schoen & Uhlenbeck, Giaquinta & Giusti, Jost & Meier and Hildebrandt & Kaul & Widman.

Martin Fuchs, Düsseldorf

Spatially Localized Free Vibrations for Certain Semilinear Wave equations in $\mathbb{R}^2$: Recent Results and Open Problems.

In our talk, we shall analyze and compare three recent results regarding the existence of spatially localized free vibrations for semilinear wave equations of the form $u_{tt} = u_{xx} - g(u)$ on $\mathbb{R}^2 \ni (x,t)$. The first one is due to J.M. Coron (1982), the second one to A. Weinstein (1985) and the third one to myself (1986). We wish to show that the class of nonlinearities $u \to g(u)$ for which such solutions exist is in fact very "small". We also wish to mention a few open problems regarding the above questions, connected with the topological methods of the calculus of variations.

Pierre Vuillermot, Arlington.

Optimal Isoperimetric Inequalities We made precise and indicated proofs for the following.

- OPTIMAL ISOPERIMETRIC INEQUALITY. Corresponding to each m dimensional closed surface in $\mathbb{R}^{n+1}$ there is an m+1 dimensional surface Q having boundary T such that

$$|Q| \leq \gamma(m+1)\,|T|^{\frac{m+1}{m}}$$

with equality if and only if T is a standard round m sphere (of some radius) and Q is the corresponding flat m+1 disk. The equality defines the optimal isoperimetric constant $\gamma(m+1)$

- OPTIMAL ISOPERIMETRIC INEQUALITY FOR MANIFOLDS WITH BOUNDARY.
- AREA- MEAN CURVATURE CHARACTERIZATION OF STANDARD SPHERES. Suppose V is an m dimensional closed surface in $\mathbb{R}^{n+1}$ without boundary. If the mean curvature vectors of V do not exceed in length those of a standard round m sphere $\mathbb{S}^m$ of unit radius, then the m area of V (actually of the extreme points of V) is not less than the m area of $\mathbb{S}^m$. Furthermore, equality holds if and only if V is such an $\mathbb{S}^m$.
- MEAN CURVATURE REGULARITY THEOREM FOR COMBINATORIAL CYCLES. Suppose $T = \underline{t}(S, \Theta, \xi)$ is a real current and there is $\varepsilon > 0$ such that the associated varifold $v(S, \Theta + \varepsilon, \tau)$ has bounded mean curvatures. Then $\operatorname{spt} T \sim \operatorname{spt} \partial T$ is almost everywhere a $C^{1,\alpha}$ submanifold.

Frederick J Almgren Jr, Princeton.

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An integrality theorem and a regularity theorem for hypersurfaces whose first variation with respect to a parametric elliptic integrand is controlled.

Consider a hypersurface S which is stationary with respect to the area integrand in $C = \{(x,y) \in \mathbb{R}^n \times \mathbb{R} : |x| < 2\}$ and which lies close to $D = \{(x,y) \in \mathbb{R}^n \times \mathbb{R} : |x| < 2$ and $y = 0\}$. The following two theorems are proved in my 1972 Annals paper.

Theorem The area of S in $\{(x,y) : |x| < 1\}$ is nearly an integer times the area of the unit disc in $\mathbb{R}^n$.

Theorem Suppose the area of S in $\{(x,y) : |x| < 1\}$ is nearly the area of the unit disc in $\mathbb{R}^n$. Then $S \cap \{(x,y) : |x| < 1\}$ equals the graph of a function whose derivative can be estimated by a constant times $\left(\int |y|^2 \, d\|S\|(x,y)\right)^{1/2}$.

In the present work we extend these theorems to arbitrary elliptic integrands.

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Minimal surfaces and variational methods in the large.

In joint work with J. H. Rubinstein, variational methods in the large and geometric measure theory are used to construct smooth minimal surfaces in manifolds. Typically one is able to bound *a priori* both the topological type and the index of instability of the minimal surfaces so obtained. For example, one has the following.

Theorem. Let Σ be a smooth, compact, connected, oriented, three dimensional Riemannian manifold with Heegard genus H. Then Σ supports a nonempty, smooth, compact, embedded, two dimensional minimal submanifold M such that $\text{genus}(M) \le H$ and $\text{index}(M) \le 1 \le \text{index}(M) + \text{nullity}(M)$.

Using these methods, one constructs many new examples of minimal surfaces of geometric and topological interest.

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Area-Minimizing Surfaces in Grassmannians

In joint work with H. Gluck and W. Ziller, we look for area-minimizing representatives of the homology of the Grassmannian $G_m\mathbb{R}^n$ of oriented unit m-planes in $\mathbb{R}^n$. SubGrassmannians, which are totally geodesic, are particularly good candidates.

Theorem. Consider $S = G_m\mathbb{R}^{m+k} \subset G_m\mathbb{R}^{m+\ell}$.
If m is odd, then S is homologous to 0 over $\mathbb{Q}$.
If m is even, then S is homologically area-minimizing.

The method of proof is to "calibrate" S by an invariant differential form φ, such as the Euler form. The hard part is to verify that the comass $\|\varphi\|^*$ is one.

However, we show that $G_2\mathbb{R}^4 \subset G_3\mathbb{R}^6$ is NOT homologically area-minimizing by presenting another surface $S \sim G_2\mathbb{R}^4$ with less area. S is not totally geodesic, and it has two conical singularities. This surface S suggests new families of canonical subvarieties of Grassmannians.

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Partial regularity for energy minimizing p-harmonic maps

A simple proof for $C^{1,\alpha}$ regularity of local minimizers for $\int |Du|^p$ among maps between riemannian manifolds outside a singular set of Hausdorff dimension at most $n-p$ is given.

The main step is to get Hölder continuity for u. That works for minimizers of $\int f(x,u,Du)$ if f is uniformly continuous, convex in Du, fulfills $c^{-1}|q|^p - 1 \le f \le c^{-1}|q|^p + 1$

and if solutions to the blow up equation $\operatorname{div}(\partial_q F(Dv)) = 0$ where $F(q) = \lim_{\alpha \to \infty} \alpha^{-p} f(x,u,\alpha q)$ are always Hölder continuous.

The proof relies on the strong convergence of blow up sequences in regular points. In order to get that one needs a lemma which says:

If $\rho^{p-n} \int_{B_\rho} |Du|^p$ is small or $\rho^{p-n} \int_{B_\rho} |Du|^p < c$ and $\rho^{-n} \int_{B_\rho} |u - \bar{u}|^p$ is small then u can be modified in the interior of B_ρ such that $\operatorname{osc}(\tilde{u}|_{B_{(1-\delta)\rho}})$ small and the energy increases by a factor $1+\delta$ only.

Stephan Luckhaus Heidelberg

On a nonlinear elliptic equation involving the critical Sobolev exponent (joint work with A. Bahri)

Let Ω be a bounded connected regular open set in $\mathbb{R}^N$. We consider the following equation

$$(*)\begin{cases} -\Delta u = u^{\frac{N+2}{N-2}} \\ u > 0 \text{ in } \Omega \\ u = 0 \text{ on } \partial\Omega \end{cases}$$

The solutions of (*) are the non trivial critical points of the functionnal I defined on $H^1_0(\Omega)$ by

$$I(u) = \frac{1}{2}\int|\nabla u|^2 - \frac{N-2}{2N}\int (u^+)^{\frac{2N}{N-2}} dx$$

I does not satisfy the Palais-Smale condition at the level pS where S is some real number which depends only on N and $p \in \mathbb{N}^*$. Following an idea introduced by A. Bahri in his paper on the Weinstein's conjecture we compute the change of topology at the level pS due to this lack of compactness
We give sufficient conditions on the topology of Ω for the existence of a solution to (*). In particular when $N = 3$ we prove that if Ω is not contractible in itself then (*) has at least a solution.

J.M. Coron
Ecole Polytechnique France

The homogeneous Dirichlet problem for the nonlinear Boussinesq equation

The point of the talk was to show that variational methods used for proving the existence of weak solutions to semilinear elliptic equations can be applied to certain nonelliptic equations. As an example the mountain pass lemma of Ambrosetti/Rabinowitz was applied to ~~certain nonellipt~~ the Boussinesq equation

$$v_{tt} - a v_{xx} - b v_{xxxx} + c v_{xx} v_x = 0, \quad b > 0, \quad a, c \in \mathbb{R}, \quad (x,t) \in G \subset \overset{\text{bold}}{\mathbb{R}}^2$$

to obtain the existence of a nontrivial weak solution on bounded domains. The solution is obtained in an anisotropic Sobolev space. It satisfies generalized homogeneous boundary conditions

G. Warnecke, FU Berlin

Constitutive inequalities in elastostatics.

Let ϕ be a diffeomorphism $\Omega \subset R^3 \to R^3$, $D\phi = F = RU$ polar decomposition of its gradient F considered as an element of the general linear group GL_+, $(\det F > 0)$. The behavior of the material is specified by the stored energy density $\mathcal{E} : GL_+ \to R$. The natural constitutive assumption of convexity of $\mathcal{E}$ is unrealistic (see rotations – $\mathcal{E}(RU) = \mathcal{E}(U)$, isochoric deformations for an almost incompresible material – Ball's papers). Usually convexity is very weakened (quasi-, poly-convexity, 1-rank convexity). We suggest another solution to this trouble: the convexity depends crucially on coordinates choosen. We propose to disregard completely the linear structure of GL_+ (as $GL_+ \subset R^9$) and instead of it to rely on the group structure of GL_+ – i.e. to consider the composition (and not the sum) of two deformations as a basic structure. The group structure induces canonically the right-invariant metric on GL_+ in which geodesics are of the form $c(t) = (\exp At) \cdot B$, $B \in GL_+$, $A \in gl$.

Then we require $\mathcal{E}$ be convex on each geodesic $c(t)$ – the global form reads $\mathcal{E}(AB) + \mathcal{E}(A^{-1}B) \geq 2\mathcal{E}(B)$, $\forall A, B \in GL_+$. The inspection of above examples shows that this is quite reasonable assumption. The square distance $\varrho^2(1, F) = \mathrm{tr}(\lg^2 U) = l_0$ on GL_+ is an invariant and is geodesically convex. The ("ideal") elasticity will be given by $\mathcal{E}(F) = f(l_0)$ with $f', f'' \geqslant 0$ – it depends only on geometry of GL_+. The usual weak convergence $F_n(x) \to F(x)$ must be changed for a new one defined in an intrinsic way in which the stored energy functional is lower semicontinuous.

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Strings

James Eells (Warwick), reporting – in the evening – on joint work just begun with Simon Salamon

Let (M,g) and (N,h) be pseudo-Riemannian manifolds, where M is a surface with signature $g=(1,1)$; and signature $h=(p,q)$. The physicists call a string a conformal harmonic map $\varphi : M \to N$. In an isothermal chart on M, conformality is expressed by $|\varphi_u|^2 + |\varphi_v|^2 = 0 = \langle \varphi_u , \varphi_v \rangle$; and harmonicity by

$$\varphi^\gamma_{uu} - \varphi^\gamma_{vv} + \Gamma^\gamma_{\alpha\beta}(\varphi^\alpha_u \varphi^\beta_u - \varphi^\alpha_v \varphi^\beta_v) = 0 \quad (1 \leq \gamma \leq p+q).$$

The Grassmannian $G_{1,1}(\mathbb{R}^{r,s})$ of subspaces of signature $(1,1)$ in $\mathbb{R}^{r,s}$ has tangential decomposition

$$T G_{1,1}(\mathbb{R}^{r,s}) = (L_+ \otimes W) \oplus (L_- \otimes W)$$

with respect to which we can define an almost product structure. With it – and in analogy with our twistor constructions in the definite case (Ann. Scuola Nor. Sup. Pisa 1986) – we can classify strings $\varphi : M \to N$ via their twistor transform into the Grassmann bundle:

$$\begin{tikzcd} \tilde{\varphi} : M \arrow[r] \arrow[dr, "\varphi"'] & G_{1,1}(N) \arrow[d] \\ & N \end{tikzcd}$$

.

In my talk I communicated the joint work with J. Stará and J. Malý. We constructed the parabolic system

$$\frac{\partial u^i}{\partial t} - D_\alpha \left(A^{ij}_{\alpha\beta}(x,u) \, D_\beta u^j \right) = 0, \quad i, j, \alpha, \beta = 1, \dots, 3$$

for which the weak solution of the initial-boundary value problem with the Lipschitzian data on the boundary develops the singularity at the interior point of the parabolic cylinder Q_T.

O. John
Charles University, Prague

Variational inequalities – eigenvalues and solvability

(joint work with P. Quittner, O. John, M. Čadek)

There were given some properties of the set $\sigma_K(A)$ of eigenvalues of a variational inequality

$$\lambda \in R, u \in K \qquad ((\lambda I - A)u, v - u) \geqq 0, \ \forall v \in K,$$

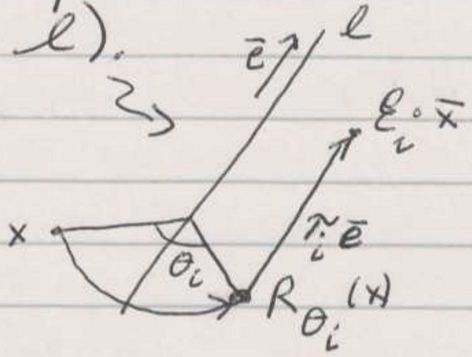
where K is a closed, convex cone in a real Hilbert space H and A a selfadjoint, positive and compact operator on H ($\sigma_K(A)$ can be one point set, can have positive limit point). Moreover, some conditions were given under which the inequality is solvable for every right hand-side and it was shown that Fredholm type theorems are not hold.

Jana Stará
Charles University
Prague

Twisted Immersed Tori of Constant Mean Curvature in R^3

Let $\omega(u,v)$ be a solution to $(*)\ \Delta\omega + \sinh\omega\cosh\omega = 0$ doubly periodic with respect to a parallogram with sides $\bar{p}_1 = \langle a, 0\rangle$ and $\bar{p}_2 = \langle c, b\rangle$. If the first fundamental form is $ds^2 = e^{2\omega}(du^2 + dv^2)$, the mean curvature is $H = 1/2$, and the lines of curvature correspond to two $\perp$ families of parallel lines in the u-v plane, then the second fundamental form is determined and there is an essentially unique map $\bar{x}(u,v): R^2 \to R^3$ which is a conformal representation of a surface of constant mean curvature $H = 1/2$ (generally not closed).

There exist Euclidean motions $\mathcal{E}_1, \mathcal{E}_2$ with $\bar{x}(w + \bar{p}_i) = \mathcal{E}_i \circ \bar{x}(w)$ $i = 1, 2$ which must commute. It follows that there exists an axis ℓ s. that $\mathcal{E}_i$ is a rotation about ℓ through an angle θ_i followed by a translation $T_i\bar{e}$ parallel to ℓ (here $\bar{e}$ is a unit vector $\parallel$ to ℓ).



There are 4 control * variables $\{a, b, c, \beta\}$ and 4 variables $\{T_1, \theta_1, T_2, \theta_2\}$ to be determined. Under suitable conditions we show that the map $\Phi(a, b, c, \beta) = (T_1, \theta_1, T_2, \theta_2)$ is smooth and locally invertible. One can then show that there exist (a_0, b_0, c_0, β_0) so that $\Phi(a_0, b_0, c_0, \beta_0) = (0, 2\pi r_1, 0, 2\pi r_2)$ where r_1, r_2 are non-zero rational nos. This gives a closed immersed torus with a twist.

Henry C. Wente (Toledo)

* β is the angle one of the family of lines of curvature makes with the u-axis.

Harmonic maps and Teichmüller theory

In this talk, we present a variational approach to Teichmüller theory. It is based on the existence and uniqueness of harmonic diffeomorphism between (topologically equivalent) closed surfaces with hyperbolic structures. We study how the harmonic map and its energy integral vary with respect to the domain and image structures, the computations for the latter case being due to M. Wolf. These calculations allow to recover the basic structures of Teichmüller's space, namely the topological and differentiable (Teichmüller's theorem) as well as the complex and metric structure. We can also give a new derivation of the Kähler property of the Weil-Petersson metric and a computation of its curvature tensor.

Jürgen Jost

The convergence of a harmonic mapping to its homogeneous limit

Consider a harmonic map $f: M^m \longrightarrow N^n$ between Riemannian manifolds, where N is compact and is considered to be isometrically embedded in Euclidean space $\mathbb{R}^d$. Let $0 \in M$ be a singular point of f: this may happen only when $m \geqslant 3$ and $n \geqslant 2$. If f minimizes energy in a neighborhood of 0, then a subsequence of the blowup sequence $f_\lambda(x) := f(\lambda x)$, as $\lambda \to 0^+$, converges weakly to a homogeneous $f_0 : \mathbb{R}^m \longrightarrow N$, called ~~the~~ a homogeneous tangent mapping of f at 0. Suppose that one such f_0 is smooth on S^{m-1}. Then Leon Simon (Annals '83) has shown that f_0 is unique and $f_\lambda \to f_0$ in $C^2(S^{m-1})$. But Simon gives no estimate on the rate of convergence. An earlier result of Allard and Almgren (Annals '81), however, may be adapted to prove that $\| f_\lambda - f_0 \| \leq c \lambda^\alpha$ for some $\alpha > 0$, under the additional hypothesis that every harmonic Jacobi field φ along $f_0 : S^{m-1} \longrightarrow N$ is actually the derivative of a one-parameter family of harmonic mappings $f^t : S^{m-1} \longrightarrow N$. We show that this hypothesis is always satisfied if the domain has dimension $m = 3$ and the target has dimension $n = 2$. The proof requires showing that f_0 is actually a conformal mapping and φ is a conformal Jacobi field. Finally, we construct explicit stationary examples for all remaining dimensions $m \geqslant 3$ and $n \geqslant 3$, for which $\| f_\lambda - f_0 \| = A (B - 2 \log \lambda)^{-1/2}$; in particular, there can be no majorant of the form $c \lambda^\alpha$. This is joint work with Brian White.

Robert Gulliver

Phase transitions of fluids and their interpretation in the Calculus of Variations

Consider a fluid, confined to a bounded container $\Omega \subset \mathbb{R}^n$ in isothermal conditions, whose Gibbs free energy per unit volume is given by a function $W(u)$ depending only on the density $u(x)$ of the fluid. According to the Van der Waals-Cahn-Hilliard theory, the equilibrium states of the fluid are the solutions of the problem

$$(*) \qquad \min_{\int_\Omega u\,dx = m} \left[\int_\Omega \left(\varepsilon |Du(x)|^2 + W(u(x)) \right) dx \right]$$

where m is the (prescribed) total mass of the fluid and $\varepsilon > 0$ is a small parameter.

Theorem. Suppose that W is continuous, non-negative, and

$$W(t) = 0 \iff t = \alpha \text{ or } t = \beta$$

with $\alpha < \beta$. Suppose also that $\partial\Omega$ is Lipschitz continuous and $m \in \left]\alpha|\Omega|, \beta|\Omega|\right[$. If u_ε is a minimizer for $(*)$ and (u_ε) converges to a function u_0 in $L^1(\Omega)$ as $\varepsilon \to 0+$, then

(a) $u_0(x) = \alpha$ or $u_0(x) = \beta$ for almost all $x \in \Omega$;

(b) the set $E_0 = \{x \in \Omega : u_0(x) = \alpha\}$ is a solution of the problem

$$\min_{E \subseteq \Omega,\ |E| = \frac{\beta|\Omega| - m}{\beta - \alpha}} \left[H_{n-1}(\partial E \cap \Omega) \right];$$

(c)

$$\lim_{\varepsilon \to 0+} \frac{1}{\sqrt{\varepsilon}} \left[\int_\Omega \left(\varepsilon |Du_\varepsilon|^2 + W(u_\varepsilon) \right) dx \right] = 2c_0\, H_{n-1}(\partial E_0 \cap \Omega)$$

where $c_0 = \int_\alpha^\beta W^{1/2}(t)\,dt$.

This theorem gives a mathematical proof of the principle of minimal interface for two-phase fluids, because u_0 describes a fluid with two phases of constant density α and β, and $H_{n-1}(\partial E_0 \cap \Omega)$ is the interface area between the phases.

LUCIANO MODICA
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Existence and finiteness results in the free boundary value problem for minimal hypersurfaces

Rugang Ye, mathamatics institute of
university Bonn

In this talk we presented the following results

Thm1 Let M^n be a cpt Riemannian manifold with $\partial M \neq 0$, and $\gamma \in H_{n-1}(M, \partial M)$, $\gamma \neq 0$. Then there exists an area-minimizing rectifiable current T in γ (γ-minimizing current). If moreover $n \leq 7$ and ∂M has non-negative mean curvature, then T is represented by regular hypersurfaces ~~with~~ meeting ∂M orthogonally along their boundaries.

Thm2 Let M^n be a cpt, connected real analytic Riemannian manifold with $\partial M \neq 0$, ∂M having non-negative mean curvature, $n \leq 7$ and $\gamma \in H_{n-1}(M, \partial M)$, $\gamma \neq 0$.
Let

$$\mathcal{S}_\gamma = \{ S \mid S \text{ is a connected hypersurface appearing in some } \gamma\text{-minimizing current} \}$$

$$\mathcal{S}^g_\gamma = \mathcal{S}_\gamma \cap \{ \text{topological type } g \} .$$

Then we have

1) If $\# \mathcal{S}_\gamma = \infty$, then M is a real analytic

bundle over S^1 all whose fibres are minimizing hypersurfaces of the same area and of the same diffeomorphical type;

2) If $\# \mathcal{S}^{g}_{\gamma} = \infty$, then we have the conclusion in 1) and we know that the type of the fibre is g. Especially, if g = disk, then $\mathcal{M}$ is either a solid torus or a Klein bottle;

3) If $n=3$ and $\# \mathcal{S} = \infty$ where $\mathcal{S}$ is the set of minimizing disks in the classical sense of Courant, then $\mathcal{M}$ either is a solid torus or a Klein bottle.

Some other finiteness results were also presented. We remark that similar results hold for γ-minimizing currents, if $\gamma \in H_{n-1}(\mathcal{M})$, $\partial \mathcal{M} = \phi$ and $\mathcal{M}$ is compact.

The proof uses ideas of geometric measure theory and elliptic systems.

RINGE und MODULN

27. 4. bis 3.5. 1986

Middle annihilator primes

Alfred Goldie   U. of Leeds

An ideal M of a ring R is a middle annihilator ideal (MA) if $M = \langle x \in R \mid AxB = 0 \rangle$ for fixed ideals $A, B \triangleleft R$ such that $AB \neq 0$. The concept, due to I. Kaplansky, has been found useful for studying quotient rings, embeddings in artin rings and in enveloping algebras. The Main problem is whether a ring R has only a finite number of prime (especially maximal) Middle annihilators. This is known for factor rings of $U(\mathfrak{g})$, $\mathfrak{g}$ f.d. Lie algebra and likewise in polycyclic-by-finite group rings and many other cases. Work of Small-Stafford showed that the set MAP is finite, except for primes which contain regular elements and gave an example of the latter case. Goldie-Krause introduced strongly regular elements to re-organise the role of MAP's and proved ~~results on~~ that there were only a finite number of maximal MA's under certain Krull symmetry restrictions. Recently, C. Dean showed that any ring embeddable in a r. artin ring has but a finite number of MAP's. Finally an example due to K.A. Brown (a factor of $U(sl_2 : \mathbb{C})$). has been shown to be non-embeddable in an artin ring. However, it has only a finite number of MAP's so the main problem remains open.

Prime Ideals in Enveloping Rings and Crossed Products

Don Passman   U of Wisconsin

We are concerned with primes in certain ring extensions. Specifically (1) Let L be a Lie algebra over K and assume that L acts as derivations on the K-algebra R. Then one can form the Lie algebra smash product $R\#U(L)$. (2) If G is a group which acts as automorphisms on R, one can form the crossed product $R*G$. In either case we relate the primes of the larger ring to those of R under appropriate Noetherian hypotheses. In either case we may assume that $P \cap R = 0$. In case (1) we show (assuming char $K = 0$) that the primes P with $P \cap R = 0$ are in one to one correspondence with certain primes of a certain twisted enveloping algebra. In case (2) we show that these primes are in one to one correspondence with certain primes of a certain twisted group algebra. In this way, arbitrary ~~p~~ rings R are essentially reduced to fields. Several corollaries are offered. The most obvious of course are related to incomparability. Another is potentially useful in trying to show that R Jacobson implies $R\#U(L)$ Jacobson. It shows that one need only consider the case of X-inner actions of L on R.

The Invariants of n×n Matrices Edward Formanek, Penn State University

Let K be a field of characteristic zero and let $C(n,r)$ be the ring of invariant functions $f: M_n(K)^r \to K$. The first fundamental theorem of matrix invariants says that $C(n,r)$ is generated by traces. The second fundamental theorem gives all multilinear relations among traces.

Let $d(n)$ be the minimal degree of a generating set for $C(n,r)$. Procesi has shown that $d(n)$ is also the minimal degree of nilpotence in the Nagata-Higman Theorem. That is, $d(n)$ is the least integer such that $(K\langle X\rangle/J)^{d(n)} = 0$, where $K\langle X\rangle$ is a free associative algebra (without unit) and J is the T-ideal generated by x^n.

Kostant used the second fundamental theorem to give a proof of the Amitsur-Levitzki Theorem, and I recently used it to prove a conjecture of Regev that a certain matrix polynomial is nonzero.

Embedding of rings, PI-rings, free products and Jordan homomorphisms, L. Bokut', Novosibirsk

I. New examples of noninvertible rings embeddable in groups (L. A. Bokut, A. I. Valitskas)

II On the embedding of ring into Jacobson radical rings (A. I. Valitskas)

III Radical PI-algebras (A. I. Valitskas)

IV On the embedding of rings into matrix algebras over commutative rings (A. Z. Anan'in)

V On the structure of a variety over a field of zero characteristics (A. R. Kemer)

VI. Free products of Λ-rings (V. N. Gerasimov)

VII Jordan homomorphisms (L. A. Lagutina)

The rational hull of a semifir, PM Cohn University College London

Let R be any ring containing a skewfield K. The tensor R-ring on a set X centralizing K is defined as the ring generated by R and X with defining relations $\alpha x = x\alpha$ ($\alpha \in K$, $x \in X$), and is denoted by $R_K\langle X\rangle$. When R is a field, $F = R_K\langle X\rangle$ has a universal field of fractions $R_K(\langle X\rangle)$, obtained by localizing F at the set of all full matrices. If R is a semifir (and X is infinite), the ring F_Σ obtained by localizing at the set Σ of all full matrices "totally coprime" to matrices over R is called the rational hull of R.

The embedding $R \to F_\Sigma$ is inert and by choosing R appropriately one obtains examples of right principal Bezout domains, first described in PM Cohn & AH. Schofield, Two examples of principal ideal domains, Bull London Math Soc 17 (1985) 25-28.

Patch-Continuity of Normalized Goldie Ranks

K. R. Goodearl, University of Utah

For a finitely generated module A over a noetherian ring R, the normalized rank of A at a prime ideal P is $\text{length}(A \otimes_R Q_P) / \text{length}(Q_P)$ where Q_P is the Goldie quotient ring of R/P. J. T. Stafford's continuity theorem, originally proved for R right and left noetherian but now proved for R right noetherian, says that the map $P \mapsto \text{length}(A \otimes_R Q_P) / \text{length}(Q_P)$ from $\text{Spec}(R)$ to $\mathbb{Q}$ is continuous provided the patch (or constructible) topology is used on $\text{Spec}(R)$. This result is used as a tool in Stafford's work on numbers of generators for finitely generated modules and on countability of cliques, along with his extension of Goodearl's + Warfield's work on identifying the extreme points of the state space of $K_0(R)$. To illustrate continuity methods, the continuity theorem may also be used to prove a generalization of Rainwater's theorem that the global dimension of a fully bounded noetherian ring equals the supremum of the projective dimensions of its simple modules.

Block Theory for Noetherian Rings

Robert B. Warfield, University of Washington

If R is an Artinian ring and S and T are simple right modules, then one can write $S \rightsquigarrow T$ if $\text{Ext}(S,T) \neq 0$. If M and N are the corresponding maximal ideals, then we write $M \rightsquigarrow N$ and this is equivalent to $M \cap N / MN \neq 0$. The connected components of the graph thus created are the "blocks" of simple modules, and R decomposes into a product of indecomposable rings, one for each block. Similarly, the spectrum of a Noetherian ring can be made into a directed graph -- the "graph of links" of R. (If R/P and R/Q are Artinian then $P \rightsquigarrow Q$ if and only if $P \cap Q / PQ \neq 0$, but the general definition is more complicated.) The components of this graph are the "cliques" of Spec R. This pair of survey talks addressed the significance of this graph for representation theory and the corresponding localization theory. Key references for this talk are ① Jategaonkar's book which has just appeared ② a beautiful survey by Ken Brown in the recent Malliavin seminar proceedings ③ forthcoming papers by Stafford, Warfield, Braun-Small, Brown-Warfield, Braun-Warfield. The talk emphasized the analysis of indecomposable injectives over rings satisfying the "strong second layer condition" and the usefulness of a recent result of J.T. Stafford's which say that certain families of primes, even if not localizable in R, are localizable in R[x].

Counterexamples to the Kač-conjecture
Lieven Le Bruyn, University of Antwerp U.I.A.

Kač conjectured that a Schur root is also an indecomposable root, i.e. ∄ ~~n~~decomposition $\alpha=\beta+\gamma$ where $\beta,\gamma \in \mathbb{N}^n \setminus \{0\}$ and $R(\beta,\gamma), R(\gamma,\beta) \geqslant 0$. Consider the quiver $\circ \xrightarrow[\longrightarrow]{\vdots n} \circ \xrightarrow[\longrightarrow]{\vdots n} \circ$ then $\alpha=(k,l,k)$ is a Schur root if $\frac{n}{n^2-1}k \leq l \leq nk$ but α is not indecomposable if $l \leq \frac{2}{n}k - \frac{2}{n}$. The reason is that whereas Schur roots are preserved under reflection functors, indecomposable roots are not.

Localization in P.I. rings   Amiram Braun (Haifa).

Goldie (1967) raised the following question,
Given a noetherian ring, P a prime ideal in R, find necessary and sufficent condition for P to be (left and/or right)

localisable. The following finite criterion is given ~~due~~ by ~~to~~ Braun-Warfield (which resolves this for prime noetherian p.i. ring. We say that P satisfies condition $\underline{\underline{(*)}}$ if the following happens:

Let $P_1, \dots, P_r$ be the set of all prime ideals in $T(R)$ contracting to P. ~~Then~~ Let $Q \in \operatorname{spec} T(R)$ satisfying $Q \cap Z(T(R)) = P_i \cap Z(T(R))$ for some i. Then $Q = P_j$ for some j, $1 < j \leq r$.

<u>Theorem 1:</u> Let R be a prime noetherian p.i and $P \in \operatorname{spec} R$. Then the following are equivalent

1) P satisfies condition $(*)$
2) P is left localisable
3) P is right localisable

The ~~following~~ same criterion is true for families of finite primes as well as for ~~set~~ infinite families of primes.

The following application is ~~not~~ ~~trivial~~ of interest:

<u>Theorem 2</u> Let R be a prime noetherian p.i. Then $J(R)$, ~~is left and right localisable~~, the Jacobson radical ~~is~~ ~~left~~ of R, is left and right localisable.

<u>The spectrum of a tubular algebra</u> C. M. Ringel (Bielefeld)

Let A be a tubular algebra, and $A \longrightarrow M_n(D)$ a (categorical) epimorphism, where $M_n(D)$ is the full $n \times n$-matrix ring over a division ring D. Since all finite-dimensional indecomposable A-modules are known, we may assume that D is infinite-dimensional over k. In this case, $D = k(t)$, the field of rational functions in one variable, and there are countably many equivalence classes of such epimorphisms, indexed in a natural way by $[0,1] \cap \mathbb{Q}$. This result follows from a general theorem which characterises a particular infinite-dimensional module over any canonical algebra.

Projective Representations of Finite Groups and the Brauer Splitting Theorem over Commutative Rings

F. Van Oystaeyen (U. of Antwerp)

We generalize the Schur-multiplier theorem by lifting a twisted group ring over a commutative connected ring to a group ring for a finite central extension up to allowing a separable free extension of finite rank of the groundring. From this we derive a version of the Brauer splotting theorem for representations in the projective case. The use of connected rings is important in order to have a nice Galois theory and the fact that one allows commutative rings also provides a tool to study infinite groups G for which RG^α is an Azumaya algebra. As a particular case we look at Clifford representations of $(\mathbb{Z}/2\mathbb{Z})^n$, these are projective representations of $(\mathbb{Z}/2\mathbb{Z})^n$ such that the center of $A = (R(\mathbb{Z}/2\mathbb{Z})^n)^\alpha$ is minimal; it is shown that such representations are exactly those for which the ring A decomposes as a direct sum of Clifford algebras over R. The first part is joined work with E. Nauwelaerts, the example for $(\mathbb{Z}/2\mathbb{Z})^n$ has been worked out in cooperation with L. Le Bruyn, the infinite case follows from some results of mine concerning graded Azumaya algebras.

From differential geometry to differential algebra.

work of Mitsuhiro Takeuchi, Tsukuba; Moss Sweedler Ithaca

Say A is a commutative K algebra of positive characteristic p. There is a pair (T, t) where T is a commutative A algebra and $t: T \to T$ is a K linear derivation with $t^p = 0$ and for any other such pair (T', t') there is a unique A algebra map $\psi: T \to T'$ with

$t'\gamma = \gamma t$. This T is graded. t is homogeneous degree 1.
$T_0 = A$. $T_1 \cong \Omega$, the Kähler module of A over K
where $t|T_0 : A \to T_1$ is the universal derivation.
T gives rise to the complex:

$$\begin{tikzcd}
T_0 \arrow[r, "t"] \arrow[d, phantom, "\shortparallel"] & T_1 \arrow[r, "t^{p-1}"] \arrow[d, phantom, "\cong"] & T_p \arrow[r, "t"] & T_{p+1} \arrow[r, "t^{p-1}"] & \cdots \\
A \arrow[r, "\partial"] & \Omega & & &
\end{tikzcd}$$

If A has a p-basis over K -- for example $A = k[X_1, \dots X_n]$
and $K = k[X_1^p, \dots X_n^p]$ or $A = K[X]/\langle X^p \rangle$ --
The complex has zero homology in positive degree
and the degree zero homology is K.
(T, t) can be used to classify intermediate
fields if A is a purely inseparable exponent
one extension field of K. In this case there is
a bijective correspondence:

$$\left\{\begin{matrix}\text{Intermediate} \\ \text{fields } B \text{ where} \\ K \subset B \subset A\end{matrix}\right\} \longleftrightarrow \left\{\begin{matrix}A \text{ subspaces } V \text{ of } T_1 \text{ with} \\ t^{p-1}(V) \subset \sum_{i=1}^{p-1} T_i\, t^{p-1-i}(V)\end{matrix}\right\}$$

$$B \longrightarrow At(B)$$

$$\{a \in A \mid t(a) \in V\} \longleftarrow V$$

This result is an analog to the differential ideal formulation of the Frobenius theorem on integral submanifolds. V corresponds to the differential ideal; B corresponds to the functions which are constant on integral submanifolds determined by the differential ideal.
The vanishing cohomology result is a positive characteristic analog to the Poincaré lemma.

Crossed Products of Hopf Algebras (S. Montgomery - USC, Los Angeles)

Let H be a finite-dimensional Hopf algebra acting on the k-algebra A. We study the relationship between A and the subring A^H of H-invariants by using the semi-direct product $A \# H$. For f a non-zero (left) integral in H, there is a "trace" function $\hat{f} : A \to A^H$ given by $\hat{f}(a) = f \cdot a$. Using this, we prove that if $A \# H$ is simple, then A is a finitely generated projective A^H module; also A^H is simple $\Leftrightarrow$ $\hat{f}(A) = A^H$ $\Leftrightarrow$ $A \# H$ is Morita equivalent to A^H. More generally, if $A \# H$ is semiprime and satisfies $(*)$: $0 \neq I \triangleleft A \# H \Rightarrow I \cap A \neq 0$, then A is Goldie $\Leftrightarrow$ A^H is Goldie, and A^H Noetherian $\Rightarrow$ A Noetherian. These results (joint work with J. Bergen) give a common generalization of known results for group actions, derivations, and group graded rings. For, $(*)$ is satisfied in the following cases: 1) G X-outer automorphisms of a semi-prime ring [Fisher-Montgomery 78] 2) L Q-outer restricted Lie algebra of derivations of a prime ring [Bergen-Montgomery 86] 3) A graded by G (so $H = (kG)^*$) with $A \# (kG)^*$ a prime ring [Cohen-Montgomery 84]. Combining 1), 2), 3) with the above gives common proofs of results of Kharchenko, Popov, Bergman, Montgomery, Cohen, and Rowen.

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New structural theory and derivations of semiprime rings (V.K. Kharchenco - USSR, Novosibirsk)

The general structural ring theory was founded by N. Jacobson in the middle of 40^{ies}. It contains the construction of the Jacobson radical, the description of semi-simple rings as subdirect products of primitive rings and presentation of primitive rings as dense rings of linear transformations of spaces over skew fields. On the other hand, the development of ring theory in the last decades has resulted in the creation of thorough structural theory concerning the Baer radical. This is due to the fact that great progress has been achived in the study of prime rings; the most developed theory of primitive rings with non-zero socle, due to Martindale theorem, occupies an impotant place in the framework of the new structural theory as well. Besides, the method of orthogonal completions, proposed by K.I. Beydar and A.V. Mikhalev not long ago, allows one to transfer theorems (with the necessary changes in formulations) almost automatically from the prime rings to semiprime ones.

In lecture we present the method of ortogonal completions and give the results on Galois theory for derivations of semiprime rings which are obtained by applying this method to the results on prime rings.

Relative Invariants of quivers (Aidan Schofield, London)

Let $\underline{d}$ and $\underline{e}$ be dimension vectors of representations of a quiver. Suppose that $\langle \underline{d}, \underline{e} \rangle = 0$, where $\langle \, , \rangle$ is the Ringel form on dimension vectors. Then we define a polynomial $f_{\underline{d},\underline{e}}$ which vanishes precisely when $\mathrm{Hom}(A_p, B_q) \neq 0$ for representations A_p, B_q for points $p \in V(\underline{d})$, $q \in V(\underline{e})$ where $V(\underline{d})$ is the representation space for the dimension vector $\underline{d}$. This gives the following two theorems.

Theorem 1 Let $\underline{d} = \Sigma \underline{d}_i$. This is the generic decomposition if and only if $\langle \underline{d}_i, \underline{d}_j \rangle \geq 0$ and $\langle \underline{d}_i, \underline{d}_j \rangle \langle \underline{d}_j, \underline{d}_i \rangle = 0$, $\forall i \neq j$, and $f_{\underline{d}_i, \underline{d}_j} \neq 0$ when $\langle d_i, d_j \rangle = 0$.

This gives an inductive calculation of the generic decomposition.

Again, let $\underline{d}$ be a dimension vector such that there is an open orbit. Then there is a module M ~~having~~ corresponding to this orbit such that $\mathrm{Ext}^1(M,M)=0$.

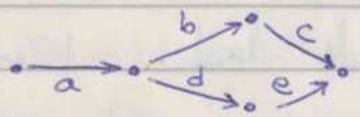
Theorem 2 Let $\underline{d}$ be a dimension vector such that there is an open orbit, and let M be ~~the~~ the corresponding representation. Let $S_1, \ldots S_t$ of the dimension vectors $e_1 \ldots e_t$ be the simple objects in the category $M\perp = \{N : \mathrm{Hom}(M,N)=0=\mathrm{Ext}^1(M,N)\}$. Then the relative invariants for $V(\underline{d})$ are the products of the polynomials $f_{\underline{d},S_i}$, where $f_{\underline{d},S_i}$ is the specialisation of $f_{\underline{d},\underline{e}_i}$ at the representation S_i.

Using Combinatorial Properties of Rings to Build Projective Resolutions

Let Γ be a directed graph on a finite number of vertices and let $A = k\Gamma/I$ be a homomorphic image of the path algebra $k\Gamma$. One approach to the algebra A is combinatorial: choose a set of paths on Γ which surjects to a basis for A, and study its properties. Another approach is algebraic: consider and classify modules over A, paying particular attention to their homological properties.

In this joint work with Ed Green, we have blended the two approaches. Combinatorial invariants derived from the map $f: k\Gamma \to A$ are assembled into projective resolutions for the vertex-associated simple A-modules. These resolutions are fairly small and they are practical for specific calculations. Applications include the construction of interesting examples of rings having finite global dimension.

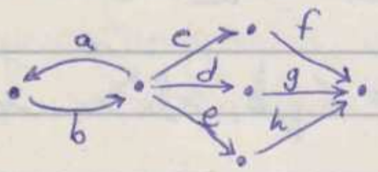
Two specific algebras which can easily be shown to have global dimension two are:



Γ

$$k\Gamma/\langle ab,\ bc-de\rangle$$

and



Δ

$$k\Delta/\langle ba,\ abcf-dg,\ cf+dg+eh\rangle\ .$$

- David J. Anick
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Symmetric Polynomials in the Free Algebra

L. Makar-Limanov (Wayne State University, Detroit 48202, U.S.A.)

Let A be a free algebra $k\langle x_1, \dots x_n\rangle$ over a field k with n generators and let G be the symmetric group of degree n. As is well known in the commutative setting when the free algebra is replaced by the polynomial ring of rank n, any element is algebraic over the subalgebra of invariants relative to the natural action of G on A (by permutation of generators) and it is not difficult to give an explicit description of the irreducible polynomial which corresponds to a given element.

It seems to be rather natural to attempt to transfer these results to the noncommutative setting where the free algebra A would be the most natural and important example. It is possible to show by comparing dimensions of some linear spaces that the elements of A are algebraic over A^G. On the other hand, the explicit description of ideals of polynomials anihilated by elements of A seems to be rather difficult. For example it would be very interesting to show that one can always find a monic polynomial in the ideal corresponding to x_1

Valuations sur les corps gauches (travail en collaboration avec [illegible] et A. Wadsworth)

J-P Tignol (Université Catholique de Louvain) (B 1348 Louvain-la-Neuve, Belgique)

Soit D une algèbre à division de rang fini sur son centre F. Il est bien connu que toute valuation hensélienne sur F se prolonge en une valuation sur D. Dans le présent exposé, on étudie le cas où D est totalement ramifié et modéré sur F, c'est-à-dire que D et F

ont même corps résiduel et que la caractéristique de F ne divise pas le degré de D.
Comme application, on donne un exemple de deux algèbres de degré impair qui n'ont "pas" de sous-corps commun, mais dont le produit tensoriel n'est pas à division.

Anwendungen der Modelltheorie in der Körpertheorie
(C. U. Jensen, Univ. Kopenhagen Dänemark)

Für einen endliche Gruppe G und einen Körper K sei $v(G,K)$ die Anzahl der nicht-K-isomorphen normalen Erweiterungen von K mit G als Galois'sche Gruppe. Resultate betreffend die Verteilung der Werte von $v(G,K)$ werden angegeben. Beispielsweise existiert (mit beliebig vorgeschriebener Charakteristik) Körper für die jede endliche einfache Gruppe, aber nicht jede endliche Gruppe als Galoisgruppe realisiert werden kann.

Graded Algebras of Finite global dimension
(W. Schelter, Dept Math, Univ of Texas Austin TX 78712 USA)

If $A = k\{x_1,\dots,x_r\}/\{f_1,\dots,f_t\}$ with $\deg x_i = 1$ and f_i homogeneous, then we say A is <u>regular</u> if

1) $\operatorname{gldim} A = d < \infty$ (on graded modules)
2) $\operatorname{gk\,dim} A < \infty$.
3) $\operatorname{Ext}_A^q(k,A) = \begin{cases} k & q = d \\ 0 & q \neq d \end{cases}$ (Gorenstein)

Regular algebras of dimension 2 and 3 are classified. A discussion of the classification in deg 3 was given. There are 2 possible cases: Let $s = \deg f_1$. Then $(r,s) = (2,3)$ or $(3,2)$ and $\deg f_i = \deg f_1$.
There are 14 types.
A discussion of type A related the algebras of this type to an elliptic curve (1 for each algebra) and an automorphism of that curve. If the automorphism had finite order the algebra is finite over its center and conversely.
A j-invariant of the algebra ~~is equivalen~~ determines when the algebra is skew polynomial. The

above work was done jointly with M. Artin, with Tate providing the relation to elliptic curves. M. Van den Bergh is responsible for part of the verifications discussed on algebras of Type A.

Periodic modules over QF algebras (Rainer Schulz, Univ. Munich)

Eisenbud proved that if $R = KG$ (K field, G finite group), then any bounded R-module is periodic. Tachikawa proved that if $R = KG$ (G finite p-group) or R of finite repr. type, then any R-module without large self-extensions (i.e. $\mathrm{Ext}^i_R(M,M) = 0$ for all $i \geq i_0$, $i_0 \in \mathbb{N}$) is projective. We show that the module $M = R_\rho(x + \alpha y)$ over the QF algebra $R_\rho = K\cdot 1 \oplus Kx \oplus Ky \oplus Kxy$ with multiplication $x^2 = y^2 = 0$, $yx = \rho xy$, $0 \neq \rho \in R$ not a root of unity is bounded, nonperiodic, nonprojective without large self-extensions ($i_0 = 2$). Also, we show

Theorem 1. For bounded M, the following are equivalent:

(a) M is periodic,

(b) $\mathrm{Ext}^*_R(M,M)$ and $\mathrm{Ext}^*_R(M,N)$ (for all simple N) are noeth. right $\mathrm{Ext}^*_R(M,M)$-modules,

(c) $\mathrm{Ext}^*_R(M,X)$ is a noeth. right $\mathrm{Ext}^*_R(M,M)$-module for all f.g. X.

Theorem 2. If the modules over a QF algebra R satisfy (c), then any module without large self-extensions is projective.

Examples for QF algebras R whose modules satisfy (c):

(1) group algebras over finite groups (by Evens' Theorem),

(2) R_ρ has (c) iff ρ is a root of unity,

(3) R is QF of finite repr. type.

We give an application to Nakayama's conjecture on algebras with infinite dominant dimension.

Localization at infinite class (Bruno J. Müller, McMaster University)

We discuss the proof of our theorem, that affine noetherian PI-algebras $R = k\{S\}$ over a field k can be Ore-localized at every clique. Technically one has to verify the intersection condition for (right) ideals I. Noetherian induction puts one into a situation where R is semiprime, and there are two elements $a, b \in I$ such that $a \in \mathcal{C}(P)$ or $b \in \mathcal{C}(P)$ for each prime P of the clique. One wants to find $\lambda \in k$ with $a\lambda + b \in \cap\, \mathcal{C}(P)$. Due to the countability of cliques, this is fairly trivially possible if k is uncountable. For countable k, the strategy is to construct $\varrho_P : R \to M_{n(P)}(K)$ with $\ker \varrho_P = P$, for every P, with an algebraically closed field K. Moreover one wants a subfield K^*, closed under extensions of degree $\leq N$ but with $k \not\subset K^*$, such that $\varrho_P(s) \in M_{n(P)}(K^*)$ holds for all $s \in S$. Once such ϱ_P exist, $a\lambda + b \notin \mathcal{C}(P)$ yields det $\varrho_P(a)\lambda + \varrho_P(b) = 0$; and since this is a polynomial of degree n $(\leq N)$ over K^*, one concludes $\lambda \in K^*$. Thus any $\lambda \in k \setminus K^*$ gives $a + \lambda b \in \cap\, \mathcal{C}(P)$.

To construct ϱ_P, one descends to $k^*\{S\} = R^*$, where $k^* = k \cap K^*$. One needs to know that all links arise from factors of finitely many ideals — here the $N_i = \cap \{P : \text{PI degree}(P) \leq i\}$ work. Taking the integer N large enough, K^* incorporates the data describing the N_i, as well as the matrix entries of $\varrho_{P_0}(s)$ for all $s \in S$ and one prime P_0. Proceeding along the links, a fairly straightforward dimension counting argument, followed by a lifting procedure from R^* to R, allows one to obtain all the ϱ_P from ϱ_{P_0}.

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The preprojective algebra of a tame hereditary algebra
(Werner Geigle, Paderborn)

Let k be an arbitrary field and Λ be a finite dimensional tame hereditary k-algebra. Then the preprojective algebra Π(Λ) is defined by $\Pi(\Lambda) = \bigoplus_{n\geq 0} \tau^{-n}\Lambda$, where τ denotes the Auslander-Reiten transformation. We obtain the following result:

Theorem: Π(Λ) is a left-noetherian prime PI-algebra of Krull-dimension 2.

The existence of a polynomial identity gives consequences on the structure of Π(Λ) and Λ. In particular, if Q denotes the unique indecomposable torsionfree divisible Λ-module and $D = \mathrm{End}_\Lambda(Q)$, then D is is finite dimensional over its center C and $\mathrm{trdeg}_k C = 1$.

Rings in the O category (A. Joseph, Paris 6 and Weizmann).

Let $\mathfrak{g}$ be a complex semisimple Lie algebra and O the category of "highest weight" modules introduced by Bernstein-Gelfand-Gelfand. A ring A is said to be an O ring if $\mathfrak{g}$ acts by derivations in A and the resulting module lies in the O category. For example,

(i) The $\mathfrak{g}$ ring End E where E is a finite dimensional $\mathfrak{g}$ module.

(ii) The ring of local sections on the big cell for the structure sheaf of the flag variety associated to a parabolic subgroup.

(iii) An appropriate combination of (i), (ii).

Any O ring satisfies a polynomial identity.

We analyse O subrings A of Fract $(U(\mathfrak{g})/P)$ for $P \in \mathrm{Spec}\, U(\mathfrak{g})$. If P is completely prime, then A is of type (ii) and this implies that P can be induced from the corresponding parabolic subalgebra. We ask if significantly different O rings, that is not of type (iii), can occur in general. Our analysis shows ~~indicates~~ that this is false, where the last steps was completed by remarks of Braun, Goodearl and Small during the meeting.

On the generalized Nakayama Conjecture and the Cartan determinant problem (Birge Zimmermann-Huisgen, Universität Passau)

(Joint work with Kent Fuller)

For Artin algebras allowing certain filtered module categories the generalized Nakayama Conjecture is confirmed with the aid of a "filtered Grothendieck module". The result applies in particular to all positively graded Artin algebras and to those Artin algebras whose radical cube is zero. For the corresponding class of left artinian rings it is shown that finite global dimension forces the determinant of the Cartan matrix to be 1.

Overrings of right chain rings and a construction method for chain rings

(Christine Bessenrodt - Timmerscheidt, Universität Duisburg)

(Joint work with G. Törner) The lattice of overrings of a classical valuation ring is easily described and this description generalizes to chain domains. If we only have a one-sided condition, the situation is much more complicated as can be seen from an example which is discussed in detail. For certain overrings T of a right chain domain R of rank 1 we obtain results relating the ideal lattices of R and T. If R and T are also assumed to be right invariant, we can describe T in terms of right invariant right chain domains contained in R.
To show how to construct chain domains a method due to Dubrovin is presented. Using this construction Dubrovin produced the first (and so far only) example of a prime chain ring with nilpotent elements. As there is a mistake in the proof of a key lemma, a more direct approach is suggested in which a kind of weak Ore condition is used.

Units in grouprings

Let P be a finite p-group and V the normalized units in the p-adic groupring $\hat{\mathbb{Z}}_p P$. We say that the Sylow theorems hold in V, provided, for every finite p-subgroup U in V,

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there exist $v \in V$ with $vUv^{-1} \leq P$.

Theorem I: If $p = 2$, then the Sylow theorems hold in V.

For a pro-p-group X we denote by $H^*(X, \mathbb{F}_p)$ the even dimensional continuous cohomology ring, and $\mathfrak{m}(X)$ its variety.

Theorem II: The following are equivalent:

(i) The Sylow theorems hold in V.

(ii) For every $H \leq P$, the variety $\mathfrak{m}(N_V(H)/H)$ is connected.

Klaus Roggenkamp
Stuttgart

Socle Filtrations of Verma modules — Ron Irving (U. of Washington Seattle, Washington, U.S.A.)

Fix a complex, semisimple Lie algebra $\mathfrak{g}$.
I reviewed basic facts on Verma modules, on Bruhat order of the Weyl group W, the BGG theorem on composition factors of Verma modules, and the Kazhdan-Lusztig conjecture. I then reviewed the Jantzen filtration and Jantzen's conjecture, along with Gabber and Joseph's formula (assuming the Jantzen conjecture):
For $y \leq w$, one has $P_{w_0w, w_0y}(q) = \sum_{j=0}^{\infty} q^j (M(w\cdot\lambda)_{\ell(w)-\ell(y)-2j} : L(y\cdot\lambda))$. [$\lambda$ antidominant]
It also follows that the Jantzen and socle filtrations coincide, so this yields multiplicity information for the socle filtration.

One would like to obtain this information without the Jantzen conjecture. Let λ be antidominant. The projective cover $P(\lambda)$ of $L(\lambda)$ in $\mathcal{O}^\lambda$ has a filtration with each $M(w\cdot\lambda)$ occurring once as quotient. One can use the socle filtration of $P(\lambda)$ to introduce a filtration on each $M(w\cdot\lambda)$, which we write $0 \subseteq {}^1M(w\cdot\lambda) \subseteq \dots \subseteq {}^{\ell(w)+1}M(w\cdot\lambda) = M(w\cdot\lambda)$, with semisimple layers. Then we prove (i) $P_{w_0w, w_0y}(q) = \sum_j q^j ({}_{\ell(y)+1+2j}M(w\cdot\lambda) : L(y\lambda))$ and (ii) ${}^jM(w\cdot\lambda) = \mathrm{soc}^j M(w\cdot\lambda)$. So one recovers the information of Gabber and Joseph without the Jantzen conjecture.

Generating Modules Efficiently
S.C. Coutinho (Leeds University, England)

An important local-global principle in commutative algebra is the so-called Förster-Swan theorem. Stafford showed that this result can be generalized to non-commutative right and left noetherian rings. Using a dimension which coincides with the Krull dimension for torsion free modules over prime right noetherian rings but is -1 for modules with non-zero annihilators, called the basic dimension, one can prove the Förster-Swan theorem for right noetherian rings. Using this approach one also obtains a basic element theorem for right noetherian rings which has Serre's splitting theorem and Bass's Cancellation theorem as easy Corollaries.

Rings of differential operators. S.P. SMITH (University of Warwick).

Problem: Find all polynomial solutions of the differential equation $(\partial_x^3 - \partial_y^2)(f) = 0$ $f \in \mathbb{C}[x,y]$, $\partial_x = \partial/(\partial x)$, $\partial_y = \partial/\partial y$.
Solution! Set $D = \partial_x^3 - \partial_y^2$, $S = \{f \in \mathbb{C}[x,y] \,/\, D(f) = 0\}$, then S is a simple $\mathcal{D}$-module where $\mathcal{D}$ denotes the differential operators over the (singular) curve $x^3 - y^2 = 0$.
This is a general fact: take X a curve (defined over $\mathbb{C}$) then we have:
Theorem: 1) Set $\mathcal{D}(X)$ for the ring of differential operators on X, $\hat{X}$ the normalization of X, $\tilde{X} \to X$ the canonical morphism. The following are equivalent:
(i) $\mathcal{D}(X)$ is a simple ring
(ii) $\tilde{X} \longrightarrow X$ is bijective.
(iii) $\mathcal{D}(X)$ is Morita equivalent to $\mathcal{D}(\tilde{X})$.
2) If $X = \{g = 0\}$ where $g = \sum a_{ij} x^i y^j$, put $D = \sum a_{ij} \partial_x^i \partial_y^j$
$S = \{f \in \mathbb{C}[x,y] \,/\, D(f) = 0\}$; S is a $\mathcal{D}(X)$-module generated by $1 \in \mathbb{C}[x,y]$.

some generalizations can be stated for other singular varieties, related to the minimal orbit ($\neq 0$) of a semi-simple Lie algebra / $\mathbb{C}$. We also discussed differential operators over a field of characteristic p.

Representation theory of solvable enveloping algebras K. Brown
(Glasgow)

The first part of my talk ~~cb~~ continues the representation theoretic part of Warfield's talk, & is joint work with him. We describe the structure of an indecomposible injective module over a Noetherian ring R with the strong second layer condition, in terms of the fundamental series of the injective module. The second part describes joint work with du Cloux (Paris), concerning the application of the above results to $U(\underline{g})$, where g is finite dimensional over $\mathbb{C}$; there are applications of this work to calculation of cohomology groups for $U(g)$, and hence for the cohomology of unitary irreducible representations for solvable Lie groups.

Allgemeine Ungleichungen

4.-10. Mai 1986

On the stability of a functional equation arising in probabilistic normed spaces

Motivated by a problem on probabilistic normed spaces we study the inequality:

$$(*) \qquad d_L\left(\tau(F(j/a), F(j/b)), F(j/(a+b))\right) \leq \varepsilon,$$

where $\varepsilon > 0$ is fixed, d_L is the modified Lévy metric, F is an arbitrary function in Δ^+, a and b are arbitrary positive real numbers and τ is a continuous nondecreasing binary operation on Δ^+ to be found. We show the following:
THEOREM. A continuous nondecreasing binary operation τ on Δ^+ satisfies $(*)$, if and only if

$$\hat{d}_L(\tau, \tau_M) := \sup\{d_L(\tau(F,G), \tau_M(F,G)) \mid F,G \in \Delta^+\} \leq \varepsilon,$$

where

$$\tau_M(F,G)(x) = \sup\{\mathrm{Min}(F(u), G(v)) \mid u+v=x\}$$

Claudi Alsina (U.P.C., Barcelona)

Linear and Nonlinear Discrete Inequalities in n Independent Variables

We introduce discrete analogue of Riemann's function and use it to study discrete Gronwall type inequalities in n independent variables. Next, we provide an estimate on Riemann's function and use it to obtain Wendroff type of estimates. As a consequence of this approach we are able to relax some of the conditions on the functions appearing in the inequalities then that of required in our earlier work.

R. P. Agarwal (Singapore).

Subadditive multifunctions and Hyers-Ulam stability

Let $(S,+)$ be an Abelian semigroup and let $(Y,\|\cdot\|)$ denote a (real or complex) Banach space. Consider a multifunction F from S into the family of all nonempty closed convex subsets of Y, fulfilling the subadditivity condition

$$F(x+y) \subset F(x) + F(y) \ , \quad x,y \in S .$$

If

$$\sup \{ \operatorname{diam} F(x) : x \in S \} < \infty ,$$

then F admits an additive selection, i.e. a homomorphism a of $(S,+)$ into the additive group $(Y,+)$ such that $a(x) \in F(x)$ for all $x \in S$.

Abstract version of this result is also possible. The problem is strictly related to the question about the behaviour of solutions to the functional inequality

$$\| f(x+y) - f(x) - f(y) \| \leq \varepsilon \ , \quad x,y \in S ,$$

considered for mappings $f: S \to Y$ (Hyers-Ulam stability problem).

The results were obtained jointly with Dr. Zbigniew Gajda from the Silesian University of Katowice (Poland).

Roman Ger
(Katowice, Poland)

POSITIVITY in ABSOLUTE SUMMABILITY

Positivity considerations are useful not only for ordinary summability, but also for absolute summability. In the latter case it is quite natural to employ two positivity concepts and two types of summing operators. Thereby one meets matrices having properties like diapositivity. Applications concern Cesàro methods (factors for absolutely summable series). Several modifications and extensions are indicated.

Karl Zeller (Tübingen)
(with W. Beekmann)

Approximation Theory in the Space of Riemann Integrable Functions

The following notion of a sequential convergence is suggested for the space $R[a,b]$ of Riemann integrable functions ($\overline{\int_a^b}\ldots$:= upper Riemann integral).
Def.: A sequence $\{f_n\} \subset R[a,b]$ is called Riemann convergent to $f \in R[a,b]$ if

(i) $\sup_{a \le x \le b} |f_n(x)| = O(1)$, (ii) $\overline{\int_a^b}\left[\sup_{k \ge n} |f_k(x) - f(x)|\right] dx = o(1)$.

It turns out that under this notion of convergence $R[a,b]$ is not only complete, but continuous functions are also dense in $R[a,b]$. This enables one to discuss approximation in $R[a,b]$. Three aspects are discussed in some detail: First convergence criteria of Banach-Steinhaus-type are developed, extending basic work of Pólya (1933) on the convergence of quadrature formulas. Then the classical approximate identity argument is extended to the present setting, and finally a measure of smoothness is suggested which may be used in error analysis, quite parallel to the standard modulus of continuity in C-spaces. The lecture is a survey of joint work with W. Dickmeis, H. Mevissen, and E. van Wickeren.

R. J. Nessel (RWTH Aachen)

Uniqueness Inequality and Best Harmonic L^1-Approximation

For a given measure space $(X, \mathcal{A}, \mu)$, let $f \in L^1(X)$ and V a subspace of $L^1(X)$. Given two best L^1-approximants $v_1, v_2 \in V$ to f, then we have the following inequality:

(*) $(f - v_1)(f - v_2) \ge 0$ a.e. in X.

This inequality is very useful in order to prove uniqueness of a best L^1-approximant in the case when the occuring functions are continuous on (an appropriate) X (e.g. if X is a compact real interval, V a Chebyshev subspace of $C(X)$).
We consider the approximation of (not necessarily continuous) subharmonic functions f by a space V of harmonic

functions and give a sufficient condition for a best L^1-approximant. Under mild assumptions on f, (*) yields the uniqueness of a best L^1-approximant.

Werner Hauptmann (Duisburg)

Rearrangements and Optimization Problems for Certain Linear Second Order Differential Equations.

Let $\Phi: [0,T] \to [0,1]$ be a given decreasing function and consider $\mathcal{F}(\Phi) = \{q \in L^\infty(0,T):$ q and Φ have the same distribution functions$\}$. Find $\sup y(T)$, $q \in \mathcal{F}(\Phi)$, where y solves the initial value problem

$$y'' - qy = 0, \quad y(0) = 1, \; y'(0) = \alpha \geq 0.$$

The problem is that $\mathcal{F}(\Phi)$ is not a convex set: therefore, we introduce $\Omega(\Phi)$ which is the weak $*$-closure of $\{\sum_{\text{finite}} c_i \Phi_i,\ \Phi_i \in \mathcal{F}(\Phi),\ c_i \geq 0,\ \sum c_i = 1\}$. This idea was suggested by L.-E. Zachrisson. Using a kind of calculus of variations, we prove that if (q_0, y_0) is an extremal pair for the problem $\sup y(T)$, $q \in \Omega(\Phi)$, and $\alpha = 0$ (the existence of y_0 such that $y_0(T)$ is maximal and the corresponding coefficient q_0 is easily proved): Th. 1 q_0 is decreasing. In the open set where $\int_0^t q_0 < \int_0^t \Phi$, we have $Q'(t) = 0$, where $Q(t) = y_0(t)^2 \int^T_t y_0(s)^{-2} ds$.

Using Theorem 1, we can solve the problem for the class $\Omega(\frac{t}{\Phi})$ when $\Phi(t) = 1$, $0 < t \leq B$; $\Phi(t) = 0$, $B < t \leq T$. (this is a simple example).

As examples of other problems which can be solved we mention.

1. Find $\sup y(T)$, $q \in \mathcal{E}_B$ where $\mathcal{E}_B = \{q \in L^1(0,T): \int_0^T |q| = B\}$ (This problem is due to Trubowitz)

2. Consider the eigenvalue problem $-y'' + qy = \lambda y$, $y(0) = y(1) = 0$. Find $q \in \mathcal{E}_B$ maximizing the first eigenvalue (the problem is due to Ramm; solved by Talenti in GI 4. We have an alternative solution).

3. Problem 2 has been solved in higher dimensions by Henrik Egnell, Uppsala (replace d^2/dx^2 by an elliptic operator).

Matts Essén (Uppsala)

Inequalities concerning convolutions of kernels of integral equations

Let $\Delta \subseteq \mathbb{R}$ be a measurable set with $0<|\Delta|<\infty$; $P;Q \in \Delta\times\Delta \rightarrow \mathbb{R}$ integralle kernel functions. Let p: $0<p\leq 2$ and q his adjoint $((1/p)+(1/q)=1)$. We define for a function $Z:\Delta\times\Delta \longrightarrow \mathbb{R}$ the following norm (if it exsists)

$$\|Z\|_p=\left\{\int_\Delta\left(\int_\Delta|Z(s,t)|^p dt\right)ds\right\}^{1/p}.$$

Under others it is proved: If $\|P\|_q<\infty$; $\|Q\|_q<\infty$, then the convolution

$$\int_\Delta P(s,r)\,Q(r,t)\,dr$$

exsists and

$$\left\|\int_\Delta P(s,r)\,Q(r,t)\,dr\right\|_p \leq \min\left\{|\Delta|^{\frac{q-2}{q}}\|P\|_p\|Q\|_q ; |\Delta|^{\frac{q-2}{q}}\|P\|_q\|Q\|_p\right\}$$

The constant $|\Delta|^{\frac{q-2}{q}}$ cannot be improved.

I. Fenyő (Budapest)

Some remarks on the Cauchy-Schwarz inequality

In order to make the Cauchy-Schwarz inequality accessible also in other cases than for elements of a real or complex inner product space, a method of proof is given which avoids divisions and the use of commutativity of the multiplication of scalars. The Cauchy-Schwarz deficite is represented as a factor in a nonnegative product rather than as the discriminant of a real quadratic form. The result is as follows.

Theorem. Let K be a ring with 1, $\bar{\cdot}: K \to K$ an involutorial antiautomorphism, and $\tilde{K} := \{\lambda \in K;\ \lambda = \bar{\lambda}\}$ a totally ordered ring contained in the center of K and having the property $\lambda\bar{\lambda} \geq 0$ $(\forall \lambda \in K)$. If X is a left-K-module and $f: X \times X \to K$ a positive semidefinite hermitian form, then $f(x,y)\overline{f(x,y)} \leq f(x,x)f(y,y)$ $(\forall x,y \in X)$. Equality occurs if and only if $\exists (\lambda,\mu) \in K \times K \setminus \{(0,0)\}$ with $f(\lambda x + \mu y, \lambda x + \mu y) = 0$.

Jürg Rätz
Bern, Switzerland

Some examples of a Hardy-Littlewood type integral inequality

This lecture reports on joint work with W. D. Evans, W. K. Hayman, and S. Ruscheweyh.

Let $q : [0,\infty) \to R$ (real field) with $q \in L_{loc}[0,\infty)$. Let the differential expression $-f'' + qf$ on $[0,\infty)$ be in the strong limit-point case in $L^2[0,\infty)$ at ∞. Let $\Delta = \{f : [0,\infty) \to R \mid f$ and $f' \in AC_{loc}[0,\infty)$, f and $-f'' + qf \in L^2[0,\infty)\}$. Then the Dirichlet integral $\int_0^\infty \{f'^2 + qf^2\}$ is (conditionally) convergent for all $f \in \Delta$ and we can consider the integral inequality

$$(*) \qquad \left(\int_0^\infty \{f'^2 + qf^2\}\right)^2 \le K \int_0^\infty f^2 \int_0^\infty \{-f'' + qf\}^2 \quad \text{all } f \in \Delta.$$

We say $(*)$ is <u>valid</u> if $0 < K < \infty$ and <u>not valid</u> if $K = \infty$; a valid inequality may or may not have non-trivial (non-null) cases of equality.

When $q = 0$ on $[0,\infty)$ (Hardy & Littlewood 1932) $K = 4$ and there is a continuum of cases of equality. If $q(x) = \tau > 0$ $(x \in [0,\infty))$ then $K = \infty$; if $q(x) = \tau < 0$ then $K = 4$ but there are no cases of equality.

When $q(x) = -x$ $(x \in [0,\infty))$ then $K = 4$ and there is a continuum of cases of equality.

When $q(x) = -x^2$ $(x \in [0,\infty))$ then $K = 4 + 2\sqrt{2}$ but there are no cases of equality.

When $q(x) = x^2 - \tau$ $(x \in [0,\infty))$ where $\tau \in R$ is a parameter then $K(\tau) = \infty$ for all $\tau \in R \setminus \{2n+1 : n = 0,1,2,\cdots\}$ and $K(\tau) = 4$ for all $\tau \in \{4n+1 : n = 0,1,2,\cdots\}$ with one case of equality; when $\tau \in \{4n+3 : n = 0,1,2,\cdots\}$ we have $K(\tau) > 4$ with two cases of equality and $K(4n+3) > K(4n+7) > 4$ for $n = 0,1,2,\cdots$; also $K(4n+3) = 4 + 3/(4\pi^2 n^2) + O(n^{-3})$ $(n \to \infty)$.

University of Birmingham, England. W. N. Everitt.

Sequential search for zeroes of $2(2^m-1)$-th derivatives

How might simple real zeroes of real valued continuous k-th derivatives $f^{(k)}$ be efficiently approximated, given that there is to be recourse solely to values of f? A standard approach to these questions entails successively selecting points to be the abscissae for sequences of k-th divided differences whose signs are then used to locate the zeroes; see Wallace [1] and the references therein to the work of S Johnson, J Kiefer, RS Booth and others. Of central importance is the particular rule (or strategy) by which these points are chosen. In this talk the speaker shows how analysis of a class of restricted subadditive inequalities has enabled him to determine the most efficient strategy for each of the special cases $k = 2, 6, 14, 30, ---, 2(2^m-1), ---$. Illustrations are given, and the suggestion made that similar analysis should lead to analogous results for other even k.

Reference

[1] R J WALLACE, Sequential search for zeroes of derivatives, in General Inequalities 4; Proceedings of the Fourth International Conference, Oberwolfach, 1983, W. Walter, ed. (Birkhauser, Basel, 1984), 151-167.

Dept. Quantitative Methods, Victoria College, Prahran, 3181, Victoria AUSTRALIA (RJWallace)

An iteration inequality of third order

The inequality is of the form

$$\psi(f^3(x)) + a_2\,\psi(f^2(x)) + a_1\,\psi(f(x)) + a_0\,\psi(x) \le 0 .$$

Form of solutions to this inequality as well as conditions under which its solutions furnish ones of a Schröder functional equation are presented. Here upper indices denote functional iterates, and a_0, a_1, a_2 are some real constants.

Results are due to Mrs. Maria Stopa from Kraków and partly to the speaker.

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Bogdan
Choczewski

An inequality for geometric means

Cochran and Lee (Math Proc Camb Phil Soc 1984) proved the inequality

$$\int_0^\infty x^{\kappa-1} \exp\left(\frac{p}{x^p}\int_0^x t^{p-1} \log f(t)\, dt\right) dx \leqslant e^{\kappa/p}\int_0^\infty x^{\kappa-1} f(x)\, dx$$

for κ real, $p > 0$, f pos measurable and $t^{p-1}\log f(t)$ locally integrable in $[0, \infty)$. The exponential on the left is a geometric mean, and I have generalized this inequality as follows. Let

$$Gf(x) = \exp\left(\int_0^x w(x,t) \log f(t)\, dt \Big/ \int_0^x w(x,t)\, dt\right),$$

where $w(x, t)$ is homogeneous and the denominator is in $(0, \infty)$ for $x = 1$. If $\sigma(x)$ is submultiplicative, $\rho(x) = x^{-1}\sigma(x^{-1})$, $\tau(x)$ is decreasing and

$$\int_0^1 w(1, t)\, \rho(t)^{1/n}\, dt < \infty \quad \text{for all positive integers } n \geqslant N,$$

then $$\int_0^\infty Gf(x)\, \sigma(x)\, \tau(x)\, dx \leqslant G\rho(1)\int_0^\infty f(x)\, \sigma(x)\, \tau(x)\, dx.$$

Cochran and Lee's Inequality is the case $w(x, t) = pt^{p-1}$, $\sigma(x) = x^{\kappa-1}$, $\tau(x) = 1$.

The constant $G\rho(1)$ is best possible if σ is multiplicative and $x^{\eta-1}\tau(x) \in L$ for all sufficiently small positive η.

The proof made use of a generalized Hardy's Inequality which was proved in GI4 (essentially).

Cochran and Lee also gave a discrete analogue of their inequality: If $\kappa \geqslant 1$, $p \geqslant 1$, and $0 < x_n \leqslant 1$, then

$$\sum_{m=1}^\infty m^{\kappa-1}\left(\prod_{n=1}^m x_n^{np-1}\right)^{p/m^p} \leqslant e^{\kappa/p}\sum_{m=1}^\infty m^{\kappa-1} x_m.$$

I have been trying to find a satisfactory generalization of this of the same kind.

University of Melbourne, Australia E. R. Love

Norm inequalities for derivatives and differences.

This work is joint with M.K. Kwong. The upper bound $U(p)$ mentioned below is due to Z.M. Franco, H.G. Kaper, M.K. Kwong and A. Zettl.

We consider the inequalities (1) $\|y'\|_p^2 \leq K\|y\|_p\|y''\|_p$ and (2) $\|\Delta x\|_p^2 \leq C\|x\|_p\|\Delta^2 x\|_p$. The norm in (1) is the classical $L^p(J)$ norm with $J=R=(-\infty,\infty)$ or $J=R^+=(0,\infty)$; in (2) the norm is the $l^p(M)$ norm with $M=Z=\{\dots,-2,-1,0,1,2,\dots\}$ or $M=N=\{0,1,2,\dots\}$. In both cases $1\leq p\leq\infty$. The constants $K=K(p,J)$ and $C=C(p,J)$ denote the smallest i.e. best constants such that (1) holds for all $y\in L^p(J)$ with y' absolutely continuous on all compact subintervals of J and $y''\in L^p(J)$ and (2) holds for all x in $l^p(M)$, respectively. It is known that $K(p,R)=C(p,Z)$ and $K(p,R^+)=C(p,N)$ for $p=1,2,$ and ∞. For other values of p none of these constants is known and it is not known if these equalities are valid for other values of p.

"Good" upper and lower bounds are given for $K(p,R)$, $2\leq p\leq\infty$: $L(p)\leq K(p,R)\leq U(p)$ with explicit formulas for $L(p)$ and $U(p)$ in terms of p. Also an elementary proof for a result of Ljubić is given showing that if u_i, $i=0,1,2$ are positive numbers satisfying $u_1^2 < K(p,J)\,u_0 u_2$ then there exists y in $L^p(J)$ such that $\|y^{(i)}\|_p = u_i$, $i=0,1,2$.

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Anton Zettl

Optimal inequalities in a semilinear BVP on a two-dimensional Riemannian manifold

A simple model describing a steady state in a reaction-diffusion process on a surface is given by

(1) $\Delta u + \lambda f(u) = 0$ in $\Omega \subset M$, $u = 0$ on $\partial\Omega$,

where Δ denotes the Laplace-Beltrami operator of M, λ is a positive parameter and only nonnegative solutions u are of interest.
If $f(s) > 0$ and $\lim\limits_{s\to\infty} \frac{s}{f(s)} < \infty$ then it is known that there are solutions u of (1) only in an interval $(0, \lambda^*)$, $\lambda^* < \infty$.
An important information in problem (1) is therefore a lower bound for λ^*.
An optimal lower bound can be obtained using a variant of a method due to L.E. PAYNE.

Set $\bar{u}(x) = X(s(\psi(x)))$ where

$X(s)$ satisfies

$$(1+ks^2)^{3/2}\left((1+ks^2)^{1/2}X'\right)' + \frac{\lambda}{K} f(X) - 0 \quad \text{in } (0, s_0)$$

$$X'(0) = 0, \; X(s_0) = 0$$

Here K is a positive lower bound for the Gaussian curvature of M in Ω, and $\psi(x)$ is the solution of

$$\Delta\psi + 1 = 0 \text{ in } \Omega \subset M, \; \psi = 0 \text{ on } \partial\Omega,$$

and finally $s(\psi) = \left[\frac{1}{K}\left(e^{2K(\psi_{max} - \psi)} - 1\right)\right]^{1/2}$.

Using a result proven in [1] one can then show that $\bar{u}(x)$ as constructed above is a supersolution to (1). This leads to a number of optimal bounds in problem (1).

The case of equality occurs when Ω is a geodesic strip on a sphere of radius $\frac{1}{\sqrt{K}}$.

R. Sperb, Seminar f. Angewandte Math.
ETH, Zürich

Ref [1] R. Sperb, Journal of Appl. Math. & Phys. ZAMP, (31), 1980, 740-753

Inequalities between norms of a function and its derivatives

A. Zettl and I have worked on extensions of the classical Landau inequality, $\|y'\|_p^2 \le K\|y\|_p \|y''\|_p$.

A discrete analog is $\|\Delta x\|_p^2 \le C\|x\|_p \|\Delta^2 x\|_p$ where $x \in l^\infty(M)$; M can be the space of semi-infinite or bi-infinite sequences. It turns out that although analogous results hold in the discrete cases, the proofs are sufficiently different from the continuous case. In the following we list several extensions of the continuous case. The same extensions exist for the discrete case, but again the proofs differ in the two cases.

The inequality has been extended to m-dissipative operators $\|Ax\|^2 \le K\|x\|\|A^2x\|$ by Kalman-Rota (Banach spaces) and Kato (Hilbert spaces). It is this theory that applies to the discrete difference operator. Higher dimension extensions have been obtained by Certain and Kurtz (Banach), Chernoff, Phong, and us (Hilbert) independently.

Everitt has extended the inequality to a Sturm-Liouville operator setting and a characterization of the existence of a finite K as well as the determination of the best K is given in terms of the m_λ function.

If the L^p norm is replaced by a weighted L^p norm the inequality still holds provided that w is non-decreasing. It is interesting to look for more general weight that preserves the inequality.

The inequality ~~can be found for~~ holds in the following for $\|y'\|_q \le K\|y\|_p^\alpha \|y''\|_r^\beta$ with p, q, r satisfying $\frac{2}{q} \le \frac{1}{p} + \frac{1}{r}$ and $\alpha+\beta=1$, α given in terms of p, q + r (Nirenberg for C_0^∞ functions and Gabushin for more general functions). Extension to the discrete case involves the construction due to Ditzian.

It has been proved that $\|y'\|_\infty^2 \le K\|y\|_\infty^{1-a} \|y''|y'|^a\|_\infty$ for $a \in [0,1)$ with $K = \frac{2}{1-a}$ for the whole line case and $K = \frac{4}{1-a}$ for the half-line case. Whether K is best possible, for $a \ne 0$, is open. Whether the inequality holds for other p norms is still not known.

M. K. Kwong (Northern Ill. Univ.)

INEQUALITIES AND MATHEMATICAL PROGRAMMING, III

(S. Iwamoto, R.J. Tomkins and Chung-lie Wang)

Three equivalent mathematical programming problems concerning monotone infinite sequences with suitable constraints are solved by the establishment of pertinent inequalities. The continuous version of the inequalities as well as some variants of discrete and continuous inequalities are also studied.

Chung-lie Wang, Regina, Canada

THE BIEBERBACH CONJECTURE

In two lectures a survey has been given on the development which lead to a proof of the Bieberbach conjecture, including: I. Historical remarks. II. Löwner Theory. III. De Branges' proof

Norbert Steinmetz, Karlsruhe

NON-NEGATIVE TRIGONOMETRIC POLYNOMIALS AND RELATED QUADRATIC INEQUALITIES

Inequalities of the form

$$\lambda \sum_{j=0}^{n} |x_j|^2 \leq \sum |x_j \pm x_{j+k}|^2 \leq \Lambda \sum_{j=0}^{n} |x_j|^2$$

are considered where $x_0, \dots, x_n \in \mathbb{R}$ (or $\mathbb{C}$), λ, Λ are constants and the sum in the middle means one of the following: (i) $\sum_{j=0}^{n-k}$, (ii) $\sum_{j=0}^{n}$ with $x_{n+1} = \dots = x_{n+k} = 0$, (iii) $\sum_{j=-k}^{n}$ with $x_{-k} = \dots = x_{-1} = x_{n+1} = \dots = x_{n+k} = 0$.
In all cases (both with positive and negative signs) the exact constants λ, Λ are given. They are minimal and maximal eigenvalues of suitable Hermitian matrices. The case $k=1$ has been known.

L. Losonczi (L. Kossuth University, Debrecen, Hungary)

A functional inequality and Schur-convexity

If $I \subset \mathbb{R}$ is an interval and $f: I \to \mathbb{R}$ is convex, then $\phi(x) = \sum_{i=1}^{n} f(x_i)$ is Schur-convex on I^n [Schur 1923 and HLP 1929].

We look for $f: I \to \mathbb{R}$ for which $\phi(x) = \sum_{i=1}^{2} f(x_i)$ is Schur-convex on I^2. This is equivalent to the determination of f satisfying the functional inequality

$$f(x)+f(y) \geq f(\lambda x+(1-\lambda)y)+f((1-\lambda)x+\lambda y)$$

for all $x, y \in I$ and all $0 \leq \lambda \leq 1$. It turns out that the inequality holds if, and only if, $f = C + A$, where C is convex on I and A is additive. Such decomposition is unique up to a linear function.

C. T. Ng
University of Waterloo
Canada.

EXPERIMENTING WITH OPERATOR INEQUALITIES

G. Pólya was not only one of the founders of inequality theory but was also very active in making the inductive process of mathematics an explicit topic.
This talk tries to report, in Pólya's spirit, on some computer experiments, done with an APL workspace, which are concerned with Pólya operators, a class of linear differential operators defined by an inverse-positivity condition.
~~I~~ Three results were given, all of which had first been found experimentally by studying examples produced with the workspace.

Thm 1. The nonzero coefficients of the basic functions of a ~~P~~ standard Pólya operator alternate.

Thm 2. The Green's kernel of a standard Pólya operator is quasiconcave

Thm 3. The ν-th eigenvalue of a Pólya operator is a ~~mono~~isotonic function of the absolute value of the second coefficient of the boundary conditions.

Conjecture: This function is also convex.

A. Clausing (Münster)

PARABOLIC MAXIMUM PRINCIPLES, DIFFUSION EQUATIONS, AND POPULATION DYNAMICS

We give general parabolic maximum principles for L-subharmonic functions u ($Lu \geq 0$) on space-time $\underline{\Omega} = \Omega \times J$ ($J =]s,T[$, $]s,T]$, etc.), L being a locally dissipative, parabolic, local operator. We consider general open sets $\underline{V}$ in $\underline{\Omega}$, and an appropriate closed boundary $B_p(\underline{V})$ for $\underline{V}$ (roughly speaking, $B_p(\underline{V})$ is obtained from $\partial\underline{V}$ by removing all the largest $\partial\underline{V}$-open horizontal parts of $\partial\underline{V}$ that can be "reached from below through $\underline{V}$"); the (linear) maximum principle then says that $\sup_{\underline{V}} u \leq \sup_{B_p(\underline{V})} u^+$ (where $u^+ = \sup\{u, 0\}$). Similarly, for the semilinear case we have comparison theorems (with lipschitz or locally lipschitz nonlinearities). There are many applications to parabolic 2nd order PDE, but also to situations where L is a more complicated object than a PD operator (for instance in transmission problems, or the construction of highly nondifferentiable extensions of PD operators as intermediate tools).

The mentioned maximum principles play an essential role in obtaining unique global solutions of problems of the type

$$Lu + Nu = 0 \quad \text{in } \underline{V}_s$$

$$u(x,s) = f(x) \quad (f \text{ initial value at } t=s),$$

$$u|\Gamma_s = 0 \quad (\Gamma_s \text{ an appropriate lateral boundary}),$$

$\underline{V}_s = \{(x,t) \in \underline{V} : t > s\}$, assuming this problem is solvable for $\underline{V} = \underline{\Omega}$ and an L-barrier can be constructed for $\underline{V}$. Such results apply in particular to second order parabolic PD operators with merely continuous coefficients (real, $c(x,t)$ indepent term ≤ 0), in general open noncylindrical domains $\underline{V}$. In particular one gets unique global solutions for generalized time-dependent Kolmogrov-Petrovskii-Piskounov equations important in population dynamics.

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Inequalities for q-factorial functions

The q-factorial function $\gamma_q : \mathbb{R}_+ \to \mathbb{R}$, given by

$$\gamma_q(x) = (1-x)\log(1-q) + \log \prod_{n=0}^{\infty} \frac{1-q^{n+1}}{1-q^{n+x}} \quad , \; q \in (0,1)$$

can be characterized as a Krull normal solution or a Nörlund principal solution of its difference equation

$$(D) \qquad f(x+1) - f(x) = \log \frac{q^x - 1}{q-1} \quad , \; x \in \mathbb{R}_+ \; .$$

Moreover, inequalities are obtained which give detailed information on the behaviour of γ_q near 1.

Theorem. a) Assume $f : \mathbb{R}_+ \to \mathbb{R}$ to be convex, to satisfy (D) for some $q \in (0,1)$ and $f(1) = 0$. Then $f = \gamma_q$.

b) Let $g(x) := f(x) + f(\frac{1}{x})$. Then $1 \leq x < y$ implies $g(x) \leq g(y)$.

c) Let $g_t(x) := g(x) + f(1-t(x-1))$, $x \in [1, 1+\frac{1}{t})$ and $\tau(x,q) := \log(2-q^{x-1})/\log q^{1-x}$.
Then $g_t(x) > 0$ if $x \in (1, 1+\frac{1}{t})$ and $t \geq \tau(x,q)$.

H. H. Kairies (Clausthal)

A Strict Inequality for Projection Constants

Let X_n be a finite-dimensional normed space, $\dim X_n = n$, imbedded into l_∞ as a subspace. Define the projection constant of X_n by

$$\lambda(X_n) = \inf \{ \|P\| \mid P : l_\infty \xrightarrow{\text{onto}} X_n \subset l_\infty ,\ P^2 = P \text{ lin. projection} \} .$$

By Kadets' result, $\lambda(X_n) \leq \sqrt{n}$. It is shown that strict inequality holds, $\lambda(X_n) < \sqrt{n}$. Thus for all $n \geq 2$ there is $\varepsilon_n > 0$ such that $\lambda(X_n) \leq \sqrt{n} - \varepsilon_n$. There are spaces X_n (over $\mathbb{C}$) showing that $\varepsilon_n \leq \frac{1}{\sqrt{n}}$.

Hermann König (Kiel)

How (to use the Hahn-Banach theorem) to make fair decisions?

Denote by Ω an abstract set of opinions (For instance, Ω can be a set of real numbers or Ω can be $\{YES, NO\}$.)
The function $d: \bigcup_{n=1}^{\infty} \Omega^n \to [-\infty, \infty]$ is called a fair decision function if

1) it is symmetric, i.e

$$d(\omega_1, \dots, \omega_n) = d(\omega_{\pi(1)}, \dots, \omega_{\pi(n)})$$

for all $n \in \mathbb{N}$, $\omega_1, \dots, \omega_n \in \Omega$, permutation π of $\{1, \dots, n\}$;

2) it is a compromise, i.e.

$$d(\omega_1, \dots, \omega_{n+m}) \in [d(\omega_1, \dots, \omega_n) d(\omega_{n+1}, \dots, \omega_m)]$$

for all $n, m \in \mathbb{N}$, $\omega_1, \dots, \omega_{n+m} \in \Omega$;

3) it neglects odd ball opinions, i.e

$$\lim_{k \to \infty} d(\underbrace{\omega_1, \dots, \omega_1}_{k}, \dots, \underbrace{\omega_n, \dots, \omega_n}_{k}, \omega_0) = d(\omega_1, \dots, \omega_n)$$

for all $n \in \mathbb{N}$, $\omega_0, \dots, \omega_n \in \Omega$.

In the lecture we prove that any fair decision function is uniquely determined by a two-variable function.

Zsolt Páles (Debrecen)

ON THE CHARECTERIZATION OF BRUHATORDER ON THE SYMMETRIC GROUP AND APPLICATIONS.

Let $\pi, \sigma \in S_n$ = symmetric group of order n. Write $\pi \leq \sigma$:$\Leftrightarrow$ There exists a sequence of order generating transpositions turning π into σ. For example ,
since

We speak of two applications of the following

THEOREM: $\pi \leq \sigma \iff$ For all k, l $|\{i : i \geq k, \pi i \geq l\}| \leq |\{i : i \geq k, \sigma i \geq l\}|$

Corollary 1: Assume $S := \sum_{i=1}^{n} x_i (y_{\sigma i} - y_{\pi i}) \geq 0$ for all $\underline{x}, \underline{y} \in M := \{\underline{u} \in \mathbb{R}^n : 0 \leq u_1 \leq \dots \leq u_n\}$
Then it is possible to write S as a sum of (the obviously positive definite) products $(x_{i'} - x_i)(y_{j'} - y_j)$ with $i' > i$, $j' > j$

Corollary 2: Let $\mathcal{W} = \{W_i\}_1^n$ be a fixed maximal flag of vectorspaces (in $\mathbb{R}^n$ say)
Let $\{U_i\}$ and $\{V_i\}$ be in relative positions σ and τ with respect to $\mathcal{W}$
Then $\tau \leq \sigma$ in Bruhat-order iff $\dim(V_i \cap W_j) \leq \dim(U_i \cap W_j)$ $(1 \leq i, j \leq n)$

Corollary 2 was proved by R. Proctor independtly and gives a positive answer to a conjecture of G. Lusztig
We also extend the Theorem 1:

THM 2. Let P, Q denote two equicardinal partially ordered sets, each of which is a finite rooted or dual rooted forest. Given two bijections $\Delta, \tilde{\Delta} : P \to Q$ the following statements are equivalent

i. $\tilde{\Delta}$ is obtainable from Δ by a sequence of switchings i.e
$$\Delta \to \Delta_1 \to \Delta_2 \to \dots \to \Delta_n \to \tilde{\Delta}$$
for some sequence $\Delta_i : P \to Q$ of bijections

ii. For all filters F of P and filters F' of Q, we have $|\Delta \cap F \times F'| \leq |\tilde{\Delta} \cap F \times F'|$

A. Kovačec (Vienna, Austria)

Refinements of norm inequalities for functions of mean value zero. Let f be a bounded measurable real-valued function on $[a, b]$ such that $\int_a^b f(x)\,dx = 0$ and $f \not\equiv 0$.
Then
$$\frac{1}{b-a} \int_a^b |f(x)|\,dx \leq \frac{1}{(b-a)^2} \int_a^b \int_a^b |f(x) - f(y)|\,dx\,dy \leq \frac{2Hh}{H+h}$$
where $H = \operatorname{ess\,sup} f$, $h = -\operatorname{ess\,inf} f$. Similar results hold for other L^p norms and for $[a, b]$ (with the measure $dx/(b-a)$) replaced by any probability space. The proof involves the non-increasing rearrangements of the functions f^+ and f^-.

B. Saffari (Orsay, France)

Entropies, generalized entropies, inequalities and the maximum entropy principle

Inequalities for the Shannon (and Hartley) entropies and their generalizations have been used in applications and they served as building blocks for their characterizations.

After a short survey of such results, the idea has been put forward that the maximum entropy principle (also an inequality) may be used not only to 'justify' probability distributions but also to 'justify' entropies.

J. Aczel (Waterloo, Ont.)

Tax progression and measurement of income inequality

Let $T: \mathbb{R}_+ \to \mathbb{R}$, $y \mapsto T(y)$, y a pre-tax income, be a feasible (i.e., $T(y) < y$ for all $y \in \mathbb{R}_+$) and incentive-preserving (i.e., $y < y^*$ implies $y - T(y) \leqq y^* - T(y^*)$) tax function, and let $\underline{I}_\mu : \mathbb{R}_+^n \to \mathbb{R}$, $\underline{x} \mapsto I_\mu(\underline{x})$, be a strictly Schur-convex "μ-measure" of income inequality, i.e., a strictly Schur-convex function that satisfies

$$I_\mu(\underline{x}) = I_\mu(\underline{x} + \tau(\mu\underline{x} + (1-\mu)\underline{1}))$$

for all $\underline{x} \in \mathbb{R}_+^n$ and $\tau \in \mathbb{R}$ satisfying $\underline{x} + \tau(\mu\underline{x} + (1-\mu)\underline{1}) \in \mathbb{R}_+^n$, where $\mu \in [0,1]$ is fixed. The functional equation shows, for which income distributions $\underline{x}$ the income inequality is preserved.

The following two statements are equivalent:

(i) $I_\mu(y_1 - T(y_1), \ldots, y_n - T(y_n)) < \underline{I}_\mu(y_1, \ldots, y_n)$ for all $\underline{y} \in \mathbb{R}_+^n$ such that $(y_1, \ldots, y_n) \neq (a, \ldots, a)$.

(ii) $T(y)/(\mu y + (1-\mu))$ is strictly increasing in $y \in \mathbb{R}_+$.

This result obtained by Andreas Pfingsten (Ph.D. student, Karlsruhe) generalizes a result ($\mu = 1$, I_μ the Lorenz measure of inequality) that I presented during the 1983 meeting on Allgemeine Ungleichungen.

<u>Corollary</u>: An inequality preserving tax function T is necessarily an affine-linear function.

Wolfgang Eichhorn, Karlsruhe, D

Gefördert durch DFG Deutsche Forschungsgemeinschaft

L'interpolation de quelques inégalités.

Supposons que $I \subset N$ (I fini et non vide) et que l'inégalité $F(I) \geq 0$ est vraie. Nous avons une interpolation de l'inégalité $F(I) \geq 0$ si l'on a $F(I) \geq F(J) \geq 0$ pour quelques $J \subset I$. Un cas important est $F(I_n) \geq F(I_{n-1}) \geq \dots \geq F(I_2) \geq F(I_1) = 0$ où $I_n = \{1, .., n\}$. Nous avons démontré plusieurs inégalités du type cité. Par example, pour l'inégalité de Levinson

$$F(I) = \Sigma p_i \left\{ f\left(\frac{\Sigma p_i x_i}{\Sigma p_i} \right) - f\left(\frac{\Sigma p_i x_i'}{\Sigma p_i} \right) \right\}$$

$$- \left\{ \Sigma p_i f(x_i) - \Sigma p_i f(x_i') \right\} \geq 0 \qquad \left(\Sigma = \sum_{i \in I} \right)$$

où $x_i + x_i' = 2a$, $0 < x_i \leq a$, $p_i > 0$, f est convex d'ordre 2 sur $(0, 2a)$, on a $F(I \cup J) \geq F(I) + F(J)$ $(I \cap J = \emptyset)$ d'où

$$F(I_n) \geq F(I_{n-1}) \geq \dots \geq F(I_2) \geq F(I_1) = 0$$

P.M. Vasić (Belgrade)

Multiplicativity and Mixed-Multiplicativity for Operator-Norms and Matrix-Norms

Let V be normed vector space over $\mathbb{C}$, let $\mathcal{B}(V)$ denote the algebra of bounded linear operators on V, and let N be an arbitrary norm on $\mathcal{B}(V)$. In this talk we discuss multiplicativity factors for N, i.e., constants $\mu > 0$ for which

$$N(AB) \leq \mu N(A) N(B), \ \forall A, B \in \mathcal{B}(V).$$

We examine several finite and infinite dimensional examples, as well as certain generalizations of the above concept.

Moshe Goldberg
Technion
Haifa, ISRAEL

Almost t-convex functions

Let $\emptyset \neq I \subset \mathbb{R}$ denote an interval and $t \in [0,1]$. A function $f : I \to \mathbb{R}$ is called almost t-convex iff

$$f(tu+(1-t)v) \leq tf(u)+(1-t)f(v) \quad \text{for almost all } (u,v) \in I^2$$

(almost all in the sense of Lebesgue measure on $\mathbb{R}^2$).

Furthermore we define

$$K_a(f) := \{t \in [0,1] : f \text{ is almost } t\text{-convex}\}.$$

__Theorem__ If $K_a(f) \neq \{0,1\}$, then

$$K_a(f) = [K_a(f)] \cap [0,1],$$

where $[K_a(f)]$ denotes the field generated by $K_a(f)$.

The proof is based on a related result for t-convex functions and on a Konstruction of Kuczma.

Norbert Kuhn, Saarbrücken, D

Ein Existenzsatz für gewöhnliche Differentialgleichungen in geordneten Banachräumen.

Es wird ein Existenzsatz für gewöhnliche Differentialgleichungen in Banachräumen gegeben, wobei die rechte Seite der Differentialgleichung eine bezüglich eines Kegels wachsende Funktion ist (gemeinsame Arbeit mit Roland Lemmert und Raymond M. Redheffer).

Peter Volkmann (Karlsruhe)

Chow's submartingale inequality for random variables taking values in a linear topological space.

Let $\mathcal{X}$ be a linear topological space and let $\mathcal{C} \subset \mathcal{X}$ be a closed convex cone with nonempty interior. For $a, b \in \mathcal{X}$, write "$a \lesssim b$" to mean $b - a \in \mathcal{C}$. Let $\mathcal{F}$ be a set of real valued functions defined on $\mathcal{X}$ with the properties (1) $x \lesssim y \Leftrightarrow f(x) \leq f(y)$ for all $f \in \mathcal{F}$, (2) $f(x) \geq 0$ for $x \in \mathcal{C}$, (3) $f(ax) = a f(x)$ for all $a \geq 0$. For an $\mathcal{X}$-valued random variable define EX in terms of an integral which, when it exists, satisfies (4) $\int (X+Y)\,dP = \int X\,dP + \int Y\,dP$, (5) if $A \subset \mathcal{X}$ is closed and convex $P\{X \in A\} = 1 \Rightarrow \int X\,dP \in A$ (6) for all $c \in \mathcal{X}$ and all events

E, $\int_E c\,dP = cP(E)$. Let $X_1, \cdots, X_m$ be $\mathcal{X}$-valued random variables which form a submartingale in the sense that $E\{X_{i+1} | X_1, \cdots, X_i\} \succsim X_i$ a.e., $i = 1, \cdots, m-1$ and suppose that $P\{X_i \in \mathcal{C}\} = 1$, $EX_i = \mu_i$, $i = 1, \cdots, m$. Then for any $\varepsilon \in \mathcal{C}$ and $a_1 \geq a_2 \geq \cdots \geq a_m > 0 = a_{m+1}$, $P\{a_k X_k \succsim \varepsilon$ for some k, $1 \leq k \leq m\}$

$\leq \inf_{f \in \mathcal{F}, f(\varepsilon) > 0} f(\sum_1^m (a_i - a_{i+1}) \mu_i) / f(\varepsilon)$. For $\mathcal{X} = R^n$ and $\mathcal{C}$ the nonnegative orthant this inequality is due to Chow [Proc. Amer Math. Soc 11 (1960) 107-111] and in that case it is known to be sharp (equality can be attained) [Birnbaum & Marshall, Ann. Math. Statist. 32 (1961) 687-703]. Only in special cases (e.g. $m=1$) is it known that the more general inequality is sharp.

Albert W. Marshall, Vancouver.

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Starting in 1972 Everitt, and later others, studied a generalisation of the wellknown inequality

$$\left(\int_0^\infty |u'|^2\right)^2 \leq 4\int_0^\infty |u|^2 \int_0^\infty |u''|^2$$

by Hardy and Littlewood. The more general problem is to decide whether there is a finite K such that

$$\left(\int_a^b (p|u'|^2 + q|u|^2)\right)^2 \leq K^2 \int_a^b |u|^2 w \int_a^b |-(pu')' + qu|^2 / w$$

holds for any function u for which the right hand side makes sense. Here p, q and $w \geq 0$ are realvalued and satisfy conditions making the differential equation $-(pu')' + qu = \lambda w u$ regular at a but in the so called strong limit point condition at b. It was thought for a number of years that no such inequality was possible when a and b are both regular points. I shall give conditions I derived recently which are fairly close to necessary and sufficient for such an inequality to hold. I will then describe a set of examples which I gave a few years ago (in "A general version of the Hardy-Littlewood-Polya-Everitt (HELP) inequality. Proc. Roy. Soc. Edinburgh 97A, 9-20, 1984").

Christer Bennewitz, Uppsala, Sweden

*) Title: The HELP inequality in a regular case.

P-estimates for ultraproducts of Banach lattices

A Banach lattice L is said to satisfy a lower p-estimate ($1 \le p < \infty$) iff there exists a constant $K > 0$ such that for each finite sequence $f_1, f_2, \dots, f_m \in L$ the inequality

$$\Big(\sum_{k=1}^{m} \| f_k \|^p \Big)^{1/p} \le K \, \Big\| \sum_{k=1}^{m} |f_k| \Big\|$$

holds. Moreover, L is said to satisfy an upper p-estimate iff there exists a constant $M > 0$ such that for each finite sequence $f_1, f_2, \dots, f_m \in L$ one has

$$\Big\| \sup_{1 \le k \le m} |f_k| \Big\| \le M \Big(\sum_{k=1}^{m} \|f_k\|^p \Big)^{1/p} .$$

The purpose of the lecture is to show in what sense these p-estimates carry over to ultraproducts of Banach lattices. An application is given to the problem of superreflexivity of Banach lattices.

Franziska Fehér (Dortmund)

<u>$\mathcal{F}$ - convexity</u>

A. Ben-Israel (Univ. of Delaware, Newark, DE, USA)
A. Ben-Tal (Technion - I.I.T., Haifa, Israel)

Let $\mathcal{F}$ be a family of functions $: R^n \to R$. A function $f : R^n \to R$ is <u>$\mathcal{F}$-convex</u> [in $S \subset \operatorname{dom} f$] if for all x [$\in S$] there is $F \in \mathcal{F}$ such that
(1) $f(x) = F(x)$, (2) $f(z) \ge F(z)$ $\forall z \ne x$ [$z \in S$],
in which case F is <u>support</u> of f at x.
f is <u>strictly $\mathcal{F}$-convex</u> if "$>$" in (2);
<u>$\mathcal{F}$-concave</u> if "$\le$" in (2), etc.
Examples: (i) $\mathcal{F} := \{ F : F(x) = \langle x^*, x \rangle - y ;\ (x^*, y) \in R^n \times R \}$

the (nonvertical) affine functions : $R^n \to R$. Here $\mathcal{F}$-convexity is convexity. (ii) $n=1$, $\mathcal{F}$ a Beckenbach family of functions : $(a, b) \to R$. The $\mathcal{F}$-convex functions are here the sub-$\mathcal{F}$ functions of Beckenbach [*Bull A.M.S.* 1937]. (iii) $\mathcal{F}$ the family of solutions of a 2nd order diff. eq. $y'' = g(y', y, x)$. Then, under certain assumptions, the $\mathcal{F}$-convex functions are the subfunctions of Peixoto [*Bull AMS*, 1949] and Bonsall [*Quart J Math Oxford*, 1950], given by $y'' \geq g(y', y, x)$.

In [*J. Austral. Math. Soc.*, 1976] we gave a theory of $\mathcal{F}$-convexity, including 1st order and 2nd order characterizations of $\mathcal{F}$-convex functions. A convex analysis for $\mathcal{F}$-convex functions was given in [*Generalized Concavity* (ed. S. Schaible & W. T. Ziemba), 1981]. Here we illustrate the need for *non-affine* supports in two applications: (i) *Constrained optimization* (following Dolecki & Kurcyusz [*SIAM J. Control Optimiz*, 1978]) (ii) *Profit maximization*

$$\sup_{x} \{ r(x) - \phi(x^*, x) \} \tag{3}$$

where r = revenue, ϕ = cost (nonlinear in the "prices" x^* and/or in x). Here (3) is the Φ-concave conjugate of r, $\Phi := \{\phi(x^*; x) - \eta\}$, and (3) is solved by $\mathcal{F}$-convex analysis.

INFORMATIONSTHEORIE

11. – 17 Mai 1986

"Self-dual binary codes and desarguesian planes of even order"

by E.F. Assmus, Jr.

Consider the code generated by the Desarguesian projective plane of order 2^e extended by an overall parity check, C say. C is self-orthogonal and we ask whether or not there is a self-dual code $S \supseteq C$ with a generator matrix of the form $(I_k \vdots M)$ where I_k is the identity matrix and M is a $k \times k$ matrix that is the incidence matrix of a biplane. (Here $k = \frac{1}{2}(2^{2e}+2^e+2)$. The answer for $e=1, 2,$ or 3 is yes and there is a general, group-theoretic construction that yields the M's in these cases.

This same construction yields matrices M for $e > 3$. These M's have the property that each row contains 2^e+1 ones and every two distinct rows have $0, 2,$ or 4 ones in common, but in every case there are two rows with 0 ones in common and hence M is <u>not</u> the incidence matrix of a biplane. For e even and $e>3$ this is predictable by the Bruck-Ryser-Chowla Theorem. For e odd and $e>3$ one needs the following Lemma: If the trace from $GF(2^e)$ to $GF(2)$ has the property that $Tr(x+x^{-1}) = 1$ whenever $x \neq 0, 1$, then $e = 1, 2,$ or 3. Although this is an immediate consequence of Weil's estimate it has an elementary proof. (Both Odlyzko and Neidereiter gave these elementary proofs at the conference.)

Some Convolutional Self-Ortogonal Codes

Torleiv Kløve, Bergen.

An (I, J) difference triangle set (DTS) is a set

$$\Delta = \{ \Delta_1, \Delta_2, \dots, \Delta_I \}$$

where $\Delta_i = \{ a_{ij} \mid 0 \le j \le J \}$, $1 \le i \le I$ are sets of integers such that all the integers $a_{ij} - a_{ij'}$, $1 \le i \le I$, $0 \le j \ne j' \le J$ are distinct. The corresponding code has generator polynomials

$$g_i(D) = \sum_{j=0}^{J} D^{a_{ij}} .$$

It is a $(I+1, I, m)$ convolutional code with $d_{min} = J+1$ where

$$m = m(\Delta) = \max \{ a_{ij} \}.$$

Let $M(I,J) = \min \{ m(\Delta) \mid \Delta \text{ is an } (I,J) \text{ DTS} \}$.

We showed that

$M(I,1) = I$ for all $I \ge 1$,

$3I \le M(I,2) \le 3I+1$ for all $I \ge 1$,

$M(I,2) = 3I+1$ for all $I \equiv 2$ or $3 \pmod 4$,

$78s + 6x \le M(13s + x, 3) \le 86s + C_x$

where C_x is given by the following table:

x	1	2	3	4	5	6	7	8	9	10	11	12	13
C_x	13	23	27	32	40	46	54	58	68	72	73	90	91.

A computer search has shown that

$M(I,2) = 3I$ for $I \equiv 0$ or $1 \pmod 4$, $I \le 25$.

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Further results on Unknown Functions

Ingemar Ingemarsson, Linköping University, Sweden

An invertible function $f(x)$ is defined and attains values from the set of integers $[1, n]$. The function is chosen from a set $\mathcal{F}$ of M such functions. An observer knows $\mathcal{F}$ but not the actual choice $f(x)$. He is however able to list the i outcomes of i input values. The question is what can he say about the value of the function for any other argument. We say that his uncertainty is maximal if equally many functions in $\mathcal{F}$ attains any sequence of values $y_1, \dots, y_i$ for any sequence of arguments $x_1, \dots, x_i$. The maximum value for i is called the security level. The maximual security level, k, satisfies

$$M = \frac{n!}{(n-k)!}$$

The Hamming distance between two functions is the number of arguments for which the functions are not equal. We have proved that a set $\mathcal{F}$ has maximal security level if and only if the minimum Hamming distance is $n-k+1$. We have also proved that the security level of a cascade (i.e. $f_1(f_2(\cdots f_2(x))\cdots)$) of functions is at least equal to the maximum of the security levels of the functions involved.

APPLICATION OF CODING THEORY TO DESIGNS

Robert Calderbank, Bell Labs Room 2C-363, 600 Mountain Ave
Murray Hill NJ 07974.

Theorems of Gleason and of Mallows and Sloane characterize the weight enumerator of maximal self-orthogonal codes with all weights divisible by 4. We apply these results to give a new necessary condition for the existence of quasi-symmetric $2\text{-}(v,k,\lambda)$ designs where the intersection numbers s,t satisfy $s \equiv t \pmod 2$. (The assumption that there are 2 intersection numbers can be weakened to intersection numbers $s_1, \ldots, s_n$ satisfying $s_1 \equiv \ldots \equiv s_n \pmod 2$)

We also apply duality in the Johnson scheme to give a very short proof of a theorem of Frankl and Füredi. We consider a family $\mathcal{F}$ of k-subsets of a v-set such that $\mathcal{F}$ is a 1-design and $|x \cap y| \geqslant \lambda > 0$ for all $x, y \in \mathcal{F}$. We prove that $v \leqslant (k^2-k+\lambda)/\lambda$ and that $v = (k^2-k+\lambda)/\lambda$ if and only if $\mathcal{F}$ is a symmetric $2\text{-}(v,k,\lambda)$ design.

Applications of algebraic coding theory to cryptography

Harald Niederreiter (Vienna)

We present two applications of algebraic coding theory to cryptography. In the first application consider a public-key cryptosystem with linear encryption function $\underline{m} \neq \underline{0} \rightarrow E\underline{m}$, where the matrix E may be known (e.g. knapsack-type systems) or unknown (e.g. FSR cryptosystems). We point out that such cryptosystems are unsafe for low-weight messages $\underline{m}$. We can avoid low-weight messages by choosing plaintexts $\underline{x} \neq \underline{0}$ of length $k < n$ (= length of $\underline{m}$), applying the coding scheme of a linear (n,k,d) code with large d to $\underline{x}$,

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and feeding the resulting code word into the cryptosystem. In this way each word entering the cryptosystem has weight $\geq d$. If E is known, then the code depends on secret data; if E is unknown, the code can be made public.

In the second application we improve on the Chor-Rivest cryptosystem by constructing cryptosystems with a much higher information rate R. Let H be a parity-check matrix of a secret t-error-correcting linear (n,k) code over F_q and let the public key K be a scrambled version of H. A plaintext $\underline{m} \in F_q^n$ of weight $\leq t$ is enciphered as $K\underline{m}$. Unique decryption is possible by means of the decoding algorithm of the secret code. If we choose a family of binary codes that meet the Gilbert-Varshamov bound and satisfy $\frac{t}{n} \to \frac{1}{4}$ as $n \to \infty$, then $R \gtrsim 0.81\ldots$, whereas for the corresponding situation in the Chor-Rivest cryptosystem we have $R \to 0$ as $n \to \infty$.

Binary transmission codes with higher order spectral zeros at zero frequency

Gerard F. M. Beenker, Philips Research Laboratories,
Eindhoven, The Netherlands.

A method is presented for designing binary transmission codes in such a way that both the power spectral density function and its low order derivatives vanish at zero frequency.
Codes are called of K-th order zero disparity if all codewords $\underline{x} = (x_1, x_2, \dots, x_n)$, $x_i \in \{-1, 1\}$, satisfy $\sum_{i=1}^{n} i^k x_i = 0$ for $k \in \{0, 1, \dots, K\}$.
The power spectral density function and its first $2K+1$ derivatives of a K-th order zero disparity code can easily be shown to vanish at zero frequency.

The maximum number of codewords of a K-th order zero disparity code of length n is determined as a coefficient of a generating function in two variables, for all $n \in \mathbb{N}$. For $K=1$ a lower as well as an upperbound for this number is derived.
It is shown that the minimum distance of a K-th order zero disparity code is at least $2K+2$.

On Binary State Symmetric Markov Channels.

R. Ahlswede & Amiram Kaspi

We study the structure of the transition matrix of binary-input binary-output Markov channels that are symmetric in the sense that the transition probability is invariant under simultaneous complementation of the input, the output and the state of the channel.

Using the structure of the transition matrix, we give bounds on the capacity of the "trap door" channel and show that the zero error capacity of this channel is 0.5.

A multi-terminal problem that arises from the "trap door" channel is presented, and it is shown that one of the extreme points in its acheivable region is $(0, \log \frac{1+\sqrt{5}}{2})$ where the second term results from the limit of the Fibonacci sequence.

Two-way channels

J. Pieter M. Schalkwijk

Shannon's (1961) model of a two-way communication channel is discussed, in particular the inner and outer bounds to the capacity region. As an example of a nontrivial dialogue we then consider Blackwell's binary multiplying channel (BMC), as does Shannon in his own two-way channel paper refered to above. We describe Schalkwijk's (1983) coding strategy for the BMC, which we subsequently show to be optimum for both fixed length strategies with vanishing probability of error, and also for variable length strategies with zero probability of error. Thus we establish for the first time the capacity region of a nontrivial (i.e. inner ≠ outer bound) two-way channel. For symmetric $R_1 = R_2$ operation the optimum rate is $R_1 = R_2 = .63056$ bit per transmission. The essential step in the converse considers the uncertainty reduction for resolutions on the initial threshold pair of the (θ_1, θ_2)-search on the unit square.

Optimal linear codes

Tor Helleseth

An $[n,k,d]$ code is a k-dimensional subspace of $GF(2^n)$ such that the minimum Hamming distance between the codewords equals d. Given k and d then $n(k,d)$ is defined as the smallest integer such that an $[n,k,d]$ code exists. For $k \leq 7$, $n(k,d)$ has been determined by H. van Tilborg. For $k = 8$ it is known that $n(8,d) = \sum_{i=0}^{7} \lceil d/2^i \rceil$ for all $d \geq 131$, where $\lceil x \rceil$ is the smallest integer $\leq x$. In a recent paper Dodunekov and Manev have determined or given the best known bounds on $n(8,d)$ for $3 \leq d \leq 130$.

We improve these bounds as follows:

$n(8,16) \geq 37$, $n(8,30) \leq 65$, $n(8,32) = 68$

$n(8,34) \leq 75$, $n(8,36) \leq 78$, $n(8,40) \geq 84$

$n(8,42) \leq 90$, $n(8,44) \in [92,93]$, $n(8,52) \leq 109$

$n(8,58) \geq 120$, $n(8,60) \geq 123$

Graph Entropy and its Relevance to Combinatorics

János Körner

The graph G is covered by the union of the graphs G_i, $i=1,2,\dots,t$ if all these graphs have the same vertex set and every edge of G is contained in at least one of the G_i's. In a graph covering problem one is given a graph G and a family of graphs $\mathcal{G}$. One then asks for the minimum number of graphs G_i, $i=1,2,\dots,t$ such that G_i is in $\mathcal{G}$ and the union of the G_i's covers G. In order to get lower bounds on t one can use a functional which is sub-additive with respect to the union of graphs. Such a functional is graph entropy, introduced by Körner, 1973. Given a distribution P on the vertex set of G, the entropy $H(G,P)$ is

$$\min_{\substack{X \in Y \in \mathcal{Y}(G) \\ P_X = P}} I(X \wedge Y)$$

where $I(X \wedge Y)$ is the mutual information and $\mathcal{Y}(G)$ is the family of independent sets of G. Graph entropy and its natural generalization hypergraph entropy were used by Körner, 1986 and Körner-Marton, 198? to improve on the Fredman-Komlós bounds for the minimum number of perfect hash functions to hash all the k-element subsets of an n-set into b classes. The analysis of the method leads to an interesting conjecture on perfect graphs that is proved here for bipartite graphs.

"Reliable transmission of two arbitrarily correlated information sources over a discrete memoryless asymmetric multiple-access channel."

by

Edward C. van der Meulen (K.U. Leuven)

(joint with K. De Bruyn (K.U.Leuven) and V.V. Prelov (IPIT, Moscow))

A discrete memoryless asymmetric multiple-access channel with two encoders is a "two sender - one receiver" multiple-access communication situation whereby messages of one source are encoded by both encoders, whereas the messages of another message set are encoded by only one of them. In this contribution necessary and sufficient conditions are given for the transmission of two arbitrarily correlated sources over such a discrete memoryless asymmetric multiple-access channel. The result shows that in this situation the so-called separation principle holds. An example is given illustrating the theorem. Furthermore it is demonstrated that the same conditions hold when feedback is available to one or both of the encoders. This research builds forth on the work by Cover, El Gamal, and Salehi (1980), Dueck (1981), and Ahlswede and Han (1983). In concreto, the theorem reads as follows:

a. A correlated source $(\mathcal{U}\times\mathcal{V}, p(u,v))$ can be transmitted reliably over a d.m. AMAC K_2, if there exists a prob. distrib. $P(x_1,x_2)$ such that

$$H(U|V) < I(X_1;Y|X_2)$$
$$H(U,V) < I(X_1,X_2;Y)$$

where $P(x_1,x_2,y) = P(x_1,x_2)\,P(y|x_1,x_2)$.

b. Conversely, if a correlated source pair $(\mathcal{U}\times\mathcal{V}, p(u,v))$ can be transmitted reliably over a given d.m. AMAC $K_2 = (\mathcal{X}_1\times\mathcal{X}_2, P(y|x_1,x_2), \mathcal{Y})$, then the following inequalities must be true for some p.d. $p(x_1,x_2)$:

$$H(U|V) \leq I(X_1;Y|X_2)$$
$$H(U,V) \leq I(X_1,X_2;Y).$$

"A new universal data compression method"

Frans M.J. Willems, T.H. Eindhoven

A new universal data compression algorithm is decribed. This algorithm encodes L source symbols at a time. The code alphabet is binary. For the class of binary stationary sources, the expected number of code symbols per source symbol is shown to be not more than

$$\frac{H(U_0.U_1,\dots,U_{L-1})+\lceil\log(L+1)\rceil}{L}.$$

In the analysis of our algorithm a result on repetition-times turns out to be crucial. The algorithm can be generalized to arbitrary source and arbitrary code alphabet sizes. Its implementation is discussed.

Burst identification codes   Henk van Tilborg

Consider a vectorspace $\mathcal{A}$ of all binary $n_1 \times n_2$ arrays. A $b_1 \times b_2$ burst is an $n_1 \times n_2$ array, all of whose non zero elements are confined to a $b_1 \times b_2$ subrectangle. A linear subspace (code) $\mathcal{C}$ is called a $b_1 \times b_2$-burst identification code, if the pattern of any single $b_1 \times b_2$ burst can be identified

Let $r_{b_1 b_2}$ be the minimal redundancy of a linear, $b_1 \times b_2$ burst identification code. It will be shown that $2b_1b_2-2 \le r_{b_1b_2} \le 2b_1b_2$

Burst identification codes combined with burst location codes can correct any $b_1 \times b_2$ burst.

Asymptotic properties of equal-weight codes

Thomas Ericson, Dep. EE, Linköping Univ., Sweden
Victor Zinoviev, Inst. probl. inf. transm., Moscow

An equal-weight code is a binary code such that all codewords have the same weight. Denote by $T(n,w,c)$ the maximum size of such a code when the length is n, the common weight is w, and the maximal correlation between codewords is c. We are interested in the asymptotic behaviour of $T(n, \lfloor \nu n \rfloor, \lfloor \nu \kappa n \rfloor)$ as $n \to \infty$, where $\nu \triangleq \frac{w}{n}$ and $\kappa \triangleq \frac{c}{w}$ are held fixed. Exponential increase of T is obtained if and only if $\kappa > \nu$. The exponent is easily lower bounded by the Gilbert bound. By combining a construction by Kautz-Singleton with a recent result by Tsfasman-Vladut-Zink we obtain an improvement of this bound in a certain range $\kappa_1 < \kappa < \kappa_2$, provided $\nu = \frac{1}{p^{2s}}$, p prime, s positive integer, and $p \geq 11$.

The simplest upper bound (for the size of an equal weight code) is Johnson bound: $T(n,w,c) \leq \binom{n}{c+1} / \binom{w}{c+1}$. For certain values of the parameters (n,w,c) this bound is satisfied with equality. The corresponding code is equivalent to Steiner system $S(n,w,c+1)$. There are a few infinite families of Steiner systems, including cyclic ones. They provide optimal protocols for multi-user channels without feedback both in the synchronious case (Steiner systems) and the asynchronious case (cyclic Steiner system). There are also special constructions of the cyclic Steiner systems $S(n,3,2)$, which for $n \equiv 1 \pmod 6$ give optimal solutions for self-orthogonal convolutional codes.

On Multipe Description Source Coding

Zhen Zhang

Suppose the source data is encoded into two codes f_1 and f_2 at rates r_1 and r_2 respectively. These two codes are sent to three decoders. The first decoder uses f_1, the second decoder uses f_2 to recover the source message, whereas the third decoder uses both of them. Let d_1, d_2, d_0 be the average distortions at the three decoders. Denote the set of all achievable quintuples $(r_1, r_2, d_0, d_1, d_2)$ by $\mathcal{R}$. To determine $\mathcal{R}$ in the general case is an extremly difficult problem. So we considered several special cases. In the no excess rate case (Zhang Berger 1983) defined by $r_1+r_2=R(d_0)$, Ahlswede determined the region $\mathcal{R}\cap\{r_1+r_2=R(d_0)\}$ (1984). He proposed another special case defined by $r_i=R(d_i)$ $i=1,2$. We obtain ~~an~~ a ~~lower~~ inner bound of $\mathcal{R}$ in this case. This lower bound suggests that the following upper bound might be tight in this case.

Th (1983) The quintuple $(r_1, r_2, d_0, d_1, d_2)$ is achievable if there exist r.v's X_1, X_2, U jointly distributed with the generic r.v X and the following conditions are satisfied

1. $\exists\ \varphi_1\ \varphi_2\ \varphi_0$ s.t.

$$Ed(X;\varphi_i(X_i,U))\leq d_i \qquad i=1,2.$$

$$Ed(X;\varphi_0(X_1,X_2,U))\leq d_0$$

$$r_1+r_2\geq 2I(X;U)+I(X_1;X_2|U)+I(X;X_1X_2|U)\ .$$

$$r_i\geq I(X;X_iU) \qquad i=1.2.$$

Our major evidence for this conjecture is that the gap between these two bounds are very small.

"The Capacity of the Permuting Relay Channel"

Kingo Kobayashi

Blackwell's trap door channel is a nice example of finite state channels. Its deterministic versions, that is, permuting channels, have been studied by Ahlswede and Kaspi (1984) in a multi-terminal information-theoretic framework. They determined the capacities of permuting jammer channels and relay channels for some special cases. In this talk, we completely solve the capacity problem for permuting relay channels. More specifically, when α is the cardinality of alphabet, and β is the number of available stock locations in channel, the capacity $C_R(\alpha, \beta)$ of the permuting relay channel is given by $\log \lambda$, where λ denotes the maximum eigenvalues of a matrix Q derived from the state transition mechanism associated with the channel.

Upper bounds for codes

Aimo Tietäväinen, University of Turku, Finland

Let $A(n,d)$ be the maximum number of code words in a binary code of length n and minimum distance at least d. We derive two asymptotical upper bounds for the number $A(2d+j, d)$ when $d \to \infty$ and j is positive and small, show that these bounds are in a sense best possible, and consider some open problems, generalizations and modifications. We also show how the second McEliece - Rodemich - Rumsey - Welch bound has been generalized to the nonbinary case.

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Decoding of generalized concatenated codes and demodulation

Zinoviev V.A., Zyablov V.V., Portnoy S.L.
[Institute for Problems of Information Transmission
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Let A, B, C correspond inner, outer and generalized concatenated (GC) code of order m correspondingly. We use m inner and m outer codes to obtain GC-code C of order m. The i-th outer code B_i, $i=1,\dots m$, over the alphabit of size q_i and with power $N_{b,i}$ can be selected independently of other outer codes only with the same length n_b. The inner codes A_i, $i=1,\dots,m$, must be the system of nested codes of length n_a. The code A_1 is partition of q_1 codes $A_2(i)$, $i=0,\dots,q_1-1$, which have the same parameters. Every code $A_2(i)$ is partition of q_2 codes $A_3(i_1,i_2)$, $i_2=0,\dots,q_2-1$, and so on. Let values of symbol of inner codes are selected from space E with Hamming d_H or Euclidian d_E metric, where d_E means square of Euclidian distance. Let for every j, $j=1,\dots,m-1$, there exizts an automorphism $\varphi_j : E^{n_a} \to E^{n_a}$ such that $\varphi_j(A_j(0,\dots,0,i_{j-1})) = A_j(0,\dots,0,0)$, $i_{j-1}=0,1,\dots,q_{j-1}-1$, and for every $x,y \in E^{n_a}$ $d(x,y)=d(\varphi_j(x),\varphi_j(y))$. Let $d_{a,i}$ and $d_{b,i}$ be the minimum distances of A_i and B_i correspondingly, where $d_{a,i}$ can be d_H or d_E. Then GC-code has parameters: $n=n_a n_b$, $d \geq \min_{1\leq i\leq m} \{d_{a,i} d_{b,i}\}$, $N=N_{b,1}\dots N_{b,m}$. The decoding algorithm consists from m steps Ψ_i, $i=1,\dots,m$. We want that i-th step Ψ_i don't depend of result of decoding Ψ_j, $j<i$. For this we want to deal only with the i-th inner code $A_i(0,\dots,0)$ and outer code B_i. After the decoding Ψ_j we'll have some word

$b^{(i)} = (b_1^{(i)}, \dots, b_{n_b}^{(i)})$ of the code B_i and therefore n_b codes $A_{i+1}(0, \dots, 0, b_s^{(i)})$, $s = 1, \dots, n_b$. Then using the automorphism φ_{i+1} we transform the code $A_{i+1}(0, \dots, 0, b_s^{(i)})$ to code $A_{i+1}(0, \dots, 0, 0)$ for every s, (see: Dumer I.I., Zinoviev V.A., Zyablov V.V. Problems of Control and Information Theory, 1983). Such decoding overally realize the minimum distance of GC-code and has complexity of decoding $\sim n^c$, where usually $c = 2$. Applications of this result are interested, when the inner codes are phase or amplitude-phase modulation, what gives the regular method demodulation and decoding simultaniously (Portnoy S.L., Problems of Inform. Transm., 1985, 21, №3, 14-22).

AN APPLICATION OF COMBINATORIAL GROUP THEORY TO CODING

Gérard COHEN, ENST, PARIS

We consider two problems in combinatorial theory and five applications to coding. Let $(G, +)$ be a finite abelian group.

Problem 1. Determine $s(G)$, defined as the smallest integer such that $\forall S, \; S \subset G, \; |S| \geqslant s(G) \Rightarrow S$ contains a subset with zero sum. Olson has solved it for p-groups: if $G = \bigotimes_i \mathbb{Z}_{p^{e_i}}$, then $s(G) = 1 + \sum_i (p^{e_i} - 1)$. This was used by Alon to prove the following conjecture for m a power of two.

Conjecture (Ito). Every binary linear $[4m, 2m+1]$ code contains a vector of weight $2m$.

Problem 2. Determine $c(G, t)$, defined as the smallest integer such that if S is a generating subset of G with cardinality $c(G,t)$, every nonzero element of G is a sum of at most t elements in S.

We consider the case $G = (\mathbb{Z}_2)^r$, which is related to coding, and prove.

Proposition. $c((\mathbb{Z}_2)^r, t) \leqslant \frac{2^r}{t}$, for t a power of two.

Problem 3. Is the proposition true for any t?

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Finally, we give an application to coding for reusing write-once memories, using Hamming codes.

On commutative groups of polynomial functions and their application in cryptography

Winfried B. Müller, Inst. f. Mathematik, Univ. Klagenfurt

During the last years the discrete exponentiation $x \to x^k$ has been used as one-way function in the Diffie-Hellman key distribution, in Shamir's three-pass algorithm and in the RSA-public key cryptosystem. Until recently, the computation of discrete logarithms, the inverse function of the discrete exponentiation, was believed to be a very hard problem. But recently progress in computing discrete logarithms has been made, especially in Galois fields of characteristic 2. In order to protect the above mentioned schemes against attacks by these recent algorithms one can replace the discrete exponentiation $x \to x^k$ by more sophisticated polynomial functions $x \to f(x)$, which also commute with respect to composition. It is shown that the so-called Dickson-polynomial functions $x \to g_k(1,x)$ and $x \to g_k(-1,x)$ can be used as cipher functions (cf. Müller W.B. and R. Nöbauer: Cryptanalysis of the Dickson-scheme. To appear in Proc. Eurocrypt 85, Lecture Notes in Computer Science).

Another group of polynomial functions on $\mathbb{Z}/(n)$ can be obtained from polynomials of the form $l^{-1} \circ x^k \circ l$ with $l = ax+b \in \mathbb{R}[x]$, $a \neq 0$. It can be proved that $l^{-1} \circ x^k \circ l$ with $k \in 2\mathbb{N}+1$ is a polynomial over $\mathbb{Z}$ iff $a^2, ab, b^2 \in \mathbb{Z}$ and $b^3 - b \in a\mathbb{Z}$. Furthermore, the function $x \to \frac{x}{a} \circ x^k \circ ax$ with $a \neq 0$, $a^2 \in \mathbb{Z}$ is a permutation of $\mathbb{Z}/(n)$ iff $(k, \varphi(n)) = 1$ and $(a^2, n) = 1$. At last, all permutations of $\mathbb{Z}/(n)$ of this form with only one fixed point are described. (If $n = p_1 p_2 \dots p_r$, any permutation of $\mathbb{Z}/(n)$ induced by polynomials x^k has at least 3^r fixed points.)

Identification via noisy channels

R. Ahlswede, G. Dueck, Universität Bielefeld

For discrete memoryless channels the notion of identification codes is introduced. In the classical transmission problem a message is to be transmitted from a sender to a receiver. In the identification model, however, the receiver wants to know only very special aspects of the message being sent.
Roughly formulated, the following result is obtained:

Identification Th. (R. Ahlswede, G. Dueck)

a. For every $\varepsilon > 0$ and large n there are randomized ID-codes of block length n, with errors $\leq \varepsilon$ and size N, such that

$$N \geq 2^{2^{n(C-\varepsilon)}},$$

where C is the capacity of the channel.

b. For every $\varepsilon > 0$ and large n there is true: Every randomized ID-code of block length n, errors $\leq 2^{-n\varepsilon}$, and size N satisfies

$$N \leq 2^{2^{n(C+\varepsilon)}}.$$

The authors see large possibilities of various applications in networks of computers or telephone-stations, in parallel computers, and in an approach to an explanation of the information processing in biological systems.

Balancing Sets of Vectors

Andrew Odlyzko, AT&T Bell Laboratories, Murray Hill, NJ, USA

Given a positive integer n, what is the minimal value of k such that there exist k vectors $\underline{v}_1, \ldots, \underline{v}_k$ of length n with entries ±1 such that for any vector $\underline{w}$ of length n with entries ±1, there is at least one i with $1 \leq i \leq k$ and $\underline{v}_i \cdot \underline{w} = 0$? A very simple construction due to Knuth shows that $k \leq n$ is possible, and a proof using commutative algebra is given that $k = n$ is best possible. This construction and its extensions have many applications to communication theory. This work was done jointly with E. Bergmann, D. Coppersmith, and P. Shor.

On the Spectrum of (d,K) Codes

Chris Heegard, Cornell University

In this talk, we present a simple method to obtain the spectrum of a (d,K) code. A (d,K) code describes a set of binary waveforms, $w(t) \in \{-1,+1\}$, that have a minimum ($T_{min} = d+1$) and maximum ($T_{max} = K+1$) length of time between transitions (note: all transition in w(t) occur at integer times). The waveform w(t) is described by several sequences: the level sequence $Z_0 = w(0^+)$, $Z_1 = w(1^+)$, $Z_2 = w(2^+)$, . . . ; the transition sequence $X_1 = (Z_1 - Z_0)/2$, $X_2 = (Z_2 - Z_1)/2$, ∘∘∘ ; (note: $X_j \in \{-1, 0, +1\}$) ; the state sequence

$$S_j = \begin{cases} 0 & X_j \neq 0 \\ S_{j-1}+1 & X_j = 0 \end{cases}$$

and the runlength sequence $T_1, T_2, \ldots$ (where

$T_i = S_{j-1}+1$ if $x_j \neq 0$). As random processes, the entropies are related by

$$H(z) = H(x) = H(S) = H(T)/E(T) .$$

Theorem: For i.i.d. runlengths (i.e., the state sequence is a Markov chain)

$$S_x(D) \equiv \sum_{j=-\infty}^{\infty} E x_j x_0 D^j = \pi_0 \frac{1-g(D)g(D^{-1})}{(1+g(D))(1+g(D^{-1}))}$$

where $g(D) = \sum_{j=d+1}^{k+1} Pr(T_1=j)\, D^j$ and $\pi_0 = Pr(S_j=0) = Pr(x_j \neq 0) = 1/E(T) = 1/g'(D)\big|_{D=1}$.

A simple derivation of this theorem is given (note: $S_z(D) = 4S_x(D)/(1-D)(1-D^{-1})$). The method is then extended to find the spectrum of a popular (d,k) code known as MFM (it satisfies $d=1, k=3$).

Bounds for Codes over the Unit Circle

Ph. Piret, Philips Res. Labs. Brussels, Belgium

Let C be a code of length n and rate R over the alphabet $A(Q) = \{\exp(2\pi i r/Q) : r = 0, 1, \ldots, Q-1\}$, and let $d(C)$ be the minimum Euclidean distance of C. For large n, lower and upper bounds are obtained in parametric form on the achievable pairs (R, δ) with

$\delta = d^2(C)/n$. For $Q \to \infty$, the bounds are expressed in terms of the modified Bessel functions of the first kind I_0 and I_1. The upper bound is compared with the Kabatyanskii-Levenshtein bound that holds for less restrictive alphabets. For $Q \to \infty$, our bound is stronger than the K-L bound in the range $0 \le \delta \le 0.93$.

Duadic Codes over arbitrary fields.

J.H. van Lint (Eindhoven)

Duadic codes over GF(2) were introduced by Leon, Masley and Pless (1984). We present results on these codes and some generalisations to GF(q) which were obtained by M.H.M Smid in his master's thesis (T.H. EINDHOVEN).

Let n be odd, $(n,q)=1$. If S_1 and S_2 are unions of cyclotomic cosets mod n, $S_1 \cap S_2 = \phi$, $S_1 \cup S_2 = \{1, 2, \ldots, n-1\}$ and if the permutation $\mu_a : x \to ax$ interchanges S_1 and S_2, then (μ_a, S_1, S_2) is called a *splitting* mod n. A *duadic code* C_i (resp. C_i') is the cyclic code with generator $g_i(x) := \prod_{j \in S_i} (x - \alpha^j)$, (resp. $(x-1)\,g_i(x)$), where α is a primitive n^{th} root of unity.

(Remark: For $q = 2$, this is *not* the definition given by Leon, Masley and Pless but the definitions are equivalent).

All QR codes and some special RS and GRM codes are duadic codes (all with μ_{-1}).

Theorem: If C is cyclic and $\bar{C}$ is self-dual, then C is duadic with splitting given by μ_{-1}.

Theorem: Let $n = p_1^{m_1} \cdots p_k^{m_k}$. A splitting mod n exists iff q is a square mod p_i for all i $(1 \le i \le k)$.

For all binary duadic codes of length < 127 the minimum distance

were determined using the "new bound" (cf. Wilson & v. Lint, IEEE-IT 1986). Several of these had been conjectured on the grounds of a computer search by Leon, Masley and Pless.
In his master's thesis (available on request) Smid proves a number of theorems on the minimum distance of q-ary duadic codes, showing that many of them have low minimum distance.

An Entropic Concept in Statistical Quality Control

E. v. Collani, Würzburg

Consider the following problem which arises in Statistical Quality Control: A lot of size N is to be inspected by means of a single sampling plan (n, c) with $0 \leq c < n \leq N$, i.e. a random sample of size n is drawn and if the number of nonconforming items in the sample is less than or equal to the acceptance number c, the lot is accepted otherwise rejected. The problem is to determine an appropriate sampling plan (n, c) given a linear cost model.
There are three sampling schemes to solve this problem, which may be classified according to their assumptions about the probability distribution $\omega(p)$ of the number of nonconforming items M in the lot:

1.) Bayes-plans, assuming complete knowledge about $\omega(p)$
2) Minimax plans, assuming that there is no knowledge at all about $\omega(p)$
3.) α-Minimax-plans, assuming that one point (for the break-even quality) of the distribution function of M is know.

To be able to compare the different concepts and to find the relevant informations on $\omega(p)$ an entropic sampling scheme is defined utilizing the principle of maximum entropy.

A Markov source model for a convolutional coding scheme

by M. R. Best

A convolutional coding scheme with maximum likelihood decoding over a discrete memoryless channel can be modelled as a Markov source. Using this model, the statistical behaviour of the errors can be analysed exactly. In effect, not only the bit and event error probability, but also e.g. the burst and gap length distribution can be computed. Moreover, for a (suboptimum) Viterbi decoder with a finite decoding delay the dependence of the error statistics on that delay can be found. This generalizes earlier results of Schalkwijk, Post, and Aarts.

Some properties of sequences over local rings

by P. Nyffeler (Bern)

The talk concerns the question: what can be saved, when generalizing periodic (or recurrence) sequences over finite fields to sequences over local rings, especially over Z_{p^r} or Galois rings $GR(p^r, k)$.
Over finite fields, the shift registers are canonical forms of finite-state machines, as representatives of companion matrices.
Over local rings, shift registers modulo a nilpotent ideal play a similar role.
The analysis of sequences can be done by an algorithm similar to the Berlekamp-Massey algorithm over Z_{p^r} and the synthesis of new sequences of higher complexity by "root combinations" is possible.

An efficient identification and signature scheme

by C.P. Schnorr (Frankfurt)

A. Shamir proposed an interactive authentication scheme that relies on the hardness of factoring large integers. For security both parties of an interaction have to use independent random numbers. However the data of the interaction cannot be later on used to convince a judge. We extend this scheme so that the identification, or message authentication can be testified by a trusted authority. For this both parties of an interactive generate pseudo random numbers which are indistinguishable from truly random numbers for the other party.

Random Access Communication and Graph Entropy,

by Katalin Marton, Budapest. (Joint work with J. Körner)

Conflict resolution in random access communication raises the following probabilistic problem. Let $u_1, \dots, u_k$ be independent random variables uniformly distributed in the unit interval [0,1]. A k-partition $\mathcal{A}$ of [0, 1] (i.e. a partition into k atoms) separates the (random) points $u_1, \dots, u_k$ if each atom contains exactly one point. For k-partitions $\mathcal{A}_1, \dots, \mathcal{A}_n$, let $P_{\mathcal{A}_1, \dots \mathcal{A}_n}(k)$ be the probability of the event that at least one $\mathcal{A}_j$ separates the points $u_1, \dots, u_k$. What is the maximum of these probabilities if $\mathcal{A}_1, \dots, \mathcal{A}_n$ vary? Hajek's conjecture (supported by the Van der Waerden - Falikman - Egorychev theorem) was

$$\min \left[1 - P_{\mathcal{A}_1, \dots \mathcal{A}_n}(k)\right] = \left(1 - \frac{k!}{k^k}\right)^n.$$

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We disprove this by showing

$$\min \left[1 - P_{A_1,\dots,A_4}(3)\right] \leq \frac{25}{81}.$$

Further, we prove the bounds

$$1 - P_{A_1,\dots,A_n}(k) \geq 2^{-n k!/k^{k-1}}$$

This is achieved by a technique for lower bounding the number of graphs of a given structure, needed to cover all edges of a given graph. This technique, developed by J. Körner, is based on the subadditivity of graph entropy - a functional of graphs.

On Weak Asymptotic Isomorphy of Correlated Sources,

by Katalin Marton (Budapest)

Isomorphy problems for correlated sources were raised in ergodic theory (Thouvenot, 1975) but the interest in them is also motivated by multi-terminal information theory. A DMSC (Discrete Memoryless Stationary Correlated) source is an i.i.d. sequence of random pairs in a finite set $\mathcal{X} \times \mathcal{Z}$. We consider weak asymptotic isomorphy of DMSC sources. Two DMSC sources $\{X_i, Z_i\}$ and $\{X'_i, Z'_i\}$ are asymptotically isomorphic in the weak sense if for every $\varepsilon > 0$ and large enough n, there exists a joint distribution of the n-length outputs of the two sources,

$$\text{dist}\,(X^n, Z^n, X'^n, Z'^n)$$

satisfying

$$\tfrac{1}{n} H(X^n | X'^n) < \varepsilon, \quad \tfrac{1}{n} H(Z^n | Z'^n) < \varepsilon,$$
$$\tfrac{1}{n} H(X'^n | X^n) < \varepsilon, \quad \tfrac{1}{n} H(Z'^n | Z^n) < \varepsilon.$$

No non-trivial cases of weak asymptotic isomorphy are known. Here we show that some spectral properties of the generic distribution $\mathrm{dist}(X_1, Z_1)$ of the source $\{X_i, Z_i\}$ are invariant for weak asymptotic isomorphy, and these properties wholly determine the generic distribution in many cases.

Arbitrarily varying channels with jamming constraints

by I. Csiszár (Budapest) and P. Narayan (College Park)

We consider AVC's in the communication situation when both the sender and the jammer are ignorant of the actual sequence selected by the other, and there is a constraint $\sum_{i=1}^{n} g(s_i) \le \alpha n$ on the jamming sequence $(s_1, \dots s_n)$. Let C_m and C_a denote the capacity for maximum and average error, respectively, and C_r be the random coding capacity, thus $C_m \le C_a \le C_r$. Then

$$C_r = \max_X \min_S I(X \wedge Y) \quad \text{for } X \text{ and } S \text{ independent, } Eg(S) \le \alpha,$$

where Y is the output random variable for input X and jamming S. Without jamming constraint, Ahlswede proved that $C_a = C_r$ unless $C_a = 0$; however, his method does not work in the present case.

We determine C_a for some deterministic AVC's, such as (i) $Y = X + S$ mod 2 (ii) $Y = X + S$ (iii) $Y = X \vee Y$, for binary input and jammer alphabets and $g(s) = s$. In case (i) $C_a = C_r = 1 - h(\alpha)$ $(0 \le \alpha \le \frac{1}{2})$ while in this case the problem of determining C_m is very hard, namely equivalent to the basic unsolved problem on the asymptotic rate of error-correcting codes. In case (ii) $C_a = C_r$ for $\alpha < \frac{1}{2}$ but $0 < C_a < C_r$ for $\alpha > 1/2$. A partial result is obtained also for Gaussian AVC's.

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On the Relation between Berlekamp-Massey and Euclidean Algorithm for Synthesizing Binary Sequences

by Zong-duo Dai and Zhe-xian Wan (Academia Sinica, Beijing)

Let

$$\underline{a} = (a_0, a_1, \cdots, a_{N-1})$$

be a binary sequence of length N. Let $a_0 = a_1 = \cdots = a_{n_0-1} = 0$, $a_{n_0} = 1$. Put

$$r_0(x) = \Sigma_{i=0}^{N-1} a_{N-1-i} x^i, \quad r_{-1}(x) = x^N + r_0(x)\varepsilon(x),$$

where $\varepsilon(x) = \Sigma_{i=0}^{n_0} \varepsilon_i x^i$ is an arbitrary polynomial of degree $\leq n_0$, and also put

$$U_0(x) = 1, \quad U_{-1} = \varepsilon(x).$$

Define $r_k(x)$ and $U_k(x)$ $(k = 1, 2, \cdots)$ inductively as follows:

$$r_k(x) = p_k(x) r_{k-1}(x) + r_{k-2}(x), \quad \deg r_k(x) < \deg r_{k-1}(x)$$

$$U_k(x) = p_k(x) U_{k-1}(x) + U_{k-2}(x).$$

If k is the smallest positive integer such that $\deg r_k(x) + \deg r_{k-1}(x) < N$, then $U_k(x)$ is a shortest LFSR generating $\underline{a}$. This is the so-called Euclidean algorithm for synthesizing binary sequence.

For $k = 1, 2, \cdots$, write

$$p_k(x) = \Sigma_{i=1}^{\omega_k} x^{\lambda_{ki}}$$

where $\lambda_{ki} > \lambda_{k\,i+1}$, $i = 1, 2, \cdots, \omega_k - 1$. Then put

$$p_{k\tau}(x) = \Sigma_{i=1}^{\tau} x^{\lambda_{ki}}$$

$$U_{k\tau}(x) = p_{k\tau}(x) U_{k-1}(x) + U_{k-2}(x).$$

Then we obtain the following sequence of polynomials

$$U_{11}, U_{12}, \cdots U_{1\omega_1} (= U_1), U_{21}, U_{22}, \cdots, U_{2\omega_2} (= U_2), \cdots.$$

Put

$$r_{k\tau}(x) = p_{k\tau}(x) r_{k-1}(x) + r_{k-2}(x),$$

and let

$$n_{k\tau} = 2N - 1 - \deg r_{k\tau}(x) - \deg r_{k-1}(x), \quad k \geq 1.$$

Then put

$$f_j = \begin{cases} 1 & 1 \leq j \leq n_0 \\ U_{11} & n_0 < j \leq n_{11} \\ U_{k1} & n_{k-1\,\omega_{k-1}} < j \leq n_{k1}, \quad k > 1 \\ U_{k\tau} & n_{k\tau-1} < j \leq n_{k\tau}, \quad \tau \geq 2 \end{cases}$$

It is proved that the sequence of polynomials $f_1, f_2, \dots f_N$ is exactly the sequence of polynomials obtained by the Berlekamp-Massey algorithm.

Information Theory and the Authentication of Digital Messages

Gustavus J. Simmons Sandia National Laboratories - Albuquerque NM, USA.

We consider the problem of a transmitter who wishes to communicate observations of a finite state source to a receiver through a channel under the control of an opponent who wishes to deceive the receiver as to the state of the source. The opponent can either impersonate the transmitter and send a forged message when none has been sent by the legitimate transmitter, or else wait and observe the legitimate message and then substitute another message it its stead. The transmitter and receiver can choose encoding rules (source states to messages) and if there is more than one message to communicate an observed state of the source in the encoding rule being used, to choose among the available messages. The opponent can either impersonate the transmitter, in which case he must choose a message to send, or wait and substitute for an observed message. In the simplest possible formulation the resulting authentication channel can be modeled in complete generality as a zero sum two person game. The "value" of the game is the probability, P_d, that the opponent succeeds in deceiving the receiver. The channel bound can be expressed in the form

$$\log_2 P_d \geq -(H(M) - H(S) - H(M/ES)) \qquad (1)$$

where $H(S)$ is the source entropy, $H(E)$ is the entropy of the strategy with which the transmitter and receiver choose an encoding rule, $H(M)$ is the induced entropy of the messages in the channel and $H(M/ES))$ is the average uncertainty of the message when the source state and encoding rule are known. If equality holds in (1), the authentication

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system is said to be perfect in the sense that all of the information in a message is used to either communicate the state of the source to the receiver or to confound the opponent, i.e. to prevent him deceiving the receiver. It was shown that there is a perfect authentication scheme for every affine resolvable block design with $P_d = \frac{k}{b}$. A more general class of "weakly" resolvable designs were defined and shown to also yield perfect systems. Examples were given showing that the "weakly" resolvable designs provide solutions not available by other means.

Sequences with Perfect Linear Complexity Profiles

– James L. Massey (Swiss Federal Institute of Technology, Zurich)

The linear complexity, $\mathcal{L}(s^n)$, of a sequence $s^n = (s_0, s_1, \dots, s_{n-1})$ of digits from a field F is the smallest nonnegative integer L for which there exist $c_1, \dots, c_L$ in F such that

$$s_j + c_1 s_{j-1} + \dots + c_L s_{j-L} = 0, \qquad L \le j < n.$$

A binary [i.e., $F = GF(2)$] sequence s^n is said to have a <u>perfect linear complexity profile</u> when

$$\mathcal{L}(s^m) = \left\lfloor \frac{m+1}{2} \right\rfloor, \qquad 1 \le m \le n.$$

The following result was obtained with (and mainly by) the author's doctoral student, M.-Z. Wang:

<u>Theorem</u>: The binary sequence s^n has a perfect linear complexity profile if and only if $s_0 = 1$ and

$$s_{2i} + s_{2i-1} + s_{i-1} = 0, \qquad 1 \le i < \lceil n/2 \rceil.$$

Hypothesis testing with multiterminal data compression

by Te Sun HAN (Senshu Univ., JAPAN)

The multiterminal hypothesis testing $H: XY$ against $\bar{H}: \bar{X}\bar{Y}$ is considered where $X^n(\bar{X}^n)$ and $Y^n(\bar{Y}^n)$ are separately encoded at rates R_1, R_2, respectively. The problem here is to determine the minimum β_n of the second kind of error probability, under the condition that the first kind of error probability $\alpha_n \leqq \varepsilon$ for a prescribed $0 < \varepsilon < 1$. We are concerned with the asymptotic behavior of β_n, so define the power exponent by

$$\theta(R_1, R_2, \varepsilon) = \limsup_{n \to \infty} \left(-\frac{1}{n} \log \beta_n\right).$$

Then, we have a good lower bound of the $\theta(R_1, R_2, \varepsilon)$:

$$\theta(R_1, R_2, \varepsilon) \geqq \sup_{\hat{V}U \in \mathcal{S}(R_1, R_2)} \min_{\tilde{V}\tilde{U}\tilde{X}\tilde{Y} \in \mathcal{L}(UV)} D(\tilde{U}\tilde{X}\tilde{Y}\tilde{V} \| \bar{U}\bar{X}\bar{Y}\bar{V}).$$

where $\mathcal{S}(R_1, R_2) = \{VU : I(U:X) \leqq R_1,\ I(V:Y) \leqq R_2,\ U \to X \to Y \to V\}$,

$$\mathcal{L}(UV) = \{\tilde{U}\tilde{X}\tilde{Y}\tilde{V} : \mathrm{dist}(\tilde{U}\tilde{X}) = \mathrm{dist}(UX),\ \mathrm{dist}(\tilde{V}\tilde{Y}) = \mathrm{dist}(VY),$$
$$\mathrm{dist}(\tilde{U}\tilde{V}) = \mathrm{dist}(UV)\},$$

and $\bar{U}\bar{V}$ satisfies conditions $\mathrm{dist}(\bar{U}/\bar{X}) = \mathrm{dist}(U/X)$, $\mathrm{dist}(\bar{V}/\bar{Y}) = \mathrm{dist}(V/Y)$ and $\bar{U} \to \bar{X} \to \bar{Y} \to \bar{V}$. It is conjectured that this lower bound is tight.

Next, consider the complete data compression case where the encoders for $X^n(\bar{X}^n)$ and $Y^n(\bar{Y}^n)$ are allowed to send only one bit information. Then, the power exponent is given by

$$\theta(\varepsilon) = \min_{\substack{\mathrm{dist}(\tilde{X}) = \mathrm{dist}(X) \\ \mathrm{dist}(\tilde{Y}) = \mathrm{dist}(Y)}} D(\tilde{X}\tilde{Y} \| \bar{X}\bar{Y}).$$

Gaussian Interference Channels

– Max H. M. Costa (Instituto de Pesquisas Espaciais – INPE, São José dos Campos, SP, Brasil)

The Gaussian interference channel, introduced by Carleial in 1975, models the communication between average power constrained senders X_1 and X_2 to their respective receivers Y_1 and Y_2 over a shared medium with additive Gaussian noise. The channel inputs and outputs are related by $Y_1 = X_1 + bX_2 + Z_1$ and $Y_2 = aX_1 + X_2 + Z_2$, where a and b are non-negative interference parameters, and Z_1 and Z_2 are unit variance normally distributed noise terms. The capacity region has been obtained when interference is strong (i.e., $a \geq 1$ and $b \geq 1$), but is yet to be established when one of the interference parameters is in the open unit interval. We examine the simpler model of the Z-Gaussian interference channel, where one of the interference parameters is zero. A signaling scheme is proposed that combines the known techniques of superposition coding and time-sharing (or frequency-sharing). This scheme is optimal within the restricted class of Gaussian signaling techniques. We motivate the conjecture that this scheme yields the capacity region of the Z-Gaussian interference channel. If true, this conjecture leads to an improved outer bound of the capacity region of the general Gaussian interference channel (with arbitrary parameters).

Optimale Steuerung mit partiellen Differentialgleichungen: Theorie und Verfahren

parallel zu

Inverse Probleme

18. – 24. Mai 1986

Über die Approximation diskreter Wahrscheinlichkeitsverteilungen in der sphärischen Stereologie

Rudolf Gorenflo, Freie Universität Berlin

Wir betrachten das Tomatensalatproblem (vgl. G. Bach: Über die Größenverteilung von Kugelschnitten in durchsichtigen Scheiben endlicher Dicke. Zeitschrift für wissenschaftliche Mikroskopie $\underline{64}$ (1959), 265–270). Aus einem festen, undurchsichtigen Medium, in welches Kugeln mit zufälligen Radien eingebettet sind, wird eine Scheibe der Dicke $s \geq 0$ herausgeschnitten, so dünn, daß sie noch durchsichtig ist und die Anzahldichte des maximalen Radius ρ einer herausgeschnittenen Kugelscheibe bestimmt werden kann (ρ gemessen parallel zu den Schnittebenen). Aus der Kenntnis der Dichte $\hat{g}(\rho)$ gewinnt man Information über die Anzahldichte $\hat{f}(r)$ des wirklichen Kugelradius r durch Lösung der Abelschen Integralgleichung

$$s\hat{f}(\rho) + 2\rho \int_{\rho}^{R} \hat{f}(r)\,(r^2-\rho^2)^{-1/2}\,dr = \hat{g}(\rho),$$

$0 \leq r \leq R$, R eine obere Schranke des möglichen Radius. Für den Fall, daß $\hat{f}(r) = \sum_{j \in J} c_j\, \delta(r - r_j)$, mit

$r_j \in (0, R)$, δ die Diracsche Deltafunktion", alle $c_j \geq 0$, $\sum_{j \in J} c_j = C < \infty$, wird ein Verfahren vorgestellt, das gestattet, aus der Kenntnis von Werten der Verteilungsfunktion $G(\varrho) = \int_0^{\varrho} \hat{g}(\hat{\varrho})\, d\hat{\varrho}$ eine Treppenfunktion als Approximation der Verteilungsfunktion $F(r) = \int_0^{r} \hat{f}(\hat{r})\, d\hat{r}$ zu gewinnen.

Die Stützpunkte werden äquidistant in r^2 genommen, diskretisiert wird mit EULER IMPLICIT, und für gegen Null strebende Schrittweite h wird Konvergenz in der L^2-Norm gezeigt (die Maximumnorm ist hier unpassend). Das Verfahren konvergiert mit der Ordnung $\sqrt{h}$ im Falle $s = 0$, h im Falle $s > 0$.

Integral Equations of the First Kind in Inverse Acoustic Scattering Problems

For the solution of the exterior Dirichlet problem for the Helmholtz equation an approximation method is described which seeks the solution in the form of an acoustic single-layer potential with a distribution extended over an internal surface. This leads to an ill-posed integral equation of the first kind which can be approximately solved by the Tikhonov regularization technique. It is illustrated how this approach can be employed to approximately solve the inverse problem: Determine the shape of a scatterer from the far-field pattern of the scattered wave for one (or more) incident (plane) waves.

Rainer Kress, Göttingen.

An Inverse Problem for an Elliptic Partial Differential Equation.

Local existence and unicity of the solution $(a=a(x),\ u=u(x,y))$ to the problem

$$\begin{cases} \Delta u - a(x)u = 0, & x>0,\ y>0 \\ u(0,y) = f(y), & y>0 \\ u_x(0,y) = g(y), & y>0 \\ u(x,0) = h(x), & x>0 \end{cases}$$

was demonstrated by reduction to an integral equation for $a(x)$ via the methods of Gelfand-Levitan. This research was joint work with Professor William Rundell.

John Rozier Cannon
Washington State University
Pullman, Wa, 99164.

Justification of necessary conditions for optimality

—Thomas I. Seidman [Univ. Md. Baltimore County / Catonsville, MD USA]

We consider minimization of an integral function of the form: $J = \int_Q f(s, w(s), x(s))$ subject to an operator condition $x = G(w)$ and perhaps a finite number of scalar constraints $\xi_j(w) = 0$ (or ≥ 0). It is assumed that a minimizer $[\bar{w}, \bar{x}]$ exists and that G is "nice" but it is not assumed that f is convex: only that it is moderately smooth at any point where it is finite. An approach is presented to the rigorous justification of the formally obtainable necessary conditions for optimality. An example is presented in the context of distributed parameter control theory in which G is given by a partial differential equation with control w.

Output Least Squares Stability for Elliptic Systems.

(Karl Kunisch, Inst. Math., Technical University of Graz, Austria)

We consider the estimation of the scalar valued diffusion coefficient $q = q(x)$ in

$$\begin{cases} -\operatorname{div}(q \operatorname{grad} u) + cu = f \text{ in } \Omega \\ \text{boundary conditions} \end{cases}$$

from an "observation" $z \in L^2(\Omega)$ in the output least squares formulation

$$(P)_{z^0} \quad \min \quad |u(q) - z^0|^2 \quad \text{over } Q_{ad},$$

where $Q_{ad} = \{q \in H^2(\Omega): q(x) \geq \alpha > 0, |q|_{H^2} \leq \gamma\}$ and $\Omega \subset \mathbb{R}^2$ or $\mathbb{R}^3$. The parameter q is called output least squares stable (OLSS) at the solution q^* of $(P)_{z^0}$ if there exist neighborhoods $V(z^0)$ and $V(q^*)$ such that for every $z \in V(z^0)$ there exists a solution $q_z \in V(q^*)$ of $(P)_z$ and all solutions of $(P)_z$ in $V(q^*)$ depend Hölder continuously on z. OLSS can only be expected and proved to hold under restrictive assumptions on $(P)_{\gamma 0}$. Therefore we subsequently consider a regularized problem for which we prove OLSS. The technical tool to obtain OLSS are lower bounds on the Lagrangian associated with $(P)_{\gamma 0}$. The necessary estimates give valuable insight in the illposed nature of $(P)_{\gamma 0}$. (This is joint work with F. Colonius, Bremen).

Optimal control of Stefan problems

Irena Pawłow, Systems Research Institute,
Polish Academy of Sciences, Warszawa

We are concerned with numerical methods of solving optimal control problems for two-phase Stefan processes, possibly of mixed elliptic-parabolic type.
The approximation method uses a variational inequality formulation of the Stefan problem.
The inequality and the associated control problem are discretized by applying piecewise linear elements in space and finite differences in time.
A gradient type algorithm is proposed to solve control problem numerically.
At the second part of the talk a computer-generated movie on the simulation of boundary control of Stefan problems is presented.

On the control of the secondary Cooling in the continuous casting

Pekka Neittaanmäki
University of Jyväskylä
Dept. of Math, SF-40100 Jyväskylä
Finland

In the continuous casting the water spray cooling is used to accelerate steel solidification and to strenghten the solified shell. The strand is to be cooled down according to the pattern which depends on steel quality, product size, casting speed and product size. The problem of optimal cooling strategy is formulated as an optimal control problem subject to a nonlinear state equation including the phase changes (solid, mushy liquid) and with certain constraints in state. Discretization, optimality conditions and numerical examples are presented.

Optimal shape control of the domain in unilateral boundary value problems

J. Haslinger(1), P. Neittaanmäki(2), D. Tiba(3)
(1) Charles University, Prague, Czechoslovakia
(2) University of Jyväskylä, Finland
(3) INCREST, Bucharest, Romania

We give a general existence theorem for an optimal control problem where the control is domain in $\mathbb{R}^n$ and where the system is governed by partial differential equations with classical or with unilateral boundary conditions. Discretization by FEM with numerical algorithm are presented. Applications for design problems in elasticity are given.

In the second part we consider an abstract optimal design problem with constraints in state and control. A variational inequality approach is given for this design problem.

Asymptotic stability of solutions of a two phase Stefan problema with flux control on the fixed boundary

Nobuyuki Kenmochi
Dept. Math. Fac. Education, Chiba Univ.
Chiba, JAPAN

A one-dimensional Stefan problem (two phase case) with the following type of flux control on the fixed boundary ($x=0$ and $x=1$):

$$u(t,0) \geq g_0(t),\quad u_x(t,0) \leq 0,\quad u_x(t,0)=0\ (u(t,0) > g_0(t)),$$

$$u(t,1)\leq g_1(t),\quad u_x(t,1)\leq 0,\quad u_x(t,1)=0\ \left(u(t,1)<g_1(t)\right).$$

In the three cases

(a) $g_i(t)\to g_{i,\infty}$ as $t\to\infty$, $i=0,1$,

(b) g_i is periodic on $\mathbb{R}$, $i=0,1$.

(c) g_i is almost periodic on $\mathbb{R}$,

the asymptotic convergence, periodicity and almost periodicity of solution are discussed.

Nobuyuki Kenmochi

On a Problem of Optimal Design in Hydromechanics

T.S. Angell, University of Delaware

In the description of bodies, either partially or totally submerged in an inviscid, irrotational fluid and subjected to a periodic vertical displacement, certain functionals dependent on the velocity potential of the wave pattern are of physical interest. These functionals, for example the "added mass", are dependent on the geometry of the body.

We report on some results concerning the choice of the shape of the body which is to be optimal in the sense of minimizing the added mass for the case that the body is totally submerged in a fluid of finite depth. The problem is cast in terms of a boundary integral equation in which the "control parameter" is the boundary of the body.

Recovering a function from a finite number of moments.

Let $\mu_0, \mu_1, \dots, \mu_N$, ~~be~~ ε, E be given numbers. Suppose a function u obeys

$$\sum_{k=0}^{N}\left(\int_0^1 x^k u(x)\,dx - \mu_k\right)^2 \leq \varepsilon^2, \quad \int_0^1 (u')^2\,dx \leq E^2.$$

We show that such a function can be recovered within the following tolerance:

$$(2E)\sqrt{(\varepsilon/E)^2\, e^{3.5(N+1)} + \frac{1}{4(N+1)^2}}\,.$$

We also present an algorithm and examples.

Giorgio Talenti
Università di Firenze

"An Isospectral Gradient Flow"
Kenneth R. Driessel, May 19, 1986

I shall consider the following problem: Given a real symmetric matrix find its eigenvalues. I shall use the theory of ordinary differential equations to solve this problem. In particular, I shall describe a spectrum preserving ("isospectral") dynamical system on symmetric matrices that flows "downhill" toward diagonal matrices.

Finite element approximations of state constraint parabolic optimal control problems
U. Mackenroth, MBB, München

A system which is governed by the following PDE is considered: $\frac{\partial y}{\partial t} + Ay = 0$, $\alpha y_{|\Sigma} + \beta \frac{\partial y}{\partial n_A} = u$, $y(0) = 0$. The problem is to find a control u such that a certain quadratic but not necessary coercive functional is minimized under a control constraint, $u \in U_{ad}$, and a state constraint, $y(t) \in C(t)$. A discretization which uses the finite element method is used and error estimates for $|\min(P_h) - \min(P)|$, $\|u_h - u_0\|$ are derived (u_h, resp u_0, is the optimal solution of the discrete problem (P_h), resp. the continuous problem (P)). The estimates for $\|u_h - u_0\|$ work only in the coercive case. The bang-bang case requires a detailed analysis in which the structure of u_0 is investigated. This leads to a further convergence result for the u_h.

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"Sensitivity analysis of convex optimal control problems for distributed parameter systems"

K. Malanowski

Systems Science Institute of the Polish Academy of Sciences, Warsaw

A family of convex optimal control problems in a Hilbert space, subject to control constraints is considered. All data of the problems depend on a vector parameter. It is assumed that the cost functional is strongly convex with respect to control variable, the mappings given by the state and adjoint equations are compact and the control constraints are of the pointwise character. It is shown that, provided that the data are regular enough, the solutions of the problems and the associated Lagrange multipliers are Lipschitz continuous and directionally differentiable functions of the parameter. The right-differentials of the solutions and the Lagrange multipliers are characterized as the solutions and the associated multipliers of an auxiliary quadratic optimal control problem. Conditions of Gâteaux differentiability are formulated.
The proof is based on stability and sensitivity results for convex programming problems.

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A sufficient condition for the uniqueness of local minima of a non-linear optimization problem.

Guy CHAVENT, Cérémade (U. of Paris IX) and INRIA.

We consider the problem:

(1) Find $\hat{x} \in C$ such that $J(\hat{x}) \leq J(x) \ \forall x \in C$

where $J(x) = \| \varphi(x) - z \|^2$, C is a connected, regular subset of a vector space E, φ is a C^2 mapping from C into a Hilbert space F, and z is a given point of F. Such problems arise in parameter estimation (x = parameter, z = data, φ = parameter $\rightsquigarrow$ output mapping, C = set of admissible parameters) or in control problems.

The purpose of the paper is to find conditions on φ and C such that (1) has at most one unique solution as soon as z is close enough to $\varphi(C)$. The exhibited condition involves evaluations of first and second derivatives of φ along paths connecting any couple of points of the boundary of C. One numerical example will be given.

Finite element discretization of parabolic boundary control problems - convergence of optimal controls

Fredi Tröltzsch (TU Karl-Marx-Stadt)

In the talk a class of boundary control problems with time-dependent control and convex objective is discussed. Using a semigroup approach

for the treatment of the boundary condition the structural behaviour of the optimal control (bang-bang properties for non-coercive objectives, switching points in the coercive case) is investigated. Combining these results with recent statements on Ritz-Galerkin approximations of parabolic equations new convergence theorems for the optimal controls of finite element approximations of the control problem are presented.
In particular, the convergence of a certain type of switching points can be proved. The possibilities of the numerical application of switching point techniques are outlined briefly.

"Some Remarks Concerning a Nonquadratic Antenna Problem"

Andreas Kirsch, Göttingen

We consider the optimization of the signal-to-noise ratio of an arbitrary cylindrical antenna array. Existence of an optimal solution and convergence of finite dimensional approximations is shown. The necessary optimality conditions are used to compute optimal solutions for some numerical examples.

Parameter Estimation for Fluid Flow Problems
— Richard Ewing - University of Wyoming

The process of determining unknown parameters, such as porosity and permeability, which are necessary for mathematical models used in reservoir simulation is very complex, especially for multiphase flow problems. A brief survey of the difficulties involved in these methods will be given emphasizing the complex interaction between various sources of error from the mathematical modeling process. Since the associated least squares minimization problem lacks uniqueness and is highly ill-conditioned, techniques for obtaining a better initial guess will be presented together with preliminary numerical results. These techniques involve a direct marching process in the spatial direction away from Cauchy data on a time boundary and require a stabilization technique. Similarly, a time series method for recursively augmenting the approximation of the unknown coefficient will be briefly discussed.

"Three-dimensional inverse scattering"

Margaret Cheney, Duke University, Durham, North Carolina, USA

We consider the problem of obtaining information about an inaccessible region of space from scattering experiments. Inverse scattering theory for the time-independent Schrödinger equation

$$\left[\Delta + k^2 - V(x)\right] \psi(k, x) = 0$$

is summarized. It is most easily understood by considering the associated hyperbolic equation

$$\left[\Delta - \partial_{tt} - V(x)\right] u(t,x) = 0 .$$

Particular attention is paid to those aspects of the theory that also hold for the wave equation

$$\left[\Delta - n^2(x)\, \partial_{tt}\right] u(t,x) = 0 .$$

Existence re-sults for the inverse problem of the newtonian potential

by Carlo Pagani, Politecnico of Milano

A classical inverse problem in potential theory consists in determining the shape of a three-dimensional homogeneous body (or a body whose density is known through a model) by measuring the newtonian potential created by it. This potential is measured: a) on the unknown surface of the body itself, or b) on the surface of a ball containing the body in its interior. We prove the existence of a local solution of the problem in both cases.

„Geometrical Representation For Dynamic System With Control (CDC)"

A. G. Butkovzkiy

Conseption of phase-space (state-space) portrait of CDC described by differential inclusion (DI) is introduced. This notion is an extension of a well-known consept of phase-space portrait for ordinary differential equation. Phase-space portrait for two-dimentional CDS are considered in more details. It is also considered connection between CDS and continious media. Formula for Laplac operator in this media is given.

"Survey Of Some Problems in the theory of Distributed Parameter Systems"

A. G. Butkovskiy

It is described some problems and results in the different part Distributed parameter systems theory; 1) Structural theory, 2) Mobile Control, 3) Control in quantum-mechanical processes.

"An inverse ~~pl~~ problem related to the Heat equation"

Salvador Pérez-Esteve

We consider the problem of determining the unknown source $F = F(x,t)$ in the heat equation from over specified data. For $F = f(t) \chi_D(x)$ we state uniqueness and continuous dependance, where $D \subset \mathbb{R}^n$ and χ_D is the characteristic function of D.

PARAbolic Inverse Problems

Paul DuChateau

Approaches to parabolic inverse problems may be roughly described as those which exactly compute a function which approximately solves the problem or else those which approximately compute

a function which exactly solves the problem.
Here we describe an approach of the 2^nd type which proceeds according to the following plan:

1° The Direct Problem — Solvability
2° The Inverse Problem — Uniqueness.
3° Approximation of the Solution to the Inverse Prob.

The approach is then illustrated with an example

Ambiguities in Reconstruction P.C. Sabatier

In the one-dimensional scattering problems governed by the Schrödinger equation or by the impedance equation, there exists a class of potentials (resp. impedances) that is bijectively related with a class of spectral data (for potentials: $L_1^1 = \{ V / \int_{-\infty}^{+\infty} dx (1+|x|)|V(x)| < \infty \}$ when there is no bound state, these data reduce to the reflection coefficient as a function of energy for all positive energies. However, this class is not the largest one consistent with scattering phenomena and authors showed examples of different potentials that are consistent with a given reflection coefficient and no true bound state. The lecture given here presents a complete study of these "ambiguities".

(1) They are related with a Transformation defined on the set of potentials (resp. impedance factors), leaves invariant the Schrödinger (resp. ..) equation, whereas the reflection coefficient is flipped, the transmission coefficient is invariant, and the transformation depends on an arbitrary parameter, c. Hence, if we start from $V(x)$ (resp. $\alpha(x)$), which yields $R^+(k)$, and apply $T(c)$, we obtain $V^T(x,c)$ (resp. $\alpha^T(x,c)$) which yield $-R^+(k)$, ie an infinity of "equivalent" potentials (resp. impedances)

(2) let us define a class $P(\ell_-, \ell_+)$ of potentials by their

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asymptotic behavior $\frac{l_\pm(l_\pm+1)}{x^2} + u\ (\in L_1^1)$ as $x \to \pm\infty$, and a similar class for impedances. The transformation takes a potential (ref...) from one class to another one

(3) the transformation introduces or suppresses zero-energy bound states or half-bound state, so that the invariance of $T(k)$ is not truly isospectral

(4) All the known ambiguities are described.

An inverse problem connected with continuous casting of steel

Heinz Engl, Linz, Austria

(Joint work with Langthaler, Linz, and P. Manselli, Firenze)

The mathematical modeling of the continuous casting process leads to a nonlinear boundary value problem for a nonlinear heat equation. If one wants to control the front of solidification by regulating the water pressure of the cooling water, this leads to an inverse problem for this boundary value problem.

In the first part of the talk, we report about the implementation of an algorithm for approximating a solution of the inverse problem in an industrial environment.

In the second part of the talk, we present some theoretical results: uniqueness proof for the inverse problem; existence, uniqueness and continuous dependence for the direct problem; restoration of stability for the inverse problem under a priori assumptions. This last result is only qualitative, no modulus of continuity is known so far.

Gel'fand-Levitan's theory and related ~~to~~ inverse problems,

Takashi Suzuki, Department of Mathematics, Faculty of Science, University of Tokyo.

My first object is to give a detailed study about the structure of Gel'fand-Levitan's theory: integral transformatión, symmetry or duality, and consistency in 1-dimensionality. Then, I will apply it to inverse spectral problems and identifiability of evolutión equatións: uniqueness and stability, exact solvability (="τ-functión"), and an extensión to multi-dimensional cases. Some phenomenon peculiar to inverse problems will be found by this method; domain of uniqueness and "well-illposedness".

Inverse Problem for the Vibrating Beam

Victor Barcilon, University of Chicago

My talk will be devoted to questions associated with the solution to the fourth order inverse eigenvalue problem

$$(r(x)u_n'')'' = \omega_n^2 \, p(x) \, u_n$$

After reviewing the question of uniqueness, I shall discuss at length the question of existence which differs greatly from that for the second order case. Indeed, whereas interlacing and simple asymptotic trends are the only conditions which two sequences of numbers must satisfy in order to qualify as the spectra of a vibrating string, the spectra for a vibrating beam must satisfy much stringer conditions. The results for the fourth order operator can be generalized to a broader class of operators.

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An Inverse Problem for Radon's Integral Equation.

In emission tomography one has to solve the integral equation

$$\int_{x\cdot\theta=s} f(x)\, e^{-D\mu(x,\theta^{\perp})}\, dx = g(\theta,s),$$

$$D\mu(x,\theta) = \int_0^{\infty} \mu(x+t\theta)\, dt .$$

If μ is unknown one has to determine μ prior to the computation of f. This can be done using the consistency conditions

$$\int_0^{2\pi}\int_{-\infty}^{+\infty} e^{\frac{1}{2}(I+iH)R\mu}\, s^m\, e^{ik\varphi}\, g(\theta,s)\, ds\, d\varphi = 0, \quad k > m \geq 0$$

where I, H, R are the identity, the Hilbert transform, the Radon transform, resp. We report on several attempts to solve these equations for μ numerically.

Frank Natterer

Quasi Newton Methods and Optimal Control Problems

Quasi-Newton methods play an important role in the numerical solution of problems in unconstrained optimization. Optimal control problems in their discretized form can be viewed as optimization problems and therefore be solved by quasi Newton methods. Since the discretized problems do not solve the original infinite-dimensional control problem but rather approximate it up to a certain accuracy, various approximations of the control problem need to be considered. It is known that an increase in the dimension of optimization problems can have a negative effect on the convergence rate of the quasi Newton method which is used to solve the problem. The purpose of this paper is to investigate this behavior and to explain how this drawback can be avoided for a class of optimal control problems. We show how to use the infinite dimensional original problem to predict the speed off convergence of the BFGS method for the finite-dimensional approximations.

Ekkehard W. Sachs

Exit theorems for stochastic infinite dimensional systems

Jerzy Zabczyk, IM Polish Academy of Sciences

Let an infinite dimensional system (1): $\dot{z} = A(z)$, $z(0)=x \in E$ evolve on a Banach space E and let a stochastic equation

(2) $$dX = A(X)dt + \varepsilon dW_t \,, \quad X(0)=x$$

be a perturbed version of (1). Let us assume that system (1) attracts an open set D to 0, $0 \in D$. Due to the additive nature of the disturbances, trajectories of (2), sooner or later, will reach the boundary ∂D of D. Exit theorems give some information about the limit behaviour of the exit time and the exit place as $\varepsilon \downarrow 0$. Theorems for finite dimensional systems are due to M. Freidlin and Wentzell and in the talk we describe some infinite dimensional extensions of their results.

Solution to the exit problem is closely linked with a minimum energy problem for the controlled system

$$\dot{y} = A(y) + Q^{1/2}u$$

Q being the covariance operator of W_1.

Jerzy Zabczyk

Operator Extremal Problems and Constrained Minimization for Linear Relations

M. Z. Nashed, University of Delaware, Newark, DE, U.S.A.

The first part of my talk deals with two problems in representation and compensation of systems (or operators). For example, let X be a Banach space and $A : X \to X$ a bounded linear operator such that each of $N(A)$ and $R(A)$ has a topological complement, say $\mathcal{M}$ and $\mathcal{S}$, respectively, in X. Let P, Q denote the induced projectors on $\mathcal{M}$ and $R(A)$, respectively. Let $A^{\dagger} := A^{\dagger}_{P,Q}$ denote the generalized inverse of A. In general (*) $(T^{-1}AT)^{\dagger} \neq T^{-1}A^{\dagger}T$ for an invertible linear operator T. However, $(T^{-1}AT)^{\dagger}_{P',Q'} = T^{-1}A^{\dagger}_{P,Q}T$, where $P' := T^{-1}PT$ and $Q' := T^{-1}QT$. The question of equality in (*) leads to several questions: e.g., given A, characterize all invertible T which commute with P & Q, as defined above. Given T, characterize all A for which equality in (*) holds.

The second part of the talk deals with characterizations and existence of restricted least-squares solutions of the inclusion $h \in L(x)$, with respect to $\mathcal{S}$, where L is a linear manifold in the direct sum $H_1 \oplus H_2$ of Hilbert spaces, $\mathcal{S} := g \dotplus \mathcal{N}$, $\mathcal{N}$ is a subspace of H_1, $g \in \operatorname{Dom} L$, $h \in H_2$. We report on recent joint work with S. J. Lee.

M. Z. Nashed

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On a minimum problem with free boundary arising in fluid mechanics.

Optimizing the shape of the blade of a turbine after a cylindrical cross-section will essentially lead to the following periodic 2-dim. problem:
Find a stream function u with
(*) $u = k$ on E^k, $u(x,y) - y$ periodic in y
(**) $\Delta u = 0$ outside E, $\nabla u(x,y) \to (s_\pm, 1)$ for $x \to \pm\infty$,
where $E = \bigcup_{k \in \mathbb{Z}} E^k$ is the cross section of the blade.

We want to prescribe the velocity distribution λ on ∂E. Taking the average over a family of flows, we are looking for a minimizer of

$$J(u,E) := \int_{\mathbb{R}^2} \left(\int_{\Omega} \left(|\nabla u_s - e_s|^2 - \lambda^2 \chi_E \right) \right) d\mu(s)$$

with side condition (*) and $E_* \subset E$, where E_* is given, and e_s a divergence free vector field satisfying $e_s(x,y) \to (s_\pm, 1)$ for $x \to \pm\infty$, $s = (s_+, s_-)$. Existence and regularity results, and numerical computations are presented.

H. W. Alt

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Optimal control of an age-dependent population
Martin Brokate

We consider optimal control of a population which depends on age and time. The dynamics of the system are defined by the Gurtin - MacCamy equations. We state the Pontryagin principle and draw some conclusions concerning the switching structure. Finally we formulate a semidiscrete version of the optimal control problem and discuss convergence of their solutions to a solution of the continuous problem.

Martin Brokate

Inverse Spectral Theory using Nodal Positions as Data
by Joyce R. McLaughlin

The present work is motivated by the inverse spectral problem for the beam. What is considered here is the use of spectral data which consists of positions of nodal points of mode shapes. A uniqueness theorem is presented to show that in a second order problem the position of a single node (judiciously chosen) from each mode shape determines a material parameter uniquely. Further existence, uniqueness results as well as constructive techniques were presented to show that nodal positions, positions of maximal deflection, and measurements of modeshapes at the midpoints can be used to reconstruct a material parameter.

Joyce R. McLaughlin
Rensselaer Polytechnic Institute
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U.S.A.

The Linear Functional Strategy for Improperly Posed Problems

R. S. Anderssen

For the solution of specific practical inverse problems, all that is required in the final analysis are simple unambiguous indicators which can be used for decision making purposes. In fact, all the practitioner requires is: confidence in the utility of the indicators; clearly defined interpretations in terms of the problem context; simple procedures for evaluating the indicators. But, because simple questions do not necessarily have equally simple answers, it is often necessary to utilize "deep" results in mathematics in order to achieve these goals.

In the talk, two different situations were examined (i) the direct use of indirect measurements; (ii) the use of indicators corresponding to bounded linear functionals on the solution. The consequences of these ideas were discussed for the Abel integral equation, the foliage angle distribution problem and the transmissivity problem, including the transformation of functionals defined on the solution into functionals defined on the data.

When the functionals required are point estimates of the solution, this often leads to the need to differentiate the available data numerically. The talk concluded with a discussion of stabilized multi-point finite difference formulas for the numerical differentiation of observational data.

Robert S. Anderssen
Division of Mathematics and Statistics, CSIRO,
Centre for Mathematical Analysis, ANU,
Canberra, ACT 2601 Australia

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A Problem in Optimal Periodic Control

Fritz Colonius

We consider optimal periodic control of functional differential equations with an average cost criterion and analyse local properness. An optimal steady state solution is called locally proper, if the system behaviour can be improved near this solution by introducing oscillations. This problem requires the use of second order necessary optimality conditions. We study also the relation to dynamic properties of the underlying equation (local properness near a controlled Hopf bifurcation) and discuss an example involving a retarded Liénard equation.

Strict total positivity and the bang-bang principle for boundary control of the heat equation:

E.J.P. Georg SCHMIDT, M^c^Gill University, Montreal, CANADA

We consider the control of the initial boundary value problem $\frac{\partial u}{\partial t} = \Delta u$, $\frac{\partial u}{\partial \nu} = g(u,f)$, $u(x,0) = u^0(x)$. One can prove the existence and uniqueness of weak solutions under the essential assumption that $g(u,f)$ be increasing in f and decreasing in u, while $g(t,u) = 0$. The problem of approximating u^1 by $u(\cdot,T)$ is ~~shown~~ to have an optimal control f and the various possibilities for obtaining a bang-bang principle are discussed. The questions which remain to be answered depend on deep theorems in unique continuation.

~~Many~~ Convergence of suboptimal controls

H.O. Fattorini, University of California, Department of Mathematics, Los Angeles, California, USA.

Many fundamental facts of control theory (such as the maximum principle) can be proved in the context of general nonlinear input-output relations, called here systems. In this degree of generality, we show that under the same conditions that make the maximum principle nontrivial, sequences of suboptimal (that is, close-to-optimal) controls are L^2-convergent. The abstract result can be used for ordinary differential equations, partial differential equations (distributed and boundary control systems) and functional differential equations, as well as some unconventional input-output systems.

Remark: I can prove Fermat's last theorem in this way but I have no space

Should be enough space here. Can't be that long!

LOKALE ALGEBRA
und
LOKALE ANALYTISCHE GEOMETRIE —
25.–31. Mai
1986

Characterizing Algebraic Cycles on Projective Hypersurfaces
(joint work with J. Steenbrink, Leiden)

① Let X be any smooth variety over $\mathbb{C}$, $K(X)$ its Grothendieck-group, $H^*(X, \mathbb{C})$ its cohomology ring. Then there is the Chern-character: $ch_X : K(X) \longrightarrow H^*(X, \mathbb{C})$. The problem is to describe its image.
Remark: By classical Hodge-theory the image is contained in $\bigoplus_p H^{p,p}(X)$, where $H^{i,j}(X) = H^j(X, \Omega^i_X)$ are the summands in the Hodge-decomposition of $H^*(X, \mathbb{C})$. The problem of "description" means (here):

(a) Find ~~(suitable)~~ generators for $K(X)$,
(b) Find a basis of $H^{p,p}(X)$ which is "suitable",
(c) Describe ch_X in terms of these data.

② If $i : X \hookrightarrow \mathbb{P}$ is a projective variety, one has the diagram:

$$\begin{tikzcd}
 & & 0 & \\
D^b(R.)/D^b S. \arrow[r] \arrow[rrr, bend left, "\gamma"] & D^b(X)/D^b(\mathbb{P}) \arrow[r] & K(X)/K(\mathbb{P}) \arrow[r] \arrow[u] & \oplus H^{p,p}(X)^\circ \\
D^b(R.) \arrow[r] \arrow[u] & D^b(X) \arrow[r] \arrow[u] & K(X) \arrow[r, "ch_X"] \arrow[u] & \oplus H^{p,p}(X) \arrow[u] \\
D^b(S.) \arrow[r] \arrow[u, "R. \overset{\mathbb{L}}{\underset{S.}{\otimes}} -"] & D^b(\mathbb{P}) \arrow[r] \arrow[u, "\mathbb{L} i^*"] & K(\mathbb{P}) \arrow[r, "ch_{\mathbb{P}}"] \arrow[u, "i^!"] & \oplus H^{p,p}(\mathbb{P}) \arrow[u, "i^*"]
\end{tikzcd}$$
,

where - $R. = \Gamma_*(\mathcal{O}_X)$, $S. = \Gamma_*(\mathcal{O}_{\mathbb{P}})$ are the homogenous coordinate rings,
- the first horizontal maps are given by sheafification ($D^b(R.(S.))$ denoting the derived categories of graded (R. or S.) modules,
- $D^b(-) \to K(-)$ is the universal function and finally
- $D^b(-)/D^b(-)$ means the quotient in the sense of triangulated categories.

③ Last year, I showed that $D^b(R.)/D^b(S.) \cong \underline{MCM}(R.)$, the category of maximal Cohen-Macaulay modules (modulo projective modules), if R. is Gorenstein.

④ Assume $X^n \hookrightarrow \mathbb{P}^{n+d}$ is a smooth, complete intersection. Then (Deligne SGA $\overline{VII}$):

- $H^i(\Omega^j_X) = 0$ unless $i=j$ or $i+j=n$
- $H^i(\Omega^i_{\mathbb{P}}) \longrightarrow H^i(\Omega^i_X)$ is an iso for $2i < n$ and injective for $2i=n$

We identify $H^i(X,\Omega^j_X)/H^i(\mathbb{P},\Omega^j_{\mathbb{P}}) =: H^i(\Omega^j_X)^\circ$ as follows:
Let N be the normal-module of R. in S. (i.e. $N = \bigoplus_{i=1}^{d} R(a_i)$),
$T^1_R = \operatorname{Cok}(\operatorname{Hom}_{S.}(\Omega^1_{S.}, R.) \longrightarrow N.)$. Then

<u>Proposition</u> $\exists$ a natural pairing $\langle\,,\rangle : H^{n-i}(X,\Omega^i_X) \otimes_{\mathbb{C}} [S_i^R(N) \otimes_{R.} \omega_{R.}]_0 \longrightarrow \mathbb{C}$ such that $\operatorname{rad}_\ell \langle\,,\rangle = H^{n-i}(\mathbb{P},\Omega^i_{\mathbb{P}})$; $\operatorname{rad}_r \langle\,,\rangle =$ image of $S_{i-1}N \otimes_R \operatorname{Hom}(\Omega^1_{S.},R) \otimes_R \omega_R$ in $S_i N$. Hence there is a perfect pairing

$$H^{n-i}(\Omega^i_X)^\circ \otimes_{\mathbb{C}} (S_i^R T^1_R \otimes \omega_R)_0 \longrightarrow \mathbb{C}.$$

(For $R = S./(f)$, $\deg f = d$, $T^1_R = R(d)/J(f)$ and one regains the description of Griffiths of $H^{n-i}(\Omega^i_X)^\circ$.)

⑤ Assume $X^{2m} \hookrightarrow \mathbb{P}^{2m+1}$ is an even-dimensional, smooth hypersurface.
An (graded) MCM over ~~R~~ R. is given by a <u>matrix-factorization</u> (ϕ,ψ)
$\phi : F \longrightarrow G$, $\psi : G(-d) \longrightarrow F$; G, F free (graded) S.-modules such that
$\phi\psi = f.\operatorname{id}_{G(-d)}$; $\psi\phi = f.\operatorname{id}_F$ (D. Eisenbud [TAMS, 1980]).

<u>Theorem</u>: $\exists$ a (universal) constant $c(d,m)$ such that $\gamma(M)$ is given by:

$$\gamma(M) = c(d,m) . \frac{\operatorname{Trace}((d\phi \wedge d\psi)^{m+1})}{\text{volume element of } S} \in (R/J(f))_{(m+1)(d-2)} = H^{m,m}(X)^\circ.$$

⑥ Applications to the variational Hodge-conjecture - using S. Bloch's semi-regularity map [Inv. math '72] are given ~~using~~ relying on B. Angéniol - M. Lejeune's explicit description of Atiyah-classes.

R.-O. Buchweitz (Hannover)

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Reflexive modules on quotient surface singularities

Jürgen Wunram (Hamburg)

Let (X,x) be a germ of an analytic quotient surface singularity. Let $\pi : \tilde{X} \to X$ be the minimal desingularization of X with exceptional system $\{E_i\}_{1 \leq i \leq r}$ and fundamental cycle $Z = \sum_{i=1}^{r} r_i E_i$. For each reflexive module M on X the sheaf $\tilde{M} := \pi^* M / \text{torsion}$ is locally free on $\tilde{X}$ and the first Chern class is represented by a divisor which is transversal to the exceptional set E of π.

The subject of this talk is a generalization of the theorem of Artin + Verdier and the multiplication formula of Esnault + Knörrer on the McKay correspondence for rational double points to the case of an arbitrary quotient surface singularity:

Thm. i) For each E_i there is exactly one indecomposable reflexive module M_i on (X,x) with $c_1(\tilde{M}_i) . E_j = \delta_{ij}$, $1 \leq i, j \leq r$, and $R^1\pi_*(\tilde{M}_i^{\vee}) = 0$. The rank of M_i is r_i.

ii) If $0 \to \tau(M) \to N_M \to M \to 0$ is an almost split exact sequence then

$$c_1(\tilde{N}_M) = \begin{cases} c_1(\widetilde{\tau(M)}) + c_1(\tilde{M}) & \text{if } M \neq M_1, \dots, M_r \\ c_1(\widetilde{\tau(M)}) + c_1(\tilde{M}) + E_i & \text{if } M = M_i \end{cases} .$$

The fundamental sequence $0 \to \omega_X \to N_{\mathcal{O}_X} \to \mathcal{O}_X \to \mathbb{C} \to 0$ induces

$$c_1(\tilde{N}_{\mathcal{O}_X}) = c_1(\omega_{\tilde{X}}) - Z .$$

J. Wunram

Surfaces arithmétiques elliptiques

L. Szpiro (Paris)

Nous considérons dans ce qui suit des courbes elliptiques semi-stables $f : X \to T$ où T est soit une courbe projective et lisse de genre g sur un corps k, soit le spectre d'un anneau d'entiers d'un corps de nombres algébriques. Nous noterons S l'ensemble -fini- de points de T dont la fibre n'est pas lisse.

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Th 1 (situation géométrique T = courbe / k) Soit Δ_X le discriminant de X sur T alors

$$\deg \Delta_X \leq 6(2g-2+\deg S)\,p^e$$

où $p = \text{char}(k)$ et p^e est le degré d'inséparabilité du morphisme f.

Th 2 (situation arithmétique $T = \text{Spec}\,\mathcal{O}_k$) Munissons le dual de l'algèbre de Lie ω_X de la métrique d'Arakelov qui satisfasse à la formule d'ajonction sur les surfaces arithmétiques. Alors on a

$$12 \deg \omega_X = \lg \text{Norme}(\Delta_X)$$

Conjecture (situation arithm.) Soit N le conducteur de la courbe elliptique semi-stable $f : X \longrightarrow \text{Spec}\,\mathcal{O}_k$ et soit ε un réel positif, alors il existe une constante $C(k,\varepsilon)$ telle que

$$\text{Norme}(\Delta_X) \leq C(k,\varepsilon)\, N^{6+\varepsilon}$$

Cette conjecture implique clairement la suivante

Conjecture' Dans la même situation il existe une constante $c(k)$ telle que

$$\text{Norme}(\Delta_X) \leq N^{c(k)}$$

L'intérêt de ces conjecture est magnifié par la construction de G. Frey :

Soient a, b, c des entiers tels que $a+b=c$ alors la courbe elliptique $y^2 = x(x-a)(x-c)$ est semi-stable si $v_2(a) \geq 4$ et $c \equiv -1\ (4)$

Comme corollaire d'une de ces conjectures et de la construction de G. Frey on voit

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que pour toute equation trinome à coefficients entiers

(*) $aX^n + bY^m = cZ^p$ telle que $a \pm b \neq \pm c$

il existe une constante $C(a,b,c)$ telle que

si $\inf(n,m,p) \geqslant C(a,b,c)$ l'équation

(*) n'a pas de solution entieres non triviale.

L. Szpiro

On the number of equations defining algebraic sets in $\mathbb{A}^n_k$.

(Gennady Lyubeznik; West Lafayette, USA).

The following theorem is proved:

Theorem: Let $V \subset \mathbb{A}^n_k$ be an algebraic set consisting of irreducible components of positive dimensions. Assume that either

(i) V is locally a complete intersection,

or

(ii) char $k = p > 0$ and V arbitrary.

Then V can be defined by $n-1$ equations

This theorem generalizes earlier results of Ferrand – Szpiro – Boratynski – – Mohan Kumar – Cowsik – Nori.

Linear Sections of determinantal Varieties

We consider linear ~~[illegible]~~ spaces of matrices, which may be described in (at least) 3 equivalent ways:

1) a linear subspace $M \subset \mathrm{Hom}(V, W)$

2) a pairing $V \otimes W^* \xrightarrow[\mu]{} M^*$

3) a $v \times w$ matrix L of linear forms in $m = \dim M$ variables,

where $v \geq w$ (say) are the dimensions of the vectorspaces V, W, over some field F.

We restrict our attention to spaces which are nice in the sense that the pairing μ of 2) is "non-degenerate". More generally we have:

<u>Proposition - Definition</u>: For a subspace $M \subset \mathrm{Hom}(V, W)$ with associated pairing μ and matrix of linear forms L, the following are equivalent:

1) The annihilator $M^\perp \subset \mathrm{Hom}(W, V) = (\mathrm{Hom}(V, W))^*$ meets the rank $\leq k$ locus only in 0.

2) No sum of $\leq k$ pure vectors in $V \otimes W^*$ goes to zero under μ

3) Even after row and column operations, any k of the elements of L are linearly independant.

When these conditions are satisfied, we say that M (or μ or L) is <u>k-generic</u>.

The most important of these conditions from the point of view of applications is 1-genericity. A first example of a 1-generic space (given as a matrix of linear forms) is

$$\mathrm{Cat}(v, w) = \begin{pmatrix} x_0 & x_1 & x_2 & \cdots & x_{w-1} \\ x_1 & x_2 & & \cdot^{\cdot^{\cdot}} & \\ x_2 & & & & \\ \vdots & & & & \\ x_{v-1} & & & & x_{v+w-2} \end{pmatrix}.$$

The first theorem we describe arose from discussions with J. Herzog, and has been used by him, with Kühl and Ulrich, in the study of special maximal Cohen-Macaulay modules and compressed algebras.

Theorem 1: Let L be an $(n-k)$-generic matrix of linear forms. The $k \times k$ minors of L generate a prime ideal of generic height, and this remains so modulo *any* $\leq k-2$ linear forms.

One may say that the determinantal ideals corresponding to L is "k-2 resilient"; it would be nice to know more such examples (beyond the theorem of Zak which says that every *smooth* projective variety X is 1-resilient so long as $\dim X \geq \operatorname{codim} X + 2$.)

1-generic matrices arise in geometry in the following situation: Let X be a reduced irreducible variety, embedded in a projective space $\mathbb{P} = \mathbb{P}(H^0(\mathcal{L}))$ by the complete linear series associated to a line bundle $\mathcal{L}$. Let $\mathcal{L}_1$, $\mathcal{L}_2$ be line bundles such that $\mathcal{L} = \mathcal{L}_1 \otimes \mathcal{L}_2$; the pairing

$$\mu : H^0\mathcal{L}_1 \otimes H^0\mathcal{L}_2 \longrightarrow H^0\mathcal{L}$$

is easily seen to be 1-generic, and the homogeneous ideal $I(X)$ contains the 2×2 minors of the associated matrix $L(\mathcal{L}_1, \mathcal{L}_2)$ of linear forms. We have:

Theorem 2: (—, Koh, Stillman): If X is a reduced, irreducible curve of genus g, and $\mathcal{L}_1$, $\mathcal{L}_2$ are line bundles of degrees $\geq 2g+1$ (and distinct if both have degree $2g+1$), then $I(X)$ is generated by the 2×2 minors of $L(\mathcal{L}_1, \mathcal{L}_2)$.

The ~~cata~~ matrix $Cat(v, w)$ arises in applying this theorem to $\mathbb{P}^1$ (with $\mathcal{L}_1 = \mathcal{O}(v-1)$, $\mathcal{L}_2 = \mathcal{O}(w-1)$). It is easy to give explicit representations of other curves as well; some elliptic cases go back to Hurwitz.

It would be nice to have an analogue of Theorem 2 for higher dimensional varieties.

David Eisenbud
(Brandeis University, Waltham MA 02254
USA).

On maximal Buchsbaum modules

Shiro Goto

Let R be a local ring and M a finitely generated R-module. Then M is said to be Buchsbaum, if the difference $I_R(M) = \ell_R(\frac{M}{\mathfrak{q}M}) - e_{\mathfrak{q}}(M)$ is independent on the choice of parameter ideals $\mathfrak{q}$ for M. (Hence M is Cohen-Macaulay if and only if M is Buchsbaum and $I_R(M) = 0$.) A Buchsbaum R-module M is called maximal, if $\dim_R M = \dim R$.

The purpose of my lecture is to show that if R is regular, then R possesses only finitely many isomorphism classes of indecomposable maximal Buchsbaum modules. In a certain special case (e.g., when $R = \mathbb{C}[[X_1, X_2, \cdots, X_n]]^G$ with G finite group), the converse is also true, which I shall discuss a little more closely in my lecture.

Koszul homology and the structure of low codimension ideals

We indicate how, in low codimension (3,4), the structure of CM ideals is mirrowed in its Koszul homology. Let R be a regular local ring and I a CM ideal; $I=(x_1,\dots,x_n)$. Denote by $H_i(I)$ the Koszul homology modules of I - using the given generating set. I is said to be linked to J if there exists a family of links $I \sim L_0 \sim \dots \sim L_r \sim J$.

There are two general theorems connecting the Koszul homology of I and J:

Theorem 1 (Peskine-Szpiro): J is Cohen-Macaulay along with I.

Theorem 2 (Huneke): The condition "$H_i(I)$ is CM for $i \leq m$" is an invariant of *even* linkage.

This result, for $m \geq 1$, does not extend to the full linkage class of I. However, one has:

Theorem 3: If I has codimension 3 then "$H_1(I)$ is Cohen-Macaulay" is an invariant of the full linkage class of I.

It does not extend to the next Koszul module, nor to higher codimension. It is fairly easy to verify by computation. Villarreal has attempted to circumscribe the class of ideals with this property. In case I has a pure resolution

$$0 \to R^{b_3}(-d-a-b) \to R^{b_2}(-d-a) \to R^{b_1}(-d) \to I \to 0;$$

Theorem 4 (Villarreal): If I is, besides, generically a complete intersection, ~~and~~ $a \geq b$ and $b_1 \geq 6$, then $H_1(I)$ is *not* Cohen-Macaulay.

For Gorenstein ideals of codimension 4

Theorem 5: $H_1(I)$ is Cohen-Macaulay $\Leftrightarrow$ I/I^2 is Cohen-Macaulay.

As a general remark, computers are easily brought in to test for CMness of these various modules.

Wolmer Vasconcelos
Rutgers Univ. / New Jersey

On the Gorensteinness of Rees algebras and associated graded rings.

Shin Ikeda (Köln).

Let (A, m, k) be a Noetherian local ring and I an ideal of A. We study the Gorensteinness of the Rees algebra $R(I) = \oplus_{n\geq 0} I^n$. Let $G(I) = \oplus_{n\geq 0} I^n/I^{n+1}$ and let K_A and $K_{G(I)}$ be the canonical modules of A and $G(I)$, respectively. Our main result is:

Theorem. Let grade$(I) \geq 2$ and $R(I)$ be CM (Cohen-Macaulay). Then the following are equivalent

1) $R(I)$ is Gorenstein
2) $K_A \cong A$ and $K_{G(I)} \cong G(I)(-2)$.

We construct an example: Let

$$A = k[\![X_1, X_2, X_3, Y_1, Y_2, Y_3, Y_4]\!]/J \quad \text{, where}$$

k is a field of $ch(k) = 2$ and

$$J = \left(\begin{array}{l} X_1Y_1 + X_2Y_2 + X_3Y_3,\ Y_1^2,\ Y_2^2,\ Y_3^2,\ Y_4^2,\ Y_1Y_4,\ Y_2Y_4,\ Y_3Y_4, \\ Y_1Y_2 - X_3Y_4,\ Y_2Y_3 - X_1Y_4,\ Y_1Y_3 - X_2Y_4 \end{array} \right).$$

Then A is not CM but R(m) is Gorenstein.

Rational surfaces in $\mathbb{P}^4$

R. Hartshorne (Berkeley CA)

Until now very little is known about surfaces in $\mathbb{P}^4$. For example, it is not known whether there are rational surfaces in $\mathbb{P}^4$ of unbounded degrees. The purpose of this talk is to report on the present state of knowledge regarding rational surfaces in $\mathbb{P}^4$. The following table lists all rational surfaces in $\mathbb{P}^4$ known at present

not linearly normal

linearly normal

	Embedding dim	degree	abstract type	hyperplane sect	comments
not linearly normal	2	1	$\mathbb{P}^2$	L	plane
	3	2	$\mathbb{P}^1\times\mathbb{P}^1$	$\mathcal{O}(1,1)$	quadric surface
	3	3	$\tilde{\mathbb{P}}^2(x_1,\dots,x_6)$	$3L-\Sigma x_i$	cubic surface
	5	4	$\mathbb{P}^2$	$2L$	Veronese surface
linearly normal	4	3	$\tilde{\mathbb{P}}^2(x)$	$2L-x$	cubic scroll
	4	4	$\tilde{\mathbb{P}}^2(x_1,\dots,x_5)$	$3L-\Sigma x_i$	del Pezzo $=F_2 . F_2'$
	4	5	$\tilde{\mathbb{P}}^2(x,y_1,\dots y_7)$	$4L-2x-\Sigma y_i$	Castelnuovo
	4	6	$\tilde{\mathbb{P}}^2(x_1,\dots,x_{10})$	$4L-\Sigma x_i$	Bordiga
	4	7	$\tilde{\mathbb{P}}^2(x_1,\dots x_6,y_1,\dots y_5)$	$6L-\Sigma 2x_i-\Sigma y_j$	Okonek
	4	8	$\tilde{\mathbb{P}}^2(x_1,\dots x_4,y_1,\dots y_{12})$	$6L-\Sigma 2x_i-\Sigma y_j$	Okonek *
	4	8	$\tilde{\mathbb{P}}^2(x_1,\dots,x_{10},y)$	$7L-\Sigma 2x_i-y$	Okonek-Alexander
	4	9	$\tilde{\mathbb{P}}^2(x_1,\dots x_{10})$	$13L-\Sigma 4x_i$	Alexander

* this one is special in the sense that $h^1(\mathcal{O}_X(1))\neq 0$. All others are nonspecial. Note also for this one the points x_i, y_j are in special position. For the others, the points are in general position.

In this table, $\tilde{\mathbb{P}}^2(x_1, \ldots)$ denotes $\mathbb{P}^2$ blown up at points $x_1, \ldots$. L denotes a line in $\mathbb{P}^2$. $3L - \Sigma x_i$ for example denotes the linear system of cubic curves passing through the points x_i.

Up to degree 6, these surfaces are known classically (see for example the book of Semple and Roth). A modern treatment of their classification and existence is given by Okonek (Math. Z., 184 (1983)). The surfaces of degree 7 and 8 were described by Okonek (Math. Z. 187 (1984) and 191 (1986)) who proved the existence of the first two but not the third. The existence of this one and of the one of degree 9 were proved by Jim Alexander, thesis Nice, 1986.

One knows [Okonek] that for degree ≤ 8 this list contains all rational surfaces in $\mathbb{P}^4$. Alexander has shown that any nonspecial rational surface (i.e. one for which $h^2(\mathcal{O}_X(1)) = 0$) is in the above list. It is yet to be seen if there are any more special ~~surfaces of~~ rational surfaces of degree ≥ 9.

In the oral presentation I gave some idea of the proofs of existence for the four surfaces of degrees 7, 8, 9.

There exist indecomposable rank two Gorenstein modules (d'après Weston).

Let (A, m) be a local noetherian ring with maximal ideal m. A fin. gen. module G is a Gorenstein module if (i) $\operatorname{injdim}_A G < \infty$ (ii) $\operatorname{Hom}_A(G, G)$ is free and (iii) $\operatorname{Ext}^i_A(G, G) = 0$ for $i > 0$. This was defined by Sharp. The following facts are known:

1) If A has a Gorenstein module, then a) A is Cohen-Macaulay and b) A has Gorenstein formal fibres.

2) If G is a Gorenstein module, then $\operatorname{rk}_A \operatorname{Hom}_A(G, G) = t^2$. The integer t is called the rank of G.

3) If G is a Gorenstein module of rank 1, then G is a dualizing mod.
4) If A has a Gor. module, then it has a unique indecomposable Gorenstein module G such that any Gor. module is a direct sum of copies of G.
(5) If A has a Gorenstein module of odd rank, then A has a dualizing module.
6) If A is a Gorenstein mod over itself, then A is Gorenstein.

CONJECTURE: If A has a Gorenstein module, then A has a dualizing mod.

THEOREM (DANA WESTON, UNIV. ILL THESIS 1986). There exists an analytically normal two dimensional factorial local domain A that has an indecomposable Gorenstein module of rank 2.

This example is produced using the techniques of Rotthaus and Ogoma, and then applying general grade reduction theory of Hochster. Such an A has Gorenstein formal fibres, but no dualizing complex.

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29. may 1986

Topology of the infinitesimal site and the Hodge Conjecture

We interpret the general problem of Hodge as to give those conditions on a C^∞ complex vector bundle E over a projective nonsingular variety X over the complex number field so that $E^m \oplus \mathbb{C}^n$ has a holomorphic structure for some integers m, n. More generally, if X is any complex manifold, we introduce a Grothendieck topology called the holomorphic topology of X, whose topos is denoted by $(X)_{hol}$, and a canonical morphism of topos $f: X \longrightarrow (X)_{hol}$, whose construction mirrors that of the crystalline site in Algebraic Geometry. It has the property that a holomorphic structure on E is equivalent to giving a crystal E_1 in $(X)_{hol}$ such that $E = f^*E_1$. The site is defined as follows: Let $\mathcal{A}$ be the sheaf of complex-valued real-analytic functions on X and $\mathcal{O}$ the subsheaf of holomorphic functions, and let $U \subset X$ be a general open set. The objects of the holomorphic site are nilpotent closed immersions of $\mathbb{C}$-locally ringed spaces $\alpha: (U, \mathcal{A}|U) \hookrightarrow (U, \mathcal{B})$ where $(U, \mathcal{B})$ is a real analytic space such that $\mathcal{B}$ is an $(\mathcal{O}|U)$-algebra, and α is a morphism of $\mathbb{C}$-ringed spaces over $(U, \mathcal{O}|U)$. Covering families, sheaves and crystals are defined in the usual manner in crystalline theory. When interpreted in the language of differential operators, the equivalence between holomorphic structures on E and crystalline structures on E is seen to be a restatement of a form of Nirenberg's complex Frobenius theorem.

It is to be hoped that one can study the Hodge problem by analyzing the "homotopy invariants" of the map $X \longrightarrow (X)_{hol}$.

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Linear Systems of Curves in $\mathbb{P}^2$ with Prescribed Multiplicity

$k=\bar{k}$, $P_1,..,P_s$ points in $\mathbb{P}^2(k)$, $R=k[x_0,x_1,x_2]$, $\wp_i\subseteq R$, $\wp_i\longleftrightarrow P_i$,

$\alpha_1\geq\alpha_2\geq\dots\geq\alpha_s\geq 1$ integers and $I=\wp_1^{\alpha_1}\cap\wp_2^{\alpha_2}\cap\dots\cap\wp_s^{\alpha_s}=\oplus I_d$. Then

the projective space based on I_d consists of all curves of deg $=d$ having a singularity at P_i with multiplicity $\geq\alpha_i$. Problem: If $A=R/I=\oplus A_t$; what is the Hilbert function of A? (Equivalently, how big is I_t?).

If $H(A,t)=\dim_k R_t-\dim_k A_t$ than $H(A,t)$ also is equal to $h^0(\mathcal{O}_{\mathbb{P}^2}(t))-h^0(\mathcal{I}_Z(t))=h^0(\mathcal{O}_Z(t))-h^1(\mathcal{I}_Z(t))$ (Z - the subscheme of $\mathbb{P}^2$ defined by $\mathcal{I}$.)

1) If all $\alpha_i=1$ it is known that, in general, $h^0(\mathcal{I}_Z(t))\cdot h^1(\mathcal{I}_Z(t))=0$, $\forall t$, ie. "most" sets of points have maximal Hilbert function, given the multiplicity.

2). If $\alpha_i\geq 1$ this is known to be false, in general, (Eg. $\wp_1^3\cap\wp_2^3\cap\wp_3^2$ - P_1,P_2,P_3 not on a line). What can be said? Harbourne (Proc. Conf. on Alg. Geo. - Vancouver 1984) answers completely if $P_1,..,P_s$ lie on a cubic.

Theorem. (Alessandro Gimigliano) - (Ph.D. thesis)

Let $P_1,..,P_s$ lie on a nonsingular curve, $\mathcal{C}$, of degree d and suppose $h^1(dE_0-E_1-\dots-E_s)=0$ on X (the blow-up of $\mathbb{P}^2$ at $P_1,..,P_s$). Let $D_t=tE_0-\sum_{i=1}^{s}\alpha_i E_i$ where $t\geq\sum_{i=1}^{s}\alpha_i$. Then

i) If $s\neq\binom{d+2}{2}-1\Rightarrow h^1(D_t)=0$.

ii) If $s=\binom{d+2}{2}-1$ and $D_t\neq mdE_0-\sum_{i=1}^{s}mE_i$ then $h^1(D_t)=0$.

iii) If $s=\binom{d+2}{2}-1$, $D_t=mdE_0-\sum_{i=1}^{s}mE_i$ then $h^1(D_t)=0$ iff the points $P_1,..,P_s$ are sufficiently general.

Cor. If $P_1,..,P_s$ are sufficiently general and lie on a nonsingular curve of degree d, I as above (so $e(I)=\sum\binom{\alpha_i+1}{2}$) then if $t\geq\sum_{i=1}^{d}\alpha_i$ $\Rightarrow H(A,t)=e(I)$.

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Unconditioned Strong d-sequences and Some Applications to Generalized Cohen-Macaulay Rings.

A sequence $a_1, a_2, \cdots, a_s$ of elements in a commutative ring A is called an unconditioned strong d-sequence on an A-module E if every power $a_1^{n_1}, a_2^{n_2}, \cdots, a_s^{n_s}$ $(n_i > 0)$ and every permutation of them form a d-sequence on E. The reason why we introduce this sequence is that we want to find a good sequence property which unifies the behaviours of s.o.p.'s for Buchsbaum rings and moreover generalized Cohen-Macaulay rings. Main conclusion is that we can describe the local cohomology modules $H^i_{\underline{a}}(E)$ $(i<s)$ in terms of quotient modules concerning a_i's and also the local cohomology modules of $R(E)$ and $G(E)$, the Rees module and the associated graded module of E w.r.t. a_j's, in terms of $H^j_{\underline{a}}(E)$'s. This sequence has a closed relation with a pS-sequence introduced by Brodmann, in fact they form a pS-sequence on E if and only if they form a u.s. d-sequence on E under suitable assumptions.

K. Yamagishi (Köln)

Topological Invariants of Quasi-ordinary Singularities

A d-dimensional irreducible hypersurface singularity $x \in X \subset \mathbb{C}^{d+1}$ is called quasi-ordinary (q.o) if it admits, locally, a finite projection π into $\mathbb{C}^d$ with discriminant locus Δ having only normal crossings. Such a singularity can be parametrized by a fractional power series, the case $d=1$ (plane curves) being the classical Puiseux parametrization. For curves one knows how the "characteristic pairs" of a parametrization control the local topology (via knot theory). In higher dimensions, one associates to fractional power series parametrizations analogues of the characteristic pairs, and naturally asks what the relation of these to the local topology of (X,x) is. →

A big step toward the answer is the following. For each component Δ_i of Δ, $(1 \leq i \leq c)$ let $Z_i = \pi^{-1}(\Delta_i)$. Then Z_i is irreducible, and we can let $m_i = \deg(\pi_{z_i})$ be the branching order of π at a generic point z_i of Z_i. <u>Lemma</u>. With suitable ordering of the i, we have $m_c | m_{c-1} | \cdots | m_2 | m_1 = \deg(\pi_x)$. ("$|$" = "divides"). ∎

<u>THEOREM</u> Locally homeomorphic q.o. singularities have the same m_i $(i = 2, 3, \cdots, c;\ m_1 \text{ omitted})$. An important role in the proof is played by the local homology group $H_{2d-2}(X, X-x)$, which is finite, of order $m_2 m_3 \cdots m_c$. (This is proved via the group of rational equivalence classes of codimension-one cycles in $\mathcal{O}_{X,x}$, which maps naturally to $H_{2d-2}(X, X-x)$: cycle $\leftrightarrow$ analytic cycle on X $\rightsquigarrow$ fundamental class. <u>Thm</u>. This map is an isomorphism.)

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Gorenstein ASL domains of dimension 3 and 4.

The concept of ASL (<u>al</u>gebras with <u>s</u>traightening <u>l</u>aws) was introduced by De Concini, Eisenbud and Procesi and proved to be very powerful to prove certain rings are Cohen-Macaulay. We want to consider the following <u>Problem</u>. Given a poset H, is there

(1) a Gorenstein ASL on H?

(2) an ASL which is an integral domain on H?

In case (1) we call H to be weakly Gorenstein and in case (2) we call H integral. We give the classification of the following posets.

(i) Integral (weighted) posets of rank 1.

(ii) weakly Gorenstein posets of rank 1.

(iii) integral, weakly Gorenstein posets of rank 2

In each case, we assume our ASL to be graded over a field and in case (iii) we assume our ASL to be homogeneous (generated by deg. 1 elem's). In particular, we can show that the coordinate rings of Del Pezzo surfaces of degree $\geqq -4$ by anti-canonical embedding are ASL. Finally, we classify H of rank 3, with unique minimal element T, which is integral and $H' = H - \{T\}$ defines a triangulation of a 2-sphere (that is the simplicial complex $\Delta(H')$ associated to H'

has S^2 as underlying topological space). There are 18 such posets.

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T^1 for quasi-homogeneous surface singularities

If A is a finitely generated (over $\mathbb{C}$) graded normal domain of dimension 2, then the module $T^1_A = \mathrm{Ext}^1_A(\Omega^1_A, A)$ is graded; one tries to compute the graded pieces in terms of the "geometry" of A (i.e, Proj A, etc.). In case A is the cone over a projectively normal embedding of a curve $C \subset \mathbb{P}^N$, there is a formula (due to M. Schlessinger – (1971)) expressing T^1_i in terms of $H^0(N_{C/\mathbb{P}^N}(i))$. It is very hard, though, to compute T^1_{-1}. Mumford (1972) showed if C non-hyperelliptic, $\deg L >> 0$ ($L = \mathcal{O}_C(1)$), then $T^1_{-1} = 0$ — but no effective bound on the degree is possible. By a different method we prove:

Theorem: If C has general moduli, $g(C) \geq 50$, then for $\deg L \geq 4g-2$, $T^1_{-1} = 0$.

If C is non-hyperelliptic, one may take the cone over the canonical embedding. With a few exceptions it is not hard to prove:

i :	2	1	0	−1	others
$\dim T^1_i$	1	g	3g-3	?	0

H. Pinkham proved that if C sits on a K-3 surface, then the cone is smoothable; in particular, $T^1_{-1} \neq 0$ (always, for $g \leq 10$; and for some C, any g).

Theorem: If C has general moduli, $g(C) \geq 50$, then for the canonical cone one has $T^1_{-1} = 0$.

One finds in particular a "generic" Gorenstein singularity — all deformations (even infinitesimal ones) are "equisingular". Further, the Jacobian algebra $\omega_A / \mathrm{Im}\, \Omega^2_A$ is as small as possible: A/m^3. The

Theorem is a consequence of the "intrinsic" description of T' in terms of C. In particular, on any $\mathbb{C}$-scheme X, with line bundle L, one defines $\phi_L : \Lambda^2 H^0(L) \rightarrow H^0(\Omega^1 \otimes L^2)$ by the "formula" $\phi(f \wedge g) = f dg - g df$. A local argument shows this is really well-defined. It is natural and functorial, and an exercise shows it is always surjective on $\mathbb{P}^N$, $L = \mathcal{O}(a)$. It is not too difficult to show ϕ_K is surjective for "most" complete intersection curves C, and a degeneration argument shows ϕ_K is surjective for the general curve of ~~degree~~ genus ≥ 50. The previous result then follows from the

<u>Theorem</u>: For a canonical cone, $(T^1_{-1})^* \simeq \operatorname{Coker} \phi_K$.

We also stated a result that (with known exceptions), every graded normal A of dimension 2 has a deformation of weight ≥ 0.

Jonathan Wahl

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Rings with doubly infinite chain condition. May 29, '86

We introduce the notion of a commutative ring with unity in which every doubly infinite chain of ideals such as: $\cdots \subseteq \underline{\mathfrak{a}}_{-1} \subseteq \underline{\mathfrak{a}}_0 \subseteq \underline{\mathfrak{a}}_1 \subseteq \underline{\mathfrak{a}}_2 \subseteq \cdots$ stabilizes either to the right side or to the left or to both sides, briefly DICC. By using the following results: Any homomorphic image or localization of a DICC ring is still so; a DICC domain is Noetherian; a DICC ring with a unique prime ideal is Noetherian; a DICC ring has only finitely many minimal prime ideals; we can prove that a reduced DICC ring is Noetherian.

We call a non-Noetherian DICC ring <u>standard</u> DICC, briefly SDICC. Then, for the nilradical $\underline{n}$ of an SDICC ring we have:

(i) $\operatorname{Ass}(\underline{n})$ consists of finitely many maximal ideals; (ii) $\underline{n}$ is nilpotent; (iii) $\underline{n}$ is DCC.

We call a <u>min/max ideal</u> a prime ideal which is both minimal

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and maximal. Then

Theorem 1. Let R be a non-Noetherian ring with no min/max ideals. Then R is SDICC iff 1. S_{red} is Noetherian; 2. $\underline{n}$ is nilpotent; 3. $\underline{n}$ is DCC; 4. $\forall x \in S - \underline{n}$: $\underline{n}/Sx \cap \underline{n}$ or equivalently $\underline{n}/x\underline{n}$ has finite length.

Theorem 2. A ring R is DICC iff either R is Noetherian or $R \cong S \times A$ where S is an SDICC with no min/max ideals and A is an Artinian ring.

Finally, we are able to exhibit a technique for constructing a DICC ring. As a corollary we get that an SDICC ring with no min/max ideals has a unique minimal frame.

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Equimultiplicity of μ-constant deformations

Almost 20 years ago Zariski asked the famous question: Does topological equisingularity of isolated hypersurface singularities imply equimultiplicity? The aim of this talk was to give a report on the ~~sta~~ present knowledge concerning this question. Actually we are mainly interested in the slightly weaker problem if the multiplicity does not change along the μ-constant stratum where μ denotes the Milnor number of the singularity. If the singularity is quasihomogeneous, then a deep result of Varchenko implies that the μ-constant stratum is linearly embedded in the base space of the semi-universal deformation. According this it is not difficult to check equimultiplicity. A completely different approach utilizes Zariski's original definition of equimultiplicity, ~~A comp~~ that of simultaneous resolution. Wahl introduced the functor ES of equitopological deformations of a good resolution which blow down to deformations of given singularity. A deep result of Laufer says that ES exactly describes the μ-constant deformations at least in dimension two. As an application one obtains some interesting partial results. But examples show that the topology of the resolution is not strong enough to control the multiplicity. Our new approach uses the concept of deformations of embedded resolutions. Then our main result characterizes equimultiplicity of μ-constant deformations of 2-dimensional isolated hypersurface singularities in terms of certain cohomological vanishing conditions. We can apply this criterion to the class of hypersurface singularities for which the Newton polyhedron gives rise to an embedded resolution via toroidal embeddings.

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Applications of homotopy to commutative algebra May 28, 1986

The usual algebraic derived functors, as $\mathrm{Tor}^R(M,N)$ or $\mathrm{Ext}_R(M,N)$, provide analogues for the homology and cohomology constructions of algebraic topology. However, although being functorial in both module arguments M and N, and in the ring argument R, Tor and Ext have no "nice" change of *rings* properties, hence are not easy to use in studying a homomorphism $\varphi : R \to S$ of commutative rings. In topology, the situation is remedied in part by the smooth behaviour of homotopy groups for a fibre sequence. The talk introduced an algebraic notion of homotopy groups, presented some basic properties of the construction, and gave some applications to problems in commutative algebra.

Definition. If R is a commutative ring, and k the residue field of $R_{\mathfrak{p}}$ for some prime $\mathfrak{p} \in R$, the graded vector space

$(I^{(2)})^{\perp} \subset \mathrm{Tor}^R(k,k)^{\vee}$ is called the homotopy of R (at $\mathfrak{p}$): here $I = \mathrm{Ker}[\mathrm{Tor}^R(k,k) \to k]$, $I^{(2)} = I^2$ + the span of the divided powers $\gamma^j(x)$ (x of even degree >0, $j \geq 2$), and $(\ )^{\vee}$ denotes vector space duals.

This is motivated by the Milnor - Moore theorem: for a 1-connected finite CW complex X, $\pi_*(\Omega X) \otimes \mathbb{Q} = (I^2)^{\vee}$, $I = \mathrm{Ker}\, H^*(\Omega X, \mathbb{Q}) \to H^0(\Omega X, \mathbb{Q})$. It was shown that - extending the category of commutative rings to that of differential graded algebras with divided powers - the basic constructions of homotopy theory (homotopy pushouts, fibres, etc.) can be carried over. For example, the fibre F of $\varphi: R \to S$ ($\varphi(\mathfrak{p}) \subset \mathfrak{q}$, $k = k(\mathfrak{p})$, $\ell = k(\mathfrak{q})$) is a DGΓ algebra whose homology is $\mathrm{Tor}^R(S,k)$. As a sample of the results obtained, we cite:

<u>Theorem</u>. Let $R \xrightarrow{\varphi} S \xrightarrow{\psi} T$ be local homomorphisms of local rings, such that φ, ψ, and $\psi\varphi$ are essentially of finite type. Then φ <u>and</u> ψ are l.c.i. maps, if and only if $\psi\varphi$ is l.c.i and flat $\dim_S T < \infty$.

<u>Theorem</u>. (with. H.-B. Foxby). Let $\varphi: R \to S$ be a homomorphism of finite flat dimension. Then the equality $I_S = I_R I_F$ holds for the Bass series ($I_R(t) = \sum \dim \mathrm{Ext}^i_R(k,k) t^i$, etc.).

<u>Theorem</u>. Let $I \subset J$ be ideals of the local ring R, such that $\mathrm{pd}_{R/I} R/J < \infty$. Then $\nu(J) = \nu(I) + \nu(J/I)$, with ν = minimal number of generators.

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Nonisolated hypersurface singularities 30 may '86.

We denote by X_f the Milnor fibre of a germ of an analytic function $f: (\mathbb{C}^{n+1}, 0) \to (\mathbb{C}, 0)$. It is a general question to compute the homotopy type of X_f and to give algebraic descriptions of the bettinumbers of X_f.

If f has an isolated singularity at 0 then $X_f \sim S^n \vee \dots \vee S^n$ a wedge of n-spheres, ~~its~~ the number of S^n's is denoted by μ, the Milnor number, one has $\mu = \dim_{\mathbb{C}} \mathbb{C}\{z\}/\left(\frac{\partial f}{\partial z}\right)$

In general the following is known:

1) X_f is homotopy equivalent with a finite CW complex of dim n. (Lê, Hamm)

2) If Σ is the singular locus of f has dim σ then X_f is $(n-\sigma-1)$-~~connected~~ connected. (~~Lê~~ Kato, Matsuoka)

3) If $f^{-1}(0)$ has only normal crossings in codim 1 then $\pi_1(X_f)$ is abelian. (Lê, Saito)

~~From now on we assume Σ is 1dimensional~~

<u>ref</u> A_∞ : $f(x, y_1, \dots, y_n) = y_1^2 + \dots + y_n^2$ $\qquad \Sigma = V(y_1, \dots, y_n)$

D_∞ : $f(x, y_1, \dots, y_n) = xy_1^2 + y_2^2 + \dots + y_n^2$ $\qquad \Sigma = V(y_1, \dots, y_n)$

A_1 : $f(y_0, y_1, \dots, y_n) = y_0^2 + \dots + y_n^2$ $\qquad \Sigma = \{0\}$.

let $J_f = \left(\frac{\partial f}{\partial z}\right)$ be the Jacobi ideal of f. let $I = \operatorname{rad}(J_f)$.

Assume from now on $\Sigma = V(I)$ is 1dimensional.

Prop. 1 If moreover Σ is a complete intersection and $\dim_{\mathbb{C}} I/J_f < \infty$ then there exists a deformation $\{f_t, \Sigma_t\}$ of (f, Σ) such that for all $t \neq 0$ and t small:

i) Σ_t is a smooth curve

ii) f_t has only A_1 singularities outside Σ_t and only A_∞ or D_∞ singularities on Σ_t.

Prop. 2 (Siersma) Under the assumptions of Prop. 1 we have

i) $X_f \sim \underbrace{S^n \vee \dots \vee S^n}_{\mu}$ with $\mu = \#A_1 + 2\#D_\infty + \mu(\Sigma, 0) - 1$

in case $\#D_\infty > 0$

ii) $X_f \sim S^{n-1} \vee \underbrace{S^n \vee \dots \vee S^n}_{\mu}$ with $\mu = \#A_1 + \mu(\Sigma, 0)$

where $\mu(\Sigma, 0)$ is the Milnor number of $(\Sigma, 0)$

Remark 1) $\mu(\Sigma, 0)$ can be computed by means of artinian local rings (Lê, Greuel)

2) In case I is generated by the regular sequence $g_1, \dots, g_n$ then $f = \sum_{ij} h_{ij} g_i g_j$ with $h_{ij} = h_{ji}$ and $\#D_\infty = \dim_{\mathbb{C}}\left(\mathbb{C}\{z\}/I + \det(h_{ij})\right)$

3) It was conjectured by Siersma that $\dim_{\mathbb{C}}(I/J_f) = \#A_1 + \#D_\infty$
This conjecture can be proved by using (iii) of the following

<u>Prop 3</u> Let R be a commutative noetherian ring. Let I and J be ideals in R. Let $J \subseteq I$.

i) If R is a local CM ring and height $I = n$. If J is generated by $n+1$ elements and $J = I \cap \mathfrak{a}$ height $\mathfrak{a} \geq n+1$ then I/J is CM.

ii) If I is a perfect ideal of grade n and grade$(I/J) \geq n+1$ and J is generated by $n+1$ elements then I/J is a perfect module.

iii) If I is generated by an R-sequence of length n and J is generated by m elements, $m \geq n$ and grade $I/J \geq m$ then I/J has a free resolution of length m.

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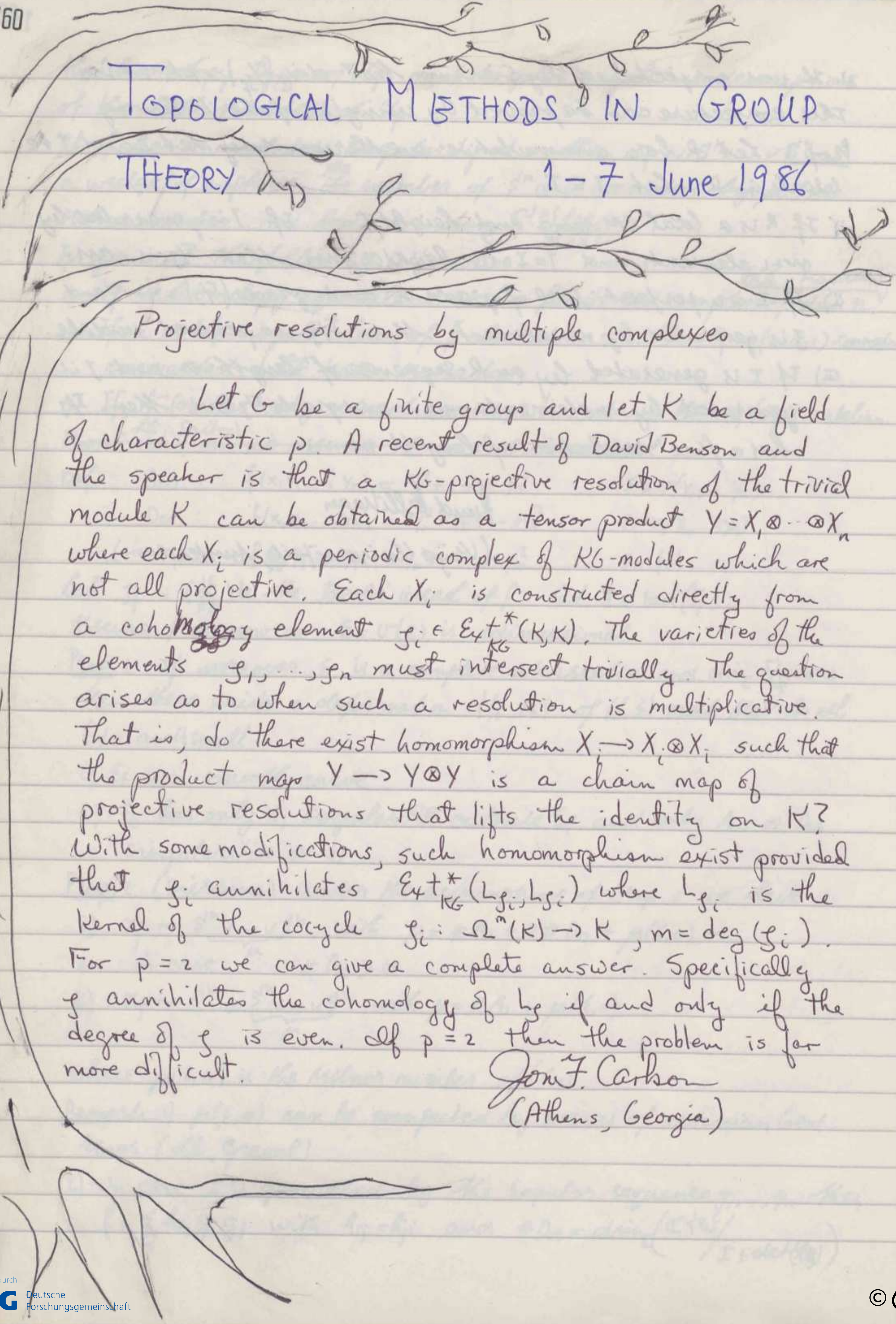

TOPOLOGICAL METHODS IN GROUP THEORY 1 – 7 June 1986

Projective resolutions by multiple complexes

Let G be a finite group and let K be a field of characteristic p. A recent result of David Benson and the speaker is that a KG-projective resolution of the trivial module K can be obtained as a tensor product $Y = X_1 \otimes \cdots \otimes X_n$ where each X_i is a periodic complex of KG-modules which are not all projective. Each X_i is constructed directly from a cohomology element $\zeta_i \in \mathrm{Ext}^*_{KG}(K,K)$. The varieties of the elements $\zeta_1, \dots, \zeta_n$ must intersect trivially. The question arises as to when such a resolution is multiplicative. That is, do there exist homomorphism $X_i \to X_i \otimes X_i$ such that the product map $Y \to Y \otimes Y$ is a chain map of projective resolutions that lifts the identity on K? With some modifications, such homomorphism exist provided that ζ_i annihilates $\mathrm{Ext}^*_{KG}(L_{\zeta_i}, L_{\zeta_i})$ where L_{ζ_i} is the kernel of the cocycle $\zeta_i : \Omega^m(K) \to K$, $m = \deg(\zeta_i)$. For $p = 2$ we can give a complete answer. Specifically ζ annihilates the cohomology of L_ζ if and only if the degree of ζ is even. If $p = 2$ then the problem is far more difficult.

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Sequences of cohomology groups.

A simplicial action of a finite group G on a simplicial complex Δ will be called admissible if for every simplex $\sigma \in \Delta$ the isotropy group G_σ fixes σ pointwise. We fix a prime p and let $\mathcal{C}$ be the class of subgroups $\{H \leq G \mid H$ has a normal p-subgroup $H_p \lhd H$ with H/H_p cyclic$\}$. Let $\mathcal{X}$ be a class of subgroups of G closed under subconjugation.

Theorem Let G act admissibly on a finite simplicial complex Δ and suppose that for all subgroups $H \in \mathcal{C}$ with $O_p(H) \notin \mathcal{X}$ the fixed points Δ^H are mod p acyclic. Then

a) For every $\mathbb{Z}G$-module M and integer $n \geq 1$ there are split exact sequences

$$0 \to H^n(G, \mathcal{X}; M)_p \longrightarrow \bigoplus_{\sigma \in \Delta_0/G} H^n(G_\sigma, \mathcal{X}_{G_\sigma}; M)_p \to \quad \cdots \quad \to \bigoplus_{\sigma \in \Delta_d/G} H^n(G_\sigma, \mathcal{X}_{G_\sigma}; M)_p \to 0$$

and another sequence with the same groups and the arrows in the reverse direction The notation indicates relative cohomology, and $\mathcal{X}_{G_\sigma} = \{H \in \mathcal{X} \mid H \leq G_\sigma\}$.

(b) In the case $\mathcal{X} = \{1\}$ the chain complex $C_\cdot(\Delta) \otimes_{\mathbb{Z}} \mathbb{Z}_p$ has a split acyclic subcomplex $D_\cdot$ so that in each dimension $C_r(\Delta) \otimes \mathbb{Z}_p = D_r \oplus P_r$ where P_r is a projective $\mathbb{Z}_p G$-module.

The above theorem applies with $\mathcal{X} = \{1\}$ to the simplicial complexes arising from the posets of p-subgroups of G, of elementary abelian p-subgroups of G, and of subgroups $H = O_p N_G(H)$.
The simplicial complexees arising from the posets
Y_k = p-subgroups of order $\geq p^{k+1}$
Z_k = subgroups $H = O_p N_G(H)$ of order $\geq p^{k+1}$
work in the theorem if we take $\mathcal{X}$ to be $\mathcal{X}_k$ = p-subgroups of order $\leq p^k$.
That's enough.

Peter Webb

Cyclic homology of groups and the Bass conjecture

For a $\mathbb{Q}$-algebra Λ we consider Hochschild homology $HH_i(\Lambda)$ and cyclic homology $HC_i(\Lambda)$, $i \geq 0$, and the Connes–Gysin exact sequence

$$(*) \quad \cdots \to HH_i(\Lambda) \to HC_i(\Lambda) \xrightarrow{S} HC_{i-2}(\Lambda) \to HH_{i-1}(\Lambda) \to \cdots$$

There exist character maps

$$ch^\ell : K_0(\Lambda) \longrightarrow H_{2\ell}(\Lambda), \quad \ell \geq 0$$

compatible with S. $ch^0 : K_0(\Lambda) \longrightarrow H_0(\Lambda) = \Lambda / \{\lambda\mu - \mu\lambda\}$ is the same as the Hattori–Stallings rank: $r_P = ch^0 P$ where P is a f.g. projective module over Λ representing an element of $K_0(\Lambda)$.

If $\Lambda = \mathbb{Q}G$ is the group algebra of a group G then $HH_i(\mathbb{Q}G)$ is easily seen to be $\bigoplus_{[x]} H_i(C_x; \mathbb{Q})$; the sum is over all conjugacy classes $[x]$ in G, and C_x the centraliser of a (fixed) $x \in [x]$. Burghelea (C.M.H. 1985) has given a similar $\bigoplus_{[x]}$-decomposition, the $[x]$-term being $H_i(C_x/\langle x\rangle; \mathbb{Q})$ if x is of infinite order (we omit here the terms for x of finite order). Moreover $(*)$ splits in exact sequences, one for each $[x]$.

We consider groups with $hd_{\mathbb{Q}} G = n < \infty$. If we can prove that for x of infinite order $H_i(C_x/\langle x\rangle; \mathbb{Q}) = 0$ for large i then it follows that all ch^ℓ, in particular the Hattori–Stallings rank r_P, has component 0 on the summand $[x]$. This can be proved for all x of infinite order in the following cases:

A) Solvable groups G with $hd_{\mathbb{Q}} G$ = Hirsch number = n finite

B) Linear groups $G \subset GL_n(F)$, char. $F = 0$, with $hd_{\mathbb{Q}} G = n$ finite

C) Groups with $cd_{\mathbb{Q}} G \leq 2$

This is a contribution towards the Bass conjecture which says that the Hattori-Stallings rank r_p always vanishes on elements of infinite order.

The method of proof for B) is to consider G_x/center which is again linear and to apply the R. Alperin-Shalen criterion to G_x/center; then one passes to $C_x/\langle x\rangle$. - In C) one uses the Bieri technique to prove that $C_x/\langle x\rangle$ has a graph-decomposition with finite edge- and vertex groups and thus is f.g. free-by-finite.

B. Eckmann

Homotopy Actions and Cohomology of Groups

Let X be a G-space, where G is a finite group, and suppose that X is connected, and the spectral sequence of the Borel constructions $E_A \times_A X \longrightarrow BA$ collapses for each p-elementary abelian subgroup $A \subseteq G$, $H^*(-;k)$-coefficients, $k = \overline{\mathbb{F}}_p$. Let $M = \bigoplus_{i>0} H^i(X;\mathbb{Z})$. <u>Theorem</u>: M is $\mathbb{Z}G$-projective iff $M|\mathbb{Z}C$ is $\mathbb{Z}C$-projective for each $C \subseteq G$ $|C| =$ prime.

<u>Applications</u>: Suppose $G \supset \mathbb{Z}_p \times \mathbb{Z}_p$ or Q_8. Then: (1) $\exists$ $\mathbb{Z}G$-module M such that M is not $\mathbb{Z}G$-isomorphic to the homology of any Moore space X with G-action (2) $\exists$ $\mathbb{Z}G$-module $M_1 \oplus M_2$ such that $M_1 \oplus M_2$ cannot be realized as H_* of Moore G-space (as in (1)) but $M_1 \oplus M_2 \cong \bigoplus_{i>0} H_i(X)$ for a G-space. (3) $\exists$ $\mathbb{Z}G$-module $M_1 \oplus M_2$, $M_i \neq 0$, such that $M_1 \oplus M_2$ is not $\cong H_*$ of any G-space at all.

The idea of the proof of the theorem is to construct a variety using J. Carlson's method for homotopy G-spaces, which coincides with the cohomological variety à la Quillen for G-spaces. Then investigation of these varieties yields the result.

A. Assadi

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Cohomological dimension of soluble groups.

If G is a countable group of finite cohomological dimension ($cd(G) < \infty$) then the homological dimension is also finite, and in fact $hd(G) \leq cd(G) \leq hd(G) + 1$. For soluble groups, a rather complete description of these dimensions can be given: If G is a soluble group then $hd(G) = cd(G)$ if and only if G is torsion-free and constructible. The constructible groups are those which can be built up from the trivial group by a series of ascending HNN-extensions or finite extensions. Their structure makes it possible to compute homology and cohomology using Meyer-Vietoris sequences, and hence to show that the two different dimensions are equal.

To show the converse, that $hd(G) = cd(G)$ forces G to belong to the very special class of constructible groups ([illegible]) requires further calculation. This can be done by computing the functor $H^{hd(G)+1}(G, -)$ on certain modules which are induced from 1-dimensional modules for nilpotent normal subgroups of G.

One reason why it is useful to understand $cd(G)$ for soluble G is the following. To determine the precise cohomological dimension of a group such as $SL_n(\mathbb{Z})$ one might use topological methods to show that $cd(SL_n(\mathbb{Z})) \leq \binom{n}{2}$ and then show that the bound is sharp by exhibiting the nilpotent subgroup $\begin{pmatrix} 1 & & * \\ & \ddots & \\ 0 & & 1 \end{pmatrix}$. It is quite common for groups Γ with $cd(\Gamma) < \infty$ to contain nilpotent or soluble subgroups S such that $cd(S) = cd(G)$. By contrast one can use the results on soluble groups to prove that if N is a <u>normal</u> nilpotent subgroup of Γ and $cd(N) = cd(\Gamma) < \infty$ then Γ must be soluble-by-finite

P. H. Kropholler

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Cyclic homology and idempotents in group rings

When a group G has an element x of order $n<\infty$, its group algebra kG (k - a field of char 0) has an idempotent: $e=\frac{1}{n}(1+x+\ldots+x^{n-1})\in kG$. On the other hand, if G has no torsion, we have the following long-standing

<u>Conjecture</u>: If a group G has no torsion then the only idempotents of kG are 0 and 1.

By Kaplansky Theorem it is enough to verify that whenever $e=e^2\in kG$ then $t_c(e)=0$ for all conjugacy classes c of G. Here $t_c: kG\longrightarrow k$, $t_c(\sum_{x\in G}a(x)x)=\sum_{x\in c}a(x)$.

To verify this condition cyclic homology of the group ring kG can be applied.

By Burghelea's Theorem

$$HC_*(kG)=\bigoplus_{c\in T_0G}H_*(G_c;k)\otimes HC_*(k)\oplus\bigoplus_{c\in T_\infty G}H_*(G_c;k)$$

where T_0G = the set of conj. classes of torsion elements

$T_\infty G$ = the set of conj. classes of elements of infinite order,

$G_c=C_G(z)/\langle z\rangle$ and z is any member of the class c.

Given an idempotent $e=e^2\in kG$ we produce a sequence of elements $e^{(n)}\in HC_{2n}(kG)$ $n=0,1,2,\ldots$ and homomorphisms $t_c^n: HC_{2n}(kG)\longrightarrow k$ such that:

i) $t_c^0=t_c$, $e^{(0)}=e$

ii) $t_c^n(e^{(n)})=t_c^{n+1}(e^{(n+1)})$ for all n.

The higher trace functions can be interpreted in terms of the Burghelea isomorphism. In particular, if $c\in T_\infty G$ then we have

$$\begin{tikzcd}
HC_{2n}(kG) \arrow[r] \arrow[dr, "t_c^n"'] & H_{2n}(G_c;k) \arrow[d] \\
 & k
\end{tikzcd}$$

Thus, if for some n we have $H_{2n}(G_c;k)=0$ then $\underset{t_c(e)}{\underset{\|}{t_c^n(e^{(n)})}}=0$ for any $e=e^2\in kG$.

As an example of an application we can give a two line proof of Formanek's Theorem: ~~torsion~~ If G is polycyclic-by-finite and torsion free then kG has no idempotents other than $0, 1$.

<u>Pf</u>: The groups G_c are polycyclic-by-finite as well. Moreover $hd_{\mathbb{Q}}G_c\le hd_{\mathbb{Q}}G=m<\infty$. For $2n>m$ we have $H_{2n}(G_c;k)=0$, hence the theorem follows $\square$

Zbigniew Marciniak

Valuations on groups

I introduce a notion of valuation on a group in order to compute the Bieri-Neumann-Strebel invariant Σ associated to a finitely generated group. A *valuation* (real-valued, for simplicity) is a function $v: G \to \mathbb{R} \cup \{+\infty\}$ such that there is a homomorphism $\chi: G \to \mathbb{R}$ satisfying: (a) $v(1) = \infty$ and $v(g) < \infty$ for some g; (b) $v(g^{-1}) = v(g) + \chi(g)$; (c) $v(gh) \geq \min\{v(g), v(h) - \chi(g)\}$. The homomorphism χ is then unique. v is *non-trivial* if it is not bounded below on ker χ.

Valuations arise naturally when one classifies abelian actions of G on $\mathbb{R}$-trees.

Theorem. If G is finitely generated, then the following conditions are equivalent for a non-zero $\chi: G \to \mathbb{R}$:

(i) $[\chi] \notin \Sigma$.

(ii) There is a non-trivial valuation v with χ as associated homomorphism.

(iii) There is a non-trivial abelian G-$\mathbb{R}$-tree (with right G-action) such that $\chi(g)$ describes the translation by g away from the fixed end.

As an application I compute Σ (and hence the finitely generated normal subgroups with abelian quotients), for an arbitrary one-relator group.

K. S. Brown

A Geometric invariant of discrete groups

This is joint work with R. Bieri and R. Strebel.
Let G be a finitely generated group, A a finitely generated G-operator group, such that G' acts on A by A-inner automorphisms. Define:

$$S(G) = (\mathrm{Hom}(G,\mathbb{R}) - \{0\}) / \mathbb{R}^*_+ \qquad (\cong S^{n-1} \text{ where } n = \mathrm{rank}_{\mathbb{Z}}(G/G'))$$

$$\Sigma_A(G) = \{[x] \in S(G) \mid A \text{ is finitely generated over some } \underline{\text{finite}} \text{ set of operators from } x^{-1}(\mathbb{R}_+)\}.$$

Theorem 1 $\Sigma^1_A(G)$ is open in $S(G)$

Theorem 2 For $H \leq G$ define $S(G,H) = \{[x] \in S(G) \mid x(H) = 0\}$. Then A is finitely generated as an H-group $\longleftrightarrow$ $S(G,H) \subseteq \Sigma_A(G)$

Corollary G finitely generated. $\Sigma^1 = \Sigma^1_{G'}(G)$. Then for $G' \leq N \leq G$, N is finitely generated $\Longleftrightarrow$ $S(G,N) \subset \Sigma^1$

Theorem 3 Let $f: H \to G$ be a homom., A a f.g H-group & G-group where the actions are compatible with f. Then

$$f^* \Sigma^{1c}_A(G) = \Sigma^{1c}_A(H)$$

where Σ^c denotes complement of Σ^1 and $f^*: S(G) - S(G, \ker(f)) \to S(H)$ is the obvious map. (Functoriality Theorem)

Theorem 4. If G finitely presented and containing no nonabelian free subgroups then $\Sigma^1 \cup -\Sigma^1 = S(G)$ $\quad (\Sigma^1 = \Sigma^1_{G'}(G))$.

Corollaries: 1) G as in Th 4 and $\mathrm{rk}_{\mathbb{Z}}(G/G') \geq 2 \Rightarrow \exists\, N \trianglelefteq G$, $G/N \cong \mathbb{Z}$ with N f.g.

2) Non finite presentability of many groups.

For abelian A. Bieri & Groves have shown that $\Sigma^1_A(G)$ is rationally polyhedral. A stronger property is true for $\Sigma = \Sigma^1_{G'}(G)$ if G is a 3-manifold group. However Σ^1 is *not* always rationally polyhedral, "most" finitely generated subgroups of $\mathrm{Homeo}_{PL}([0,1])$ have $\Sigma_1^c = S(G) - \Sigma_1$ consisting of two <u>irrational</u> points. This is true in particular for a large class of finitely <u>presented</u> such subgroups studied by Ken Brown (who showed finite presentation). It is still open whether Σ_1 is always polyhedral for f.g or f.p groups.

Walter Neumann

Accessibility of Finitely Presented Groups and Related Topics

If K is a 2-complex, a pattern is a subset P of $|K|$ such that for each 2-simplex σ of K, $|\sigma| \cap P$ is a union of finitely many line segments joining distinct faces of σ, and if γ is a 1-simplex, $|\gamma| \cap P$ is finitely many points in the interior of $|\gamma|$.

Let P be a pattern and let D_P be its dual graph in K. If $H^1(K; \mathbb{Z}_2) = 0$, D_P is a tree. Corresponding to a pattern P is a map $f_P : K^1 \to \mathbb{Z}^+$ (non-negative integers) which satisfies simple numerical conditions on the faces of each 2-simplex. The map f_P uniquely determines P up to a homeomorphism of $|K|$ which maps each simplex into itself. Any ~~map which~~ map $f : K^1 \to \mathbb{Z}^+$ which satisfies these conditions is of the form f_P for some pattern P.

<u>Defn</u> A group G is almost finitely presented (a.f.p) if it acts freely on a 2-complex K so that $G \backslash K = L$ is a finite complex and $H^1(K; \mathbb{Z}_2) = 0$

Let G, K, L be as in this definition and let T be a G-tree. Choose a G-map $\alpha : K^0 \to VT$. For $\gamma \in K^1$ let $f(\gamma) = d(\alpha(u), \alpha(v))$ where u, v are the vertices of γ. Then $f = f_P$ where P is a G-pattern. Thus D_p is a G-tree T'. This gives a generalization of a result of Bieri-Strebel.

THEOREM. If T is a G-tree G a.f.p, then $\exists$ a G-tree T' and a G-morphism

$$\alpha : T' \to T$$

and the following conditions are satisfied

The edge stabilizers of T' are finitely generated

T' has at most $n(L)$ G-orbits of vertices with valency > 2. Here $n(L)$ is a constant associated with L.

One can deduce from this theorem that afp are accessible. Also a knot $(S^n \to S^{n+2}, n = 1, n \geq 3)$ can be written as a sum of indecomposables.

A sketch of a proof of the Stallings Structure Theorem for afp groups using patterns was also given

MJ Dunwoody

End invariants of finitely presented groups

This talk discusses two geometric invariants of finitely presented (f.p.) groups, and their relations to group cohomology. They are simple connectivity at ∞ and semistability at ∞. If a f.p. group G, is simply connected at ∞, then $H^2(G;\mathbb{Z}G) = 0$. Semistability at $\infty \Rightarrow H^2(G;\mathbb{Z}G)$ is free abelian.

A basic unsolved problem in cohomological group theory is: Are all f.p. groups G, such that $H^2(G;\mathbb{Z}G)$ is free abelian? The two results discussed are:

Theorem A: If all 1-ended f.p. groups are semistable at ∞, then all f.p. groups are semistable at ∞.

Theorem B. Assume G is f.p. solvable with derived series $G \rhd G^{(1)} \rhd \cdots \rhd G^{(n)} \rhd G^{(n+1)} = 1$. If $G^{(n)}$ contains an element of infinite order, then either G is simply connected at ∞ or G contains a subgroup A, of finite index in G, and A contains a finite normal subgroup N such that A/N is one of the groups $< x, y : x^{-1} y x = y^{P} >$ for some integer P.

Mike Mihalik

On the homology of the special linear group over a number field

Let F be a number field and $SL(F) := \lim_{\overrightarrow{n}} SL_n(F)$ its infinite special linear group. The integral homology groups $H_i(SL(F);\mathbb{Z})$ are in general not finitely generated.

Theorem 1. $H_i(SL(F);\mathbb{Z}) =$ (torsion group) $\oplus$ (free abelian group of finite rank), $\forall i \geq 0$.

The rank of $H_i(SL(F);\mathbb{Z})$ / torsion is given by Borel's computation of $H_*(SL(F);\mathbb{Q})$.

In order to prove this theorem we look at the simply connected ∞-loop space $BSL(F)^+$, which has the same homology as the group $SL(F)$. The assertion of the theorem follows from the description of the homotopy groups of $BSL(F)^+$ and from the comparison of $BSL(F)^+$ with a product of Eilenberg - MacLane spaces (using Postnikov k-invariants). The basic point of the proof is the fact that the k-invariants of $BSL(F)^+$ are cohomology classes of finite order. We prove actually a more general result:

Theorem 2. $\exists$ integers S_{n+1} $(n \geq 1)$ such that, $\forall$ connected ∞-loop space X one has: $S_{n+1}\, k^{n+1}(X) = 0$.

A consequence of Theorem 1 is the following

Corollary 3. $\forall i \geq 0$, $H^i(SL(F); \mathbb{Z})$ contains no ∞-divisible element except 0.

We then deduce from this corollary and results by Eckmann-Mislin the following result on Chern classes of representations of discrete groups:

Theorem 4. $\exists$ integers $E_F(i)$ such that, for any representation $\rho: G \to GL(F)$ of a discrete group G over a number field $F \subset \mathbb{C}$, the Chern classes $c_i(\rho)$ of the representation ρ satisfy: $E_F(i)\, c_i(\rho) = 0$ in $H^{2i}(G; \mathbb{Z})$, $\forall i \geq 1$.

This is true without any finiteness condition on the group G.

Dominique Arlettaz

Deficiency of Free Products and Homotopy Type of 2-Complexes

Generators of a free product can be transformed into the factors (Grushko). A corresponding question for 2-complexes is whether $K_\pi \simeq K_{\pi_1} \vee K_{\pi_2}$ for $\pi = \pi_1 * \pi_2$. Counterexamples to this splitting arise by a construction which also yields that the deficiency in general is not additive under the operation of forming the free product of groups. The factors may even be chosen to be finite abelian. There exist many presentations for the same examples which may contribute to the homotopy theory of 2-complexes (for instance whether homotopy type and simple homotopy type always coincide. (Joint work with Cynthia Hog & Martin Lustig)

W. Metzler

Torsion in the Homology of the Mapping Class Groups

Let $S_{g,r}$ be an oriented surface of genus g with r boundary components and let $\Gamma_{g,r}$ be the mapping class group of $S_{g,r}$, i.e.

$$\Gamma_{g,r} = \begin{array}{l}\text{isotopy classes of orientation preserving}\\ \text{homeomorphisms } S_{g,r}\circlearrowleft \text{ pointwise fixing } \partial S_{g,r}\end{array}$$

One can define a limit group $\Gamma = \varinjlim_{g} \Gamma_{g,1}$ which, by a theorem of J. Harer, satisfies $H_*(\Gamma) \cong H_*(\Gamma_{g,r})$ for all $r \geq 0$ providing $g >> *$. There is a natural homomorphism $\Gamma_{g,1} \longrightarrow GL_{2g}(\mathbb{F}_p)$ which takes a homeomorphism of $S_{g,1}$ to the induced map on $H_1(S_{g,1};\mathbb{F}_p) \cong \mathbb{F}_p^{2g}$. Passing to the limit groups and applying Quillen's plus-construction gives rise to a map $f_p : B\Gamma^+ \to BGL(\mathbb{F}_p)^+$. We prove

<u>Theorem</u>: If ℓ and p are odd primes such that p generates $(\mathbb{Z}/\ell^2)^*$, then f_p induces split surjections on the ℓ-primary torsion in the homology and homotopy. Hence $H_*(\Gamma;\mathbb{Z})_\ell$ (resp. $\pi_*(B\Gamma^+)_\ell$) contains a direct summand isomorphic to $H_*(GL(\mathbb{F}_p);\mathbb{Z})_\ell$ (resp. $K_*(\mathbb{F}_p)_\ell$).

The latter groups have been completely computed by Huebschmann (resp. Quillen).

R. Charney (joint work with R. Lee)

Homology and cohomology of locally supersoluble groups

There has been a good deal of work on the (co)homology of nilpotent groups, and more recently of locally nilpotent groups. The theorems assert the vanishing of (co)homology groups in all dimensions provided they vanish in dimension 0 and the module satisfies appropriate finiteness conditions. Corresponding results are announced for locally supersoluble groups G where the module has no non-zero cyclic $\mathbb{Z}G$-submodules or $\mathbb{Z}G$-quotients.

Derek Robinson

Automorphism Groups of Free Products

Let $G = \mathop{\ast}\limits_{i=1}^{n} G_i$ be the free product of groups G_i which are indecomposable with respect to free products and are not infinite cyclic. We examine the structure of Aut G and present evidence for the conjectures:

1) If all G_i and all Aut G_i are torsion free then Aut G is torsion-free-by finite.

2). If all G_i are finite then [illegible] Aut G is again torsion free-by-finite and vcd(Aut G) = n - 1.

These conjectures are easily seen to be valid when n = 2. and ~~Ra~~ rewriting processes for subgroup presentations show that 2) is valid when the groups G_i are finite abelian.

Donald Collins (joint work with N.D. Gilbert).

Preferred points on hyperbolic surfaces

Finitely many preferred points on an orientable surface of small genus g (≤ 2), equipped with a hyperbolic structure, are exhibited as the only possible intersection points of simple geodesics with $\overset{\text{geometric}}{\text{intersection}}$ number 1. No analogous statement can be true for surfaces of higher genus. For M_2 (closed, genus = 2) the preferred points coincide with the six Weierstraß points on M_2 (interpreted as Riemann surface).

As algebraic consequence one obtains (beside a new criterion for simple curves on M_2 and for primitive elements in $F(a,b)$) the following "virtual" splitting:

$$\mathrm{Aut}\,\pi_1 M \overset{\text{finite index}}{>} i\,\mathrm{Aut}\,\pi_1 M \rtimes \mathcal{OS}(\pi_1 M)$$

with $M \in \left\{ \substack{\text{closed} \\ \text{genus } 2}, \substack{\text{1-bounded} \\ \text{genus } 1} \right\}$, $\mathcal{OS}$ subgroup of finite index in $\mathrm{Out}\,\pi_1 M$.

M. Lustig

On the nonvanishing of Ext between simple modules over a finite group. (joint work with Peter Linnell)

Let G be a finite group, p a prime with $p / |G|$, k a field with char $k = p$, and let kG denote the groupalgebra.

<u>Thm.</u> A Let G be p-constrained. Then for any simple kG-modules M_1, M_2 in the principal block of kG there exists $n \geq 0$ with $\mathrm{Ext}^n_{kG}(M_1, M_2) \neq 0$.

<u>Thm</u> B Let G be p-solvable. Then for any simple kG-modules in the same block there exists $n \geq 0$ with $\mathrm{Ext}^n_{kG}(M_1, M_2) \neq 0$

As an application we obtain:

<u>Prop.</u> Let G be p-solvable with $O_{p'}G = e$, and let φ a nontrivial automorphism of G with $(|\varphi|, |G|) = 1$. Then

$$1 \neq \varphi^* : H^*(G, \mathbb{Z}/p\mathbb{Z}) \to H^*(G, \mathbb{Z}/p\mathbb{Z}).$$

Urs Stammbach
ETH · Zürich

Growth functions of amalgams

Suppose that a group G contains a finite subset S that generates it as a semigroup and doesn't contain 1. Such an S defines a (word) length function ℓ on G, and we let $G(z) = \sum_{g \in G} z^{\ell(g)}$ denote the corresponding growth function of G (with respect to S). The length function ℓ filters G so that the group algebra RG inherits the structure of a filtered R-algebra (R is any commutative ring with unit). gr RG will denote the corresponding (connected) associated graded R-algebra.

Suppose now that $G = G_1 *_A G_2$. We give conditions on the inclusions $A \hookrightarrow G_i$ $(i = 1, 2)$ which ensure that:

(a) $\mathrm{gr}\, RG \cong \mathrm{gr}\, RG_1 \coprod_{\mathrm{gr}\, RA} \mathrm{gr}\, RG_2$ as graded R-algebras

(b) $$\frac{1}{G(z)} = \frac{1}{G_1(z)} + \frac{1}{G_2(z)} - \frac{1}{A(z)}$$

One could expect that (b) might hold (under some restrictions) by recalling the (mysterious) fact that (sometimes!!) $\frac{1}{G(1)} = \chi(G)$, the Euler characteristic of G.

Then (b) evaluated at $z=1$ would give the valid formula: $\chi(G) = \chi(G_1) + \chi(G_2) - \chi(A)$. We discussed briefly how (b) is used to compute the growth function of the fundam. group of the orientable surface of genus g. I believe I'll soon have a formula for arbitrary genus.

Juan M. Alonso (Stockholm)

Semidihedral groups and Bott periodicity

Let $\mathcal{A}_n$ be the subalgebra of the Steenrod algebra generated by $Sq^1, Sq^2, \ldots, Sq^{2^n}$, and let SD_{16} be the semidihedral group of order 16, $SD_{16} = \langle x, y \mid x^8 = y^2 = 1, y^{-1}xy = x^3 \rangle$.

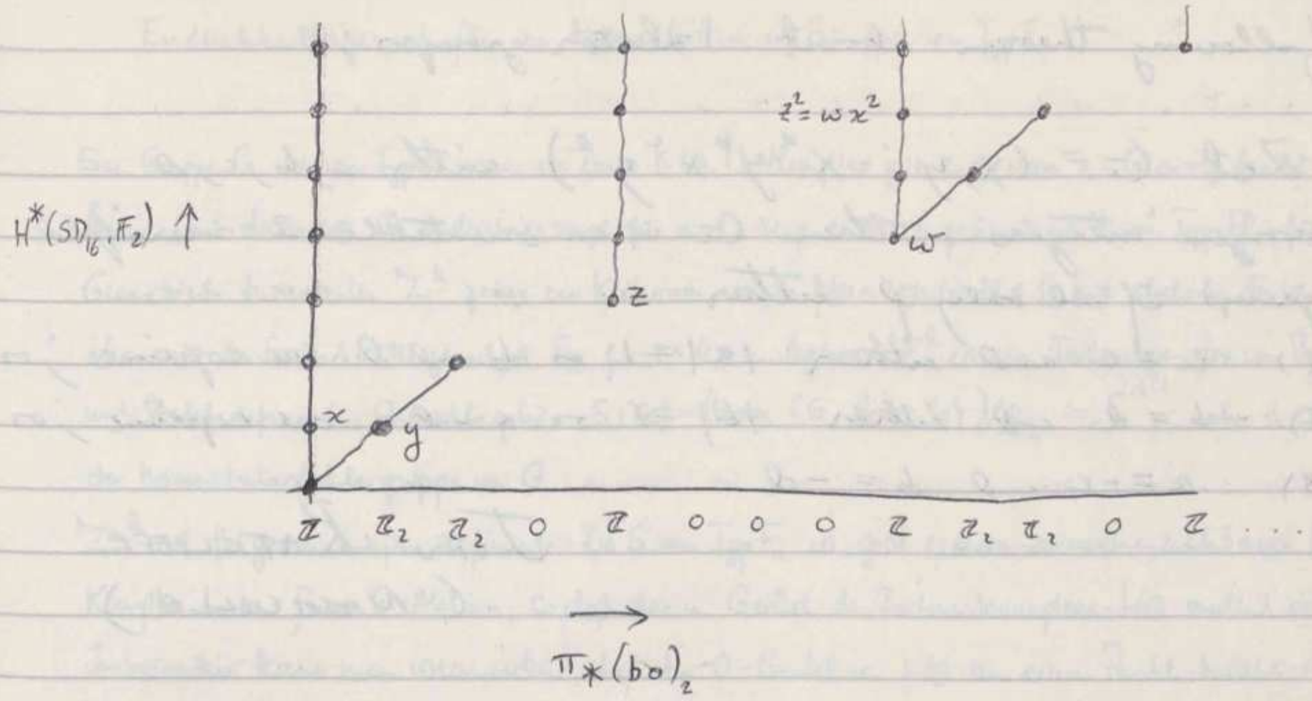
① Observation $\boxed{\mathcal{E}xt^*_{\mathcal{A}_1}(\mathbb{F}_2, \mathbb{F}_2) \cong \mathcal{E}xt^*_{SD_{16}}(\mathbb{F}_2, \mathbb{F}_2)}$

$$\cong \mathbb{F}_2[x, y, z, w]/(y^3, xy, yz, z^2 + wx^2)$$

Since $H^*(bo)_{(2)} \cong \mathcal{A} \otimes_{\mathcal{A}_1} \mathbb{F}_2$, we have an Adams spectral sequence

$$\mathcal{E}xt^{**}_{\mathcal{A}_1}(\mathbb{F}_2, \mathbb{F}_2) \cong \mathcal{E}xt^{**}_{\mathcal{A}}(H^*(bo)_{(2)}, \mathbb{F}_2) \Rightarrow \pi_*(bo)_2$$

This spectral sequence collapses ($E_2 = E_\infty$) and so we have a picture which horizontally depicts real Bott periodicity and vertically depicts cohomology of SD_{16}.



We conjecture that in some sense the same thing happens with $\mathcal{A}_n$, $n > 1$. In particular there should be a finite 2-group with the same cohomology as $\mathcal{A}_n$, and with the upper triangular $(n+2) \times (n+2)$-matrices as a quotient group.

Dave Benson

A uniqueness Theorem for amalgamated product decompositions for a groups

Sufficient conditions on a group G are given so that if G has two amalgamated product decompositions

$$G = A *_C B \quad \text{and} \quad G = A' *_{C'} B',$$

then the triple (A', C', B') is conjugate to (A, C, B) or (B, C, A). This uniqueness theorem is applied to prove a splitting theorem for a 3-manifold along an annulus. This splitting theorem is applied in turn to prove the following theorem about 1relator groups:

Thm Let $G = (x, y ; x^a y^b x^c y^d)$ with a, b, c, d nonzero integers. then G is an orientable 3-manifold group if and only if either

(1) $a = c$ and either $|a| = 1$ or b and d are coprime; or
(2) $b = d$ and either $|b| = 1$ or a and c are coprime; or
(3) $a = -c$ and $b = -d$.

JOHN RATCLIFFE
(NASHVILLE)

On groups with property $\mathcal{P}_1$

We say that a group G has $\mathcal{P}_1$ if there exists a $\mathbb{Z}G$-module A which satisfies i) $\text{p.d}_{\mathbb{Z}G} A \leq 1$, ii) $H^0(G, A) \neq 0$ and A is torsion-free as a $\mathbb{Z}$-module.

It is clear that if G has $\mathcal{P}_1$ then so does every subgroup K of G.

We show that if an infinite group G has $\mathcal{P}_1$ then $H^1(G, P) \neq 0$ for every projective $\mathbb{Z}G$-module P. We then prove:

A: If $G = \varinjlim_{i \in I} G_i$ where G_i are fin. gen. accessible subgrps of G, $|I| = \aleph_n$ and G has $\mathcal{P}_1$ then G is the fundamental group of a graph of finite grps.

As a corollary we obtain that a torsion free group G with $|G| = \aleph_n$ and $\mathcal{P}_1$ is free.

B: If a torsion group G has $\mathcal{P}_1$ then G is a countable locally finite group.

C Countable locally finite grps have $\mathcal{P}_1$

D Certain (may be all?) groups of period q after 1-step have $\mathcal{P}_1$.

Note that B + C implies that a torsion group G is a countable locally finite group iff it has $\mathcal{P}_1$.

Olympia Talelli
(Athens - Greece).

Endlichkeitseigenschaften von Normalteilern in Gruppen vom Typ F_n.

Eine Gruppe G ist vom Typ F_n, wenn sie einen $K(G,1)$-Komplex mit endlichem n-Gerüst besitzt. Äquivalent dazu ist G endlich erzeugt für $n=1$, resp. endlich präsentiert & vom Typ FP_n für $n \geq 2$. Geometrische Invarianten ${}^*\Sigma^k$ geben ein Kriterium, welche Normalteiler $N \trianglelefteq G$ mit abelscher Faktorgruppe ihrerseits die Endlichkeitseigenschaft F_k für $1 \leq k \leq n$ haben. ${}^*\Sigma^k$ ist eine Teilmenge der von Bieri und Strebel definierten Charaktersphäre $S(G) = (\mathrm{Hom}(G, \mathbb{R}) \setminus \{0\})/\mathbb{R}_+ \simeq S^{d-1}$, wobei $d = \mathbb{Z}$-Rang G/G', der Kommutatorfaktorgruppe von G.

${}^*\Sigma^k$ ist folgendermaßen definiert. Da G vom Typ F_n ist, gibt es einen zusammenziehbaren CW-Komplex L mit freier G-Aktion, so daß das n-Gerüst des Bahnenkomplexes L/G endlich ist. Insbesondere kann man voraussetzen, daß das 0-Gerüst von L/G aus einem Punkt besteht, d.h. $L^0 = \{v_g \mid g \in G\}$. Sei nun $\chi : G \to \mathbb{R}$ Repräsentant eines Charakters $[\chi] \in S(G)$. $L_{\chi,r}$ ist dann definiert als der volle Unterkomplex von L erzeugt durch $\{v_g / \chi(g) \geq -r\}$. $L_{\chi,0}$ heiße im wesentlichen k-zusammenhängend, wenn es ein $r > 0$ gibt, so daß die durch die Inklusion induzierte Abbildungen $\pi_0(L_{\chi,0}) \to \pi_0(L_{\chi,r})$ und $\pi_i(L_{\chi,0}, v_1) \to \pi_i(L_{\chi,r}, v_1)$ für $1 \leq i \leq k$ trivial sind. ${}^*\Sigma^k$ ist dann $\{[\chi] \in S(G) / L_{\chi,0}$ ist im wesentlichen $(k-1)$ zusammenhängend$\}$

Die Definition ist unabhängig von der Wahl von L. Es gilt dann: ${}^*\Sigma^k$ ist offene Teilmenge von $S(G)$ und der Satz: Sei $N \trianglelefteq G$, $G' \leq N \leq G$. Dann gilt:

$$N \text{ ist vom Typ } F_k \Leftrightarrow {}^*\Sigma^k \supseteq S(G,N) = \{[\chi] \in S(G) / \chi(N) = 0\}.$$

${}^*\Sigma^k$ verallgemeinert die geometrische Invariante Σ, definiert von Bieri, Neumann und Strebel. Als Korollar des Satzes ergibt sich, daß die Endlichkeitseigenschaft F_k für Kerne von Epimorphismen von G auf $\mathbb{Z}^j$ eine offene Bedingung ist. Dies wurde (mit anderen Methoden) für F_2 von Fried und Lee bereits bewiesen.

Burkhardt Renz

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Finiteness properties of S-arithmetic groups

A group Γ is called of type FP_n if the trivial $\mathbb{Z}\Gamma$-module $\mathbb{Z}$ has a projective resolution $\to P_n \to \cdots \to P_0 \to \mathbb{Z} \to 0$ with P_i finitely generated $\mathbb{Z}\Gamma$-modules for $i \le n$. A group is FP_1 iff it is finitely generated. It is FP_2 if it is finitely presented. The converse is an open problem. So a group Γ is defined to be of type F_n if, for $n=1$ it is of type FP_1, and for $n \ge 2$ if it is finitely presented and FP_n. Existing results concerning the question which S-arithmetic groups are F_n where surveyed. The following new results where presented

- Let G be a k-split solvable linear algebraic group over a number field k. An S-arithmetic subgroup Γ of G_k is finitely presented iff $\Sigma^c \cap -\Sigma^c = \emptyset$ for the Bieri-Strebel invariant Σ and $H_2(\Gamma,\mathbb{Z})$ is finitely generated (Abels).
- $SL_n(\mathbb{F}_q[t])$ is F_{n-2} not F_{n-1} for any n and any finite field with $q \ge 2^{n-2}$ (Abels, for Abramenko's result see below).

Length functions of group actions on Λ-trees

This reports on work of Chiswell, Culler, Shalen, Morgan, R. Alperin & Bass, expanding the themes of Serre's book, Trees. It is motivated by applications to hyperbolic geometry (Morgan-Shalen).

Let Λ be a tot. ordered abelian group. A Λ-metric space X, $d: X\times X \to \Lambda$, is a Λ-tree if (1) given $x,y \in X$, $\exists$ metric $\alpha: [0,d(x,y)] \to X$ s.t. $\alpha(0)=x$, $\alpha(d(x,y))=y$; (2) If $\alpha:[0,a]\to X$ and $\beta:[0,b]\to X$ are metric and $\alpha(0)=\beta(0)$ then $\{t \mid \alpha(t)=\beta(t)\}=[0,c]$ for some c (hence α in (1) is unique; denote its image $[x,y]$); (3) If $[x,y]\cap[y,z]=\{y\}$ then $[x,y]\cup[y,z]=[x,z]$.

Example. $\Lambda=\mathbb{Z}$, X is a simplicial tree. Then $V(X)$, with path length distance, is a $\mathbb{Z}$-tree.

The tree X_v: Let $v: F^\times \to \Lambda$ be a valuation on a field F. Let $X=X_v$ be the set of homothety classes (L) of $\mathcal{O}_v$-lattices L in $V=F^2$. If L and L' are lattices $\exists$ a basis e,f of L and $a,b\in F^\times$, $v(a)\le v(b)$, s.t. ae, bf is a basis of L'. Then $d(L,L')=v(b)-v(a)$ depends only on $(L),(L')$. Theorem (X,d) is a Λ-tree, on which $GL_2(F)$ acts. In $SL_2(F)$ the point stabilizers are the $GL_2(F)$-conjugates of $SL_2(\mathcal{O}_v)$.

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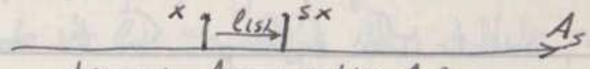
Lyndon length functions on a group Γ are functions $L:\Gamma\to\Lambda$ s.t. (0) $L(1)=0$; (1) $L(s)=L(s^{-1})$; and, putting $s\wedge t=\frac{1}{2}(L(s)+L(t)-L(s^{-1}t))$; (3) $s\wedge t\in\Lambda$; and (4) $s\wedge u\geq\min(s\wedge t, t\wedge u)$.

Theorem Let Γ act on a Λ-tree X, and $x_0\in X$. Put $L_{x_0}(s)=d(x_0,sx_0)$. Then L_{x_0} is a Lyndon length function, and every such arises from an essentially unique (X,x_0) as above.

Automorphisms Let X be a Λ-tree, $s\in\mathrm{Aut}(X)$. Call s elliptic if s has a fixed point, an inversion if s^2 is elliptic but s isn't, and hyperbolic if s^2 isn't elliptic. Suppose that s is not an inversion. Then $l(s)=\min\limits_{x\in X} d(x,sx)$ exists, and we put $A_s=\{x\mid d(x,sx)=l(s)\}$. For s elliptic, $l(s)=0$, $A_s=$ the tree of fixed points. For s hyperbolic, $l(s)>0$, and A_s is a linear tree ($\cong$ a subtree of Λ) on which s induces a translation of amplitude $l(s)$:



$d(x,sx)=l(s)+2d(x,A_s)$

For inversions s we agree to put $l(s)=0$, $A_s=\phi$.

Theorem If s,t are not inversions then $d(A_s,A_t)=\max(0,\frac{1}{2}(l(st)-l(s)-l(t)))$

Hyperbolic length functions. Let a group Γ act on a Λ-tree X. Then we have $l=l_X:\Gamma\to\Lambda$. This is the analogue for tree actions of the character of a linear representation. Example. Let $h:\Gamma\to\Lambda$ be a homomorphism. Then Γ acts on $X=\Lambda$ by translation via h: $s:x\to x+h(s)$, and $l_X(s)=|h(s)|$, clearly. Such length functions are called abelian. They can arise from tree actions that are geometrically much more complicated (cf. work of Ken Brown). In contrast, non abelian length functions essentially determine the geometry uniquely.

Theorem Let X be a Λ-tree on which Γ acts w/o inversions and with l_X non-abelian

(a) $\exists$ a unique minimal Γ-invariant subtree X_{min} (and $l_{X_{min}}=l_X$).

(b) Let Y be a Λ-tree on which Γ acts without inversions. If $l_Y=l_X$ then there is a unique Γ-equivariant isomorphism $X_{min}\to Y_{min}$.

The proof proceeds by carefully choosing a base point x_0, expressing L_{x_0} in terms of l_X, and then invoking the theorem on Lyndon length functions.

When is a function $l:\Gamma\to\Lambda$ of the form l_X as above? Necessary conditions: (I) $l(s)=l(s^{-1})$; (II) $l(sts^{-1})=l(t)$; (III) Either $l(st)=l(st^{-1})$ or $\max(l(st),l(st^{-1}))\leq l(s)+l(t)$; (IV) If $l(s)>0$ and $l(t)>0$ then $l(st)=l(st^{-1})\geq l(s)+l(t)$ or $\max(l(st),l(st^{-1}))=l(s)+l(t)$. Culler-Morgan conjectured (for $\Lambda=\mathbb{R}$) that these conds. are also sufficient. This was recently established by Walter Parry. Thus we can speak of "the space of hyperbolic length functions" on Γ, and this space parametrizes Λ-tree actions of Γ in some sense. A projectivized version of this space (for $\Lambda=\mathbb{R}$) is central in the work of Morgan-Shalen and Culler-Morgan.

Group theoretically, the fundamental problem is to find the group theoretic information carried by a Λ-tree action, analogous to Ch. I of Trees. E.g. what can be said about groups acting freely on $\mathbb{R}$-trees? This question, modulo language, goes back to Lyndon.

Hyman Bass
Columbia University

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FP_* - properties of $SL_n(\mathbb{F}_q[t])$

The following result can be proved:

Theorem: If $q \geq \binom{n-2}{[\frac{n-2}{2}]}$ then $SL_n(\mathbb{F}_q[t])$ is of type FP_{n-2}, but not of type FP_{n-1}.
(for the definition of FP_* see Abels above)

The basic tool in order to get some information about the FP_*-properties of S-arithmetic groups in the function field-case is the action of these groups on certain Bruhat-Tits buildings. In the case of $\Gamma = SL_n(\mathbb{F}_q[t])$ one can describe a very concrete model of the corresponding building. Furthermore, it exists a nice fundamental domain $\mathcal{F}$ for the action of Γ.

In order to use these facts to get finiteness properties of Γ, one has to filter X according to the following

Lemma 1 Suppose X is a Γ-complex with an increasing filtration $X = \bigcup_{j \in \mathbb{N}} X_j$ such that

1) X is contractible 2) The stabilizers Γ_σ are finite $\forall$ simplices $\sigma < X$

3) All X_j are Γ-invariant and the X_j / Γ are finite

4) All $X_{j+1} \supsetneqq X_j$ and v is not adjacent to v' $\forall\, v, v' \in X_{j+1} - X_j$ (v, v' are vertices)

5) $\forall\, v \in X_{j+1} - X_j$ $lk\, v \cap X_j$ is l-spherical and not contractable. ($\Rightarrow lk\, v \cap X_j \sim \bigvee S^l$)

Then the group Γ is of type FP_l, but not of type FP_{l+1}.

The proof of this lemma is given by applying a much more general necessary and sufficient criterion of Ken Brown. The crucial point in this lemma is condition 5) which is often hard to prove for a given filtration. In the case of $\Gamma = SL_n(\mathbb{F}_q[t])$ a Γ-invariant filtration can be given corresponding to a filtration of $\mathcal{F}$. The occurring links can be described as subcomplexes of the finite Tits building $\mathcal{T}(V)$, where $V \cong \mathbb{F}_q^n$.

They are all $(n-2)$-spherical, if the following subcomplexes of $\mathcal{T}(V)$ are $(n-2)$-spherical:

$T(E,V)$ is for a given $0 < E < V$ the subcomplex generated by the vertices $\{0 < U < V \mid U \cap E = 0 \vee U + E = V\}$

Lemma 2 $\dim E = k \wedge q \geq \binom{n-2}{k-1} \Rightarrow T(E,V)$ is $(n-2)$-spherical

The conclusion of Lemma 2 is not always true if $q < \binom{n-2}{k-1}$, for example $H_2(T(\mathbb{F}_2^2, \mathbb{F}_2^5)) = \mathbb{Z}/2\mathbb{Z}$

Now the Theorem is a consequence of Lemmas 1, 2 and the strange requirement $q \geq \binom{n-2}{[\frac{n-2}{2}]}$ in the Theorem is ~~explained~~ motivated.

Problem Is the statement of the Theorem true without restriction?

Peter Abramenko
(Frankfurt)

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A Kan-Thurston Theorem for duality groups:

A group G is in the class $\mathbb{D}_n$ if there exists a $\mathbb{Z}G$-module $\mathcal{D}$ and $e \in H_n(G;\mathcal{D})$ so that $-\cap e$: $H^k(G;B) \longrightarrow H_{n-k}(G; B\otimes_{\mathbb{Z}}\mathcal{D})$ is an isomorphism for any $\mathbb{Z}G$-module B and $BG = K(G,1)$ is homotopy equivalent to a finite complex of dim n. This is essentially the definition of a 'duality group' of R. Bieri and B. Eckmann. Let $\mathbb{D} = \cup_n \mathbb{D}_n$. We prove:

Theorem For any finite complex X, there exists $G_X \in \mathbb{D}$ and a map $f: BG_X \longrightarrow X$ such that $f_*: H_*(BG;B) \longrightarrow H_*(X;B)$ is an isomorphism for any ~~and~~ $\mathbb{Z}\pi_1(X)$-module B.

Problem Can ~~one~~ find G_X as above with $G_X \in \mathbb{D}(n)$, with $n =$ homotopy dim of X. ?

Jean-Claude Hausmann
(Genf)

IA-Outer Automorphisms of Free Groups

The IA-Outer automorphisms of a group G are those which act trivially on the abelianization of G. We denote this group $IAO(G)$. Let F_n denote the free group of rank n. We show:

Theorem. $IAO(F_3)$ is not finitely presented. In fact, $H_2(IAO(F_3);\mathbb{Q})$ and $H_3(IAO(F_3);\mathbb{Q})$ are both of infinite rank.
(Magnus showed that $IAO(F_n)$ is finitely generated for all n). This fact was suggested by the euler characteristic

calculation of Smillie and Vogtmann which revealed that $\chi(GL_3(\mathbb{Z})) \cdot \chi(IAO(F_3)) = 0 \neq \chi(Out(F_3))$ in spite of the short exact sequence $IAO(F_3) \rightarrowtail Out(F_3) \to GL_3(\mathbb{Z})$.

This is of interest because it suggests that the groups $IAO(F_n)$ may be examples of groups which are of type FP_m but not FP_{m+1}, where m increases with n. This phenomenon is known to occur among groups of arithmetic type. (See Abels, Abramenko, Bieri, Brown, Stuhler). It is suspected to occur for the Torelli groups $\mathcal{I}_n = IAO(\pi_1(S_n))$, S_n a surface of genus n. McCullogh-Miller and Mess have shown that $rk(H_1(\mathcal{I}_2)) = \infty$ and Mess showed that $rk(H_2(\mathcal{I}_2)) = \infty$.

Our method, inspired by Mess' work, involves a contractible space $\mathbb{X}_n$ upon which $Out(F_n)$ acts with finite stabilizers. $\mathbb{X}_n$ equivariantly deformation retracts to a complex K_n upon which $Out(F_n)$ acts with finite quotient. ~~We define~~ The quotients $\mathbb{X}_n/IAO(F_n)$ and $K_n/IAO(F_n)$ are classifying spaces for $IAO(F_n)$ and are $GL_n(\mathbb{Z})$-spaces. We define a $GL_n(\mathbb{Z})$-equivariant map from $\mathbb{X}_n/IAO(F_n)$ to Q_n, the space of $n \times n$ positive definite real symmetric matrices. The retract K_n maps to Soulé's retract of Q_n. For $n=3$, and no other n, the dimensions of these spaces are equal and the map is surjective. By analyzing the inverse images of points we are able to produce a nice filtration of $K_n/IAO(n)$ and compute its homology.

Marc Culler

Joint work with Karen Vogtmann

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Equations over groups and pictures

A picture arises in the following way: K, L are CW-complexes with $L = K \cup \{$1- and 2-cells$\}$; F is a compact orientable surface, and $f:(F, \partial F) \to (L, K \cup L')$ is a map of pairs. Up to homotopy, $f^{-1}(\text{int (2-cells in } L \setminus K))$ is ~~a~~ int(disjoint union of small discs in int F) — Each small disc is called a vertex of the picture P, and is mapped homeomorphically onto a 2-cell of $L \setminus K$. Similarly, $f^{-1}(\text{int (1-cells in } L \setminus K))$ is int (regular neighbourhood of a compact 1-manifold in $F \setminus$ (vertices)).

P contains other information also: orientations and labellings by elements of $\pi_1 K$. This allows information to be ~~app~~ obtained about (for example) the relationship between the groups $\pi_1 K$ and $\pi_1 L$. For example, if K is connected, it can sometimes be deduced that a system of equations over $\pi_1 K$ has a solution in an overgroup.

Application: Let $G = \frac{A * B}{s^m}$, a 1-relator product of groups A, B, where s is cyclically reduced of length ≥ 2, and $m \geq 4$. Then:

① (Freiheitssatz) $A \to G \leftarrow B$ are injective
② ~~A~~ The word problem for ~~A~~ G reduces to those for A, B.
③ A $K(G,1)$-space can be explicitly constructed.
④ $H^*(G)$ can be explicitly computed.
etc.

Jim Howie
Glasgow

Acyclic and abelian groups: McLain groups and Eilenberg-MacLane spaces

Baumslag, Dyer & Heller (1980) applied the Kan-Thurston theorem to the Eilenberg-Mac Lane space $K(G,2)$ to deduce the existence of an acyclic group (that is, one having the integral homology of the trivial group) whose centre is a given abelian group G. By a modification of McLain's unitriangular matrix groups over a linearly ordered set, an explicit construction of such an acyclic group is now given. In consequence one is able, by means of Quillen's plus-construction, to provide a "group-theoretic" model for $K(G,2)$. One also has a natural construction of a perfect group whose Schur multiplicator is G.

Two questions suggested by this work are:–

1. Which groups can be normal subgroups of an acyclic group?
2. Can one construct, for $n \geqslant 3$, groups G_n such that the plus-construction gives $K(G_n,1)^+$ as a $K(G,n)$?

Jon Berrick

Singapur

Cohomology of nilpotent groups

Let G be a nilpotent group and let $G = G_0 \supset G_1 \supset \ldots \supset G_n = e$ be a central series so that the associated graded group (say) A is abelian. Then G may be viewed as A together with a perturbation of its multiplication law. Accordingly, an appropriate perturbation applied to a free resolution for A yields a free resolution for G. Given a~~n~~ suitable free resolution for A together with a contracting homotopy, it is in fact possible to make this perturbation explicit. For small nilpotency class and few generators ~~is in fact so explicit~~ the resulting resolution is in fact so explicit that computations can be done by hand.

Johannes Huebschmann

Cohomology of moduli spaces of K3 surfaces of degree 2

A K3 surface S is by definition a nonsingular complex analytic variety of dimension 2 whose first Betti number and first Chern class are both zero. When S is projective, there is a hyperplane section class L in the second cohomology $H^2(S;\mathbb{Z})$ such that the valuation of the square L^2 of this class against the fundamental cycle $[S]$ is always a positive even integer, known as the degree of the surface S. One of the striking results in algebraic geometry was the result of Piatetski-Shapiro and Shafarevich that the moduli space K_d of K3 surfaces of degree d is isomorphic to the arithmetic quotient space D/Γ_d. Here D is the bounded hermitian domain associated to $SO(2,19;\mathbb{R})$ and Γ_d is the automorphism group of $H^2(S;\mathbb{Z})$ which preserves the intersection pairing and the Chern class L. From this, it follows that the real cohomology of K_d is the same as the cohomology of the discrete group Γ_d. In the case $d=2$ this moduli space K_2 can be constructed explicitly. Using the work of Frances C. Kirwan on "Cohomology of quotients in symplectic and algebraic geometry", we computed the cohomology of this moduli space.

Ronnie Lee

(Joint work with F. Kirwan)

Group automorphisms inducing the identity map on cohomology.

Let G be a finite group. The automorphisms of G which induce the identity map on integral cohomology of G form a subgroup $\mathrm{Aut}^*(G)$ of the group of all automorphisms $\mathrm{Aut}(G)$. The Atiyah spectral sequence which relates cohomology of a group to its complex representation ring suggests that $\mathrm{Aut}^*(G)$ is related to the subgroup $\mathrm{Aut}_c(G) \subset \mathrm{Aut}(G)$ consisting of all automorphisms preserving conjugacy classes of elements of G. Unfortunately $\mathrm{Aut}^*(G) \not\subset \mathrm{Aut}_c(G)$. However we proved that any $\varphi \in \mathrm{Aut}^*(G)$ preserves conjugacy classes of elements of prime order and conjugacy classes of elementary abelian subgroups. Moreover assuming that $\varphi^n = \mathrm{Id}$ ~~where~~ $(n, |G|) = 1$ then φ preserves conjugacy classes of elements of prime power order. For solvable groups we deduce from that $|\mathrm{Aut}^*(G)|$ is divisible only by primes dividing also $|G|$. Same result for the group $\mathrm{Aut}_c(G)$ (without solvability assumption) goes back to Burnside.

A proof of the main theorem uses the Atiyah spectral sequence mentioned above and Quillen's description of prime ideals in the cohomology ring.

The results will appear in a joint paper with Zbigniew Marciniak.

Stefan Jackowski

The Euler Characteristic of the outer automorphism group of a free group

Let $\Gamma_n = \mathrm{Out}(F_n)$ be the group of outer automorphisms of a free group on n generators. Baumslag and Taylor have shown that the natural map from $\mathrm{Out}(F_n)$ onto $GL_n(\mathbb{Z})$ has torsion-free kernel, so any torsion-free subgroup of finite index in $GL_n\mathbb{Z}$ pulls back to a torsion-free subgroup of finite index in Γ_n. Culler and Vogtmann have shown that Γ_n has finite virtual cohomological dimension by producing a $(2n-3)$-dimensional contractible simplicial complex of "marked graphs" G with $\pi_1 G \cong F_n$, on which Γ_n acts with finite stabilizers and finite quotient. Thus the rational Euler characteristic $\chi(\Gamma_n)$ is defined, and can be computed by choosing a set of representatives S for the simplices of K_n modulo Γ_n. Then

$$\chi(\Gamma_n) = \sum_{\sigma \in S} \frac{(-1)^{\dim \sigma}}{|\mathrm{stab}\,\sigma|}.$$

This talk was a report on joint work with John Smillie. We first use the above formula to produce a generating function for $\chi(\Gamma_n)$ by using techniques from combinatorial enumeration, starting from Cayley's theorem that there are n^{n-2} vertex-labelled trees on n vertices. The generating function can then be fed to a computer. Examination of the results shows patterns reminiscent of Bernoulli numbers. By studying automorphisms of graphs G with $\pi_1 G \cong F_n$, we find a bound on the power of p which appears in the denominator of $\chi(\Gamma_n)$, which is exact if $(p, n-1) = 1$ and $n \equiv 0 \pmod{p-1}$.

Karen Vogtmann

(joint with John Smillie)

Groups, Graphs and Property τ

For a homogeneous graph $X=(V,E)$ consider $\lambda_1(X)$ the first non-zero eigenvalue of $kI-\Delta$ where $k=$ degree of homogeneity and $\Delta f(v)=\sum_{(w,v)\in E} f(w)$ and also $\mu_1(X)$ the next largest eigenvalue of Δ. The graph X is an expander if $|\partial A|\geq c|A|$ for all $A\subset V$, $|A|\leq \frac{n}{2}$ where $\partial A=\{v \mid d(v,A)=1\}$

Proposition (Alon) : $c\geq \frac{\lambda_1}{2k}$

Suppose that $X_{n,k}$ is any valence k-graph on n vertices

Theorem (Alon-Boppana) $\varliminf_{n\to\infty} \mu_1(X_{n,k})\geq 2\sqrt{k-1}$

def: (Lubotzky-Phillips-Sarnak) A family of valence k-graphs $\{X_{n,k}\}_{n\to\infty}$ is a Ramanujan family if $\mu_1(K_{n,k})\leq 2\sqrt{k-1}$ all n

~~Proposition~~ Theorem: Let $\Lambda(2q_0)$ be the congruence subgp of level $2q$ in $\Lambda=\text{im } \mathbb{H}(\mathbb{Z}[\frac{1}{p}])^* \to GL_2(\mathbb{Q}_p)$ and T_p the Bruhat-Tits Tree of valance $p+1$ Then $\{T_p/\Lambda(2q)\}_q$ is a Ramanujan family of graphs

In this case these graphs are the same as the Cayley graphs of the finite groups $\Lambda(2)/\Lambda(2q_0)$

Proposition: (Alon-Milman) Γ a group with generating set $S=S^{-1}$, $|S|=k$, $\{N_i\}$ a family of normal subgps of finite index. Then the following are equivalent

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1) $\{\overset{G_i}{\overset{\shortparallel}{\text{Cayley Graph}}}(\Gamma/N_i, S)\}$ is a family of expanders with $c > 0$

2) $\lambda_1(G_i) \geq c_2 > 0 \quad \forall i$

3) Γ has property τ with respect to the family of unitary reps induced from the trivial rep. on N_i

i.e. $\pi: \Gamma \to \Gamma/N_j \to$ Unitary Group $\exists j$.

defn: Γ has property τ with respect to a family of unitary reps if the 1 rep is isolated with respect to the family in the topology of $\hat{\Gamma}$

Problem: Can a solvable group Γ admit a family of finite index subgps N_i, $|\Gamma/N_i| \xrightarrow[i\to\infty]{} \infty$ so that Γ has property τ with respect to this family

Problem: Give other interesting examples of Ramanujan families.

Roger Alperin

Higher geometric invariants for groups

Joint Work with Burkhardt Renz.

Let G be a group. The set $S(G) = \text{Hom}(G, \mathbb{R}_{ab}) \setminus \{0\}$ / mult with $r > 0$ is isomorphic to the sphere S^{n-1}, $n = \text{rk}\, G/G'$. If G is of type FP_n (i.e., admits a free resolution $\underline{\underline{F}} \twoheadrightarrow \mathbb{Z}$ which is finitely generated in all dimensions $\leq n$) we can associate to it a certain subset $\Sigma^n \subseteq S(G)$ which is defined in terms of $\underline{\underline{F}}$ but is independant of all choices.

Theorem 1 Σ^1 coincides with the invariant $\Sigma_{G'}$ (see p. 167)

Theorem 2 Σ^n is open in $S(G)$

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Theorem 3. If N is a normal subgroup in G with Abelian factor group G/N and $S(G,N)$ stands for the subsphere $\{[\chi] \in S(G) \mid \chi(N)=0\}$ of $S(G)$ then N is of type $(FP)_n \Leftrightarrow S(G,N) \subseteq \Sigma^n$.

Corollary: The set of all normal subgroups of type $(FP)_n$ with $G/N \cong \mathbb{Z}^m$ is an open subset of the set of *all* normal subgroups of G with factor group $\cong \mathbb{Z}^m$.

Robert Bieri (Frankfurt a.M.)

GAMBLING AND OPTIMAL STOPPING

June 8-14, 1986

Competing Research Projects

A policy which gives priority on the basis of dynamic allocation indices is known to be optimal for a family of alternative bandit processes. This includes the case of a set of jobs which yield discounted rewards on completion after a random service time. With an obvious notation, the index for a job which has so far been served for a time x is

$$V \sup_{t>x} \frac{\int_x^t f(s) e^{-\gamma s} ds}{\int_x^t [1-F(s)] e^{-\gamma s} ds}. \qquad (1)$$

The result extends to the situation of jobs subject to precedence constraints in the form of an out-tree, which includes the possibility of a Poisson arrival process for further jobs. In general the form of the index in this case depends in a complicated way on the arrival rates and other parameters of the different types of job. In the limit as $\gamma \to 0$, however, the index may be shown to be monotone in the limit of (1), which may be used, therefore, to determine priorities.

For research projects, however, we certainly need a positive discount rate, and must face up to the complications of the resulting indices. Some judicious use of generating functions, and some queueing-theory like arguments, lead, at least in a particular rather restricted case, with just two types of job (project), with completion rates which have unique local maxima, to a more general, and just about tractable, version of (1).

John Gittins. Oxford

Stochastic control of two-parameter processes and application to the two-armed bandit problem

By developing a compactification method, we study a control problem for two-parameter processes which generalizes the two-armed bandit problem.
Given an upper semi continuous stochastic process X indexed on $\Pi^2 = \mathbb{N}^2$ or $\mathbb{R}^2_+$, say $(X_z ; z \in \overline{\Pi}^2)$, such that $E(\sup_z |X_z|) < \infty$, and given a fixed bounded random measure (dV_u) on $\overline{\Pi}$, we associate to an arbitrary optimal increasing path $Z = (Z_u ; u \in \overline{\Pi})$ the average pay-off

$$C_{X,V}(Z) = E(\int_0^\infty X_{Z_u} \, dV_u)$$

The notion of randomized optimal increasing paths, which is the analog of the one-parameter notion of randomized stopping times, is introduced. The set of randomized optimal increasing paths is compact, convex and its extreme elements are the optimal increasing paths. Then, we prove the existence of such Z^* verifying

$$C_{X,V}(Z^*) = \sup\{C_{X,V}(Z); Z \text{ optimal increasing paths}\}$$

In the discrete case ($\Pi = \mathbb{N}$), an explicit construction of the optimal solution is obtained.

Gérald Mazziotto, Paris.
O Miller, Angers

Prophet-type inequalities for Multi-Choice Optimal Stopping

For independent random variables $\{X_n, n \geq 1\}$, let $v_r = \sup E(X_{\tau_1} + \dots + X_{\tau_r})$ where the supremum extends over all stopping times $\tau_1 < \dots < \tau_r$. It is shown that for each r there exists a (best) C_r, $1 < C_r \leq 2$ with $E \sup_n X_n \leq C_r v_r$, and a recursive formula for computing the C_r is given. This extends the prophet inequality which is the case $r=1$, when $C_1 = 2$. In addition, if the random variables take values in $[0,1]$ it may be shown that $E\sup_n X_n \le F_r(v_r)$ where the functions F_r are defined by

$$F_r(x) = \sup_{x/r \le y \le x\wedge 1} \left[y + y(1-y)F_{r-1}\left(\frac{x-y}{y}\right)\right],\ r>1,$$

$0 \le x \le r$, with $F_0(x) \equiv 1$.

D P Kennedy, Cambridge.

A PROPHET INEQUALITY WITH ORDER SELECTION FOR INDEPENDENT RANDOM VARIABLES.

For $X_1, \dots, X_n$ ~~be~~ independent with finite means, let $m = E(X_1 \vee \cdots \vee X_n)$ and let $v_\pi = V(X_{\pi(1)}, \dots, X_{\pi(n)})$
$= \sup\{EX_\tau : \tau$ a stop rule for $X_{\pi(1)}, \dots, X_{\pi(n)}\}$
where π is a permutation of $(1, \dots, n)$;
let $w = \max_\pi v_\pi$.

Theorem. The set of all possible values of the ordered pairs (w, m) for two independent random variables with values in $[0,1]$, is precisely the closed convex set S in R^2 given by

$$S = \left\{ (x,y) : x \le y \le x + \frac{x(1-x)}{(1+\sqrt{1-x})^2}\ ;\ 0 < x \le 1 \right\}.$$

Furthermore, every point in S can be generated by X_1, X_2 which are order-indifferent (but not necessarily identically distributed). The upper boundary of S cannot be attained by equally distributed X_1 and X_2.

Corollary 1 For $n=2$ and X_1, X_2 indep. with values in $[0,1]$,

$$m - w \leq \frac{w(1-w)}{(1+\sqrt{1-w})^2} \leq \frac{5\sqrt{5}-11}{2} \approx 0.09$$

Corollary 2. For X_1, X_2 independent non-negative $m \leq \frac{5}{4} w$. The constant $\frac{5}{4}$ is best possible though not attained. It can be approached by bounded X_1, X_2 for which w is close to 0.

Some information (mostly negative) on the optimal ordering of a given finite collection of distributions is also discussed.

David Gilat, Tel-Aviv

Optimal Stopping values and Prophet inequalities for negatively dependent r.v.'s, Y. Rinott & Ester Samuel Cahn.

Let $\underset{\sim}{Y} = (Y_1, \ldots, Y_n)$ be a sequence of r.v.'s satisfying the negative dependence condition $P(X_i < a_i | X_1 < a_1, \ldots, X_{i-1} < a_{i-1}) < P(X_i < a_i)$.

(e.g., $Y_1 \ldots Y_n$ obtained by sampling without replacement from a finite population, normals with correlations ≤ 0, multinomial, Dirichlet, etc.)

Let $X_1, \ldots, X_n$ be independent and $X_i \sim Y_i$, $i = 1, \ldots, n$. We show that the following comparison of values holds:

$\sup_t E X_t \leq \sup_t EY$ (sup taken over stop rule for the sequence)

and obtain a prophet inequality.

$$E\{\max(Y_1,..,Y_n)\} \le 2 \sup_t EY_t \;.$$

The prophet inequality is obtained by showing that there exists a value b such that

$$E\max(Y_1,...,Y_n) \le 2 EY_{t(b)}$$

where t(b) is the threshold rule : stop first time $Y_i \geqslant b$.

We also study partial replacement schemes, i.e., sampling from a finite population and randomly replacing or removing sampled elements, and obtain value comparisons and prophet inequalities.

Y. Rinott, Jerusalem.

A probabilistic approach of the reduite (with N. El Karoui, J. P. Lepeltier)

We use randomized stopping times to study the reduite $R^{\alpha} f(x) = \sup_{T \in \mathcal{C}} \mathbb{E}_x (e^{-\alpha T} f(X_T))$ of a function f for a strongly Markov process (X_t). We obtain a unified treatment of several known results: independence of the realization, continuity, connection with the Snell envelope

A. Millet, Angers

Bandit Problems and Optimal Stopping

Consider two Bernoulli sequences $X_1, X_2, \ldots, Y_1, Y_2, \ldots$ where $P(X_i = 1) = \theta$, $P(Y_i = 1) = \lambda$; given (θ, λ), all X's and Y's are independent. A "strategy" indicates at each stage i whether to observe X_i or Y_i (call the resulting observation Z_i), depending on all previous selections and observations. Suppose λ is known, and θ is unknown with known distribution F. The objective is to maximize $E[\sum_1^\infty \alpha_i Z_i \mid F, \lambda]$ where $A = (\alpha_1, \alpha_2, \ldots)$ with $\alpha_i \geqslant 0$ and $\sum_1^\infty \alpha_i < \infty$ is a discount sequence. Necessary and sufficieny conditions on A are given for the bandit to be an optimal stopping problem for all (F, λ): when is the problem such that the decision maker need only decide when to stop observing the X's and switch permanently to the Y's?

"Index strategies" are examined when there are k processes with unknown characteristics.

D. A. Berry, Minneapolis, USA

Continuous-time gambling

After certain results from discrete-time gambling have been recalled, two approaches to ~~the~~ continuous-time are considered. The first approach is global in time while the second is local and defines the gambling problem by specifying a controll set of infinitesimal parameters for ~~an~~ Ito processes. The theory is illustrated by a continuous-time ~~[illegible]~~ version of red-and-black.

W. Sudderth, Minneapolis, USA

On the chance to visit a goal set infinitely often

The probability of visiting a goal set infinitely often is a typical criterion in the theory of gambling founded by Dubins and Savage in their book "How to gamble if you must". This criterion is more difficult to handle than the usual criteria in dynamic programming (total return and average return per unit time). So the excistence of optimal strategies was known only for a model with finite state space and finite action space. In the present paper, that excistence result will be extended to the case of a compact action space under the continuity assumptions known from the average return criterion. Also the methods of proof are borrowed from dynamic programming with the average return criterion.

Manfred Schäl, Bonn

Prophet Problems in Optimal Stopping and Stochastic Control

For independent r.v.'s $X_1, X_2, \ldots$ taking values in $[0,1]$, exact comparisons of $V(X_1, X_2, \ldots) = \sup\{EX_t : t \text{ is a stop rule for } X_1, X_2, \ldots\}$ and $E(\sup_{j\geq 1} X_j)$ have been given by Krengel and Sucheston, Hill, and others.

First, an extension is given in which $V(X_1, X_2, \ldots)$ is compared to $E(k^{-1}\Sigma_{i=1}^{k} M_i(X_1, X_2, \ldots))$, where $M_i(X_1, X_2, \ldots)$ is the i^{th} largest order statistic of the sequence $X_1, X_2, \ldots$. Second, Krengel and Sucheston's variation of the original comparison results, in the setting of transforms of sequences of independent r.v.'s, is discussed.

Throughout, reduction techniques are emphasized, and extremal distributions are given. This extension and variation provide insights into the original prophet comparison.

Robert P. Kertz, Atlanta, U.S.A.

Minimizing the expected time to reach a goal

An object moves on the negative half-line according to an Ito process, where the infinitesimal mean and standard deviation at each point are chosen from a given control set. The problem of minimizing the expected time to reach zero is formulated as a continuous-time gambling problem, and the standard Bellman-equation approach from optimal control theory is seen to be inadequate. Necessary and sufficient conditions are established on the control set for zero to be attainable in arbitrarily small expected time. A new "verification lemma" serves as a tool for characterizing the optimal return function. Examples are discussed, and the theory is extended to cover certain situations where the set of available controls depends on the position of the object. The talk is based largely on joint work with D. Heath, S. Orey, and W. Sudderth.

Victor C. Pestien, Miami, USA

Stationary Decision Strategies

F. Thomas Bruss, Namur, Belgium

Suppose that one or more decisions have to be made on a given time interval [0,t], and that neither the number nor the qualities of options are known in advance. Decisions have to be spontaneous, and they are irrevocable. – The aim is to find a strategy which maximizes the expected payoff as a function of a certain number of top-options.

We shall investigate the possibilities of modelizing this type of decision problems and briefly discuss the advantages and disadvantages of existing Best-Choice-Models. A 'waiting time' model will be selected for its simplicity, and, in particular, for the performance of the corresponding optimal strategies. We shall see that in many cases the optimal stopping rule does not depend on sequential observation of the arrival process and we will show how to pick out such (stationary) strategies which allow for a simple mecanism of self-teaching strategies to deal with repetitive tasks under weak information. □.

Sharp inequalities for martingale transforms and the optimal control of martingales

Donald L. Burkholder, University of Illinois, Urbana

Let $M = (M_t)_{t \geq 0}$ be a martingale with right-continuous paths having limits from the left. Let $M^*(\omega) = \sup_{t \geq 0} |M_t(\omega)|$ and

$\|M\|_p = \sup_{t \geq 0} \|M_t\|_p$. Recall the classical inequality

(1) $$\lambda P(M^* \geq \lambda) \leq \|M\|_1$$

due to Ville (1939), in the case $M \geq 0$, and Doob (1940) with antecedents in the work of Kolmogorov, S. Bernstein, and Lévy. This inequality can be improved as follows: Let $N = V \cdot M$ where $V = (V_t)_{t \geq 0}$ is predictable (relative to M) and $0 \leq V_t(\omega) \leq 1$ for all t, ω. That is, $N_t = \int_{[0,t]} V_s \, dM_s$ a.s. Then

(2) $$\lambda P(N^* \geq \lambda) \leq \|M\|_1 .$$

Note that (2) contains (1). In fact, if $a \leq V \leq b$ where $a \leq 0 \leq b$, then

(3) $$\lambda P(N^* \geq \lambda) \leq (b-a) \|M\|_1$$

and the constant $b - a$ is best possible (where M and N vary over all possibilities). Furthermore, if $-1 \leq V \leq 1$, $1 < p < \infty$, and $p^* = p \vee q$ where $1/p + 1/q = 1$, then

(4) $$\|N\|_p \leq (p^* - 1) \|M\|_p .$$

The constant $p^* - 1$ is best possible and strict inequality holds if $p \neq 2$ and $0 < \|M\|_p < \infty$.

The best constants in these and a large

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number of other inequalities can be obtained by similar methods. One of the underlying ideas (see my 1981 and 1984 papers in the Annals of Probability) is the following: Suppose that B is a Banach space and S is a biconvex subset of $B \times B$. Let $S_\infty \subset S$ and $F: S_\infty \to \mathbb{R}$. Let $Z = (Z_1, Z_2, \ldots)$ be a simple martingale (Z_k is a simple function and, for some n, $Z_n = Z_{n+1} = \cdots = Z_\infty$, say) with values in S such that $Z_1 \equiv (x,y) \in S$ and $Z_\infty(\omega) \in S_\infty$ for all ω. Also assume that Z is a zigzag martingale: $Z_n = (X_n, Y_n)$ and, for all n, either $X_{n+1} - X_n \equiv 0$ or $Y_{n+1} - Y_n \equiv 0$. Then

$$(5) \qquad L_F(x,y) \le E\,F(Z_\infty) \le U_F(x,y)$$

where U_F is the least biconcave function $u: S \to \mathbb{R}$ such that $u \ge F$ on S_∞ with L_F defined in a dual way. The bounds for $E\,F(Z_\infty)$ in (5) are best possible.

These results have applications to the optimal control of martingales and to problems in functional and harmonic analysis.

Prophet compared to gambler: case of transforms.

X_i, $i = 0, \ldots, r$ are random variables $EX_i = e_i$, $\mu(X) = EX - E(X - EX)^+$, The maximal gain of the gambler is defined as

$$G = \sup_u (u * X) = \sup_u \sum_{0 \le i < r} u_{i+1}(X_{i+1} - X_i)$$

where $U_{i+1} \in \sigma(X_i)$, $0 \le U_i \le 1$. The maximal gain P of the <u>prophet</u> differs only in that U_i are arbitrary, $0 \le U_i \le 1$. The corresponding "signed" expressions G_s and P_s are obtained allowing $-1 \le U_i \le 1$. Assume $E(X_i | X_{i-1}) = e_i$.

<u>Theorem 1</u> Assume $\mu(X_0) \le \mu(X_r)$, which is not a loss of generality if all $X_i \geq 0$. Then $P \le 3G$. If $e_0 = \dots = e_r$ then $P \le 2G$.

<u>Theorem 2</u> Assume $\mu(X_0) \le \mu(X_r)$ and $e_0 \geq e_r$. (Both conditions hold if $e_0 = X_r$.) Then $P_s \le 3G_s$. The constant 3 is optimal in both theorems.

Louis Sucheston, Columbus, Ohio.
(joint work with Ulrich Krengel)

Nonexistence of Uniformly adequate Stationary Plans for leavable gambling problems on a Fortune space of cardinality c

S. Ramakrishnan, University of Miami, Coral Gables, USA.

There exists a leavable gambling problem, with fortune space of cardinality c ~~which~~ with at most three gambles available at each fortune, all gambles having atmost two points in their support, where the objective is to reach a goal, where stationary plans are <u>not</u> uniformly adequate.

Sequential Detection of a Change-point

The problem of sequentially detecting a change of distribution is introduced. The procedures of Page (1954) and Shiryayev (1963) are described, and the optimality properties obtained in the case of two completely specified distributions by Shiryayev (1963), Lorden (1971), and Pollak (1985) are reviewed. For detecting an increase in the drift of Brownian motion, approximations to the average run length are given and used to compare the Page and Shiryayev procedures numerically. Some results indicating how one can make similar comparisons for discrete time processes are given. Extensions to more complex situations are briefly discussed.

David Siegmund

On Optimal Stopping with Concave Costs of Observation

For optimality results in sequential analysis which contain explicit description of optimal strategies, the assumption of linear costs of observation $c \cdot t$ has been of major importance. In this talk we investigate the question how the shape of nonlinear cost functions influences the shape of optimal stopping boundaries. We do this for an optimal stopping problem which arises in the derivation of locally most powerful sequential tests for the Wiener process. It is found that costs of observation of the form $t^{\frac{1}{2}+a}$, $0 < a < \frac{1}{2}$, lead to optimal stopping boundaries which grow in the order of $t^{\frac{1}{2}-a}$ as $t \to \infty$.

Albrecht Irle, Universität Kiel

Potential theory for a gambling house

We show how nice are the analytic gambling houses through an exposition of the most important results included in the third volume of "Probabilités et Potentiel" (joint work with P.A. Meyer): Mokobodzki's theorem on analytic sublinear functionals, analyticity of balayage order, extension of Straßen's theorem, etc.

Dellacherie, Université de Rouen

Sensitive optimal policies in denumerable Markov decision chains.

In this talk we consider a discrete-time Markov decision chain with a denumerable state space and compact action sets and we assume that for all states the rewards and transition probabilities depend continuously on the actions.

An analysis for average and more sensitive optimality without assuming a special Markov chain structure is presented. In doing so, new conditions which include the finite state and action model, are given.

Arie Hordijk, Leiden

Optimal stopping rules for processes in semimartingale representation

Let $(\Omega, \mathcal{F}, P)$ be a probability space with a filtration $(\mathcal{F}_t), t\in\mathbb{R}_+$ and $(Z_t), t\in\mathbb{R}_+$ a stochastic process adapted to this filtration. The problem of optimal stopping is then to maximize $EZ_\tau, \tau\in C$ in a given class C of stopping times. This problem is considered for processes Z which admit a semi-martingale representation. The monotone case in continuous time is introduced and conditions are given under which the so called infinitesimal-look-ahead stop-ping rule is optimal. Furthermore it is shown that the reduction of the semimartingale representation to discrete time leads to the well known discrete monotone case. But the integrability conditions which are necessary to prove the optimality of the one-step-look-ahead stopping rule, differ slightly from the classical ones.

Uwe Jensen, Stuttgart-Hohenheim

Prophet Problems: Results, Applications and Open Problems
Theodore P. Hill Georgia Institute of Technology

Theorem If $X_1, X_2, \ldots$ are nonnegative independent random variables, then
$$E\left(\sup_n \frac{X_1+\cdots+X_n}{n}\right) \leq 2 \sup\left\{E\left(\frac{X_1+\cdots+X_t}{t}\right) : t \text{ is a stop rule}\right\},$$ and this bound is best possible.

Such an inequality is called a prophet inequality; applications of the original prophet inequalities of Krengel + Sucheston are made to problems of order selection, nonmeasurable stop rules, look-ahead stop rules, iterated maps of random variables, and a double process problem. A number of open problems are mentioned, including the question of the best universal constant in the analog of the above theorem for iid X's.

On existence of optimal policies in stochastic scheduling

A general model of stochastic scheduling for a project with n jobs is presented. Durations of the jobs are distributed according to some joint probability measure P. A particular realization is not known to the decision maker, but becomes increasingly known during the excution of the project. The problem is to find a schedule plan which minimizes the expected costs, which are continuous functions of job completion times. Constraints, such as precedence, are allowed. Jobs may be interrupted or not (nonpreemptive case). For this model it is shown that there always exists an optimal schedule plan.

Dieter Kadelka, Universität Karlsruhe

Betting to Leave an Interval: In treating a problem in combinatorial optimization, Joel Spencer conjectured that a gambler betting on the outcome of a coin toss, restricted to bets of size 1 or smaller and attempting to win or lose at least G (an integer) in n bets, should always bet 1 until he reaches +/-G. Using the framework of Dubins and Savage we construct two other gambling houses; one smaller, and one larger, and find optimal policies for these houses. The optimal return functions agree at the integers; this allows us to deduce that Spencer's conjecture is correct.

David Heath

Roulette as a ruin game: Optimal Strategies for Gambling.

Roulette is considered as a ruin game: the gambler starts with the amount z and plays until he is either ruined (loss z) or has reached a pre-fixed capital a (gain $a-z$). If he is only allowed to bet on even chances, this model is treated in various textbooks. However, if he is allowed to bet on different combinations of numbers, the model can only be discussed by theory combined with computer calculations.

If the strategy is fixed for every capital z, the ruin probabilities, the mean duration time, of the game and the gamblers mean stakes can be calculated recursively by a Kind of Gauß-Seidel iteration. The same is true for higher moments.

Furthermore, optimal strategies which minimize the ruin probabilities, can be calculated if the set of possible strategies is finite. The numerical values are close to the ones given by the Dubins-Savage theory for an idealized roulette game. In European casinos, where the prison rule is is usually valid, gambling on even chances (red, black, manque, pass, ...) is preferred if $z \geq a/3$. At American casinos betting on single numbers has more advantages. This empirical observation was confirmed by computer calculations.

U. Dieter (TU Graz Austria)

A sequential estimation procedure for the parameter of an exponential distribution

n "units" have unknown lifetimes $\xi_1, \dots, \xi_n$ which are assumed to be i.i.d. with density $\theta e^{-\theta x}$, $x > 0$, with unknown $\theta > 0$. θ is to be estimated on the basis of the following observation process: At time t it is known, how many units have failed up to t and when this happened. θ is assumed to have a gamma prior distribution. An ~~Bayesian~~ optimal ~~B~~ sequential Bayes estimate is found explicitly. It is of the following form: There are constants $\gamma_n = 0, \gamma_{n-1}, \dots, \gamma_0$ such that ~~at~~ one stops at time t iff $\xi_{(j)} \leq t < \xi_{(j+1)}$ and $\xi_{(1)} + \dots + \xi_{(j)} + (n-j)t \geq \gamma_j$ for some $j \in \{0, 1, \dots, n\}$. Some monotonicity results on $\gamma_0, \dots, \gamma_n$ are also presented. Further the minimal Bayes risk is computed in closed form. – The loss function ~~ist~~ is given by $(\hat{\theta}_t - \theta)^2 + a N_0(t) + bt$, where $a, b > 0$, $\hat{\theta}_t$ is the estimator used at time t, and $N_0(t)$ is the number of failures up to time t.

W. Stadje (Univ. Osnabrück)

Macroscopic models for processes with interaction

(Joint work with M. Alecoglu)

Assume that a large number N of particles are distributed among d states, such that $f_i N$ particles are in state i, $(1 \le i \le d)$. The particles move independently of each other, but the probability of a transition of a particle in state i to state j may depend on $f = (f_i)$. E.g. i leads to $i+1$ with probability $\frac{1}{4}(2 - f_{i+1})$ and to i with probability $\frac{1}{4}(2 + f_{i+1})$, (Addition mod d). If N is very large, the distribution Tf of the particles is described by the frequencies

$$(Tf)_i = f_i\left(\tfrac{1}{4}(2 + f_{i+1})\right) + f_{i-1}\left(\tfrac{1}{4}(2 - f_i)\right).$$

The operator T can be continued to operate in $(\mathbb{R}^+)^d = L$ with the following properties: $T0 = 0$; $\int Tf = \int f$, where $\int f = \Sigma f_i$; and $f \le g \Rightarrow Tf \le Tg$. Under an aperiodicity condition (fulfilled in the above example) one obtains convergence of $T^n f$ to an element $\tilde{f}$ with $\int f = \int \tilde{f}$ and $\tilde{f} = T\tilde{f}$. (There is only one such $\tilde{f}$). This extends a basic convergence result for Markov chains to the nonlinear case.

M. Krengel (Göttingen).

Three related problems were treated in my talk.

1. A problem of sufficency of Markov strategy in a problem of $\min P(\liminf (X_n \in D_n))$ (starting from result of T. Hill)

2. A mathematical model regarding asympthotic behaviour of the

nonhomogeneous solution in a system or vessels (discrete stream) is considered. One of the main results – a theorem on separation of jets – states that every stream with a bounded number of vessels can be decomposed into such jets that stabilization of volume and concentration takes place in every jet and the full flows between different jets are finite on an infinite time interval. This th. may be reformulated also in terms of nonhom. Markov chains.

3. The investigation of above problems uses the thm. about the existence of nonrand. sequences (barriers) such that expected number of intersections between ~~these~~ such sequences and martingale type random sequences is finite on infinite time interval.

Sonin I.M.

Optimalsteuerung und Variationsrechnung - Optimal Control

15. - 21. Juni 1986

On the synthesis of optimal nonlinear feedback laws

The paper is concerned with the automatic computation of the optimal nonlinear feedback control law, starting with a short review of the theory. The synthesis of a control system constitutes the main part of an optimization problem.
We have shown, in previous papers, that the optimal nonlinear feedback control law satisfies a set of partial differential equations. The knowledge of the feedback law can therefore be considered as equivalent to the computation of the hypersurface corresponding to the solution of these equations.
This hypersurface is computed off-line.
Attractive features for real time implementation are discussed.

Houria Bourdache-Siguerdidjane
LSS CNRS/ESE 91 Gif. (France)
بوردا شي حورية

„Sensibilität und optimale Steuerung elastischer Strukturen
mit verteilten Parametern"

Die Entwicklungstendenzen der modernen Optimierungstheorie in der Mechanik führen zur Kopplung von drei Formalismen: der Variationsrechnung, der Steuerungstheorie und der mechanischen Theorie der Energieprinzipien. In der Arbeit wird ein Verfahren der Optimierung elastischer Systeme mit verteilten Parametern präsentiert. Es werden sowohl lineare als auch nichtlineare Operatoren des Zustandes betrachtet. Die Zustandsgleichung des Systems wird in variationeller Form dargestellt. Unter der Voraussetzung, daß alle in dem Problem auftretenden Funktionale Gateaux-differenzierbar sind, werden die Sensibilitätsoperatoren effektiv konstruiert. Dabei spielt das Konzept der adjungierten Gleichung die entscheidende Rolle. Beispiele aus der nichtlinearen Plattentheorie illustrieren die Effektivität der Methode.

G. Szefer
Technische Universität Krakau
(Polen)

„Generalized conjugate functions and nonconvex
optimization"

Since more than three decades, duality theorems formulated by FENCHEL-conjugate functions are considered as an essential and today as a classical part of the theory of convex optimization. Forthcoming numerous papers were devoted to extend the concept of conjugate mappings for treating nonconvex problems. In the paper the concept of Φ-conjugation (DEUMLICH/ELSTER) will be handled and compared with several other new corresponding concepts. Moreover, applications to a class of nonconvex optimization problems will be discussed

Karl-Heinz Elster
(Technische Hochschule Ilmenau, DDR)

„Analytische und rechentechnische Aspekte zur Dualität bei Steuerungsproblemen"

Es werden Grundlagen einer durch W.F. Krotow und den Autor entwickelten allgemeinen Dualitätstheorie bei Steuerungsproblemen vorgestellt. Daran schließen sich einige Übersichten über alte und neuere Optimierungsprobleme an, die über diesen Zugang gelöst werden konnten. Abschließend wird ein Ausblick auf rechentechnische Nutzungsmöglichkeiten dieser Theorie gegeben.

R. Klötzler
Karl-Marx-Universität Leipzig, DDR

"Control of a free boundary problem with hysteresis"

We consider the problem of controlling the free boundary of the 2 - phase Stefan problem by means of boundary hysteresis control based on the Preisach model. It is proved that for each control μ there is a corresponding solution of the Stefan problem and that there exists an optimal control. The asymptotic behaviour of the free boundary is investigated as well. Numerical work complements the theoretical results and gives some hints for further research.

K.-H. Hoffmann, Univ. Augsburg
(joint work with A. Friedman)

Endogeneous Optimal Oscillations

It is known that for one-state, continuous-time, autonomous, infinite time optimal control models the state variable is monotonic. Endogeneous cyclical solutions in such models only occur if there are at least two states.

Two methods to prove the existence of periodic solutions for optimal control problems are discussed. The first approach provides sufficient conditions for the existence of a cycle which is reached in finite time. Chattering is prevented by fixed transitional costs. The second method is based on Hopf's bifurcation theorem and deals with limit cycles. In both methods the oscillations are due to non-concavities in the Hamiltonian.

Both procedures are illustrated by economic examples (inventory/production planning: how can constant demand lead to cyclical production?; advertising: continuous ADPULS, environmental planning).

Gustav Feichtinger

Techn. Univ. Vienna/Austria

A Pointwise Quasi-Newton-Method for Unconstrained Optimal Control Problems

For a class of unconstrained optimal control problems we propose a quasi-Newton method that exploits the structure of the problem. We define a new type of superlinear convergence for sequences in function spaces and prove superlinear convergence of the iterates generated by the quasi-Newton method in this sense. The method is applied to a simple unconstrained optimal control problem and numerical results are presented.

Ekkehard Sachs, Universität Trier

Systems with hysteresis

We consider time optimal control of a system whose dynamics consist of a system of ordinary differential equations and a nonlinearity of hysteresis type in the sense of Krasnoselski. We discuss existence and necessary optimality conditions and present numerical results, which are obtained by the multiple shooting algorithm

Martin Brokate

Computational Strategies for the Tension Parameters of the Exponential Spline

Three different strategies to determine the tension parameters p_i of the exponential spline (or spline under tension) are discussed. A first heuristic strategy is based on the knowledge of the interpolating cubic spline and p_i-values are proposed in order to eliminate undesired inflection points. Convexity or monotonicity of the interpolant cannot be guaranteed. A convexity preserving C^2-interpolant (if possible) is constructed by solving a constrained nonlinear optimization problem for the tension parameters p_i. This second strategy characterizes an "optimal" set of p_i-values. The optimization problem is the base to derive 'a priori' estimates for the p_i in a third strategy. The convexity arguments are supplemented by monotonicity constraints. The performance of all strategies is demonstrated in several examples.

Peter Rentrop, Univ. Kaiserslautern
(joint work with U. Wever)

"Singular Perturbations in Nonlinear Optimal Control"

We first briefly review both singular perturbation theory for nonlinear ordinary differential equations and the development of asymptotic methods in fluid mechanics and other branches of applied mathematics. These approaches are then applied to the nonlinear optimal control problem. The key requirement is found to be the stability properties of the singular points of the boundary layer equations. These singular points are saddle-points of type n_f, where n_f is the number of fast, or boundary-layer, state variables. The use of the theory is illustrated by representative applications to aircraft flight path optimization problems.

Mark Ardema
Santa Clara University
California, U.S.A.

"Optimal Control of distributed-parameter system with boundery condition involving a time deley and initial state not a priori given"

The purpose of this paper is to show the use of Milutin - Dubovicki's method in solving some non-typical control problems for distributed-parameter systems. As an example, an optimal control problem for the system described by a linear partial differential equation of parabolic type with time delay in the boundary condition is considered. Also the initial condition is not given by a known function, but it belongs to a certain set (the initial state is not a priori given).

In our problem the time delay in the boundary condition is constant. The performance functional has the integral form. The control time is fixed. Finally, we impose some constraints on the control. Making use of the Milutin-Dubovicki's theorem necessary and sufficient conditions of optimality with the convex performance functional and constrained control are derived for the Neumann problem. We also present a particular example in which the set of admissible controls and the one of initial conditions are given by means of the norm constraints.

Adam KOWALEWSKI Institute of Automatic Control
Academy of Mining and Metallurgy, Kraków (Poland)

Topics in Fixed Order Controller Design

The implementation of the linear quadratic results and of the pole placement results of modern control theory usually require on-line reconstruction of the state using, for example, a Kalman filter, and this produces a controller whose dimension is equal to that of the system. Such a high order controller is often unnecessary, with essentially equivalent performance being obtainable from lower order controllers. This paper addresses the question of how one can design optimal controllers of any prescribed dimension. First, a theorem on stabilizability of a system by a controller of a chosen dimension is presented. Then algorithms to obtain fixed order controllers that are optimal with respect to a quadratic cost are discussed, and pole placement methods are presented where the remaining freedom after placement is used for optimization with respect to a quadratic cost. The issue of robustness with respect to system uncertainty is also considered for both approaches in terms of penalization of cost functional or eigenvalue sensitivity.

Richard W. Longman
Columbia University, New York

Echtzeitberechnung fastoptimaler Rückkopplungssteuerungen bei Steuerungsproblemen mit Beschränkungen

Viele Optimierungsprobleme in Naturwissenschaft und Technik können mathematisch durch optimale Steuerungsprobleme beschrieben werden wie z. B. die Steuerung eines Raumfahrzeuges oder eines Flugzeuges, einer chemischen Reaktion oder eines Industrieroboters. Sollen diese optimalen Lösungen praktisch realisiert werden, genügt es nicht Anfangsdaten vorzugeben und den Prozeß sich selbst zu überlassen. Vielmehr benötigt man schnellste Rechenverfahren, die den zukünftigen Verlauf der optimalen Steuerungen eines gestörten Prozesses noch während seines zeitlichen Ablaufs berechnen und einstellen, damit Optimalitätsbedingungen und vorgeschriebene Beschränkungen trotz auftretender Störungen erfüllt bleiben.

Das in dieser Arbeit entwickelte Rechenverfahren zur schnellen numerischen Berechnung fast-optimaler Rückkopplungssteuerungen ist sehr allgemein auf beschränkte Optimal-Steuerungsprobleme anwendbar. Alle Beschränkungen können vor der Rückführung überprüft werden. Bezüglich Rechenzeit und Speicherplatzbedarf ist das Verfahren für den Einsatz in Bordrechnern geeignet. Die erfolgreich korrigierbaren Abweichungen liegen weit über den bei Raumfahrtunternehmungen auftretenden Störungen.

Wer fliegt mit?

H.-Josef Pesch
(Techn. Universität München)

Numerische Berechnung singulärer Steuerungen für die Bewegung eines Roboterarms

Es wird das Modell eines reibungsfrei gelagerten zweigliedrigen

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Roboterarms betrachtet. Die Zustandsgrößen sind: Ellbogenwinkel α, Winkel Θ des Oberarms bezogen auf seine Ausgangslage sowie die beiden Winkelgeschwindigkeiten $\omega_{1,2}$ von Ober- und Unterarm. Die Bewegung wird gesteuert mittels der an Ober- und Unterarm angreifenden Drehmomente $Q_{1,2}$. Aufgabe ist es, die Steuerung so vorzunehmen, daß die Spitze des Roboterarms eine vorgegebene Entfernung x_L in kürzester Zeit zurücklegt. Dabei soll der Arm zu Beginn und zu Ende der Bewegung in Ruhe ist.

Für dieses Problem haben Bryson, Weinreb (1985) Lösungen vom bang-bang Typ angegeben. Ziel dieser Arbeit ist es nun, zu zeigen, daß für gewisse Entfernungsvorgaben x_L singuläre Steuerungen für das Oberarm-Drehmoment auftreten können. Hierzu werden die notwendigen Bedingungen der Variationsrechnung aufgestellt. Durch zweimalige Differentiation der Schaltfunktion gelingt es, einen expliziten Ausdruck für eine singuläre Steuerung für Q_1 zu erhalten. Somit läßt sich – in Abhängigkeit der Schaltstruktur – ein Randwertproblem mit Schaltbedingungen aufstellen, welches mit der Mehrzielmethode numerisch gelöst wird. In Abhängigkeit der Reichweite x_L erhält man so zwei Lösungszweige, wobei der Lösungszweig mit asymmetischen Steuerungen für mittlere Reichweiten singuläre Teilstücke aufweist.

H. J. Oberle
(Universität Hamburg)

Boundary Value Problems for Differential Inclusions and the Computation of Optimal Trajectories

Simplicial algorithms allow the computation of fixed points – even for set valued mappings in $\mathbb{R}^n$. The extension of known convergence theorems to Banach spaces enable constructive fixed point results, based only on compactness and continuity principles.

The application to boundary value problems for differential inclusions will be presented, deriving thus an extension of numerically treatable problems to Peano-type dynamics.

This result is of particular interest for optimal control theory, as the necessary conditions of the Pontryagin Maximum Principle can be interpreted in this context. There results a new indirect method for the computation of optimal trajectories with advantages for bounded control domains, for problems with singular control arcs or without differentiability properties.

A numerical example about optimal harvesting of fish populations is given as illustration.

Klaus Schilling
(Dornier System GmbH)

Oscillatory Cruise - A Short History

An expression is obtained for the 2nd variation in the case of periodic variations about minimum-fuel cruise, the minimum being assumed to occur at less than maximum thrust.

Sinusoidal variations in speed and altitude will generally produce some cost reduction for wave-lengths within a certain range, the reduction being due to 2 causes: (i) δv in phase with $\delta\gamma$, implying a negative time-average for the flight-path-angle γ, and hence assistance from gravity; (ii) the increase of induced drag with altitude, so that $\delta^2 D/\partial h \partial L \; \delta h \, \delta L$ (δh being 180° out-of-phase with δL) rectifies to a negative value.

Improvements indicated by reduced-order analyses, "energy-state" and "intermediate order", are related to cause (i)

Extrapolation of the one-variation approximation to cost increase up to thrust variations δT of ± 60%, in the case of an aircraft studied by Grimm & Well, gives close agreement with their optimal wave-length and δh, δv histories fairly comparable with theirs. Better agreement is obtained by allowing a square-wave pattern for δT and solving for the optimal δL, retaining the linearized dynamics and quadratic cost increase.

J. V. Breakwell (Stanford University)

A New Approach for Optimizing Hydropower System Operation with a Quadratic Model.

This paper is devoted to the development and application of a reservoir optimization model that yields monthly release policies. The generalization consists of capability to handle nonlinear energy generation rates in the objective function (maximization of system annual energy generation). A quadratic model for the elevation-storage (average storage) is used.

The optimization problem is described and formulated as the optimal control of a multivariable state-space model in which the state and control vectors are constrained by sets of equality and inequality relations. Lagrange and Kuhn-Tucker multipliers are used to adjoin these constraints to the objective function. The resulting cost functional is maximized by using the minimum norm formulation of functional analysis. Numerical results are reported for a system consisting of three rivers; each river has two series reservoirs.

Christensen (University of Alberta).

Periodic Cruise Models, Uday Shankar, Eugene Cliff and Henry Kelley, Virginia Polytechnic Institute & State University, Blacksburg, Virginia, U.S.A.
Fuel-optimal periodic cruise is studied with attention to modelling simplifications. One simplified model has specific energy frozen for study of the fast motions, altitude and airspeed; these are averaged over a period. Another is a relaxation oscillation model in which slow (energy-gaining and energy-losing) intervals are interspersed with fast altitude-airspeed transitions.

Henry J. Kelley, 19 June 1986

Nonlinear System Analysis by Direct Collocation
A heuristic analysis approach of N.L.S. is proposed which is based on solving a sequence of optimal control problems with varied problem parameters. Thus an engineering compromise as trade-off between constraint satisfaction and cost efficiency is possible. Processes are considered which are controlled by a two-degree-of-freedom controller. Various aspects of design efficiency lead to multicriteria problems for which Pareto-optimal solutions are sought.
The core of the algorithm deals with the solution of constrained optimal control problems by discretization and direct collocation. This leads to a nonlinear programming formulation with structured Jacobian and Hessian matrices. This is treated by sequential quadratic programming.
The procedure is exemplified by the controller design for a cryogenic wind tunnel with three state variables and four control variables, three of which exhibit state depen-

dant transportation time lags. Considerable performance increase is gained compared to heuristic engineering design approaches based on non-optimal design procedures.

Dieter Kraft, DFVLR
Oberpfaffenhofen

Deterministic control of uncertain systems

Many systems in the "real" world are subject to human intervention and control. The first step in devising a control policy for the accomplishment of the desired end is the abstraction of the salient features of the system — usually embodied in a mathematical model. Mathematical models are always uncertain because they involve unknown or partially known elements, either in the model itself (uncertain parameters) or in the description an uncertain environment (input uncertainty). In place of the classical stochastic approach, we propose a deterministic one which assures the desired behavior (practical stability). Applications to robotics, ecology and seismic control are given.

George Leitmann
Univ. of California
Berkeley, CA.

Control of a Slewing Beam,

E.M. Cliff, J.A. Burns, & F.H. Lutze

A control problem is studied for a system consisting of rigid-masses connected by a flexible beam. A semigroup formulation is employed to show that the resulting system of ordinary and partial differential equations is well-posed on a certain space. The control problem involves regulation over an infinite horizon of a finite number of outputs using a single control torque. The Trotter-Kato Thm is used to motivate an approximation procedure and it is shown that the sequence of feedback operators for the approximating problems converges to that for the original problem. Numerical results are presented.

Eugene M Cliff
Virginia Poly. Inst.
Blacksburg, VA

Direct and Indirect Approach for Real-Time Optimization of Aircraft

Two methods for real-time optimization of aircraft are presented, which attempt to satisfy time- and reliability requirements for on-line algorithms.

One method is based on the direct approach, i.e. it starts with a parametrization of the control functions. The computational workload is reduced by use of an effective integration scheme particularly designed for the underlying ODE-system. The robustness is enhanced by an active set strategy based on the elimination of parameters by active

(nonlinear) constraints.
The second method is based on the indirect approach, i.e. it solves the BVP derived from variational calculus. The state and adjoint equations are solved by a collocation method. Extensions to constrained problems are also possible.
Both methods are taken as part of a feedback algorithm. Simulations are performed to test the accuracy of the feedback-guidance. The test problem is to maximize range in fixed time. The results are compared to the optimal solutions for the respective boundary conditions. Generally, a close agreement of optimal and suboptimal control is observed.

Werner Grimm / Peter Hiltmann, DFVLR, Oberpfaffenhofen

A Pursuit-Evasion with a Chattering Junction of Non-Singular and Singular Subarcs

A simple two dimensional constant velocity pursuit-evasion scenario is considered. From that scenario a time-optimal control problem is posed which is linear in the control. The solution consists of a bang-bang control with possible singular arcs. It is shown that a junction between non-singular and singular arcs must be of a chattering type

Klaus Schnepper, DFVLR, Oberpfaffenhofen

BVP methods for direct and indirect solution of constrained optimal control problems

— Hans Georg Bock, Universität Bonn —

Boundary value problem methods and theory are a very helpful tool for the constuction of effective numerical solution procedures for constrained optimal control problems. Two approaches are presented: the direct BVP approach parameterizes the control function by finite dimensional function spaces, Discretization of the ODE system by multiple shooting collocation or finite differences yields large but sparse constrained nonlinear optimization problems. A separation property is introduced which leads to block diagonalization of the Lagrange-Hessian and allows high rank update formulas which speed up the asymptotic convergence rate. The direct multiple-shooting approach is completely derivative free, generates gradient information by (adjoint) ~~diff~~ internal numerical differentiation techniques. The indirect BVP approach first transformes optimal control problem into a multipoint BVP which is solved by an adequate BVP solver. New developments in multiple shooting and collocation are described. The indirect approach is more complicated and requires substantial information about the rough structure, but it is very general and includes state-constraints, Chebyshev-problems and disconnected control regions. In addition, it can be extended to yield a neighboring feedback control valid for the constrained case as well. All BVP approaches are numerically stable. Several application to demonstrate the performance are given.

"Necessary Conditions for Optimal Pulse Control"
A. J. Calise, Georgia Inst. of Technology, Atlanta,
GA, 30332, Dept. of Aerospace Engineering.

This presentation considers the control problem for a missile with a pulse rocket motor. The characteristics of such a motor are that the pulse heights and widths are fixed by design, and the only control variables are the pulse firing times. This fact gives rise to a constrained variational problems for which the usual necessary conditions are no longer valid. For the case in which the control (thrust as a function of time) enters linearly it is shown that the first order necessary condition becomes $H_u(t_i) = H_u(t_i+\Delta_i)$ where t_i is the pulsing time and Δ_i is the fixed pulse duration. It is also shown that a form of the first integral of the motion exists which states that $H_0(t_i) = H_0(t_i+\Delta_i)$ where $H_0 = H(u=0)$. These necessary conditions are used to derive a pulse triggering algorithm for a simplified model of the missile dynamics. For constant altitude and mass it is shown that an exact analytic solution exists for the problem of maximizing final velocity for a specified final range. The pulsing condition is simply $(V/D)_{t_i} = (V/D)_{t_i+\Delta_i}$ where V is velocity and D is drag. In a sense, optimal pulsing results in maximizing the average value of V/D. A complete characterization for the minimum time problem is also given.

Anthony Calise

"A MODEL COMPARISON OF A SUPERSONIC AIRCRAFT MINIMUM TIME-TO-CLIMB PROBLEM"

BION L. PIERSON, IOWA STATE UNIVERSITY, AMES, IOWA 50011, U.S.A.

A minimum time-to-climb problem is formulated as a parametrized optimal control problem and is solved using sequential quadratic programming. Five dynamic models are treated. Each involves a single control function and between one and five states. The five-state model features the usual point-mass translational equations of motion for flight in a vertical plane. Time is the independent variable. For the remaining four models, range is used to replace time as independent variable. The last model is the well-known energy-state approximation with specific energy $E = \frac{1}{2}V^2 + gh$ as the only state variable and speed V as the control function. Numerical results are presented for an early representation of the F-4 fighter aircraft. For each of the five dynamic models, the solutions are compared with regard to accuracy and computational effort.

Bion L. Pierson
June 20, 1986

"Aircraft Trajectory Optimization by Curvature Control"

Rainer Walden, Gesamthochschule Paderborn, Paderborn, W.-Germ.

The trajectory of an aircraft is described by the Frenet equations known from differential geometry and a parametrizising equation for the aircraft acceleration, which includes all aircraft data. Control functions are the curvature and torsion of the trajectory and the power setting. Hamiltonian Theory is used to derive necessary conditions for the optimal control for minimal-time problems

R. Walden
20.6.86

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Steuerung eines Roboterarms auf einer vorgegebenen Bahn unter verschiedenen Optimalitätsbedingungen

Ulrich Leiner, TU München

Für ein drei-dimensionales Robotermodell werden Steuerungen gesucht, die bestimmte Optimalitätsbedingungen erfüllen. Dabei soll eine vorher bekannte Bahn von der Roboterhand nachgefahren werden. Als Optimalitätskriterium werden Minimierung der Fahrzeit und des Energieverbrauchs betrachtet. Die Vorgabe der Bahn wird zur Reduzierung der Anzahl der Differentialgleichungen verwendet. Lösungen wurden mit Hilfe der Mehrzielmethode gefunden. Für zeitoptimale Bahnen ergibt sich eine 'bang-bang' ähnliche Steuerstruktur mit unter Umständen mehreren Schaltpkten. Bei energieoptimalen Bahnen ergeben sich, bei Hinzunahme von Steuerbeschränkungen, sowohl unbeschränkte wie auch beschränkte Teilstücke der Steuerung.

20.6.86

Optimale Gestaltung von elastischen Balken.

L. MIKULSKI, TU MÜNCHEN (AvH-Stiftung)

Das Thema dieser Arbeit ist die Bestimmung der optimalen Form von statisch bestimmten oder unbestimmten Balken unter Berücksichtigung des Eigengewichts. Die Balken werden unter verschiedenen Restriktionen so gestaltet, daß erstens die Durchbiegung und zweitens das Volumen minimiert wird.
Als Steuerung wählen wir die Breite des I-Profils.
Zur Lösung der Aufgabe benutzen wir das Maximumprinzip.
Zur numerische Lösung wurde die Mehrzilmethode verwendet (BNDSCO) Die gewählten Beispile sind von praktischer Bedeutung für die Baumechanik.

L. Mikulski –
Humboldtstipendiaten 85/86

20.06.1986.

Reduction of deterministic differential games to problems of optimization. The approximate strategy method.

C. Marchal _ D.E.S _ ONERA _ 92320 Chatillon _ FRANCE

Deterministic differential games are necessarily competitive two-players zero-sum games and they require many cautions in order to avoid all hidden sources of undeterminism.

These games have been studied by many people since the early studies of R. Isaacs and they are characterized by a very large variety of singularities such as universal surfaces, dispersion surfaces, focal lines, barrier, equivocal lines etc... and it is generally extremly difficult to obtain the full solution of a game with many parameters.

A strategy, also called closed-loop strategy, is a choice of the control of one player in terms of the state of the game, the time and, if possible, the control of the other player. For a given strategy of one player the opponent faces an ordinary problem of optimization with one or several optimal solutions and thus a given value of the performance index.

It is almost hopeless to find directly the best strategies of both players, but approximate strategies can be found easily and can be improved step by step systematically.

The strategies of the minimizer give upper bounds of the value of the game and those of the maximizer give lower bounds. These bounds may converge to the same value, the value of game; however, most game studies are terminated before this convergence takes place.

C. Marchal   1986_ June 20.

Some Problems Associated with the Control of Distributed Structures

Leonard Meirovitch, Virginia Poly. Inst., Blacksburg, Va.

Control of structures can be carried out conveniently by modal control, whereby the structure is controlled by controlling its modes. Modal control requires estimation of the modal states for feedback, which can present a problem for two- and three-dimensional structures. One approach that does not require modal state estimation is direct feedback control, which implies collocated sensors and actuators. This paper examines some problems encountered in direct feedback control of distributed structures in conjunction with pole placement. A perturbation technique permits the computation of control gains for multi-input systems. The paper demonstrates that the difficulties experienced in using direct feedback in conjunction with pole placement are endemic to the approach.

Presented by L. Meirovitch

Ⓡ Interpolation of Subspaces and Quotient Spaces. Guido Weiss, Washington University, St. Louis
In the spirit of the theory of interpolation developed by Coifman - Cwikel - Rochberg - Sagher - Weiss, in a joint work with E. Hernandez & R. Rochberg we develop a theory of interpolation of "subspaces & quotient spaces." In the finite dimensional case, where it is easiest to explain the theory, the CCRSW theory solves the following Dirichlet problem: Given a domain $\Delta \subset \mathbb{C}^2$ with smooth bdry $\partial\Delta$ (think of the unit disk) and a family of spaces $D_\varsigma \subset (\mathbb{C}^n, N_\varsigma)$, $\varsigma \in \partial\Delta$, where N_ς is a norm, how does one obtain a family $D_z \subset (\mathbb{C}^n, N_z)$, $z \in \Delta$, of intermediate spaces in the the domain Δ, consistent with the theory of interpolation of operators. The answer is in the formula for the norm $N_z(v)$, $z \in \Delta$:

$$\circledast \qquad N_z(v) = \inf\left\{\left(\int_{\partial\Delta} N_\varsigma(F(\varsigma))^2 P_z(\varsigma)\,d\varsigma\right)^{1/2} : F \text{ holomorphic}, \overset{(F(\varsigma)\in D_\varsigma)}{F(z)=v}\right\},$$

where $P_z(\varsigma)$ is the Poisson kernel. ⊛ makes sense if on the boundary D_ς is only a subspace of $\mathbb{C}^n$ & if N_ς's are seminorms. The existence of extremal functions is established and a duality between the subspace theory & the seminorm (quotient space) theory is established. One of the consequences is an extension of the Wiener - Masani theorem for semi-definite positive operators.

Reelle Methoden der Analysis

22. - 28. Juni 1986

Ⓡ

Eigenvalues in potential theory
The investigation of eigenvalues in the framework of harmonic spaces introduced in a joint paper with H. Hueber is motivated by the generalized Schrödinger equation $(\Delta-\mu)u=0$. Assuming that the signed measure μ satisfies a local Kato condition or, equivalently that the potentials $G_U^{|\mu|}$ are continuous and real for every relatively compact

open subset U of $\mathbb{R}^n$, each potential operator K_U^μ, defined by $K_U^\mu f(x) = \int_U G_U(x,z) f(z) \mu(dz)$, maps the space $\mathcal{B}_b(U)$ into the space $\mathcal{C}_0(U)$ of all continuous bounded functions on U tending to zero along regular sequences in U.
In the general situation of a Bauer space $(X, \mathcal{H})$ the measure μ is replaced by a compatible family $M = (M_U)$ of differences of continuous real potentials leading to potential operators K_U^M. The perturbed space $(X, {}^M\mathcal{H})$ is given by ${}^M\mathcal{H}(V) = \{h \in \mathcal{C}(V): h + K_U^M h \in \mathcal{H}(U) \text{ if } \bar{U} \subset V\}$ (Walsh 1970). Obviously, ${}^\mu\mathcal{H}(V) = \{h \in \mathcal{C}(V): (\Delta - \mu) h = 0\}$ in the classical case.
Given a compatible family (N_U) of continuous real strict potentials, a real number α is called an N-eigenvalue if $\mathcal{E}_N^\alpha := {}^{M+\alpha N}\mathcal{H}(U) \cap \mathcal{C}_0(U) \neq \{0\}$. The set E_N of all N-eigenvalues is upper bounded and has no accumulation points. All eigenspaces have finite dimension. If $H_U 1 > 0$ and U has no proper absorbing subsets then $E_N \neq \emptyset$, the eigenspace $\mathcal{E}_N^{\alpha_0}$ of the greatest eigenvalue α_0 consists of multiples of a strictly positive function, and there are no positive eigenfunctions for $\alpha < \alpha_0$.

25.6.1986 W. Hansen (Bielefeld)

The Dirichlet problem for sub-Laplacians

After a short introduction into the potential theory of hypoelliptic differential operators which are sums of squares of vectorfields, I present a very general version of Wiener's criterion which covers such situations. The following applications to sub-Laplacians are joint work with W. Hansen: Consider a finite dimensional real Lie algebra $\mathcal{N}$ which has a decomposition $\mathcal{N} = V^1 \oplus \cdots \oplus V^r$, $[V^i, V^j] = V^{i+j} \neq \{0\}$ iff $i+j \leq r$. Fix a basis $Y_{i1}, \dots Y_{in_i}$ for each V^i. On the simply connected Lie group N belonging to $\mathcal{N}$ we use the coordinates given by the exponential map and the basis of $\mathcal{N}$ introduced above. Hence each Y_{ij} may be regarded as a left invariant vectorfield on N and $L = Y_{11} + \cdots + Y_{1n_1}$ is a sub-Laplace operator. Let

$$A = \{ (y_{ij})_{ij} \mid \Big(\sum_{ij \neq rn_r} y_{ij}^2 \Big)^{\frac{\alpha}{2}} \leq \varkappa_{rn_r} \}$$

Then we have:

Theorem 1: $n_1 \geq 3, \mathcal{N} \neq R^3 \implies A$ is thin at 0 iff $\alpha < r$

Theorem 2: $n_1 = 2, r \geqslant 3 \implies A$ is thin at 0 iff $\alpha < \frac{r}{2}$

Theorem 3: $r \leq 2 \implies$ bounded domains with smooth boundary are regular

$n_1 = 2, r \leq 4 \nearrow$

In all other cases there exist smooth domains which are not regular.

H. Hueber

There exists an orthonormal basis for $L^2(\mathbb{R})$ which is, at the same time, an unconditional basis for most of the classical functional spaces (roughly speaking, those on which the Hilbert transform is bounded). This allows to construct new Calderón-Zygmund operators and to understand better their symbolic calculus. Y Meyer

Let $A \in S^m_{1\delta}(r,N)$ be a ΨDO such that for all $\alpha \in \mathbb{N}_0^n$ with $|\alpha| \leq N$ (i) $|\partial_\xi^\alpha a(x,\xi)| \leq C(1+|\xi|)^{m-|\alpha|}$, (ii) $\|\partial_\xi^\alpha a(\cdot,\xi)\|_{\Lambda^r} \leq C(1+|\xi|)^{m+\delta r-|\alpha|}$, $0 \leq \delta \leq 1$, where Λ^r denotes the Hölder-Zygmund space of order $r > 0$. Denote by $H^{sp}(w)$, $0 < p < \infty$ and $-\infty < s < \infty$, the weighted Hardy-Sobolev space. The weight may belong to the Muckenhaupt class A_∞. Define $p_w := \inf\{q : w \in A_q\}$. Then if $N > n \max\{\frac{1}{2}, \frac{p_w}{p}\}$ and $n(\max\{1, \frac{p_w}{p}\} - 1) - (1-\delta)r < s < r$ one has $A: H^{s+m,p}(w) \to H^{sp}(w)$ boundedly,

Jürg Marschall

"Harmonic functions on trees"

A geometric description of the Martin boundary of some denumerable

Markov chains is given in terms of the asymptotic behaviour of the associated graph. Particular attention is devoted to the case of nearest neighbour transition operator on trees, where the Martin boundary is known to be isomorphic to the space of ends of the tree. An analogous description is obtained for non-nearest-neighbour operators on trees whose transition probabilities satisfy some natural bounds. For more general graphs, ends are defined as classes of equivalence of paths which cannot be separated at infinity. If the graph admits a uniformly spanning tree, the same geometrical characterization holds. More generally, similar results are valid when the transition probabilities satisfy local bounds which vary slowly in moving to infinity

M.A. Picardello

'The oblique derivative problem on Lipschitz domains'

I described the L^p solvability results for boundary value problems for harmonic functions on Lipschitz domains. The previously known, (and optimal) results for the Dirichlet and Neumann problem are on the ranges $2-\varepsilon<p<\infty$ and $1<p<2+\varepsilon$ respectively. A.P. Calderón established the corresponding results for oblique derivative problems with continuous, transverse vector fields, in the range $2-\varepsilon<p<2+\varepsilon$. In joint work with J. Pipher, we showed that the optimal range of p's for this problem is $2-\varepsilon<p<\infty$.

Carlos E. Kenig

"On the existence of singular integrals in L^1"

For a homogeneous singular integral T in $\mathbb{R}^n$, $n \geq 2$, defined by convolution with $K = \text{p.v. } \Omega(x')|x|^{-n}$, the method of rotations gives L^p boundedness, $1<p<\infty$, with no regularity assumptions on Ω (only a size condition: $\Omega \in L^{1+\varepsilon}(S^{n-1})$), but no result was known for $p=1$. I described part of a joint work with M. Christ in which it is proved that T is of weak type $(1,1)$ under the same hypothesis on Ω. The same result holds for the maximal operator $Mf(x) = \sup_{t>0} |k_t * f(x)|$ where $k(x) = \Omega(x')\chi_{\{|x|\leq 1\}}$.

José L. Rubio de Francia

"Convergence for the square root of the Poisson kernel"

Let P be the Poisson kernel in the bidisk or in a general symmetric space. If f is a function on the maximal distinguished boundary, define $P_0 f$ by integrating f against $\sqrt{P}$. This is a special case of the generalized Poisson integrals $P_\lambda f$. The normalized Poisson integral $P_0 f / P_0 1$ converges to f at the boundary. For $f \in L^1$ this convergence turns out to be better than for other λ, in the sense that wider approach regions can be used.

Peter Sjögren (Göteborg)

"The Beltrami equation on the Heisenberg group"

Fourier analysis on the Heisenberg group is used for the study of the Beltrami equation

$$\bar{Z} f = \mu Z f \qquad Z = \frac{\partial}{\partial z} + i\bar{z}\frac{\partial}{\partial t} \qquad \bar{Z} = \frac{\partial}{\partial \bar{z}} - iz\frac{\partial}{\partial t}$$

the transformation B corresponding to the Hilbert-Beurling transformation in $\mathbb{C}$ is given by

$$Bg(z) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty} \mathrm{tr}\, \pi_\lambda^*(z,t)\, \pi_\lambda(g)\, B_\pm |\lambda|\, d\lambda$$

where π_λ is the Bargmann representation and $\pi_\lambda(g)$ the Fourier transform of g. The operators B_+ (for $\lambda > 0$) and B_- (for $\lambda < 0$) can be realized as matrix multiplications

$$B_+ = \begin{pmatrix} 0 & 0 & \sqrt{\frac{2}{1}} & & \\ & 0 & 0 & \sqrt{\frac{3}{2}} & \\ & & 0 & 0 & . \\ & & & \cdots & \end{pmatrix} \qquad B_- = \begin{pmatrix} 0 & & & \\ 0 & 0 & & \\ \sqrt{\frac{1}{2}} & 0 & 0 & \\ & \sqrt{\frac{2}{3}} & 0 & \\ & & \cdots & \end{pmatrix}$$

The equation $\bar{Z} f = \mu Z f$ with prescribed asymptotics for f cannot be solved for every (smooth) μ with $\|\mu\|_\infty < 1$. Various sufficient conditions on μ are established, which guarantee that this equation has a unique normalized solution.

HM Reimann, Bern

Andrzej Hulanicki

Schrödinger operators and nilpotent groups.

For operators of the form

$$L = -\Delta + \Sigma |P_j|^{\alpha_j} \qquad 0 \le \alpha_j < 1,$$

where P_j are polynomials the following fact is true.

Let $Lf = \int_0^\infty \lambda \, dE(\lambda) f$ on $L^2(\mathbb{R}^n)$ and let $K \in C^\infty(\mathbb{R}^+)$ satisfy the following condition

$$\sup_{\lambda > 0} \lambda^n |K^{(j)}(\lambda)| \le R^n (n!)^\gamma$$

$j \le n$, $n = 0, 1, 2, \dots$ for some $R > 0$ and $\gamma > 0$ independent of n, $K(0) = 1$.

Then

$$\lim_{t \to 0} \left\| \int_0^\infty K(t\lambda) \, dE(\lambda) f - f \right\|_{L^p} = 0$$

for every $f \in L^p(\mathbb{R}^n)$, $1 \le p < \infty$.

The proof uses functional calculus on nilpotent groups and a recent theorem of P. Głowacki

And Hulanicki

Singular integrals on general nilpotent Lie groups

Fulvio Ricci

The subject of this talk is joint work with E.M. Stein. Singular integral operators on nilpotent Lie groups that are related to non-automorphic dilations are considered.

Even though these operators do not have nice invariance property and cannot be treated by the usual techniques available on spaces of homogeneous type, it is possible to prove that they are bounded on L^p-spaces, for $1<p<\infty$.

The proof consists in lifting the operators on appropriate manifolds in a free group and use the method of transference.

Maximal operators related to non-automorphic dilations can be treated similarly.

FRicci

"Averages over hypersurfaces"

This talk concerns joint work with E.M. Stein. We study averages of the form:

$$(M_\varepsilon f)(x) = \int_{S_{\varepsilon,x}} f(x+\varepsilon y)\, d\sigma_{\varepsilon,x} ,$$

where $S_{\varepsilon,x} \subset \mathbb{R}^n$ is a compact hypersurface with Lebesgue measure $d\sigma_{\varepsilon,x}$. If $n \geq 6$ and if the surfaces depend smoothly on the parameters $(\varepsilon,x) \in \mathbb{R} \times \mathbb{R}^n$ and if, furthermore, the surfaces $S_{0,x}$ have the property that their Gaussain curvatures do not vanish of infinite order then we prove that there are numbers $0 < \varepsilon_0, p_0 < \infty$ depending on the family for which

$$\Big\| \sup_{0<\varepsilon<\varepsilon_0} |(M_\varepsilon f)(x)| \Big\|_{L^p(\mathbb{R}^n)} \leq C \|f\|_{L^p(\mathbb{R}^n)} , \quad p > p_0 .$$

Christopher Sogge

Pointwise multiplication in Bessel potential spaces

Results of Strichartz 1967 are carried over to a more general situation and elementary proofs are given: To P, a real $n \times n$ matrix with eigenvalues λ_j, $\operatorname{Re} \lambda_j > 0$, $\nu = \operatorname{tr} P$, associate the dilation $A_t = t^P$, $t>0$, and a continuous, positive definite, A_t-homogeneous distance function r; let P', A_t', ϱ be the adjoint notions. Consider generalized Bessel potential spaces

$$\mathcal{L}^p_{\sigma,\varrho} = \{ f : f = G_{\sigma,\varrho} * g , \ \|f\|_{p,\sigma,\varrho} = \|g\|_p < \infty \} , \quad 1 \le p \le \infty, \ \sigma > 0,$$

where $G_{\sigma,\varrho}$ is given by its Fourier transform

$$G^{\wedge}_{\sigma,\varrho}(\xi) = (1+\varrho(\xi))^{-\sigma} , \quad \varrho \in C^{[\frac{n}{2}+1]}(\mathbb{R}^n \setminus \{0\}).$$

There holds: $\mathcal{L}^p_{\sigma,\varrho}$, $1<p<\infty$, $\sigma > \nu/p$, is a multiplication algebra: $\mathcal{L}^p_{\sigma,\varrho} \mathcal{L}^p_{\sigma,\varrho} \subset \mathcal{L}^p_{\sigma,\varrho}$. Main tool in the proof is the following equivalent norm on $\mathcal{L}^p_{\sigma,\varrho}$ $(1<p<\infty, \sigma>0, m$ suff. large$)$

$$\|f\|_p + \sup_{\varepsilon>0} \Big\| \int_{\varepsilon \le r(y) \le 1} r(y)^{-\sigma-\nu} \Delta_y^m f \, dy \Big\|_p$$

and the Leibniz formula for central differences Δ_y^m.
A characterization of the pointwise multipliers on $\mathcal{L}^p_{\sigma,\varrho}$, $\sigma > \nu/p$, also follows along the lines of Strichartz.

W. Trebels

Quasiradial Fourier multipliers

We consider criteria for multipliers $m \circ \varrho$; where $m : (0,\infty) \to \mathbb{C}$, and $\varrho \in C^\infty(\mathbb{R}^n \setminus 0) \cap C(\mathbb{R}^n)$, $\varrho(x) > 0$, $x \neq 0$; $\varrho(t^P x) = t \varrho(x)$ $(t>0)$. Then we have the necessary conditions

$$\sup_{t>0} \| \varphi \, m(t \cdot) \|_{B^{p'}_{\alpha p}} \le c \, \| m \circ \varrho \|_{M^p} , \quad 1 \le p \le 2, \quad \alpha = (n-1)(1/p - 1/2).$$

If $\Sigma_\varrho = \{\xi, \varrho(\xi) = 1\}$ is strictly convex, then we have the sufficient condition

$$\| m \circ \varrho \|_{M_p} \le c \sup_{t>0} \| \varphi \, m(t \cdot) \|_{L^2_\gamma} , \quad \gamma > n\left(\frac{1}{p} - \frac{1}{2}\right), \quad 1 < p \le \frac{2(n+1)}{n+3}.$$

(φ some bump function supp. in $(0,\infty)$)

Both are sharp. Essential tool for the sufficient conditions is the Restriction theorem for the Fourier transform and some extension of Calderón-Zygmund-singular-integral-theory to L^p, $p>1$.

Andreas Seeger

L^2-boundedness of Singular Integral Operators.

This talk presents recent joint work with Michael Christ. Let T be a singular integral operator on $\mathbb{R}^n$, that is, an operator defined from $\mathcal{C}_0^\infty(\mathbb{R}^n)$ to $[\mathcal{C}_0^\infty(\mathbb{R}^n)]'$ so that $U(g,f) = \langle g, Tf\rangle$ is defined a priori for f and g in $\mathcal{C}_0^\infty(\mathbb{R}^n)$. Suppose that T is given by a kernel satisfying $|K(x,y)| \le \frac{C}{|x-y|^n}$ and $|\nabla_x K(x,y)| + |\nabla_y K(x,y)| \le \frac{C}{|x-y|^{n+1}}$. A general L^2-boundedness criterion is that $T1$ and T^*1 lie in BMO and T has the Weak Boundedness Property, that is if g and f are $\mathcal{C}_0^\infty(\mathbb{R}^n)$, then

$$|\langle g, Tf\rangle| \le C\, t^{n+2} \|\nabla g\|_\infty \|\nabla f\|_\infty$$

where $t = \operatorname{diam}\{\operatorname{supp} g \cup \operatorname{supp} f\}$. We generalize this criterion to treat the case of the Cauchy-kernel on Lipschitz curves and extend it to multilinear actions of Calderón-Zygmund type.

(a.k.a. The Clone)

Convex Hypersurfaces and Fourier Transforms

Let $S \subset \mathbb{R}^{n+1}$ be a smooth compact hypersurface bounding a convex domain. Suppose S is of finite type m; ie. every tangent line makes order of contact at most m with S. For $x \in S$ let T_x be the affine tangent hyperplane to S at x, and for $\delta > 0$ define

$$B(x,\delta) = \{y \in S \mid \operatorname{dist}(y, T_x) < \delta\}$$

In joint work with Joaquim Bruna and Stephen Wainger we prove:

Theorem: There is a constant $C = C(S)$ so that

(a) if $x_1, x_2 \in S$, $\delta > 0$ and $B(x_1,\delta) \cap B(x_2,\delta) \neq \emptyset$ then $B(x,\delta) \subset B(x_2, C\delta)$.

(b) if $x \in S$, $\delta > 0$ then $\sigma(B(x, 2\delta)) \le C\, \sigma(B(x,\delta))$ where $d\sigma$ is surface measure on S.

Theorem 2: let $\eta \in \mathbb{R}^{n+1}_+$, $|\eta| = 1$ and let $x_0 \in S$ with η orthogonal to T_{x_0}. Let $\chi \in C_0^\infty(\mathbb{R}^{n+1})$ have sufficiently small support. Put

$$H(\lambda) = \int_S e^{i\lambda \langle x,\eta \rangle} \chi(x - x_0)\, d\sigma(x)$$

Then $H(\lambda) = e^{i\lambda\langle x_0,\eta\rangle} F(\lambda)$, and there are constants $C_j = C_j(S)$ independent of x_0 so that

$$|F^{(j)}(\lambda)| \leq C_j \lambda^{-1} \sigma(B(x_0, 1/\lambda))$$

Alexander Nagel

Singular Integrals Associated to curves

We present an extension of the theory in the following sense: we consider a C^1-curve in $\mathbb{R}^2$, $\Gamma = \{(t, \gamma(t)) : -\infty < t < +\infty\}$, $\gamma(0) = \gamma'(0) = 0$, with the following properties:

a) γ is bicmvex, i.e. $|\gamma'(t)|$ is decreasing in $(-\infty, 0)$ and increasing in $(0, +\infty)$.

b) γ has doubling time, i.e. there exists a constant $C > 1$ s. t. $|\gamma'(Ct)| \geq 2|\gamma'(t)|$, $\forall t$.

c) γ is balanced, by which we mean the following: there exists $K > 1$ s. t. $|\gamma(K^{-1}t)| \leq |\gamma(-t)| \leq |\gamma(Kt)|$ for every $t > 0$.

Let us consider the operators:

$$Hf(x) = \text{P.V.} \int f(x - \Gamma(t)) \frac{dt}{t}, \quad H^*f(x) = \sup_{\varepsilon > 0} \left| \int_{|t| \geq \varepsilon} f(x - \Gamma(t)) \frac{dt}{t} \right|$$

$$Mf(x) = \sup_{h > 0} \frac{1}{2h} \int_{-h}^{+h} |f(x - \Gamma(t))|\, dt$$

We have:

Theorem. Under the assumptions a) b) and c) on the curve Γ, the singular integral H and the maximal functions M and H^* are bounded operators on $L^p(\mathbb{R}^2)$, $1 < p < \infty$.

A Cordoba

Oscillatory singular integrals and convergence of Fourier integrals and Fourier series

Some results on convergence and summability almost everywhere of Fourier series and Fourier integrals were discussed. Related results on oscillatory singular integrals and regularity of solutions to the Schrödinger equation $\Delta_x u(x,t) = i \frac{\partial u}{\partial t}(x,t)$ were mentioned.

Per Sjölin

Operators which have an H^∞ functional calculus

Alan McIntosh

Let T be a closed linear operator in a Hilbert space $\mathcal{H}$ which is one-one and has spectrum $\sigma(T)$ contained in a double sector $S_\omega = \{z \in \mathbb{C} \mid |\arg z| \leq \omega$ or $|\arg(-z)| \leq \omega\}$ where $0 \leq \omega < \frac{\pi}{2}$. Suppose also that $\|(T-\lambda I)^{-1}\| \leq c\,[\operatorname{dist}(\lambda, S_\omega)]^{-1}$. Then T has an H^∞-functional calculus if & only if it satisfies square-function estimates. In particular $\operatorname{sgn}(T) \in \mathcal{L}(\mathcal{H})$ where $\operatorname{sgn}(\zeta) = +1$ if $\operatorname{Re} \zeta > 0$ and $\operatorname{sgn}(\zeta) = -1$ if $\operatorname{Re} \zeta < 0$. An example of such an operator is differentiation on a Lipschitz graph, in which case $\operatorname{sgn}(T)$ is the Cauchy integral.

H^p Spaces on Lipschitz Domains

Let D be a bounded Lipschitz domain in $\mathbb{R}^n$, $n \geq 3$. Define $H^p(D, d\sigma)$ as the space of harmonic functions whose nontangential maximal function belongs to $L^p(\partial D, d\sigma)$, where $d\sigma$ denotes surface measure on the boundary. In joint work with C. Kenig it is shown that $H^1(D, d\sigma)$ has an atomic decomposition and its dual is identified with a weighted BMO space. The fact that atoms belong to H^1 is a result of B.E.J. Dahlberg (1979). Finally, an atomic decomposition and characterization of the dual is given for $H^p(D, d\sigma)$, $1 < p < 2$, in the range of p in which H^p cannot be identified with $L^p(d\sigma)$.

Jill Pipher

Some Remarks on the Calderón-Zygmund Theory

Tony Carbery

The Cotlar-Stein Lemma and the Littlewood-Paley Theory teach us to think of the different dyadic annuli $\{|x| \sim 2^k\}$, $k \in \mathbb{Z}$ as being "independent" as far as operators like singular integrals are concerned. In this talk we present some new results (some also obtained by A. Seeger) in this vein which apply to a wide variety of operators.

Concrete Integral Representations,

Let C be a cap in a closed lattice cone $H \subset \mathscr{C}_+(X)$, X LCCB and that $1 \in C$.
Then C is a Choquet simplex, and every $h \in C$ has a unique minimal representing measure μ_h.
Assume furthermore the following minimum principle: $h \in H-H$, $h \geq 0$ outside some compact set $\Rightarrow h \geq 0$ on X.
The following results are joint work with P. Loeb.

Theorem: There exist a compactification $\hat{X}$ of X such that $\Delta = \hat{X} \setminus X$ is metrizable, and a "nice" kernel $Q : X \times \Delta \to \mathbb{R}_+$ such that

(1) $z \mapsto Q(\cdot, z)$ is a homeomorph. from Δ into C

(2) $h \in \mathscr{C}$ bounded $\Rightarrow h(x) = \int Q(x,z)\, \mu_h(dz)$, $x \in X$

(3) $\mu_h \ll \mu_1$

(4) $h \in C$: $h \in \overline{C}_b \iff \exists \mu$: $h(x) = \int Q(x,z)\, \mu(dz)$ for some $\mu \in M_+(\Delta)$, $\operatorname{supp} \mu \subset \operatorname{supp} \mu_1$

Consequences of this theorem, especially the treatment of the Dirichlet problem and connection to the Martin compactification are discussed.

On two sides of a curve: Let Γ be a Jordan curve in the plane, let Ω^+ and Ω^- be the complementary domains and let $\omega^\pm$ be harmonic measure on Γ relative to $\Omega^\pm$ (for fixed $z^\pm \in \Omega^\pm$). If Γ is rectifiable, then by F. + M. Riesz ω^+ and ω^- are each mutually absolutely continuous to arc length. If Γ is the Von Koch snowflake, ω^+ and ω^- are mutually singular $\omega^+ \perp \omega^-$. Thm:

$$\omega^+ \perp \omega^- \Longleftrightarrow \{p \in \Gamma : \Gamma \text{ has a tangent at } p\} \text{ has one-dimensional measure zero.}$$

This is due to the author, Christopher Bishop, L. Carleson and P. Jones (jointly). The proof uses Pommerenke's extension of the fundamental work of N. G. Makarov on harmonic measure in simply connected plane domains

John Garnett

Harmonic measure and the law of the iterated logarithm

Peter W. Jones, Yale University

Let B denote the Bloch space on the unit disk D. Let $\|\varphi\|_B \leq 1$ and

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suppose that on every hyperbolic ball of radius 1 there is a point where $|\varphi'(z)|(1-|z|) > \varepsilon$. $(*)$ Then

$$\varlimsup_{r \nearrow 1} \frac{\operatorname{Re}\varphi(re^{i\theta})}{\sqrt{\log\frac{1}{1-r}\,\log\log\log\frac{1}{1-r}}} \geq c(\varepsilon) > 0$$

for almost every θ. (The reverse inequality holds with constant C_0 a.e. $(d\theta)$ for all $\|\varphi\|_B \leq 1$. This is due to Makarov.) Let $f: D \to \Omega$ be any univalent function and let $\varphi = \log f'$. A characterization is given of all domains Ω so that $(*)$ holds; the desciption is in terms of the geometry of the boundary of Ω and says that, in a strong way, $\partial\Omega$ has no tangents. As a corollary one obtains for these domains that harmonic measure is supported on a set of Hausdorff measure zero with respect to the measure function $t\exp\left\{c\sqrt{\log\frac{1}{t}\log\log\log\frac{1}{t}}\right\}$.

(Makarov has shown that, exept for the value of c, this size is smallest possible.)

Thinness in nonlinear potential theory.

Lars Inge Hedberg, University of Linköping, Sweden

Classical potential theory is closely related to the study of the Sobolev space $W^{1,2}(R^N)$. In a similar way there is a nonlinear potential theory associated to the Sobolev spaces $W^{m,p}(R^N)$. Several of the classical definitions ~~of~~ thinness have natural generalizations to this nonlinear situation, but these generalizations are in general not equivalent. It is a consequence of an inequality of T. Wolff that the most inclusive of these definitions is the "right one", in the sense that the classical Kellogg and Choquet properties, and a variant of the Wiener criterion generalize.

On prescribing curvature on S^2.

In this talk, we discussed the following problem posed by L. Nirenberg: what function K is allowed to be the Gaussian curvature function of a metric conformally to the standard metric? or equivalently for what function K does the equation

$$(*) \qquad \Delta u + K e^{2u} = 1 \qquad \text{has solution on } S^2 ?$$

We started with Moser's Theorem that even function K (i.e. $K(\xi)=K(\xi)$)) allows an even solution for $(*)$ and posed the connection of this problem to the best exponent in a sharp Sobolev inequality. We then gave various sufficient condition on K for $(*)$ to be solvable.

Sun-Yung A. Chang.

Graphentheorie, 86-06-29 ≈ 07-05

On some generalizations of outerplanar graphs

(Maciej M. Sysło, Inst. Comput. Sci., U. of Wrocław, Poland)

Outerplanar graphs have been recently generalized in many directions. For instance: A graph G is W-outerplanar, $W \subseteq V(G)$, if G can be embedded in the plane so that all vertices of W lie on the boundary of the same face. G is k-outerplanar, if G has a planar embedding with k vertices on the boundary of one face and k is maximum. A number of generalizations deal with covering the vertex set of a graph (planar) with a smallest number of faces, disjoint or not. In particular, A planar graph G is of disk dimension $d_0(G)$ if G admits an embedding on the sphere (S_0) minus $d_0(G)$ open discs $\mathcal{D}$ with every vertex in $\partial\mathcal{D}$ and $d_0(G)$ is minimum. Note, that the disc dimension $d_k(G)$ can be defined for an arbitrary graph G embedded on the surface S_k of genus k.

Almost all generalizations of outerplanar graphs have been introduced to parameterize the family of planar graphs so that in consequence some of the decision problems which are NP-complete for planar graphs and trivial for outerplanar graphs can be solved in polynomial time for every fixed value of a parameter.

In this talk, we review some generalizations of outerplanar graphs, present their characterizations and recognition algorithms, and discuss complexity of some of the problems which are NP-complete for arbitrary planar graphs.

Maciej M. Sysło
(U. of Wrocław, Poland)

Graphen und nichtkommutative Geometrie

Eine *Färbung* des vollständigen ~~Graphen~~ (schlichten) Digraphen K_X mit der Eckenmenge X ist eine Abbildung $\langle\,,\rangle : X^2 \to F$, $(x,y) \mapsto \langle x,y\rangle$ mit $\langle x,y\rangle \neq \langle z,z\rangle$ für $x \neq y$. Ein vollständiger Digraph mit Färbung heißt (nichtkommutativer) *P-Raum* oder kurz *Raum* (J. Pfalzgraf, J. Geometry $\underline{25}$, 147-165 (1985)). Die Elemente von X bzw. $R := \{\langle x,y\rangle \in F \mid x \neq y\}$ des Raumes $R = (X, \langle,\rangle, F)$ heißen *eigentliche* bzw. *uneigentliche* Punkte. Die *Verbindungslinie* von x nach y $(x, y \in X)$ ist definiert durch: $x \square y := \{x\} \cup \{z \in X \mid \langle x,y\rangle = \langle x,z\rangle\} \cup \{\langle x,y\rangle\}$; hierbei ist $\langle x,y\rangle$ der uneigentliche Punkt dieser Linie. Zwei Linien mit gleichem uneigentlichen Punkt heißen *parallel*. Die q-*Simplex-Aussage* Sim_q $(q \in \mathbb{N})$ ist definiert durch: Zu $x_0, \ldots, x_q, x_0', x_1' \in X$ mit $\langle x_0, x_1\rangle = \langle x_0', x_1'\rangle$ gibt es $x_2', \ldots, x_q' \in X$ mit $\langle x_i, x_j\rangle = \langle x_i', x_j'\rangle$ $(i, j \in \{0, \ldots, q\})$. Die desarguessche Schließungsaussage ist Sim_3 mit $x_0 = x_0'$. Ist $g = (X, Y)$ ein stark zusammenhängender (schlichter) Digraph, so ist der *Graphenraum* $\bar{g}$ durch $\langle x,y\rangle := d(x,y)$ (d = Eckenabstand im Graph g) definiert. U.a. wird gezeigt: Genau dann gilt Sim_2 in $\bar{g}$ (*schiefaffiner Raum*), wenn in g das *Prinzip der freien Beweglichkeit* gilt: Zu Ecken $x_0, x_1, x_2, x_0', x_1'$ mit $d(x_0, x_1) = d(x_0', x_1') = a$, $d(x_0, x_2) = b$ und $d(x_1, x_2) = c$ gibt es eine Ecke x_2' mit $d(x_0', x_1') = b$ und $d(x_1', x_2') = c$. Zum Schluß werden Beziehungen zur algebraischen Graphentheorie aufgestellt: Ist der Raum $R = (X, \langle,\rangle, F)$ endlich und ist N_f $(f \in F)$ die Nachbarschaftsmatrix zu dem durch $f \in F$ gegebenen Teilgraphen von K_X (d.h. $N_f(x,y) = 1$ für $\langle x,y\rangle = f$ und $= 0$ sonst), so ist der durch diese N_f $(f \in F)$ aufgespannte Unterraum genau dann hinsichtlich der Matrizenmultiplikation abgeschlossen (Verbindungsalgebra, Nachbarschaftsalgebra oder Hecke-Algebra), wenn R stark schiefaffin (Verschärfung von Sim_2) ist.

Johannes André (Saarbrücken)

Clique covers and coloring problems of graphs

Classical theorems of Vizing and Shannon give upper bounds for the chromatic index $\chi'(G)$ of a graph, resp. multigraph G. The chromatic index of G coincides with the chromatic number of its line graph. If we characterize line graphs by clique covers, the theorems of Vizing and Shannon take the following form.

Vizing: Let the cliques $C_1, \dots, C_m$ cover G, $k = \max_i |C_i|$. If $|C_i \cap C_j| \leq 1$ for all $i \neq j$ and every vertex is covered at most twice, then $\chi(G) \leq k+1$

Shannon: Let the cliques $C_1, \dots, C_m$ cover G, $k = \max_i |C_i|$. If every vertex is covered at most twice, then $\chi(G) \leq \frac{3}{2} k$.

We establish sharp upper bounds for the clique number $\omega(G)$, if the cliques covering G may cover the vertices of G more than twice. In some special cases these bounds turn out to be also upper bounds for $\chi(G)$. Some open problems are stated as an invitation to prove the same coincidence in some other cases.

Walter Klotz, Clausthal-Zellerfeld

Decompositions of K_9 and solving Bagemihl's conjecture on neighborly tetrahedra.

We proved that $K_9 \neq 2K_{2,3} + 6K_{2,2}$, and with the aid of a computer that $K_9 \neq 3K_{2,3} + 4K_{2,2} + K_{1,2}$, $K_9 \neq 3K_{3,3} + 4K_{3,2} + 2K_{1,1}$, $K_9 \neq 4K_{2,3} + 2K_{3,2} + 2K_{1,2}$ and $K_9 \neq 5K_{3,3} + 3K_{1,2}$.

These are parts in our proof of Bagemihl's conjecture that the maximum number of neighborly tetrahedra is eight, where a family of convex d-polytopes in E^d is called neighborly if every pair meets in a $(d-1)$-polytope.

The assertion that there exists a neighborly family of nine tetrahedra leads to a 0,±1 matrix, and a system of Diophantine equations $\Sigma(i+j)x_{ij}=36$, $\Sigma ij x_{ij}=36$, $x_{0,1}\leq 2$ and $x_{ij}\neq 0 \Rightarrow i\leq j\leq 3$, $(i,j)\neq(3,3)$. In addition there should be a decomposition of K_9 into $\Sigma x_{ij}K_{ij}$. The system has 24 solutions, some of them lead directly to contradictions; in all the remaining cases, we found with computer aid all the combinatorially different decompositions of K_9 and proved that the corresponding 0±1 matrix is not related to a neighborly family of 9 tetrahedra.

A collection of tetrahedra in E^3 is called <u>nearly-neighborly</u> if each pair is separated by a hyperplane which contains a facet of each. By Perles' result there can be at most 16 nearly neighborly tetrahedra; we can prove that there are at most 15 nearly neighborly tetrahedra, by using similar tools to the above, plus a lemma that states: The multigraph obtained from K_{16} by taking the edges of a 1-factor as multiple edges can be decomposed to not fewer than 9 complete bipartite graphs. The correct max. # of nearly neighborly tetrahedra is probably just eight.

Joseph Zaks
HAIFA, ISRAEL

<u>The Ramsey number of K_5-e</u>

The Ramsey number $r=r(K_5-e)$ is defined to be the smallest complete graph K_r, such that every 2-coloring of its edges contains a monochromatic K_5-e. After $r(K_5-e)\leqq 23$ has been proved (J. Graph Theory <u>9</u> (1985) 483-485), we now use this result to prove $r(K_5-e)\leqq 22$. A recent coloring of K_{21} without a monochromatic K_5-e, which was given by G. Exoo, finishes the proof of $r(K_5-e)=22$. — Now K_5 remains the only graph with five vertices, and with its Ramsey number unknown.

Heiko Harborth
BRAUNSCHWEIG

Partitions of graphs

Let $G = (V, E)$ be a graph and let $\Lambda : V \to P(\mathbb{N})$. A Λ-colouring of G is a vx colouring $\varphi : V \to \mathbb{N}$ s.t. $\varphi(x) \in \Lambda(x)$ $\forall x \in V$. The list-chromatic number of G is $\chi_\ell(G) = \min\{k :$ G has a Λ-colouring $\forall \Lambda : V \to \mathbb{N}^{(k)}\}$. It's very easy to see (though surprising) that $\sup\{\chi_\ell(G) : \chi(G) = 2\} = \infty$.

The list-edge-chromatic number $\chi'_\ell(G)$ is defined analogously. Clearly $\chi'(G) \le \chi'_\ell(G) \le 2\Delta - 1$. It was conjectured by Albertson and Tucker that $\chi'_\ell(G) = \chi'(G)$ $\forall G$; in particular, $\chi'_\ell(G) \le \Delta + 1$.

Recently A. J. Harris and I proved that

(1) if $c > 11/6$ and $\Delta(G)$ is suff. large then $\chi'_\ell(G) < c\,\Delta(G)$.

Some years ago I proved that $\exists\, S : \mathbb{N} \to \mathbb{N}$ s.t. if $\delta(G) \ge S(k)$ then $\forall \ell$ G contains a cycle of length $\equiv 2\ell \pmod{k}$. Thomassen proved that (2) $\exists\, \sigma : \mathbb{N} \to \mathbb{N}$ s.t. if $\delta(G) \ge \sigma(k)$, G is 2-conn^{d} and not bipartite then $\forall \ell$ G contains a cycle of length $\equiv \ell \pmod{k}$.

Both (1) & (2) are based on partitions of our graph which judiciously split the degrees. Namely, let $f(s) = \min\{r :$ if $\delta(G) \ge r$ then $V = V_1 \cup V_2$, $\delta(G[V_i]) \ge s$, $i = 1, 2\}$. Thomassen proved that $f(s) \le 12s$ and Häggkvist showed that $f(s) \le 3s$. Among others, we showed that $\exists\, c > 0$ s.t.

$$2s + cs^{1/2} \le f(s) \le 2s + 2(s \log s)^{1/2}$$

if s is suff. large. There are several related conjectures, including the following. Let $g(s) = \min\{k :$ if G is a k-connected k-regular graph then $V = V_1 \cup V_2$ s.t.

$\varkappa(G[V_i]) \geq s$, $i = 1, 2$, and $\varkappa(G[V_1, V_2]) \geq s\}$.

Conjecture. $g(s) < \infty$ $\forall s$; in fact, $g(s) = (2 + o(1))s$.

Béla Bollobás

TREE-LIKE PROPERTIES OF INFINITE GRAPHS

- $\mathcal{G}(X,E)$: locally finite, infinite, connected. $d_{\mathcal{G}}$ – graph metric.

? : Properties, that $\mathcal{G}$ looks faintly like a tree. In particular for transitive graphs / Cayley graphs.

- Tree-like property (a) $\mathcal{G}$ has a uniformly spanning tree (UST) : $\exists$ tree T, vertex set X, s.t. $d_{\mathcal{G}} \sim d_T$ (i.e. $L^{-1} d_T \leq d_{\mathcal{G}} \leq L d_T$, $0 < L < \infty$)

- Ω : space of ends of $\mathcal{G}$. (End: equivalence class of one-sided infinite simple paths in $\mathcal{G}$) [Freudenthal, Halin]
$\omega \in \Omega$, $U \subseteq X$ finite: Denote by $C(U, \omega)$ the component of ω in $(\mathcal{G} \setminus U) \cup \Omega$.
$\{U_n\}$: sequence of finite subsets of X contracting toward ω ($U_n \rightsquigarrow \omega$) $:\Longleftrightarrow$ (1) $C(U_n, \omega) \supseteq U_{n+1} \cup C(U_{n+1}, \omega)$ and (2) $\{C(U_n, \omega)\}$ neighbourhood basis at ω (for any point in $X \cup \Omega$, different from ω, there is some n s.t. U_n separates ω from this point)

Def: $\operatorname{diam}_{\mathcal{G}}(\omega) = \inf \{ \liminf_{n \to \infty} \operatorname{diam}_{\mathcal{G}}(U_n) \mid U_n \rightsquigarrow \omega \}$

Tree-like property (b) $\operatorname{diam}_{\mathcal{G}}(\omega) < \infty$ $\forall \omega \in \Omega$ (All ends have finite diameter)

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- In general: Lemma (a) implies (b), and ends have uniform bound for diameters.

- But in general: (b) $\not\Rightarrow$ (a). G transitive?

Thm 1: If G is transitive, then (b) $\Rightarrow$ (a).
In particular: if all ends have finite diameters, then these are uniformly bounded.
Thm 2. G Cayley graph of group Γ, then
(a) holds $\Leftrightarrow$ Γ contains free subgroup of finite index.

- Thm. 2 uses results of [Muller-Schupp; Dunwoody]

- Further tree-like properties: triangulation prop. of [Muller-Schupp], adapted for graphs ($\Leftrightarrow$ (a), basic tool of proofs of Th$_s$ 1, 2)
Planar geodesic graphs: all ends have diameter zero by [Watkins].

Wolfgang Woess (Leoben, Austria)

On minimal graphs

Let Γ_0 resp. F be the set of all finite undirected graphs without loops and multiple edges resp. an arbitrary orientable or non-orientable surface or the spindle-surface (pseudo-surface) F. Furthermore, let be $\Gamma_F = \{ G \in \Gamma_0 \mid G$ is non-embeddable in $F\}$, $\geq$ the subdivision-relation, and

$M_1(\Gamma_f) = \{G \in \Gamma_f \mid G$ is $\geq_1$-minimal$\}$ the minimal basis of Γ_f in relation to $\geq_1$. For $f = f_0$ = sphere (plane) Kuratowski proved in 1930 that $M_1(\Gamma_{f_0}) = \{K_5, K_{3,3}\}$

1936 D. König asks the questions:

a) How does the minimal basis $M_1(\Gamma_f)$ look like for every orientable surface f?

b) Is $M_1(\Gamma_f)$ finite for every orientable surface f?

Klaus Wagner and myself settled b) totally and a) only partially by proving:

Ad a) With the help of the partial order-relation $\geq_4$ fulfilling the properties (i) $\geq_4 \not\subseteq \geq_1$, (ii) $M_4(\Gamma_f) \not\subseteq M_1(\Gamma_f)$ we prove

1. $M_4(\Gamma_{f_1}) \geq \{G_1', \ldots, G_{12}'\}$

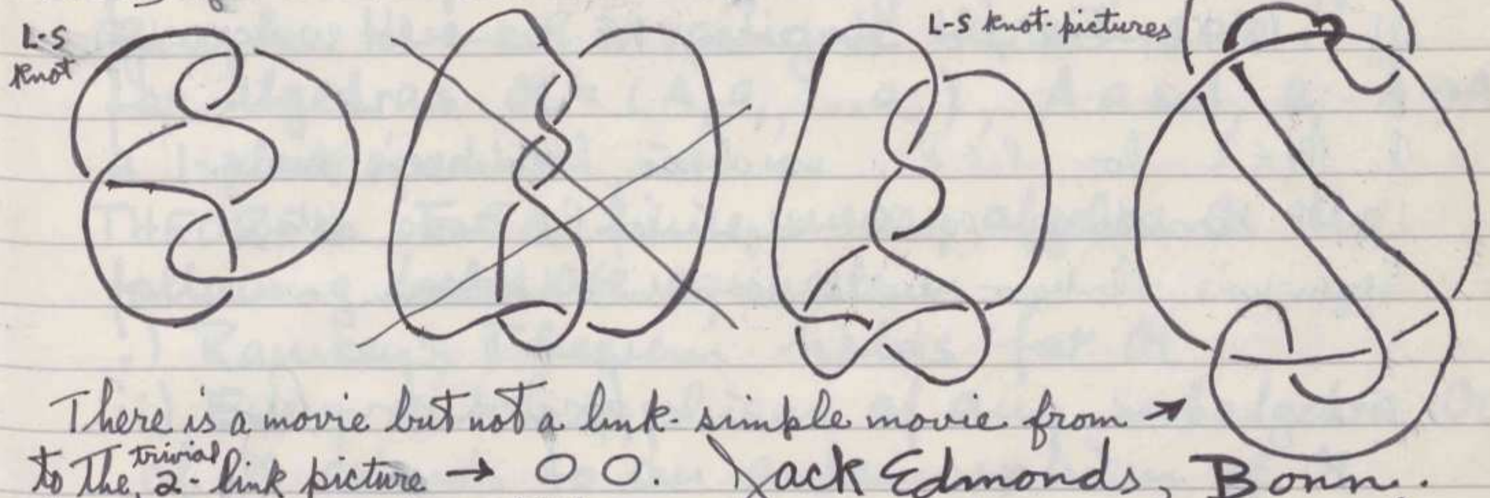
2. $M_4(\Gamma_{Torus}) \geq \{G_1, \ldots, G_{23}\}$

Ad b) 1. $|M_4(\Gamma_{f_1})| < \infty$

2. $|M_4(\Gamma_f)| < \infty$ for every orientable surface f

Rainer Bodendiek (Kiel) joint with Klaus Wagner (Köln)

A knot-movie is a sequence of knot-pictures related consecutively by Reidemeister moves. Conjecture: If there is a movie from knot-picture K_1 with m crossings to knot-picture K_2 with n crossings then there is a movie from K_1 to K_2 which is not longer than some fixed polynomial in m and n. ¶ A simpler theory: (We use the natural terminology "knot having several (S^1) links" rather than the more usual terminology "link having several (S^1) knots".) A knot in $\mathbb{R}^3$ is called link-simple if each of its links is by itself trivial (unknotted). A knot-picture (in $\mathbb{R}^2$) is called link-simple if each of its links is by itself a simple closed curve. A knot-movie is called link-simple if each of its pictures is link simple. ¶ (1) Theorem: Every link-simple knot has a link simple knot-picture. (2) We describe a theory of link-simple knot-movies.



There is a movie but not a link-simple movie from → to the trivial 2-link picture → O O. Jack Edmonds, Bonn.
(P.S. I also spoke about some concepts of "algorithm" for graph problems.)

On cycles in sparse graphs
Erdös and Hajnal suggested to measure the richness of a graph w.r.t. cycles by $L(G) := \{\frac{1}{k} \mid \exists$ cycle of length k in $G\}$.
Gyárfás, Komlós and Szemerédi showed that $f(\alpha) \geq a \cdot \log \alpha$ for some positive a and $\alpha > b$ sufficiently large. They raised the question to determine $f(1+\varepsilon)$ for small ε.
In a joint paper Gyárfás, Prömel, Szemerédi and Voigt showed that $f(\frac{k+1}{k}) \geq (300k \cdot \log k)^{-1}$, where
$f(\alpha) = \inf\{L(G) \mid |E(G)|/|V(G)| \geq \alpha\}$.

Bernd Voigt Bielefeld

Gefördert durch Deutsche Forschungsgemeinschaft

On circuit decomposition of Eulerian graphs

(András FRANK – Herbert FLEISCHNER)

Generalizing earlier results of the second author and of P. Seymour, we prove the following theorem:

THM. Let G be a planar Eulerian graph. At every node v a partition $P(v)$ of the incident edges is specified. Let $\mathcal{P} = \cup P(v)$. There exists a circuit decomposition $\mathcal{D}$ of the edge set of G such that $|P \cap C| \leq 1$ for $P \in \mathcal{P}, C \in \mathcal{D}$ iff $|P \cap D| \leq |D|/2$ for every $P \in \mathcal{P}$ and for every cut D.

If $|P| \leq 2$ for $P \in \mathcal{P}$, we obtain Fleischner's theorem. If P consists of parallel edges for $P \in \mathcal{P}$ we obtain Seymour's ("integer sum of circuits") theorem.

András Frank

Some applications of graph factorizations to design theory

Some recent applications of 1-factorizations to the existence problem for certain types of design-like objects are discussed. The results on factorizations required are:

(i) 1-factorizations of complete and of regular bipartite graphs
(ii) 1-factorizations of cyclic graphs (the Stern–Lenz Lemma).

These can be used to give the constructive part of the proof of the following results on designs:

(i) The Existence problem for Steiner triple systems with subsystems,
(the Doyen-Wilson theorem)
(ii) The existence problem for resolvable designs $S_3(3,4;v)$
(iii) The determination of minimal linear spaces

Dieter Jungnickel (Gießen)

Ramsey's Theorem for Unary Algebras

Ramsey Theory at the beginning investigated partition problems for graphs and hypergraphs. Here we investigate the situation for algebras $\mathfrak{A} = (A, a_1, \ldots a_n)$, A a set, $a_i : A \to A$ a 1-place operation $i = 1 \ldots n$.

THEOREM. For a finite unary algebra $\mathfrak{A}$ the following ~~holds~~ are equivalent:
i) Ramsey's Theorem holds for $\mathfrak{A}$
ii) Every automorphism of any subalgebra $\mathfrak{A}'$ of $\mathfrak{A}$ extends to an automorphism of $\mathfrak{A}$.

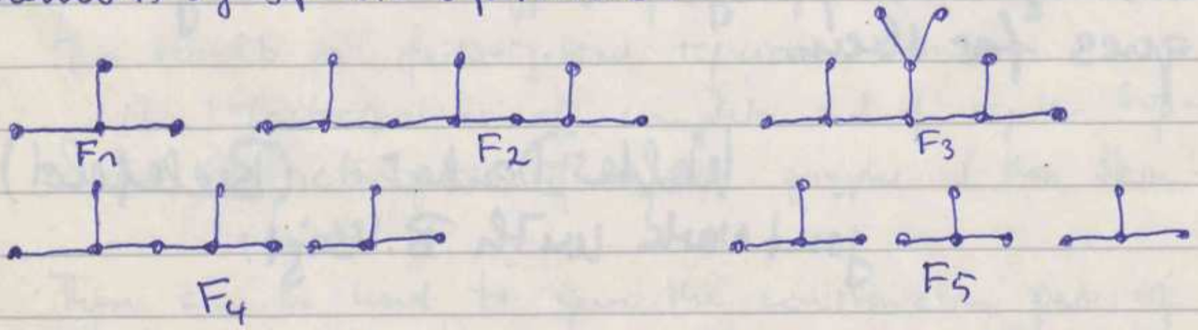
The necessity of (ii) for Ramsey's Theorem is well known, the sufficiency is the interesting part, which may be proved by a structural analysis of unary algebra and amalgamation techniques for them.

Walter Deuber (Bielefeld)
joint work with B. Voigt.

A Ternary Search Problem on Graphs

M. Aigner studied the following search problem on graphs. For a graph G, let $e^* \in E(G)$ be an unknown edge. In order to find e^*, we choose a sequence of test-sets $A \subseteq V(G)$ where after every test we are told whether e^* has both end-vertices in A, one end-vertex, or none. Find the minimum $c(G)$ of tests required. Since in this problem ternary tests are performed, we have the usual information theoretic bound $\lceil \log_3 |E(G)| \rceil \leq c(G)$. Beside his main results which are on complete and complete bipartite graphs, Aigner proved that each forest F with maximum degree at most two is optimal, i.e., the information theoretic bound is achieved. Here we consider the more general question, how close we can come to achieving the information theoretic bound for forests with maximum degree at most r, $r = 1, 2, \ldots$. Let $\mathfrak{F}_r$ be the class of forests with non-empty edge-set and maximum degree at most r. We shall investigate the function $f(r) = \max\{c(F) - \lceil \log_3 |E(F)| \rceil : F \in \mathfrak{F}_r\}$ and obtain that $f(r) = t+1 - \lceil \log_3 (2^t+1) \rceil$ for $2^t < r \leq 2^{t+1}$, $t = 0, 1, \ldots$. In addition, we show that, with the exception of five small graphs, all members of $\mathfrak{F}_3$ are optimal, and we conjecture that a similar result holds for $\mathfrak{F}_r$, $r \geq 4$.

Conjecture: With only a finite number of exceptions, all members of $\mathfrak{F}_r$ are optimal.



The figure shows the non-optimal forests with maximum degree 3.

Thomas Andreae (Freie Universität Berlin)

Über Fortschritte in der Theorie der färbungskritischen Graphen.

Die betrachteten Graphen sind endlich, ungerichtet und schlicht. Ist G ein Graph, so sei $\chi(G)$ seine chromatische Zahl. G heißt k-kritisch, wenn $\chi(G) = k$ ist und $\chi(G') < k$ gilt für jeden echten Unter-graphen G' von G. Ein hoher (niederer) Knotenpunkt – auch als Haupt- (Neben-) Knotenpunkt bezeichnet – eines k-kritischen Graphen ist ein Knotenpunkt der Valenz $\geqq k$ $(= k-1)$.

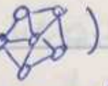
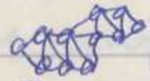
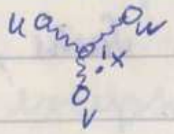
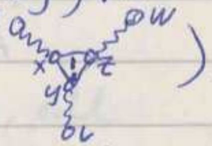
Probleme, die mit niederen Knotenpunkten verbunden sind, tendieren dazu, sich weit leichter handhaben zu lassen, als Fragen, die hohe Knotenpunkte betreffen.

Der Vortragende berichtet über gewisse Fortschritte in der Theorie der färbungskritischen Graphen, welche niedere Knotenpunkte besitzen, unter Hervorhebung von konstruktiven Methoden und Charakterisierungssätzen. Hierbei stützt er sich insbesondere auf Untersuchungen, die Michael Stiebitz und er in den vergangenen Jahren angestellt haben.

Horst Sachs, Technische Hochschule Ilmenau, DDR

Amalgamation and expansion procedures in graphs

The amalgamation of two graphs G_1 and G_2 is obtained by glueing them together along a common (nonempty) subgraph $G_1 \cap G_2$. Then the expansion is obtained by "pulling" the two graphs apart: thus we get the disjoint union of G_1 and G_2 together with a matching between

corresponding vertices of the two copies of $G_1 \cap G_2$ in G_1 and G_2 resp.
A 'multiple' expansion is obtained similarly from $G_1, \dots, G_n$, where $G_i \cap G_j = \bigcap_k G_k$ $(i \neq j)$. Now we have matching K_n's between the n copies of $\bigcap_k G_k$ in $G_1, \dots, G_n$ resp.
A very general problem is: given an initial set of graphs, and given conditions for $G_1 \cap G_2$, characterize the graphs obtainable from the initial set by amalgamation (or expansion) under these conditions. Here we consider some particular instances.
A subgraph H of a graph G is (geodesically) convex if, for all vertices u,v of H, all (u,v)-geodesics in G are in H. And H is Δ-convex if it is convex and whenever H contains an edge of a triangle Δ, then it contains all of Δ.
Thm 1. (Keszthely, July 1, 1976) G is obtainable by convex expansions from K_1 iff G is obtainable by convex amalgamations from hypercubes (Q_n) iff G is a median graph (a median graph being a connected graph such that every triple $u,v,w \in V(G)$ has a unique vertex $x = x(u,v,w)$ minimizing $d(u,x)+d(v,x)+d(w,x)$).
Thm 2 (1979/1980) G is obtainable by Δ-convex multiple expansions from K_1 iff G is obtainable by Δ-convex amalgamations from products of complete graphs iff G is a quasi-median graph.
Thm 3 (1985/1986) G is obtainable by Δ-convex amalgamations from $W_{m\geq 0} \times Q_{n\geq 0}$ (W_m wheels: ), $S_{m\geq 0} \times Q_{n \geq 0}$ (S some snake, for example ), $K^-_{m\geq 0} \times Q_{n\geq 0}$ (K^-_m complete graph minus a matching) iff G is a pseudo-median graph ($u,v,w \in V \Rightarrow$  or ).

Thm 4 (1986) The geodesic convexity of a pseudo-median graph is S_4 (a Hausdorff-type seperation property for convex sets).
This last thm can be proven by the same techniques as developed for the proofs of thms 1-3...

Henry Martyn Mulder
July 1, 1986 (Vrije Universiteit, Amsterdam)

Large Cycles in Graphs

A graph G has E_r $(r \geq 3)$ iff G is 2-connected, has minimum degree $\geq r$ and $\geq 2r$ vertices. A well-known result of G.A. Dirac is: each graph with E_r has a cycle of length $\geq 2r$. C. Zuluaga and I proved: each non-bipartite graph with E_r contains both an odd and an even cycle of length $\geq 2r-1$. Taking into account the girth O. Ore showed: each graph of girth g, $g \geq 5$, with E_r, $r \geq 4$, contains a cycle of length $\geq (g-2)(r-2)+5$. I improved and generalized this result.

(I) Each graph (non-bipartite graph) of girth g, $g \geq 3$, with E_r contains a cycle (both an odd and an even cycle) of length $\geq 2^{c_1 g}$ with $\geq 2^{c_2 g}$ diagonals, where c_i is constant.

(II) Let G be a k-connected graph with E_r, $r \geq k \geq 2$, or a cyclically k-vertex-connected graph (in the sense of C. Thomassen) with E_3. Then each set of $k-1$ edges of G lying on a cycle of length $\geq k$ is on a cycle of length $\geq 2^{c_1 g}$ with $\geq 2^{c_2 g}$ diagonals.

Heinz-Jürgen Voss,
Pädagogische Hochschule Dresden, DDR

On large induced trees and paths in graphs
(M. Saks, P. Erdős, V.T. Sós)

Let $t(G)$ be ~~p~~ resp $p(G)$ be the max size of an induced tree resp. path in G_n. We investigate the relationships of $t(G)$ resp $p(G)$ to other parameters associated with G: to the number of edges, to the radius, independence number, max. clique size or connectivity. E.g. we proved the following

<u>Thm</u> Let $g(G) = e(G) - n(G) + 1$, $\ell(n;p) = \min\{t(G) \mid |V| = n, |E(G)| = g+n-1$ Suppose G is connected$\}$ Then

$$\ell(n;g) = \frac{2n}{g+2} + o\left(\frac{n}{g+2}\right) \quad \text{if } g = o\left(\frac{n}{\log\log n}\right)$$

$$\ell(n;cn) = 2\log\log n + O(\log\log n) \quad \forall c > 0$$

$$\ell(n;n^{1+\delta}) = 2\log\left(1+\frac{1}{\delta}\right) + \varepsilon \quad |\varepsilon| < 2.$$

(So very sparse graphs can have relatively small $t(G)$!)

Vera T. Sós
ELTE, Budapest

Adjacency characterizations and diameters of polytopal graphs

Let E be a finite set and $\mathcal{F} \subseteq 2^E$; for each $F \subseteq E$, let $x^F \in \mathbb{R}^E$ denote the incidence vector of F. Obviously, the vertices of $P(\mathcal{F}) := \operatorname{conv}\{x^F \in \mathbb{R}^E \mid F \in \mathcal{F}\}$, i.e., the polytope associated with $\mathcal{F}$, are in 1-1-correspondence with the elements of $\mathcal{F}$. Define a graph $S(\mathcal{F})$, the so-called skeleton of $P(\mathcal{F})$, whose nodes are the vertices of $P(\mathcal{F})$ and where two nodes u, v are linked by an edge if and only if they are (geometrically) adjacent on $P(\mathcal{F})$. Polytopes of type $P(\mathcal{F})$ come up naturally in many applications (like the matching polytope, the travelling salesman polytope, the stable set polytope). Two questions of interest are: Is there a characterization of adjacency in $S(\mathcal{F})$ in terms of properties of $\mathcal{F}$? What is the diameter of $S(\mathcal{F})$? The latter question is related to the Hirsch conjecture of linear programming. In this talk we give a survey of some results of the literature and present new adjacency criteria and diameter estimations for the polytopes associated with clique partitionings, cycles in binary matroids, Eulerian subgraphs and cuts of a graph.

Martin Grötschel, Augsburg

Some other results on Ramsey-Turán numbers of Graphs

This is from a joint work with Erdös, Simonovits, Vera Sós and Szemerédi ~~[illegible]~~.

Let $r, \ell \geq 1, v \geq 1$. Define $RT_v(n, r, \ell) = \max\{e(\mathcal{G}) : \mathcal{G} \text{ has } n \text{ vertices} \wedge K_r \not\subset \mathcal{G} \wedge \alpha_v(\mathcal{G}_n) \leq \ell\}$

where $\alpha_v(\mathcal{G}_n) = \max\{|X| : X \subset V(\mathcal{G}) \wedge X \text{ does not contain a } K_{v+1} \text{ of } \mathcal{G}\}$.

We are looking for the constants a_r^v satisfying

$$RT_v(n, r, o(n)) = (a_r^v + o(1))n^2.$$

For $v = 1$ we proved (see Combinatorica 1983) $a_r^1 = \frac{1}{2}\left(\frac{r-3}{r-1}\right)$ for r odd

and $a_r^1 = \frac{1}{2}\left(\frac{3r-10}{3r-4}\right)$ for r even.

Th 1. $a_r^v \leq \frac{1}{2}\left(\frac{r-v-2}{r-1}\right)$ and this is best possible if $r \equiv 1 \mod (v+1)$, $(r > v+1)$

Th 2. $a_5^2 \leq \frac{1}{12}$, $a_6^2 \leq \frac{1}{6}$. Th 3: $a_6^2 > 0 \Leftrightarrow a_8^2 > \frac{1}{4}$

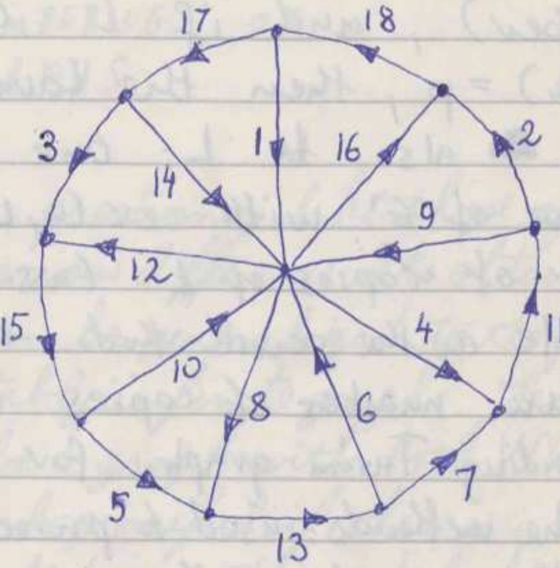
Problem: Is $a_5^2 > 0$?

To establish $a_4^1 \geq \frac{1}{8}$ Bollobás and Erdös proved that there is a sequence $\mathcal{G}_n$ of graphs on n-vertices such that $\alpha(\mathcal{G}_n) = o(n)$, $K_4 \not\subset \mathcal{G}_n \wedge \lim_n \frac{e(\mathcal{G}_n)}{n^2} = \frac{1}{8}$ and $V(\mathcal{G}_n) = A_n \cup B_n$ where A_n, B_n contain $o(n^2)$ edges and no triangle of $\mathcal{G}_n$

Th. 4: The above sequence can be chosen so that the trace of G_n has large girth on A_n and B_n

András Hajnal, Budapest

Numerierbare Graphen



Ein Graph G mit q Kanten heiße rigoros, wenn die Kanten mit 1, 2, 3, …q so durchnumeriert und gleichzeitig gerichtet werden können, daß in jeder Ecke vom Grade $\neq 1$ das Kirchhoff'sche Gesetz gilt. Wenn G n Ecken besitzt und sich in t Hamiltonische Kreise zerlegen läßt und $t(n-1)$ eine gerade Zahl ist, so ist G rigoros. Aus diesem Satz und aus weiteren Konstruktionsprinzipien lassen sich folgende Graphen als rigoros erkennen: Der vollständige Graph K_n für $n \neq 3$; der vollständige paare Graph $K_{n,n}$ für $n \equiv 0 \pmod 4$; das Kantensystem Q_n des n-dimensionalen Würfels für $n \neq 2$; das Rad R_n mit $n-1$ Speichen für $n \geq 3$; jeder Baum, der nur Ecken vom Grade 1 oder 4 besitzt. Aber z.B. der paare Graph $K_{n,n}$ ist nicht rigoros, wenn n ungerade ist $(n \neq 1)$.

Gerhard Ringel, Santa Cruz,
Californien

Supersaturated graphs, extremal graphs with large forbidden subgraphs.

Miklós Simonovits (Budapest)

Let ex(n, L) denote the maximum number of edges a graph G^n of order n can have without containing L.

(a) If $\chi(L)=p+1$ (where χ denotes the chromatic number), and if L has a critical edge e, i.e. $\chi(L-e)=p$, then the Lovász–Simonovits theorems on K_{p+1} generalize also ~~to~~ L: One can describe the structure of G^n with ex(n, L) edges, containing the least number of copies of L (assumed $k<cn$). This structure is stable in the sense that the graphs with almost minimum number of copies are "very similar" to the corresponding Turán graph, for $k=o(n^2)$.

(b) The methods used to prove these results apply to many similar situations. I mentioned ~~the~~ some sieve formulae and the Szemerédi uniformization theorem.

Partitioning Nodes into Directed Paths

Kathie Cameron (Waterloo)

For certain weightings w(P) of the simple dipaths P in a digraph G, we consider the problem of partitioning the node-set V of G by a set of dipaths whose weight-sum is minimum. In particular, where G is acyclic, k is a positive integer, and w(P) is the minimum of k and the number of nodes in P, we give a min-max equality and note structure of a dual optimum: It yields a sequence $S=(S(1), S(2) \ldots, S(k))$ of independent sets of nodes, such that each dipath P of every optimum partition intersects w(P) members of S, in order. For non-acyclic digraphs, it may be true that each optimum partition intersects in this way some S, though there is no single S which will do for every optimum partition.

LABELLINGS AND NUMBERINGS OF INFINITE GRAPHS

Some number theoretic questions, like those of M. Hall and R. Entringer involving various difference sets obtained from the set of natural numbers, can be generalized to the problem of gracefully labelling countably infinite graphs. Let $N_0 = \{0, 1, 2, 3, \ldots\}$, and let G be a graph with both V(G) and E(G) countably infinite. Call G k-graceful if there is a one-to-one function $h: V(G) \to N_0$ such that $g_h: E(G) \to \{k, k+1, k+2, \ldots\}$ is a bijection where for uv in E(G) we have $g_h(uv) = |h(u) - h(v)|$. Call G bijectively-k-graceful if h can be chosen to be a bijection.

The countably infinite version of the Ringel-Kotzig conjecture that all finite trees are graceful is settled as follows. Let $\beta_1(T)$ denote the maximum number of independent edges in tree T. All countably infinite trees are k-graceful for each $k \geq 1$; any countably infinite tree with $\beta_1(T) = \infty$ is bijectively-k-graceful for each $k \geq 1$; and a countably infinite tree with $\beta_1(T) < \infty$ is bijectively-k-graceful if and only if the number of vertices of infinite degree is one and $k = 1$.

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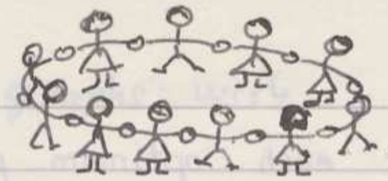
Cycles in Orsay



A survey of some of the recent results obtained by people working in Orsay on cycles (about 16 people) In particular results on:

- pancyclism in graphs, digraphs and bipartite digraphs (Amar, Flandrin, Fournier, Germa, Chacroun)
- D_λ-cycles (Fraisse)
- Cycles in cubic graphs (Fouquet-Thuillier)
- Double loop graphs (Maheo-Favaron)
- Girth (Bond, Homobono, Peyrat, Thuillier)

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ON GRAPHS WHICH ARE LOCALLY HOMOGENEOUS.

V. NEUMANN-LARA (MEXICO)

Let G be a graph. For $u \in V(G)$ denote by $N(u,G)$ the subgraph of G induced by $\{w \in V(G) \mid uw \in E(G)\}$.

G is said to be <u>locally</u> H if $N(u,G) \cong H$ for every $u \in V(G)$.

Define $G_1 \times G_2$ by $V(G_1 \times G_2) = V(G_1) \times V(G_2)$; $(u_1,u_2)(v_1,v_2) \in E(G_1 \times G_2)$ iff $u_1 v_1 \in E(G_1)$ and $u_2 v_2 \in E(G_2)$.

<u>Theorem</u>. - There exists a number $n(n)$ such that $K_{m_1+1} \times K_{m_2+1} \times \dots \times K_{m_n+1}$ is the only graph which is locally $K_{m_1} \times K_{m_2} \times \dots \times K_{m_n}$, provided $m_i \geq n(n)$ for $i = 1, 2, \dots, n$.

How to play von Neumann's Hackenbush.

Played on a forest of rooted trees. A move is to select a node and delete all nodes on the (unique) path from that node to the root, together with incident edges. The roots of the resulting (sub)trees are the nodes incident with deleted edge. von Neumann gave a non-constructive proof that, when played starting with a non-empty tree, the game was always a first-player win. Úlehla found a constructive strategy, as did Conway & Berlekamp. In Berlekamp's version the value of the position is given by its genetic code, a ternary number, which, when the digits 2 are replaced by zeros, becomes the (binary) nim-value, or size of the equivalent heap of beans in the game of Nim.

Calculation of the genetic code involves

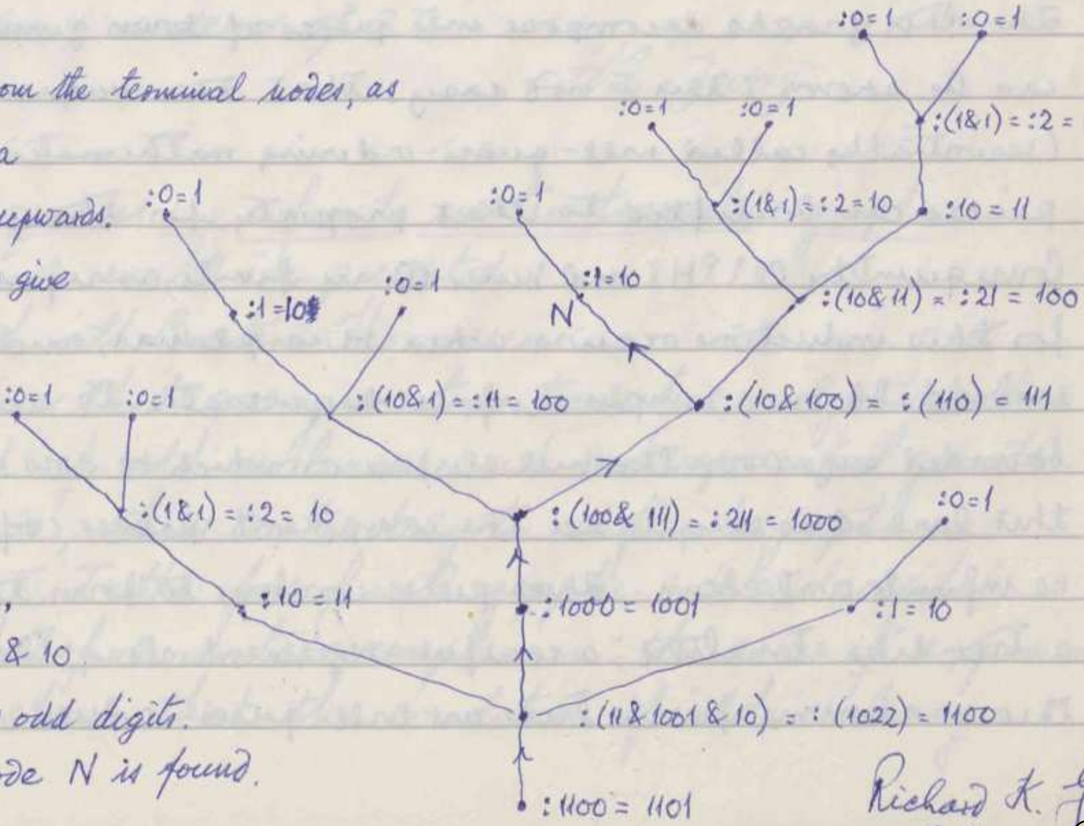
&	0	1	2
0	0	1	2
1	1	2	1
2	2	1	2

(a) agglomeration: grouping a set of trees together to form a forest. The code digits are added according to the table opposite, without carrying. Note that if 2 is replaced by 0 the agglomeration table this is just nim-addition.

(b) colonization: forming a single tree from a forest. Take a new root, join by an edge to each old root. In the code, change the rightmost 0 to a 1, and each digit to its right to a 0: :102101221 = 102110000

:1200 = 1201 :122121 = 1000000

Work downwards from the terminal nodes, as shown. To find a good move, work upwards. The lowest two nodes give 1100 and 1022, which are no good because they have a 1-digit. Proceed up the middle, leading to 11 & 1000 & 10 which still contains odd digits. Continue until node N is found.



Richard K. Guy
86:07:04 Calgary,

On the Proof of Wagner's Conjecture

Neil Robertson

In the 1960's Kurt Wagner conjectured that all antichains (sets of pairwise unrelated elements) of finite graphs are finite in the minor inclusion relation. The minors of a graph are formed by taking a subgraph, then contracting pairwise disjoint connected subgraphs of it to single vertices while maintaining the edge incidences. This extremely general conjecture includes an earlier one of König, that the set of topologically minimal graphs not embedding on any fixed surface is finite; and one of Vázsonyi that all antichains of graphs with maximum valency 3 are finite. These conjectures were well-known and date to 1935 and the 1940's, respectively. This talk reported on joint work with Paul Seymour in which Wagner's conjecture is proved, via an elaborate structure theory of graphs G not including a fixed graph as a minor. Roughly speaking G breaks down into pieces each of which is essentially embeddable onto a surface in which H is not embeddable. Essentially means that ≤ f(H) vertices may not be on the surface; and that there are irregularities of the embedding at the boundary components "cuffs" of the surface. The proof of Wagner's theorem is by induction on the genus of a graph H contained in a presumably infinite antichain $\mathcal{A}$. The other graphs decompose into pieces of lower genus than H, and it can be shown (this is not easy) that the Wagner finiteness property (essentially called well-quasi-ordering mathematically) for the pieces can be lifted to that property for the graphs of $\mathcal{A} \setminus \{H\}$. Consequently $\mathcal{A} \setminus \{H\}$ and hence $\mathcal{A}$ are finite, as required. The basis for this induction occurs when H is planar, and being essentially embeddable on a simpler surface degenerates to simply being of bounded size (on the null surface on which H does not embed). At this level it is easy to see the component pieces (of bound size) have no infinite antichain. These pieces combine to form the graphs G in a tree-like structure; a careful argument along the lines of Kruskal's theorem showing finite trees are well-quasi-ordered lifts this property to the tree-structures of graphs of bounded size. In a sense the exclusion method, using the fact that H is not a minor of G for all $G \in \mathcal{Q} \setminus \{H\}$, and the standard methods of well-quasi-ordering theory win the day.

Longest circuits and paths in regular graphs of large degree

B. Jackson proved that a 2-connected d-regular graph on $n \geq 3d$ vertices has a hamiltonian circuit. As a "twin" result G. Fan obtained that in a 3-connected d-regular graph a longest circuit has n or at least 3d vertices. As a supplement one has that a non-bipartite graph, which is d-regular and 3-connected, on at most 3d-2 vertices is hamiltonian connected; in a 4-connected d-regular graph any two vertices are joined by a path of length at least 3d-3 or n-1. Analogous results hold for dominant cycles and paths involving the factor 4 instead of 3

H.A. Jung

Distance sequences in infinite vertex-transitive graphs

Two recent results are presented:

1. Joint work with Carsten Thomassen:

We show that every vertex-transitive, edge transitive infinite graph of odd valence and subexponential growth is 1-transitive, thus extending to infinite graphs a result of W.T. Tutte for finite graphs. We describe several families of counterexamples in the case of exponential growth and show that the condition of odd valency

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cannot be relaxed in the infinite case, as I. Z. Bouwer had shown in the finite case.

2. Joint work with James B. Shearer:

It is shown that contrary to a pair of well-known conjectures (presented, for example, by L. Babai, Burnaby, B.C., 1979), there exist finite and infinite examples of (1) vertex-transitive graphs whose distance sequences are not unimodular and (2) graphs with primitive automorphism group whose distance sequences are not logarithmically concave. In particular, a family of graphs is presented (the smallest has $3721 = 61^2$ vertices) whose automorphism groups are primitive and whose distance sequences are not unimodular.

Mark E. Watkins

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and Syracuse University, Syracuse, USA.

Edge-disjoint paths in planar graphs

We discuss the following theorem, conjectured by K. Mehlhorn. Let $G = (V,E)$ be a planar graph, embedded in the plane $\mathbb{R}^2$; let O denote the unbounded face and let I be some other fixed face; let $C_1, \ldots, C_k$ be curves in $\mathbb{R}^2 \setminus (I \cup O)$ starting and ending in vertices on $\delta(I \cup O)$ (the boundary of $I \cup O$), so that for each vertex v we have: $\deg_G(v)$ + (# of C_i beginning or ending in v) is even. Then there exist pairwise edge-disjoint paths $P_1, \ldots, P_k$ so that P_i is homotopic to C_i in the space $\mathbb{R}^2 \setminus (I \cup O)$ $(i = 1, \ldots, k)$, if and only if for each dual path Q from

$I \cup O$ to $I \cup O$ we have: # of edges intersected by Q $\geq \sum_{i=1}^{k}$ # of necessary intersections of C_i and Q).
The theorem generalizes a theorem of Okamura & Seymour and its proof gives a polynomial-time algorithm. The result is joint work with C. van Hoesel and M. Kaufmann.

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Alternating cycles in 2-connected graphs with applications to graphs with unique f-factors.

We show that if the edges of a 2-connected finite graph are 2-coloured such that each vertex is incident with edges of both colours then it contains a cycle whose edges alternate in colour. We deduce that if G is a graph with a unique f-factor then G contains a vertex x with $d_G(x) = f(x)$. The work is joint with Robin Whitty

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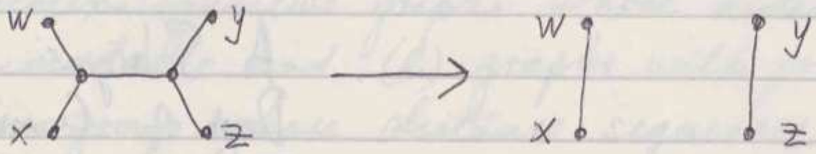
On k-critically n-connected graphs.

A well known conjecture of P. J. Slater says that there is no non-complete $(k+1)$-critically $(2k+1)$-connected graph. It is proved that such a graph has to contain less than $3k$ vertices.

W. Mader
(Hannover)

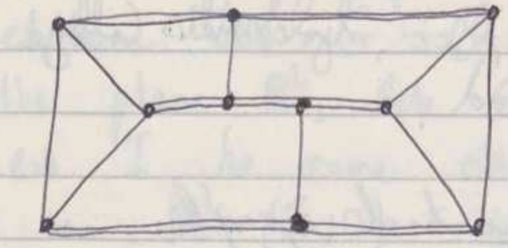
Connectivity preserving edge reductions in cubic graphs.

An edge reduction in a cubic graph is the following reduction.



We give a sharp lower bound on the number of edge reductions in a 3-connected (respectively, cyclically-4-connected) cubic graph which give a smaller 3-connected (respectively, cyclically-4-connected) cubic graph. For 3-connectivity the lower bound is $\frac{1}{2}|V(G)|+3$, and for cyclic-4-connectivity it is roughly $\frac{3}{10}|V(G)|$.

A method of generating all cyclically-5-connected cubic graphs is given.



A forest with at least 3 trees.

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