MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 21/1976

Finite Geometries

16.5. bis 22.5.1976

This years' Finite Geometries conference was held under the leadership of Professor D.R. Hughes (London) and Professor H. Lüneburg (Kaiserslautern). An innovation and major feature this year was two series of three lectures, one presented by Professor J. van Lint (Eindhoven) and the other by Dr. P. Cameron (Oxford). Professor van Lint described some of the recent results obtained by members of the Combinatorial Theory Seminars in Eindhoven and Amsterdam, whilst Dr. Cameron discussed various kinds of configurations which have proved useful in the study of multiply transitive groups. Mainstreams of the conference were Designs and Codes, with some attention being given to other combinatorial problems and graph theory. Amongst many other fine lectures, particular attention was paid to those of Professor Buekenhout (Brussels), who presented an elegent generalisation of Dynkin diagrams, Professor Seidel (Eindhoven), who discussed codes and designs on the unit sphere, and Professor Ott (Giessen) who established that a flag transitive finite projective plane is of prime power order.

Participants

Baer, R	Zürich
Beker, H	London
Bermond, J.C.	Gif-sur-Yvette
Beutelspacher, A	Mainz
Brouwer, A.E.	Amsterdam
Buekenhout, F	Brüssel
Cameron, P.J.	Oxford
Cofman, Judita	Mainz
Dehon, M	Brussels
Dorber, G.H.	London
Doyen, J.	Brüssel
Dugas, M.	Essen
Erbach, D.W.	Cambridge
Fanning, D.	London
Fisher, J.C.	Bologna
Freeman, J.	Bologna
Ganter, B.	Darmstadt
Goethals, J.M.	Brüssel
Grundhöfer, Th.	Neustadt
Hall, J.I.	Eindhoven
Hering, Ch.	Tübingen
Higman, D.G.	Ann Arbor
Hirschfeld, J.W.P	Brighton
Hubaut, X.	Brüssel
Hughes, D.R.	London
Ito, N.	Chicago
Jonsson, W.	Montreal
Jungnickel, D.	Berlin
Kallaher, M.J.	Kaiserslautern
Key, J.D.	Birmingham
Köhler, E.	Hamburg
Laskar, R.	Paris
van Lint, J.H.	Eindhoven
Livingstone, D.	Birmingham
Lüneburg, H.	Kaiserslautern
Metz, R	Darmstadt

Mortimer, B.	London
Norman, C.W.	London
Ott, U.	Giessen
Prohaska, O.	London
Rahilly, A.	Bologna
Rink, Rosemarie	Kaiserslautern
Röhmel, J.	Berlin
Rosa, A.	Hamilton
Saxl, J.	Oxford
Schulz, R.H.	Berlin
Seidel, J.J.	Eindhoven
Sloane, N.J.A.	Murray Hill
Sprague, A.P.	London
Teirlinck, L.	Brüssel
Tilborg, H.C.A. van	Eindhoven
Totten, J.	Tübingen
Unkelbach, H.	Mainz
Walker, M.	Tübingen
Werner, H.	Darmstadt
Wille, R.	Darmstadt

Vortragsauszüge

H. BEKER : On Strong Tactical Decompositions

R. Harris generalised the concept of affine 2-designs to a class of strongly resolvable 2-designs, which includes all affine designs and is closed under complementation.

I shall define a class of 1-designs, called strongly divisible, which includes all symmetric and strongly resolvable 2-designs and is closed under both complementation and duality.

I shall give a brief summary of some of the results that have been proved for strongly divisible designs.

Jean-Claude BERMOND : Hypergraphs and Designs

We study relations between hypergraphs and designs. Let us denote by : a $(v,k,1)$ t-design a system of subsets called blocks of a given set X of v vertices, satisfying : (i) each block of cardinality k and (ii) each subset of t elements of X belongs to exactly one block.

K_v^t the complete t-uniform hypergraph (the edges are all the t-subsets of a given set of n vertices).

$L_t(H)$ the graph whose vertices represent the edges of H, two vertices being joined if the corresponding edges intersect in at least t elements.

A first relation is to consider a $(v,k,1)$ t-design as a partition of the edges of K_v^t into hypergraphs isomorphic to K_k^t. This suggest generalizations. An other way consists to relate the existence of a $(v,k,1)$ t-design to the determination of the stability number of $L_t(K_v^k)$. The determination of the chromatic number of this graph is related to other problems of designs. Determinations of stability and chromatic number of $L_t(H)$ for other hypergraphs are related to the existence of orthogonal arrays and new configurations.

A.E. BROUWER : Optimal constant weight codes and related designs.

In order to determine the optimal constant weight codes with parameters $d = 6$ and $w = 4$ one has to construct certain designs.

The following theorem says that these designs almost always exist :

Thm.

(i) $S(2,4,v)$ exists iff $v \equiv 1$ or $4 \pmod{12}$ [HANANI]

(ii) $B(\{4,7^*\};v)$ exists iff $v \equiv 7$ or $10 \pmod{12}$, $v \neq 10,19$

(iii) $GD(4,1,2;v)$ exists iff $v \equiv 2 \pmod 6$, $v \neq 8$

(iv) $GD(4,1,\{2,5^*\};v)$ exists iff $v \equiv 5 \pmod 6$, $v \neq 11,17$ (with 5 temporary exceptions).

F. BUEKENHOUT : DIAGRAMS FOR INCIDENCE STRUCTURES.

A generalization of the Dynkin diagrams and the associated incidence geometries discovered by Tits was discussed. Examples are $o\overset{L}{-}o\overset{L}{-}o \ldots o\overset{L}{-}o$ for the geometric lattices, $o\overset{Af}{-}o-o-o \ldots o-o$ for the affine spaces, $o\overset{c}{-}o\overset{Af}{-}o$ for the inversive planes. A series of sporadic simple groups do have a diagram build on the classical strokes and one additional stroke $o\overset{c}{-}o$ for instance, M_{22} : $o\overset{c}{-}o-o$, M_{23} : $o\overset{c}{-}o\overset{c}{-}o-o$, M_{24} : $o\overset{c}{-}o\overset{c}{-}o\overset{c}{-}o-o$, J_1 : $o=o\overset{c}{-}o$, HS : $o\overset{c}{-}o-o\overset{3}{-}o$, F_{24} : $o\overset{c}{-}o\overset{c}{-}o\overset{c}{-}o-o=o$; etc.

P.J. CAMERON : SOME COMBINATORIAL STRUCTURES AND THEIR AUTOMORPHISM GROUPS : TWO-GRAPHS, ORDERINGS, PARALLELISMS. I.

This was the first of a series of three talks on various kinds of configuration which have proved useful in the study of multiply transitive groups. A two-graph is a collection T of 3-subsets of a set X such that any 4-subset of X contains an even number of members of T. Equivalent concepts include : Double coverings of complete graphs; switching classes of graphs; sets of equiangular lines in $\mathbb{R}^d$. A two-graph is regular if it is also a $2-(|X|, 3, \lambda)$ design. Eigenvalue arguments give necessary conditions on $|X|$ and λ_o for the existence of regular two-graphs. A theorem of Shult and Seidel on graphs with the "triangle property" can be translated into the language of two-graphs.

P.J. CAMERON : SOME COMBINATORIAL STRUCTURES AND THEIR AUTOMORPHISM GROUPS : TWO-GRAPHS, ORDERINGS, PARALLELISMS. II.

(1) If a group G acts on X (not necessarily finite or faithful) so that G is 2-transitive and G_x has a strongly closed subgroup $N(x)$ of index 2 (i.e. $N(x) \cap G_y \leq N(y)$) which is transitive on the remaining points, then either G has a subgroup N of index 2 with $N \cap G_x = N(x)$, or G acts on a nontrivial two-graph or an "oriented two-graph" on X.

(2) If X is infinite and a group G is t-homogeneous on X for all $t \geq 0$ and r- but not $(r + 1)$-transitive, then $r \leq 3$ and there is a linear or circular order on X preserved or reversed by G.

(3) A t-$(v,k,1)$ design is basis-transitive if its automorphism group is transitive on ordered $(t + 1)$-tuples of points not contained in a block. Such designs arise from Jordan groups and from $(t + 1)$-transitive groups which act imprimitively on t-subsets. Some partial classifications exist.

P.J. CAMERON : SOME COMBINATORIAL STRUCTURES AND THEIR AUTOMORPHISM GROUPS : TWO-GRAPHS, ORDERINGS, PARALLELISMS. III.

A parallelism of the set $\binom{X}{t}$ of t-subsets of the n-set X is a partition of $\binom{X}{t}$ into "parallel classes", each of which partitions X.

(1) The necessary condition $t|n$ for the existence of parallelisms was shown to be sufficient by Baranyai, using the Integrity Theorem for network flows.

(2) Some results on enumeration exist for $t=2$. They are comparable with results on Latin squares and Steiner triple systems.

(3) Defining a subspace in the natural way, it can be shown that, if $n > t$, then a subspace Y has $|Y| \leq \frac{1}{2}n$, with equality if and only if $X - Y$ is a subspace. Many examples meet the bound for $t=2$.

(4) The parallelogram property asserts that any $(t+1)$-subset is contained in a subspace of cardinality $2t$. Apart from trivial cases ($t=1$, $t=n$, $n=2t$), the only systems with this property are affine spaces over GF(2) and a unique example with $t=4$, $n=24$.

(5) A parallelism with $(t+1)$-transitive automorphism group either has the parallelogram property or $t=2$, $n=6$. There is a purely group-theoretical consequence : any $(t+1)$-transitive group which is imprimitive on $\binom{X}{t}$ is one of these.

(6) A final question concerned graphs defined from parallelisms.

M. DEHON : Planar Steiner triple systems.

A regular planar Steiner triple system is a Steiner triple system provided with a family of non-trivial subsystems (called planes) such that :

(i) all the planes have the same cardinality.

(ii) every set of 3 non-collinear points is contained in exactly one plane.

(iii) for every plane $\underline{P}$ and every line ℓ such that $\underline{P} \cap \ell = \phi$, there are exactly α planes P' such that $\ell \subset \underline{P}'$ and $P \cap P'$ is a line.

Theorem : A regular planar Steiner triple system is necessarely one of the following

(1) a projective space of dimension greater than 2 over G F(2).

(2) the 3-dimensional affine space over G F(3).

(3) an $S(2,3,2(6m+7)(3m^2+3m+1)+1)$ with $m \geq 1$; the planes are $\delta(2,3,6m+7)$ and $\alpha = 1$.

(4) an $S(2,3,171)$; the planes are $S(2,3,15)$ and $\alpha = 2$.

(5) an $S(2,3,183)$; the planes are $S(2,3,21)$ and $\alpha = 7$.

(6) an $S(2,3,2055)$; the planes are $S(2,3,39)$ and $\alpha = 4$.

J. DOYEN : DESIGNS, GAMES AND TRANSVERSALS

Given a design $\mathcal{D}$, two players A and B color alternately a point of $\mathcal{D}$ (in red for A, in green for B, say). The winner is the first player who succeeds in coloring all points of a block of $\mathcal{D}$ with his own color. If A plays first and if A and B play as well as possible, then either A has a winning strategy or every game on $\mathcal{D}$ ends in a draw. For example, A has a winning strategy on every Steiner triple system of order ≥ 7, but every game on a finite projective plane ends in a draw (except on the plane of order 2). It is not known what happens if $\mathcal{D}$ is a finite projective space PG(d,n) with $d > 2$ and $n > 2$.

A transversal of $\mathcal{D}$ is a set of points of $\mathcal{D}$ which has a non empty intersection with every block of $\mathcal{D}$. If B succeeds in coloring all points of a transversal in green, he can obviously force a draw. Thus, if A plays on $\mathcal{D}$, B is actually playing on the space Tr $\mathcal{D}$ of all minimal transversals of $\mathcal{D}$ (note that $Tr(Tr\mathcal{D}) = \mathcal{D}$). The transversals of minimum cardinality in PG(d, n) are the hyperplanes. In a finite affine plane AG(2,n), we conjecture that every transversal is of cardinality $\geq 2n - 1$ (this is known to be true for n = 2,3,4,5 and 7).

BERNHARD GANTER : t-covers

A t-cover is the design of the t-generated substructures of a structure, i.e. the system of the t-generated closed sets of a closure system. More precisely, a t-cover of a set P is a set $B \subseteq \mathcal{P}(P)$ satisfying :

(i) $\forall_{b \in B} \ |b| \geq t$

(ii) $\forall_{x \in \mathcal{P}_t(p)} \ \exists_{b \in B} \ x \subseteq b$

(iii) $\forall_{x \in \mathcal{P}_t(p)} \ \bigcap_{x \subseteq b \in B} b \in B$

t-partitions are t-covers in which any set of t distinct points is contained in a unique block. The well known divisibility conditions for the existence of a t-partition with given block sizes are also necessary for t-covers.

t-partitions and t-covers have been used to construct combinatorial and algebraic structures. We give a general approach to these applications by introducing a language for combinatorial and algebraic structures and characterize the axiom systems of those classes of structures, for which the construction methods using t-partitions and t-covers apply.

J.M. GOETHALS : THE EXTENDED NADLER CODE IS UNIQUE.

Constructions for a 32-word binary code of length 12 and minimum distance 5 were published in 1962 by Nadler and in 1972 by van Lint. These codes are not equivalent, but their extended codes are. By use of the results of Delsarte and of the fact that this code is optimal, we show that, up to a permutation of the coordinates, there is a unique way to construct the extended code.

J. HALL : CONFIGURATIONS RELATED TO EQUIDISTANT CODES

An (r, λ)-system is an incidence structure whose incidence matrix satisfies $AA^t = (r - \lambda) I + \lambda J$, and so gives a special type of equidistant code. Stanton and Mullin (Ann. Math. 37 (1966)) conjectured that if no block is on all points then the number of points v satisfies $v \leq \frac{r(r-1)}{\lambda} + 1$, and further if equality holds the system must be a symmetric design. This is true for $\lambda = 1$ but not true for $\lambda \geq 2$. For $\lambda = 2$, $v \leq \frac{r(r-1)}{2} + 1$ but equality can be achieved for systems other than biplanes. These examples are related to the Hadamard 3-design on 8 points and to projective planes containing complete ovals.

D.G. HIGMAN : PARTITIONS OF X^2

We define the properties of stability, n-stability, coherence and orbitality for a partition V of X^2, X a finite set. For each of these properties we show the existence of a unique coarsest refinement of V with that property. In particular, we obtain in this way a coherent configuration based on the maximal flags of each geometry having a generalized Dynkin diagram as defined by Buekenhout.

J.W.P. Hirschfeld : The twisted cubic

In $PG(3,q)$, $q = p^h$, every twisted cubic C can be written as $C = \{P(t) = (t^3, t^2, t, 1) \mid t \in GF(q) \cup \infty\}$. At each point $P(t)$ of C, there is an osculating plane $\pi(t)$ with equation $x_0 - 3tx_1 + 3t^2x_2 - t^3x_3 = 0$. For $h \neq 3$, there is a null polarity A interchanging $P(t)$ and $\pi(t)$. The line coordinates of $P(r)P(s)$ are $(S^2, RS, R^2-S, S, -R, 1)$, where $S = rs$ and $R = r + s$. A line with these coordinates is called a real chord, a tangent or an imaginary chord as $x^2 - Rx + S$ has 2, 1 or 0 roots in $GF(q)$. Under A, these lines become respectively a real axis, a generator and an imaginary axis of the osculating developable. For $(q + 1, 3) = 1$, the imaginary chords and the tangents form a set of $(q^2 + q + 2)/2$ mutually skew lines such that every other line meets one of these. For $(q + 1,3) = 3$, the imaginary chords, the tangents and the imaginary axes form a spread. For $q > 2$, the spread is not regular and so defines a non-Desarguesian plane, which was previously found by Hering for q odd. The construction also carries through for $q = 2^h$ to the $(q + 1)$-arc $\{(t^{k+1}, t^k, t, 1) \mid t \in GF(q) \cup \infty\}$, where $k = 2^n$ and $(n,h) = 1$. So, for h odd, this defines $\Phi(h)$ distinct translation planes.

X. HUBAUT: GEOMETRIES ASSOCIATED WITH GENERALIZED DYNKIN DIAGRAMS.

We use the usual strokes appearing in Dynkin diagrams with a supplementery one introduced by Buekenhout, i.e. $o\overset{c}{-}o$ which means the trivial incidence structure of a circle. If one assumes that the right groups are acting on a classical buildings one may obtain the extensions by $o\overset{c}{-}o$ of $P\Omega^{+}_{2n}(2)$ as the 2-tr. representation of $Sp_{2n}(2)$ and of $P\Omega_{2n+1}(2)$ with a regular normal subgroup (RNS). Also extensions of $P\Omega_5(3)$, $PSU_4(4)$, $PSU_6(4)$, $P\Omega^-_6(3)$ give $PSU_5(4)$, $P\Omega^-_6(3)$, Fi22 and McL; the 2 first have infinitely many extensions by $\circ\overset{c}{-}\circ$, Fi22 has Fi23 and Fi24.

$\circ\overset{c}{-}\circ\overset{c}{-}\circ$ is the diagram of a 3-trans-group.

$\circ\overset{\text{ↄ}}{-}\circ\overset{c}{-}\circ$ is the extension of a lattice $n \times m$. If one assumes that Alt(n)x Alt(n) is acting on that lattice then one gets Alt(2n).

$\circ\overset{c}{-}\circ\overset{\text{ↄ}}{-}\circ$ is the extension of a triangular graph. If one assumes that the aut. group is Alt(n) one gets E_2^{n-1}. Alt(n) or E_2^{n-2} Alt(n) when n is even.

n=3, 4 and 6 give biplanes.

NOBORU ITO : <u>3-DESIGNS WITH BLOCK RANK 3 AUTOMORPHISM GROUPS.</u>

Ito has a plan to attack the following problem : "classify 3-designs with 2 intersection numbers". The first step is to solve the conjecture of C. Norman concerning Hadamard designs affirmatively. Ito is making progress about this and hoping that he will complete it in the near future.

DIETER JUNGNICKEL : HJELMSLEVEBENEN MIT REGULÄRER ABELSCHER KOLLINEATIONSGRUPPE.

Wir betrachten endliche projektive Hjelmslevebenen (PH-Ebenen, vgl. DEMBOWSKI). Bisher ist für Parameter (t,r), wo t keine Potenz von r ist, nichts über die Existenz nichttrivialer Kollineationen überhaupt und für Parameter (r^n, r) nichts über die Existenz von regulären Kollineationsgruppen bekannt. Sei S das Spektrum aller (t, r), für die eine (t, r)-PH-Ebene mit regulärer abelscher Kollineationsgruppe $G = Z \oplus N$ existiert, so daß N jede Nachbarklasse (von Punkten bzw. Geraden) regulär auf sich abbildet. Wir zeigen mit Differenzenmethoden und direkten Konstruktionen:

(i) $(q^n, q) \in S$ für jede Primzahlpotenz q.

(ii) Es seien q, r Primzahlpotenzen mit $(r^{n-1} + 1)(r + 1) \leq q + 1 \leq r^n(r + 1)$ und es sei $(s, q) \in S$; dann ist auch $(sr^n, r) \in S$, insbesondere stets $(q^k r^n, r) \in S$ für jede nat. Zahl k.

(iii) Es seien (t, r), $(s, q) \in S$ mit $q := t(r + 1)-1$. Dann ist auch $(st, r) \in S$.

MICHAEL J. KALLAHER : A SUFFICIENT CONDITION FOR TRANSLATION PLANES.

Let $\mathfrak{A}$ be a finite affine plane and G a collineation group of $\mathfrak{A}$ which is transitive on the affine points of $\mathfrak{A}$. Let θ be an affine point of $\mathfrak{A}$.

A block orbit of G_θ is an (affine) point orbit Γ of G_θ such that $\Gamma \cup \{\theta\} = \bigcup_{m \in \Omega} m$, where Ω is a set of affine lines through θ.

THEOREM: Let θ be an affine point of $\mathfrak{A}$. If G_θ has three block orbits, then $\mathfrak{A}$ is a translation plane and $G > T$, the group of translations of $\mathfrak{A}$.

The group G has rank (r,s) if r is the rank of G as a permutation group on the affine points of $\mathfrak{A}$ and s is the number of orbits of G on ℓ_∞, the line at infinity.

THEOREM: If G has rank (r,s) with $r + 1 < 2s$, then $\mathfrak{A}$ is a translation plane and $G > T$, the group of translations of $\mathfrak{A}$.

E. KÖHLER : THE OBERWOLFACH PROBLEM

Some connections between the Oberwolfach problem and LANGFORD-(SKOLEM-) sequences were exhibited. These methods give some new solutions : $OP(3, 8k-2)$ $OP(3, 4k, 4\lambda)$ $(\lambda \in \mathbb{N})$. Also with the help of finite geometries some other cases can be solved.

A paper containing the proof will appear in the vol. of the proceedings of the conference on geometric algebra at Duisburg 1976.

RENU LASKAR : Finite Nets

A finite (k,n)-net is a system of points, lines, together with an incidence relation subject to the conditions : 1) lines are partitioned into k parallel classes such that lines belonging to different classes have exactly one point in common. Each point is incident with exactly one line in each class.
(ii) each line contains exactly n points. This concept is due to Bruck. A generalization of Bruck-nets is given consisting of points, lines, planes, together with an incidence relation. An affine 3-space is such a 3-net. A construction is given which is not an affine 3-space, but constructed from projective and affine spaces.

J.H. van LINT : CODING AND DESIGNS

We present a number of results obtained recently by members of the combinatorial theory seminars in Eindhoven (T.H.E.) and Amsterdam (Math. Centre).

I. Special Codes

a) A simple proof of Lloyd's Theorem using Block's lemma (D. Cuethovič, J.H. van Lint).

b) By a result of M. Deza and J.H. van Lint an equidistant code C with $d = 2k$ which is nontrivial, has $\leq k^2 + k + 2$ words and equality is possible iff a proj. plane of order k exists. For $k=6$ it was known that $|C| = 32$ is possible. We prove : Theorem. If $k = 6$ then $|C| \leq 32$. (J.I. Hall, A.J.E.M. Janssen, A.W.J. Kolen, J.H. van Lint).

c) If the supports of the words of weight 3 in a single-error-correcting linear code C form a Steiner Triple System on n points we say that C is supported by STS(n). The situation for STS(7) was known. We prove Theorem: there is a lin. code C over GF(q) supported by STS(g) iff $q \not\equiv 2 \pmod 3$.

Theorem: There is a nonlinear code C supported by a STS(13) (L.M.H.E. Driessen, G.H.M. Frederix and J.H. van Lint).

J.H. van LINT : BOUNDS ON CODES

The following results usually are related to the linear programming bound

a) Theorem: The triply shortened Hamming code is optimal (M. Best, A.E. Brouwer).

b) Theorem: The best possible bound obtainable using a method like Rankin's is far larger than the Wax bound. Conclusion: the Wax bound is false (M. Best).

c) The Johnson bound yields $A(14,6,7) \leq 52$. Equality would yield an interesting design which is shown not to exist, i.e. $A(14,6,7) < 52$ (Math. Centre Seminar).

d) $A(10,4,5) = 36$. The corresponding code is unique. It is a 3-design but this design is not unique. (H.C.A. van Tilborg).

J.H. van LINT : DESIGNS

a) Associative Block Designs. We present definition and some elementary results obtained by R. Rivest and the motivation.

Def. ABD(k,w) is a rectancular array with $b := 2^w$ rows, k columns and entries from {0,1,*} such that the *'s form a 1-design with k-w *'s in a row and such that for each possible 0,1 sequence of length k there is a row of the array differing only in the *-positions.

Theorems obtained by the group (Brouwer, van Ende Boas, Schijver) are

Th. 1: If ABD(k,w) exists, $\alpha \geq 1$ and αk and αw are integers, then ABD(αk, αw) exists.

Th. 2: If ABD(k,w) exists, (w > o) and $k = k_o 2^e$ (k_o odd), then ABD(k, w + k_o) exists.

b) Steiner Triple Systems:

Th. 1: If $v \equiv 3 \pmod 6$, $v \geq 9$ then there is a pair of Kirkman systems of order v with intersection ϕ (resp. 1 block.)

Th. 2: If $q \equiv 1$ or $3 \pmod 6$, $q \geq 12v + 7$, T = a partial triple system on v points, then there is a pair of STS(q) with intersection T (J.I. Hall and J.T. Udding).

c) STS(7): There are 35 triples on 7 points. It is known (Cayley) that these do not form 5 STS(7)'s.

Theorem: There are 10 STS(7)'s such that they cover each triple twice. (A.E. Brouwer).

R. METZ : <u>NON HERMITIAN UNITALS IN PG(2,q^2)</u>

Using a construction method of Buekenhout one can show :

For any q > 2, there exists a non hermitian unital in PG(2,q^2).

It can also be seen, that these unitals as designs are non hermitian.

U. OTT : Fahnentransitive Ebenen

Es wird über Beweise folgender Sätze berichtet:

Satz Eine fahnentransitive Ebene gerader Ordnung ist von Primzahlpotenzordnung, oder sie hat eine scharf Fahnentransitive Automorphismengruppe.

Satz Fahnentransitive Ebenen ungerader Ordnung sind von Primzahlpotenzordnung.

Satz Sei π eine projektive Ebene der Ordnung n und G eine Kollineationsgruppe derart, daß jede Fahne von einer involutorischen Homologie aus G fixiert wird. Dann liegt einer der folgenden Fälle vor :

I. π ist desarguessch, $G \geq PSL(3,n)$

II. G läßt ein Unital fest, und n ist eine Primzahlpotenz. Enthält G Keine Baer-Involutionen, dann ist π desarguessch und es gilt $G \geq PSU(3,n)$.

III G läßt ein Oval fest, $G \geq PSL(2,n)$. π ist desarguessch.

IV. G läßt eine Antifahne fest, $G \geq SL^{\pm}(2,n)$. π ist desarguessch.

V. π ist eine verallgemeinente Hughesebene (einschiesslich des desarguesschen Falles), $G \geq PSL(3,\sqrt{n})$ bzw $G \geq SL(3,7)$ für n = 49.

VI. G läßt eine Gerade ℓ fest, π^{ℓ} ist Translationsebene, und G enthält die Translationsgruppe der Ebene.

VII. Dual zu VI.

J. RÖHMEL : DIRECTED DESIGNS

Directed designs are introduced as an example of a complete comb. system, i.e. (M, I, ϕ), where M, I are ordered sets (partial) $\phi : M \to I$ strongly monotone and the properties

1) Given X with $\phi(X) = i$. Then $\#\{Y \mid \phi(Y) = j, Y \leq X\} = c_{ij}$ = const. for all X with $\phi(X) = i$.

2) Given $X \leq Z$, $\phi(X) = i$, $\phi(Z) = k$, then $\#\{Y \mid \phi(Y) = j, X \leq Y \leq Z\} = c_{ijk}$ = const.

Then the complete directed design is given by $M = \{X \mid X : \{1, \ldots, j\} \to \{1, \ldots, v\}$ injekt , $j = 1, \ldots, v\}$, $I = \{1, \ldots, v\}$ $X \in \{X : \{1, \ldots, j\} \to \{1, \ldots, v\}$, $Y \in \{Y : \{1, \ldots, i\} \to \{1, \ldots, v\}$ then

$X \leq Y :\Leftrightarrow$ there exists a strongly monotone $\alpha : \{1, \ldots, j\} \to \{1, \ldots, i\}$ with $X = Y_o \alpha$

Incomplete structures are introduced as follows $(M, I, \phi, \mathcal{B})$ is called incomplete if $\mathcal{B} \subsetneq M_R = \{X \mid \phi(X) = k\}$ and for every X with $\phi(X) = t$ there are exactly λ elements $Y \in \mathcal{B}$ with $X \leq Y$.

Examples of incomplete directed designs are constructed, especially those with a point and block transitive automorphism groups. For triple systems a binary relation is introduced and their properties are investigated.

ALEXANDER ROSA : NONISOMORPHIC STEINER QUADRUPLE SYSTEMS

Let $N(v)$ and $N^*(v)$ denote the number of nonisomorphic Steiner quadruple systems, and the number of nonisomorphic automorphism-free Steiner quadruple systems of order v respectively.
A result of Lindner, E. Mendelsohn and myself on the number of nonisomorphic 1-factorizations and nonisomorphic automorphism free 1-factorizations of the complete graph, together with a generalized direct product type construction for Steiner quadruple systems is used to show that for $v \equiv 4$ or $8 \pmod{12}$, $N(v) \to \infty$ $[N^*(v) \to \infty]$ as $v \to \infty$. For small values of v, the knowledge of all 1-factorizations of the complete graph K_v yields improved lower bounds on $N(v)$.

J. SAXL : PRIMITIVE PERMUTATION GROUPS OF SOME MORE SPECIAL DEGREES.

According to a theorem of N. Ito, transitive permutation groups of degree $p = 2q + 1 > 11$ with both p and q prime numbers are either soluble or "almost" 4-transitive. Using this, we prove the following

Theorem (P.M. Neumann & J.S.) Let G be primitive of degree kp, where $p = 2q + 1$, p and q primes.

If $k = 2$ then G is 7-transitive.

If $k = 3$ then G is 10-transitive.

If $k = 4$ then G is known.

J.J. SEIDEL : SPHERICAL CODES AND DESIGNS.

Let X be a finite set of n points on the unit sphere in Euclidean space $\mathbb{R}^d$.

X is a spherical s-code, if its points have at most s distinct distances.

X is a spherical t-design, if its k^{th} moments are constants w.r.t. orthogonal trfm. of $\mathbb{R}^d$, k=1,2,...,t.

Subject of the talk : relations between n,d,s,t, and examples of extreme configurations X.

Methods : discrete mathematics and special functions.

Reference : P. Delsarte, J.M. Goethals, J.J. Seidel, Spherical codes and designs, Geometriae Dedicata, to appear.

N.J.A. SLOANE : BOUNDS FOR CODES

Let A(n,d) be the maximum number of binary vectors of length n such that any two vectors differ in a least d places, and let A(n,d,w) be the maximum number of binary vectors of length n, each containing w 1's, such that any two vectors differ in at least d places. The purpose of this talk is to announce a number of new values and bounds for A(n,d) and A(n,d,w); tables have been constructed for $n \leq 24$ and $d \leq 10$.

This is a joint work with M.R. Best, A.E. Brouwer, F.J. MacWilliams and A.M. Odlyzko.

L. TEIRLINCK : ON LINEAR SPACES IN WHICH ALL PLANES ARE AFFINO-PROJECTIVE.

An affino-projective plane is a projective plane from which a part of a line is deleted. An affino-projective space is a projective space from which a part of a hyperplane is deleted. The dimension of a linear space S is the smallest cardinal number n for which there exists a set of n + 1 points generating S. If S is a linear space of finite dimension in which all planes are affino-projective and if there is at least one plane of order ≥ 4, then S is an affino-projective space. We also describe all infinite dimensional linear spaces in which all planes are affino-projective and in which there is at least one plane of order ≥ 4.

H.C.A. van TILBORG : UNIFORMLY PACKED CODES

An e-error correcting code C in V(n,q), a n-dim. vector space over GF(q) is called uniformly packed iff

(i) $\forall_{\underline{x}\in V(n,q)} [d(\underline{x},C) = e \Rightarrow |\{\underline{c}\in C|\ d(\underline{x},\underline{c})\} = \lambda]$.

(ii) $\forall_{\underline{x}\in V(n,q)} [d(\underline{x},C) \geq e+1 \Rightarrow |\{\underline{c}\in C|\ d(\underline{x},\underline{c})\}| = \mu]$.

Giving λ and μ the appropriate values shows that uniformly packed codes contain the perfect, nearly perfect and strongly uniformly packed codes. An interesting property of u.p. codes is the fact that the words of fixed weight form an e-design (if $\underline{o}\in C$). For $q=2$, $\mu=\lambda+1$ the words of fixed weight in the extended code form an $(e+1)$-design. Many infinite sequences of u.p. codes are known, however we do now know that u.p. codes with $e \geq 4$ do not exist. For $e=3$, $q=2$ and for $e=1$ or 2, $q=2$, $\mu=\lambda+1$ we have obtained a full classification.

J. TOTTEN : A CLASSIFICATION OF LINEAR SPACES BASED ON QUADRANGLES.

A linear space L is a set of elements, called points, together with distinguished subsets of cardinality at least two, called lines, such that every pair of distinct points is contained in a unique line. A quadrangle Q of L is a set of four points any three of which are non-collinear. A diagonal point of Q in L is a point $p \in L \setminus Q$ such that any line joining p to a point of Q intersects Q in exactly two points. Clearly Q can have at most three diagonal points. Let $d(Q)$ denote the number of diagonal points of Q. We shall say that L is a linear space of type T, where $T \subseteq \{0,1,2,3\}$, if $T = \{d(Q) \mid Q \in L\}$. The type of a linear space determines a classification of the class of all linear spaces into 16 subclasses corresponding to the 16 subsets of $\{0,1,2,3\}$. We now consider the simpler classes. Type Φ and types $\{i\}$, $i \in \{0,1,2,3\}$ are settled and are not difficult, and type $\{i,j\}$, $i \neq j \in \{0,1,2,3\}$ is discussed giving solutions for $\{0,3\}$, $\{2,3\}$, $\{1,3\}$, $\{0,2\}$ (partial only). The last three above are solved only when every line has finite cardinality.

MICHAEL WALKER : FINITE COLLINEATION GROUPS CONTAINING SYMMETRIES OF GENERALIZED QUADRANGLES.

Let $(K,\mathcal{L})$ be a generalized quadrangle and $G \leq \mathrm{Aut}\,(K,\mathcal{L})$ a finite group of collineations of $(K,\mathcal{L})$. Assume G contains non-identity symmetries and let $(\bar{K},\bar{\mathcal{L}})$ be the minimal substructure of $(K,\mathcal{L})$ to contain all axes of axial symmetries in G. Let $\bar{G} \leq \mathrm{Aut}(\bar{K},\bar{\mathcal{L}})$ be the collineation group of $(\bar{K},\bar{\mathcal{L}})$ induced by G and denote the kernel of the representation $G \mapsto \bar{G}$ by k. If $(\bar{K},\bar{\mathcal{L}})$ is a subquadrangle then :

Theorem: $z(G) = k$ and one of the following holds:

1) $\bar{G}$ contains a unique minimal normal subgroup M, M is simple non-abelian and $M \trianglelefteq \bar{G} \leq \mathrm{Aut}(M)$

2) $\bar{G} \cong SL(2,2^n) \wr S_2$ for $n \geq 2$.

3) $O_{2'}(\bar{G}) \neq 1$ and $\bar{G}/_{O_{2'}(\bar{G})} \cong S_4$.

Furthermore if case 2) occurs then $(\bar{K},\bar{\mathcal{L}})$ is isomorphic to the symptotic geometry of a projective space of dimension 3 over a finite field of order $q = 2^n$.

RUDOLF WILLE : WHAT CAN ONE DO WITH JOIN AND MEET IN FINITE GEOMETRIES?

The subspace lattice $S(G)$ of allmost all finite geometries G have the properties that the greatest element is the join of atoms and $S(G)$ is simple. For finite lattices with these properties it is shown that every (n-ary) order-preserving map can be described by a lattice polynomial. This is a consequence of a more general theorem which states that in a finite lattice L every (n-ary) order-preserving map can be described by a lattice polynomial if and only if the identity and the constant map to θ are the only join-preserving maps $\delta : L \to L$ with $\delta x \leq x$ for all $x \in L$.

G.H. Dorber
(Westfield College, London)

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