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Ringe, Moduln und homologische Methoden

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Die von R. Baer ins Leben gerufene traditionsreiche Tagung wird im Jahre 1977 zum letztenmal stattfinden. Die vorletzte Veranstaltung unter der allgemeinen Thematik: "Ringe, Moduln und homologische Methoden" fand entsprechend ein sehr starkes internationales Interesse.

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Vortragsauszüge

P. Gabriel: Group representations without groups

For any k-algebra A define by $\underline{\text{mod}}\ A$ the residue category of mod A having as objects the A-modules and as groups of morphisms the residue groups

$$\text{Hom}_A(M,N)\ /\ \{M \to \text{projective} \to N\}\ .$$

(Kill the A-linear maps factorizing through a projective!). Now call two algebras A and B Green-equivalent, if there is an exact k-linear functor $L : \text{mod}\ B \to \text{mod}\ A$ mapping projectives into projectives and inducing an equivalence of categories $\underline{\text{mod}}\ B \xrightarrow{\sim} \underline{\text{mod}}\ A$. It is shown that the algebras, which are Green-equivalent to some symmetric (generalized uniserial) Nakayama algebra, are precisely the algebras which Janusz and Kupisch construct from Brauer trees.
The statement has its origin in a group theoretical situation: Let G be a finite group having a cyclic p-Sylow subgroup $D = \sigma^{\mathbb{Z}/p^n\mathbb{Z}}$. Set $S = (\sigma^{n-1})^{\mathbb{Z}/p\mathbb{Z}}$ and $N = \text{Norm}_G S$, so that $S \subset D \subset N \subset G$. Then $k[N]$ is Nakayama by Michler and $k[G]$ is Green-equivalent to $k[N]$ by Feit-Green-Thompson.

M. A. Knus: Computation of some Brauer groups

Let X be a real algebraic variety with coordinate ring R. The Brauer group Br(X) of X is the Brauer group of R, Br(R). One also defines a topological Brauer group $Br_{top}(X)$ of X by considering all real continuous functions on X (not just polynomial functions). It is known that $Br_{top}(X) = H^0(X,\mathbb{Z}/2) \oplus H^2(X,\mathbb{Z}/2)$ (Karoubi). For curves the two groups are canonically isomorphic (DeMeyer-Knus). It is wrong in general. One can prove for example that $Br(S^2) \cong \mathbb{Z}/2 \oplus \mathbb{Z}/2$ for S^2 the real algebraic sphere $\mathbb{R}[X,Y,Z]\ /\ (X^2+Y^2+Z^2-1)$, as in the topological case but the isomorphism is not canonical! Some reducible surfaces are also considered, for example 4 planes in 3-space, i.e. $R = K[X,Y,Z]/XYZ(X+Y+Z-1)$. It can be shown in this case that $Br(R) = Br(K) \oplus \mu(K)$ where $\mu(K) =$

$= \{$roots of 1 in $K\}$. This gives the topological group for $K = \mathbb{R}$ and $\mathbb{Q}/\mathbb{Z}$ for $K = \mathbb{C}$ which answers a question of Grothendieck in Groupes de Brauer II, § 2. The formula remains valid for certain rings taken as coefficients K. For example $K = \mathbb{Z}[\xi]$, $\xi = \frac{2\pi i}{n}$ gives $Br(R) = \mathbb{Z}/n$. In this way one can construct rings with prescribed finite abelian groups as Brauer groups.

B. Pareigis: Morita equivalence over monoidal categories

Let $(\underline{C}, \otimes : \underline{C} \times \underline{C} \longrightarrow C, I \in \underline{C})$ be a monoidal category (i. e. there are natural coherent isomorphisms $A \otimes (B \otimes C) \cong (A \otimes B) \otimes C$ and $A \otimes I \cong A \cong I \otimes A$). A monoid in $\underline{C}$ is an object A with an associative unitary multiplication $\mu: A \otimes A \longrightarrow A$. An A-object is an object M together with an associative unitary operation $\nu: A \otimes M \longrightarrow M$. The A-objects form a category ${}_A\underline{C}$ in the obvious way.
Define inner hom functors by ${}_A\underline{C}(M \otimes X, N) \cong \underline{C}(X, {}_A[M,N])$ for $X \in \underline{C}$ and $M,N \in {}_A\underline{C}$. Observe that usually ${}_A[M,N]$ does not exist, e.g. in $(K\text{-Mod}^{op}, \otimes, K)$ for a commutative ring K and $A = K$. If ${}_A[P,A]$ and ${}_A[P,P]$ exist then there is a morphism ${}_A[P,A] \otimes P \rightarrow {}_A[P,P]$ defined in a natural way. If this is a difference cokernel of ${}_A[P,A] \otimes A \otimes P \rightrightarrows {}_A[P,A] \otimes P$ then P is called finite. If in addition the evaluation $P \otimes {}_A[P,A] \longrightarrow A$ is a difference cokernel of $P \otimes {}_A[P,P] \otimes {}_A[P,A] \rightrightarrows P \otimes {}_A[P,A]$ then P is called faithfully projective.
A functor $F: {}_A\underline{C} \rightarrow {}_B\underline{C}$ is a $\underline{C}$-functor if $F(M \otimes X) \cong F(M) \otimes X$ for $X \in \underline{C}$ and $M \in {}_A\underline{C}$ (coherent and natural).
A $\underline{C}$-functor $F : {}_A\underline{C} \rightarrow {}_B\underline{C}$ is an equivalence if and only if there is a faithfully projective $P \in {}_A\underline{C}$ with ${}_A[P,P] \cong B$ such that $F \cong {}_A[P,-]$. In this case the "centers" of A and B are isomorphic. This theorem may be used to introduce the notion of the Brauer group of $\underline{C}$.

A. M. Ostrowski: On Kronecker's extension of ideals and modules in commutative rings

A module M is considered in the commutative ring Ω with coefficients from an underring ω . If A satisfies an equation of the form

$$A^m + \sum_{\mu=1}^{m} K_\mu A^{m-\mu} = 0, \; K_\mu \subset M^\mu \quad (\mu = 1,\ldots,m),$$

A is called isobarically dependent on M. The set of all elements of Ω isobarically dependent on M is called Kronecker's extension $\bar{M}$, of M in Ω . Different properties of $\bar{M}$ will be discussed, in particular an application to Kronecker's Elimination Theory.

G. Azumaya: Conditions for flatness of character modules

A left R-module M is called locally injective if given a left R-module A and a finitely generated submodule B of A every homomorphism $B \to M$ can be extended to a homomorphism $A \to M$. It is proved that if R is left coherent and M is locally injective then its character module $M^* =$ $= Hom_Z(M, \mathbb{Q}/\mathbb{Z})$ is flat as a right R-module. Using this fact, it is further shown that, when R is left coherent, M is absolutely pure (i.e. M is pure in every its extension module) if and only if M^* is flat.

M. Harada: Exchange property in a directsum

Let R be a ring with identity. An R-module M is called completely indecomposable if $End_R(M)$ is a local ring. We take a set $\{M_\alpha\}_I$ of completely indecomposable modules M and put $M = \Sigma_I^{\oplus} M_\alpha$. Let N be a direct summand of M. We shall give some criteria for N to have the following property: for any decomposition $M = \Sigma_I^{\oplus} T_\alpha$ with T_α completely indecomposable, $M = N \oplus \Sigma_{I'}^{\oplus} T_\alpha$ for some subset I' of I.

C. U. Jensen: Einige Bemerkungen über Ringe vom endlichen Darstellungstypus

R werde ein links U-Ring genannt, falls jeder links R-Modul als direkte Summe von endlich präsentierbaren Moduln dargestellt werden kann. Es werden u. a. die folgenden Sätze bewiesen:

Satz 1. R sei ein links und rechts U-Ring. Dann gibt es nur endlich viele Isomorphieklassen von unzerlegbaren links R-Moduln.

Satz 2. R sei ein Ring für den eine Kardinalzahl $\aleph$ existiert derart, daß jeder links R-Modul die direkte Summe von durch höchstens $\aleph$ Elemente erzeugten Moduln ist. Dann ist R ein links U-Ring.

Der letztere Satz findet sich in:

L. Gruson et C.U. Jensen. Deux applications de la notion de L-dimension, C.R.Acad. Sci. Paris Sér A-B 282 (1976),A 23-25.

H. Lenzing: Left and right pure global dimension zero implies finite representation type

As usual, R is of finite representation type if R is right artinian and R has only a finite number of indecomposables of finite length. R has (right) pure global dimension 0 if any right R-module is a direct sum of finitely presented ones. Let F be the ringoid (=small additive category) of finitely presented right R-modules.

Proposition 1. Right pure global dimension (R) = 0

$\Leftrightarrow$ F is right perfect

Proposition 2. Left pure global dimension (R) = 0

$\Leftrightarrow$ F is left noetherian.

Proposition 3. The following are equivalent for any ring R

(a) R has finite representation type

(b) Left and right pure global dimension (R) = 0.

C M. Ringel: The second Brauer Thrall conjecture

Theorem: Let k be an infinite field, and A a finite dimensional k-algebra. Then either there are only finitely many indecomposable A-modules, or there is a natural number d such that there are $|k|$ different isomorphism classes of indecomposable A-modules of dimension $n \cdot d$, for any natural number d.

In the case of a perfect field k, this theorem is due to Nazarova and Roiter. The proof uses vectorspace categories and their subspace categories.

A. R. Magid: Factorality in quotients of linear G_m-actions

Let k be an algebraically closed field of characteristic 0, and let $G_m = GL_1(k)$ act rationally on the k-vector space V. Let $R = k\,V^{G_m}$. We calculate the divisor class group $Cl(R)$. Choose a basis x_i, $i = 1, \ldots, n$ of V such that the action is diagonal, so that t in G_m sends x_i to $t^{b_i} x_i$. Without loss of generality we may assume that all b_i are non-zero. Suppose that exactly p are positive. If both p and $n-p$ are greater that 1, $Cl(R)$ is infinite cyclic. If not, $Cl(R)$ is finite cyclic, and we indicate how to compute its order. The above calculations proceed via a study of the geometric quotient prevariety $W = (V-0)/G_m$.

R. A. Morris: Extension of formal groups

Let $\eta : 0 \to G^m \to G \to G^n \to 0$ describe a finite commutative local groupscheme G over a field k of characteristic $p \neq 0$, with G^m multiplicative and G^n unipotent. Suppose G^m is contained in a local multiplicative p-divisible formal group Γ^m and G^n in a local unipotent p-divisible group Γ^n (in each case we mean Ker p has the given type).

When can we find an extension $\eta' \in \mathrm{Ext}'(\Gamma^n, \Gamma^m)$ containing η? After finite separable extension one may embed G^m in a torus $T = G_m^n$ and without loss of generality $T = \mathbb{G}_m$. The obstruction to embedding η is the image, θ, of η in $\mathrm{Ext}^2(\Gamma^n/_{G^n}, \mathbb{G}_m)$. η, and hence θ is p-torsion. We prove that $\mathrm{Ext}^2(Q, \mathbb{G}_m)$ is

torsion free for any local unipotent p-divisible group Q, whence $\Theta = 0$ and η is always embeddable.
The proof proceeds by examination of the Roos spectral sequence
$E_2^{p,q} = R^p \varprojlim \{ \mathrm{Ext}^q (\mathrm{Ker}\ p_Q^n, \mathbb{G}_m)\} \Rightarrow \mathrm{Ext}^{p+q} (Q, \mathbb{G}_m)$.
All Ext's are for sheaves in the f.p.p.f. site and are analysed by f.p.p.f. cohomology.

D.A.R. Wallace: Intersection Theorems

Let G be a group, let R be a ring and let R(G) be the group ring. The primitivity of R(G) is considered when R is not a field and theorems analogous to those of Passman an Martha Smith are obtained. Some generalizations of work of Zilesskii are obtained and a more immediate characterisation of his group-theoretic work is obtained.

E. G. Evans: General Position and Special Position in commutative Ring Theory

Let R be a regular local ring with maximal ideal $\mathfrak{m}$,M a finitely generated module, and $x \in \mathfrak{m}M$, then $\{f(x) \mid f \in \mathrm{Hom}(M,R)\}$ has height at most equal to the rank of $M = \dim_{R_{(o)}} M_{(o)}$. With a suitable definition of rank this theorem remains true for any local ring containing a field. The case M a free module is Krull's altitude theorem. This theorem was proved along with applications to k-syzygies an determinental ideals. This theorem begins the investigation of determining to what extent the theorems about basic elements are the best possible.

D. Eisenbud: Quadrativ forms on finite dimensional algebras and the topological degree of infinitely differentiable mappings

Let $f:(\mathbb{R}^n,0) \to (\mathbb{R}^n,0)$ be a C^∞ map germ which is finite in the sense of Mathes; i.e. the algebra $Q(f)=C^\infty(\mathbb{R}^n,0)/f^*\mathfrak{m}_n$ is a finite dimensional real vectorspace, where $\mathfrak{m}_n$ is the ideal of germs of C^∞ functions on the target of f which are 0 at 0. Let $\mathcal{J} = \mathcal{J}(f)$ be the determinant of the determinant of the jacobian matrix $\left|\frac{\partial f_i}{\partial x_j}\right|$ of f, regarded as a germ on $\mathbb{R}^n$.

Theorem (essentially due to R. Berger): The image $\mathcal{J}_o$ of $\mathcal{J}$ in Q(f) generates the unique minimal ident of Q(f).

In particular, the above result implies that there is a linear functional $\varphi: Q(f) \to R$ such that $\varphi(\mathcal{J}) = 1$. Define a symmetric real bilinear form $\langle\ ,\ \rangle$ on Q(f) by the formula $\langle p,q\rangle =$ $= \varphi(pq)$.

Theorem (H.Levine and the author): The topological degree of f at 0 is the signature of the form $\langle\ ,\ \rangle$.

W. Bruns: On some applications of basic elements

In constructing basic elements in a finitely generated module over a commutative noetherian ring one usually starts with a certain set of generators $x_1,\ldots,x_n$ and gets a basic element $\Sigma a_i x_i$, $a_i \in R$, by an induction machinery. Sometimes it is essential to choose the coefficients a_i in some suitable subring, let us say: an infinite field $K \subset R$. In fact all theorems on basic elements, which apply to general noetherian rings may be strengthened in this way. An application: Let A be an analytic K-Algebra, K infinite perfect field. Then there exists an element $x \in A$ such that the inclusion Spec A/Ax $\to$ Spec A, which is incluced by the natural epimorphism $A \to A/Ax$, maps Sing A/Ax into Sing A. (H. Flenner proved this for all local rings) - Similar ideas lead to theorems on basic elements in Stein modules. There are just the right analogies between noetherian Rings, their finitely generated modules, and their spectrum on one side and Stein algebras, their Stein modules, and their spectrum of closed prime ideals on the other one.
(For the terminology "basic element" compare D. Eisenbud and E.G. Evans, J. Algebra 27).

F. Szász: A second almost subidempotent radical

Mit elementenfreien Methoden wird bewiesen, daß die Klasse K der Ringe A mit $a \in aA+Aa+AaA$ für jedes $a \in A$ eine Amitsur-Kurošsche Radikalklasse bildet. Wir geben äquivalente Bedingungen, die diese Klasse charakterisieren. Für dieses Radikal R ist jeder lokal nilpotente Ring streng R-halbeinfach, und jeder streng R-halbeinfache Ring ist antieinfach. Gewisse algebraische Anwendungen (z.B. zwei Charakterisierungen der Artinschen nilpotenten Ringe) und sechs offene Probleme werden erwähnt.

M. P. Malliavin: Euler Poincaré Characteristics and cohomology of nilpotent Lie algebras

Let G be a nilpotent Lie algebra, $U(G)$ its envelopping algebra, $\mathfrak{m}$ the augmentation ideal of $U(G)$ and $A = U(G)_{\mathfrak{m}}$ the localization at $\mathfrak{m}$ of $U(G)$.
Let $\mathcal{J} = Ax_1 + \ldots + Ax_r$ an ideal of A generated by a A-centralizing sequence. If M is a (left) A-module of finite type such that $lg_A\ (Tor_i^A\ (\frac{A}{g}, M))$ is finite for some $i \geq 0$, then $lg_A(Tor_j^A(\frac{A}{g}, M))$ is finite for every $j \geq i$ and is null for $j \gg 0$. One let as in the vommutative case:

$$\chi_i^A\ (\frac{A}{g}, M) = \sum_{j=i}^{\infty} (-1)^{j-i} lg_A\ (Tor_i^A(\frac{A}{g}, M))$$

Then all the results known is commutative algebra are true (it suffices to take for Krull dimension that of Gabriel-Rentschler) These results extend one of Dixmier about cohomology of nilpotent Lie algebra. Also they extend a result of Nomezu about Euler-Poincaré (topologically) characteristic of Nilpotent Lie groups. Then they give some combinatorial inequalities for the number of elements of length j $(j=1,\ldots,n)$ of the Weyl group of a semisimple complex Lie algebra of dimension n.

G. J. Hauptfleisch: Semi-flat modules

A characterization of flatness in terms of commutative diagrams is utilized for a certain generalization called semi-flatness. Whereas Von Neumann-regular rings R are characterized by the property that all R-modules are flat, the class of rings R for which all injective R-modules are flat, the class of rings R for which all injective R-modules are flat (called IF-rings) is characterized by the property that all R-modules are semi-flat. The concepts of semi-flatness and flatness coincide for left hereditary rings. Submodules of flat modules over left hereditary rings are flat. The finitely presented semi-flat modules are precisely the submodules of finitely generated free modules.

L. Robbiano: Normal flatness and primary powers of prime ideals

Let B be a commutative noetherian ring, with identity, $\mathfrak{p}$ a prime ideal, $A=B/\mathfrak{p}$, the problem is to find conditions on $B,\mathfrak{p},A$ in order to get the following property: all the powers of $\mathfrak{p}$ are primary ideals. Using the fact that the property we are looking for is equivalent to saying that $G(\mathfrak{p}) = \bigoplus_{n=0}^{\infty} \mathfrak{p}^n/\mathfrak{p}^{n+1}$ is a torsion-free A-module, we get the following results.

1) Let $(R,\mathfrak{m})$ be a local ring, $\mathfrak{a} \subset \mathfrak{P}$ ideals which are generated by regular sequences, $\mathfrak{P}$ a prime ideal, assume that $V(\mathfrak{P}/\mathfrak{a})-\{\mathfrak{m}/\mathfrak{a}\}$ is regular in $\mathrm{Spec}(R/\mathfrak{a})$ and $\mathrm{Spec}(R/\mathfrak{P})-\{\mathfrak{m}/\mathfrak{P}\}$ is regular, and depth $(R/\mathfrak{a})=d \geq 2$.
Then $(\mathfrak{P}/\mathfrak{a})^2$ is primary.
If, in addition, we assume $d > \mathrm{gr}(\mathfrak{a})$ then $(\mathfrak{P}/\mathfrak{a})^n$ is primary for every n.

2) Let $(R,\mathfrak{m})$ be a local ring, $\mathfrak{a} \subset \mathfrak{b}$ ideals such that $\mathfrak{b}=(\mathfrak{a},f_1,\dots,f_t)$ and $f_1,\dots,f_t$ is a $R/\mathfrak{a}$-sequence. Then if $G(\mathfrak{b})$ is torsion-free and $G(\mathfrak{a}R_{\mathfrak{p}})$ is free for every $\mathfrak{p} \in \mathrm{Ass}(R/\mathfrak{b})$, $G(\mathfrak{a})$ is torsion-free.

For proving the second result we prove the following theorem, which generalizes some theorems on the transitisity of the normal flatness.

Theorem: Let $(R,\mathfrak{m})$ be a local ring, $\mathfrak{a} \subset \mathfrak{b}$ ideals such that $\mathfrak{b}=(\mathfrak{a},f_1,\dots f_t)$ and $f_1,\dots,f_t$ is a $R/\mathfrak{a}$-sequence. Then the following conditions are equivalent

a) $G(\mathfrak{b})$ is free and $G(\mathfrak{a}R_{\mathfrak{p}})$ is free for every $\mathfrak{p} \in \mathrm{Ass}(R/\mathfrak{b})$

b) $G(\mathfrak{a})$ is free.

K. R. Goodearl: Completions of regular rings

This talk is concerned with completions of (von Neumann) regular rings with respect to pseudo-rank functions, and with the relationship between such completions of a regular ring R and the compact convex set ℙ(R) of all pseudo-rank functions on R. For example, (1) The completion of R with respect to a pseudo-rank function P is a simple ring if and only if P is an extreme point of ℙ(R). (2) In any completion of R, the Boolean algebra of central idempotents is naturally isomorphic to a certain sublattice of the lattice of faces of ℙ(R).

M. B. Wischnewsky: On the Relevance of Module Theory to Computation and Control of Technical Systems

Zur Beschreibung technischer Systeme, wie Übertragungsanlagen, Verstärkerschaltungen, Datenverarbeitungsanlagen usw. wurden bisher hauptsächlich funktionalanalytische bzw. funktionentheoretische Techniken verwendet. In diesem Vortrag soll gezeigt werden, daß viele Begriffe der linearen Systemtheorie im Rahmen der Modultheorie besser beschrieben werden können. Diese modultheoretische Beschreibung hat viele praktische Vorteile: Die Laplace-Transformation, der Schlüssel zur klassisch analytischen Theorie, und die Beschreibung durch Zustandsvariablen werden in einen einzigen Rahmen gebettet. Man erhält neue Methoden zur effektiven Berechnung der Realisierungen. Weiter werden die Faltungstheorie, die Dualität von Systemen, sowie die Erreichbarkeit und die Beobachtkeit behandelt.

D. Handelman: Equivalence of projections in Finite Rickart C^* algebras

By purely algebraic means, a problem of Kaplansky in functional analysis is almost entirely resolved. A Rickart *-ring is an involutive ring such that the right annihilator of any element is generated by a projection. A Rickart C^* algebra is a C^* algebra that is also a Rickart *-ring. To each element of a Rickart *-ring are associated left and right projections and the 1951 question of Kaplansky asks whether these two projections are always linked by a partial isometry. We answer in the affirmative if the C^* algebra is additionally finite ($xx^* = 1$ implies $x^*x = 1$). This technical result admits a number of interesting consequences, and the proof is entirely algebraic.

H. Bass: Locally polynomial algebras are symmetric

Let K be a commutative ring. Let A be a finitely presented K-algebra. Suppose that, for all maximal ideals $\mathfrak{m}$ of K, $A_{\mathfrak{m}}$ is a polynomial algebra over $K_{\mathfrak{m}}$. Then A is the symmetric algebra S(P) of a projective module P. (This comes from joint work with David Wright and Ed Connell).

E. A. Rutter: Coherence and weak global dimension of R [[X]] when R is von Neumann regular

Let R be a commutative (von Neumann) regular ring. This paper is concerned with determining necessary and sufficient conditions on R for the ring of formal power series R [[X]] to be coherent (equivalently, semi-hereditary) and also conditions for R [[X]] to have weak global dimension one. An example is given to show that R [[X]] can have weak global dimension one without being coherent even when R is a Boolean ring.

R. Raphael: An arithmetic in a class of rings

Rings are fully idempotent if $I=I^2$ for all two-sided ideals I. For fully idempotent rings in which each non-zero ideal is a finite product of primes we show that a form of unique factorization holds. Examples and counterexamples from the theory of regular rings are given.

C. Procesi: A remark on Capelli's identity

The classical identity of Capelli is interpreted and generalized in terms of the computation of the eigenvalues of a certain polynomial which in turn are interpreted in the language of Young Tableau.

L. W. Small: Converses of some theorems in ring theory

The following results were proved:

Thm 1: If R is semi-prime, $e^2=e$, eRe and $(1-e)R(1-e)$ are right noetherian then R is right noetherian.

Thm 2: Let R be prime and P.I. The centrally generated ideals of R are projective if and only if R is a finite module over its center which is a Dedekind domain.

Thm 3: $R \subset \Delta n$, Δ a division ring. If the classical ring of quotients, $Q(R)$, exists, then $Q(R) \subset \Delta_t$, $t \leq n$.

F. Kasch (München)