MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 23/1976

Gruppentheorie

30.5. bis 5.6.1976

Die diesjährige Gruppentheorietagung stand wieder unter der Leitung von Prof. W. Gaschütz (Kiel), Prof. K. W. Gruenberg (London) und Prof. B. Huppert (Mainz). Es haben 48 Mathematiker teilgenommen; die Hälfte davon kam aus dem Ausland. 31 Vorträge wurden gehalten, die sich mit neuesten Ergebnissen aus verschiedenen Gebieten der Theorie der endlichen sowie der unendlichen Gruppen befaßten. Die vortragsfreie Zeit wurde zu intensivem Gedankenaustausch und angeregten Diskussionen genutzt.

Teilnehmer

J.L.Alperin, Chicago, z.Z.Bonn
B.Amberg, Mainz
Annie Astié, Toulouse
R.Baer, Zürich
D.W.Barnes, New Haven (USA)
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R.M.Bryant, Manchester
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K.Doerk, Mainz
Joan L.Dyer. Bronx (USA)
B.Fischer, Bielefeld
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J. Neubüser, Aachen
D.Perrin, Paris
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D.J.S.Robinson, Urbana (USA)
K.W.Roggenkamp, Stuttgart
G.Rosenberger, Hamburg
T.Schmid-Leißler, Tübingen
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L.L.Scott, Charlottesville (USA)
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D.Segal, London
R.Strebel, Heidelberg
O.Tamaschke, Tübingen
J.Tappe, Aachen
Mary Tyrer-Jones, Cambridge
B.A.F.Wehrfritz, London
H.Wielandt, Tübingen
J.S.Wilson, Cambridge

Vortragsauszüge

J.L.ALPERIN: On the Brauer correspondence

Let G be a finite group, F an algebraically closed field of prime characteristic p, Q a p-subgroup of G and H a subgroup of $N(Q)$ containing $Q\,C(Q)$. Let b and B be block ideals of the group algebras FH and FG, respectively.

Theorem: B corresponds to b under the Brauer correspondence if and only if b as an $F[H\times H]$-module is isomorphic with a direct summand of the restriction to $H\times H$ of the $F[G\times G]$-module B.

A. ASTIE: Characterisation of transitive permutation groups of odd order which have minimum rank

A permutation group is of rank r if it is transitive and the stabilizer of a point has exactly r orbits. For any odd integer n, the minimum rank, denoted r_n, of a (transitive) subgroup of odd order of S_n is computed; and the whole set of subgroups of odd order and minimum rank r_n which are maximal in S_n is determined.

D. W. BARNES: First cohomology groups of p-soluble groups

I shall prove the following two theorems:

Theorem 1: Let G be a finite p-soluble group and let A be an irreducible $\mathbb{Z}_p G$-module. Then $H^1(G,A) = 0$ if and only if G has no complemented chief factor isomorphic to A.

Theorem 2: Let G be a finite group. Then G is p-supersoluble (that is, every chief factor of G has order equal to p or relatively prime to p) if and only if $H^1(G,A) = 0$ for every irreducible $\mathbb{Z}_p G$-module A of dimension greater than 1.

A. M. BRUNNER: Nielsen inequivalent presentations of one-relator groups

Let G be a group. Two presentations F/R, F/S given as factor groups of a free group F are Nielsen equivalent if there is an automorphism α of F with $R\alpha = S$. The main purpose of this lecture is to publicise an example of a one-relator group with an infinite number of Nielsen inequivalent one-relator presentations. This answers a question raised by J.McCool and A. Pietrowski. The example is a group G defined by the presentation $\langle x,y\ ;\ x^{x^y} = x^2 \rangle$ (where w^v denotes $v^{-1}wv$) , and the presentations are $P_k : \langle x,y\ ;\ x^{x^{2^k y}} = x^2 \rangle$, $k = 0, 1\ldots$ The group G has other Nielsen inequivalent presentations as well.

In another direction there is the following problem: Let BS denote the group defined by the presentation $\langle x,y\ ;\ y^{-1}x^2y = x^3 \rangle$. Is the presentation associated with the generating pair $(x^4\ ,\ y^{-1}xy^2)$ three-relator?

R. M. BRYANT: The verbal topology of a group

Let $w_1\ ,\ w_2\ ,\ldots,\ w_n$ be words from a free group F. A group G is said to satisfy the disjunction $w_1 \vee w_2 \vee\ldots\vee w_n$ if, for all $f \in \mathrm{Hom}(F,G)$, $w_i f = 1$ for some i .

Question: If G satisfies the disjunction $w_1 \vee w_2 \vee\ldots\vee w_n$, must some w_i be a law of some section H/K with $|G:H|$ and K both finite?

Theorem: If G is a linear group or a finitely generated abelian-by-nilpotent group, then G has a 'discriminating' subgroup of finite index.

(A group G is discriminating if the only disjunctions that G satisfies have the form $w_1 \vee w_2 \vee\ldots\vee w_n$, where some w_i is a law of G.) The proof uses the verbal topology defined for any group G , which is similar to the Zariski topology for a linear group.

J. D. DIXON: Rigid embedding of simple Lie groups in GL(n,$\mathbb{C}$)

Let $E := \mathrm{Mat}(n,\mathbb{C})$ (nxn-matrices over $\mathbb{C}$) and let $S \subset E$. Define $\mathrm{Fix}(S) := \{\sigma \in GL_{\mathbb{C}}(E) \mid S^{\sigma} \subseteq S\}$. If $S = E \setminus GL(n,\mathbb{C})$, then Dieudonné (1949) has shown that Fix(S) equals the group R ('rigid mappings') generated by 'translations'

$x \mapsto xa$, $x \mapsto ax$ $(a \in GL(n,\mathbb{C}))$ and 'reflections' $x \mapsto x'$ (transpose). Since then it has been proved for a number of different subsets S and various fields in plan of $\mathbb{C}$ that Fix(S) is a subgroup of R . Using results of Dynkin (1952) we can prove the following

Theorem: Let G be an irreducible Lie subgroup of $GL(n,\mathbb{C})$. Then Fix(G) is a subgroup of R except (possibly) in the cases: G is of type A_1 and $n = 4$ or G is of type B_m and $n = 2^m$ $(m \geq 2)$.

J. L. DYER: Finite Automorphism Sequences

Let A(G) denote the automorphism group of a group G , $A^2(G) := A(A(G))$, ... and let $I: G \longrightarrow A(G)$ be defined by $I(g)(x) = gxg^{-1}$ (g,x G). If I is an isomorphism, G is termed complete. An automorphism sequence $G \xrightarrow{I} A(G) \xrightarrow{I} A^2(G) \longrightarrow \ldots$ is finite if $A^r(G)$ is complete. A theorem of Wielandt states that the automorphism sequence of a finite centerless group is finite; and there are two conjectures, due to G. Baumslag, concerning possible extensions of Wielandt's theorem:

Conjecture A: A sufficiently symmetric group has a short automorphism sequence.

Conjecture B: A torsion-free finitely generated nilpotent group has a periodic automorphism sequence.

Results supporting these conjectures are as follows (henceforth, F is a free group of rank n , with $2 \leq n < \infty$):

1. (Dyer, Formanek) A(F) is complete.
2. (D,F) Let C be a characteristic subgroup of F , s.t. $G := F/C$ is centerless and residually torsion free nilpotent. Put $K(G) := \mathrm{Ker}(A(G) \longrightarrow A(G/G'))$. Then A(G) is complete if K(G)/I(G) is centerless and residually nilp.
3. (D,F) If R is characteristic in F and F/R is residually torsion free nilpotent, then A(F/R') is complete.

4. (D,F) With $G := F/[F,F']$, $A^2(G)$ is complete.
5. (D) The groups $GL(n,\mathbb{Z})$ and $SL(n,\mathbb{Z})$ have finite automorphism sequences.
6. (D) With $G := \mathbb{Z} * \mathbb{Z}_3$, $A^i(G) \cong A^j(G)$ implies $i = j$.

R. FOOTE: Finite groups of component type

A survey of recent results in the theory of finite groups of component type is presented, and in this perspective the following result of the author is discussed:
Assume G is a finite group, L a 2-component of $C_G(t)$ for some involution t in G with $L/O(L) \cong L_2(q)$ or A_7, for some odd integer q , and L is maximal (in a modified form of the component ordering); assume the unbalanced 2-components of the centralizers of involutions in G satisfy the abelian core property; then one of the following holds:

i) $L \trianglelefteq L(G)$
ii) $\exists A \trianglelefteq L(G)$ with $A \neq A^t$ and $L = C_{AA^t}(t)^{(\infty)}$
iii) $F^*(G)$ is simple of sectional 2-rank at most 4 or $F^*(G) \cong S_4(4)$, $L_5(2)$, $U_5(2)$, HiS or He .

W.GASCHÜTZ: Untergruppenkriterien für abelsche Gruppen

Es sei F die freie abelsche Gruppe auf $\{x_1, x_2, \ldots\}$ und $w = \sum w_i x_i \in F$, $w_i \in \mathbb{Z}$, $w_i = 0$ für fast alle i . w heiße Untergruppenkriterium für eine abelsche Gruppe G , wenn - wie z.B. bei $w = x_1 - x_2 + 0x_3 + \ldots$ für alle G - aus $\emptyset \neq K \leq G$ und $\sum w_i k_i \in K$ für alle $k_i \in K$ folgt: K ist Untergruppe von G. Es wird bewiesen: G habe den Exponenten e und es sei $G \neq \mathbb{Z}_2$. Dann ist w Untergruppenkriterium für G genau dann, wenn $(\sum w_i - 1, e) = (e, w_1, \ldots, \hat{w}_i, \ldots) = 1$ ($\hat{w}_i$: w_i auslassen) für alle i und für $e = 0$ $w \neq x_1 + x_2$ und $w \neq -x_1 - x_2$ ist.

J. A. GREEN: Rational blocks for linear groups

Let F be a field, $G \leq GL(n,F)$ a linear group, and $A = A(G)$ the algebra of polynomial functions (= regular functions) of G into F. The group multiplication map $G\times G \longrightarrow G$ induces a map $\mu : A \longrightarrow A \otimes A$, with respect to which A becomes an F-coalgebra, with co-unit ε_1 (in general, for $x \in \Gamma = FG$, $\varepsilon_x : A \longrightarrow F$ is the evaluation map $f \longmapsto f(x)$). Then $\Gamma_o = \mathrm{Hom}(A,F)$ becomes naturally an F-algebra, and Γ is embedded in Γ_o by $x \longmapsto \varepsilon_x$. Every rational Γ-module V (i.e. V is a locally finite Γ-module, whose coefficient functions all lie in A) can be made into a Γ_o-module; for any two rational Γ-modules V, V' one has $\mathrm{Hom}_{\Gamma}(V,V') = \mathrm{Hom}_{\Gamma_o}(V,V')$. In this sense Γ_o is a kind of 'closure' of the group algebra Γ, which seems useful in the study of rational Γ-representations. A itself is, as everyone knows, a 2-sided rational Γ-module; blocks $Bl(i)$ are defined in terms of the (unique) decomposition $A = \bigoplus A_i$ into indecomposable 2-sided summands A_i. Each such A_i corresponds to a primitive idempotent E_i in the centre of Γ_o which is uniquely determined by $A_i = AE_i = E_iA$.

K. W. GRUENBERG: Arithmetic questions connected with free presentations of finite groups

The lecture is a survey of known results concerning the number of generators and the number of defining relations of a finite group. This material is part of chapters 5, 6, 7 of my CBMS booklet on relation modules (AMS 1976). The lecture ends with some unpublished material concerning $H^1(G,M)$.

N. D. GUPTA: A problem of R. H. Fox

Let R be a normal subgroup of a non-cyclic finitely generated free group F and let $\underline{f}$, $\underline{r}$ denote, respectively, the augmentation ideals of the integral group rings $\mathbb{Z}F$, $\mathbb{Z}R$. Then $F(n,R) := F \cap (1 + \underline{f}^n \underline{r})$ $(n \geq 1)$ is a normal subgroup of F whose identification is a longstanding problem of R. H. Fox (1952). $F(n,R)$ is known in the following cases:

i) $n = 1$

ii) $R = F$ (Magnus)

iii) $n = 2$ (Enrigat, Hurley)

iv) $R = F'$ (Gupta, Gupta)

Here we are able to prove the

Theorem: If rank F = rank $R/R \cap \gamma_2(F)$, then $F(n,R) = I_R([R \cap \gamma_n(F), R \cap \gamma_n(F)] \gamma_{n+1}(R))$, where $I_R(S)$ is the isolator of S in R . In particular, the identification problem is solved for finitely generated periodic F/R .

B. HARTLEY: Soluble groups satisfying Min-n

It has been known for some time that metabelian groups satisfying Min-n (minimal condition on normal subgroups) are countable. An example of a soluble group of derived length 3 which is uncountable but satisfies Min-n will be discussed.

C. R. LEEDHAM-GREEN: On p-groups of maximal class

This is joint work with Susan McKay. We produce a classification of groups of order p^n and class $n-1$ subject to certain small edge effects. We construct an infinite class X of groups of order p^n and class $n-r$ and ask whether these groups are 'dense' in the class of groups of order p^n and class $n-r$; more precisely, does every group P of order p^n and class $n-r$ have subgroups H and K with $H \lhd K$, and $|K|$ and $|P:K|$ of order bounded by functions of p and r alone, such that $K/H \in X$? Since the groups in X have solubility length ≤ 3 , this would bound the solubility length of P in terms of p and r alone.

We have affirmative results for $r = 1$ and partial results for $r = 2$. The groups in X are constructed using easy results in number theory, difference equations, and homological algebra.

J. C. LENNOX: <u>On groups in which every subgroup is nearly subnormal</u>

The following results are discussed:

<u>Theorem A:</u> Suppose G is a group and there exist m , n with $|H_n:H| \leq m$ for all finitely generated subgroups H of G . Then G is a finite by nilpotent group and there exists a function μ such that $|\gamma_{\mu(m+n)}(G)| \leq m!$

<u>Theorem A*:</u> Suppose G is a group and there exist r , s such that each finitely generated subgroup H of G is subnormal of defect at most r in a subgroup of index at most s . Then G is finite by nilpotent and $|G : \zeta_{\mu(r+s)}(G)| \leq (s!)^{\lambda(s)}$.

Here, for a group G , $\gamma_n(G)$, $\zeta_n(G)$ denote the n-th terms of the lower and upper central series respectively, and if H is a subgroup of G , H_n denotes the n-th term of the normal closure series of H in G . $\lambda(s)$ is the number of primes $\leq s$.

F. LEVIN: Residually free one related groups

G. Baumslag has shown that if F_2 is free of rank two and $\varphi(F_2)$ is an isomorphic image of F_2, then the free product of F_2 and $\varphi(F_2)$ with $u \in F_2$ amalgamated with $\varphi(u) \in F_2$ is residually free when u is not a proper power. He then asked if this free product with $u \in F_2$ and $v \in \varphi(F_2)$ amalgamated were residually free, where u, v are arbitrary elements which are neither powers nor primitive. In this report we answer this question in the negative by constructing an example. This is a joint work with Benjamin Baumslag.

J. MENNICKE: On Picard's modular group

Consider the group $G = PSL_2(\mathbb{Z}[i])$, and its congruence subgroups $N_{\mathfrak{p}}$, where $\mathfrak{p}$ is a prime ideal of $\mathbb{Z}[i]$. For the study of the groups $N_{\mathfrak{p}}^{ab}$, some geometric techniques are developed. As a by-result, the multiplicity formulae of Hecke-Eichler for the operation of $PSL_2(p) \cong \langle 2, 3, p \rangle / N_p$ on N_p^{ab} are recovered.

G. MICHLER: On vertices of simple modules in p-solvable groups

Theorem 1: Let M be a simple FG-module of the finite p-solvable group of characteristic $p > 0$ with vertex $vx(M) =_G V$. Suppose M is contained in the block B of G with defect group $\delta(B) =_G D$. If $Z(D)$ denotes the centre of D, then

a) $Z(D) \leq_G V$

b) $Z(D) =_G V$ if and only if D is abelian.

If n is a positive integer, then $\nu(n)$ is the exponent of the highest p-power dividing n.

Theorem 2: Let G be a p-solvable group with $\nu(|G|) = a$, and let F be a splitting field of characteristic $p > 0$ for all subgroups of G. Then

$$\nu(\dim_F M) = a - \nu(|vx(M)|)$$

for any simple FG-module M.

Both results are obtained in joint work with W. Hamernik (Giessen) and strengthen classical results due to P. Fong.

D. PERRIN: Codes and permutation groups

A (biprefix) code is a set of words over a finite alphabeth X which is the basis of a submonoid P of the free monoid X^* satisfying:

i) $uv, u \in P \Longrightarrow v \in P$

ii) $vu, u \in P \Longrightarrow v \in P$

iii) $\forall u \in X^* \exists n \in \mathbb{N} : u^n \in P$

One defines the degree of a finite code C to be the common integer n such that $x^n \in C$ for every $x \in X$. The group of the code may then be defined as the group of all permutations γf of $\{0, 1, \ldots, n-1\}$ such that $f \in X^*$ and $i\gamma f = j$ if and only if $x^i f x^{n-j} \in C^*$.

One shows that this definition does not depend on the letter x chosen; the main results on the group $G(C)$ of a code C are as follows:

Theorem (Perrot): $G(C)$ is imprimitive if and only if there exists a code D with $C^* \subset D^* \subset X^*$.

Theorem: $G(C)$ is 2-fold transitive whenever it is primitive and $C \neq X^p$.

One may show by means of examples that $G(C)$ may happen not to be more than 2-transitive and that such highly transitive groups as the Mathieu group M_{11} may be the group of a code.

A. RAE: Elementary abelian p-groups acting on p-soluble groups

This is a generalisation of Hall-Higman's theorem 'B' to non cyclic operator groups.

Let p and q be primes and a be the order of p mod q. The we say that q is bad (with respect to p) if the lowest common multiple of a and p is $q-1$.

Theorem: Let $p = 2$ and $G = O_{p'pp'}$. Let A be abelian of type $p - p$ acting on G and $[A,G] \nleq O_{p'p}$. Suppose that no bad primes divide $|G/O_{p'p}|$ and that G has abelian Sylow p-subgroups. Then, if V is a faithful irreducible $\mathfrak{K}AG$-module where char $\mathfrak{K} = p$, V_A contains a copy of $\mathfrak{K}A$.

An example is given to show that if a bad prime divides $|G/O_{p'p}|$, then (for any prime p) V_A need contain no indecomposable submodule of dimension greater than p.

D. J. S. ROBINSON: Groups with finite automorphism groups

If C is the centre of a group G and $Q := G/C$ there is an exact sequence $\mathrm{Hom}(Q_{ab}, C) \rightarrowtail \mathrm{Aut}\ G \twoheadrightarrow C_{\mathrm{Aut}\ C \times \mathrm{Aut}\ C}(\Delta)$ where Δ is the cohomology class of the central extension $C \rightarrowtail G \twoheadrightarrow Q$. This can be used together with standard theorems in the cohomology of groups to prove some new results about groups whose automorphism groups are finite. For example: necessary and suffidient conditions are found for an abelian group to be isomorphic with the centre of a group whose automorphism group is finite; a group is constructed such that Aut G is finite but Aut C is infinite.

K. W. ROGGENKAMP: Projective geometries for group extensions

Let G be a (finite) group and S a simple $\mathbb{Z}G$-module. Let $\binom{G}{S}^e$ be the isomorphism classes of group extensions $1 \longrightarrow S^n \longrightarrow E \longrightarrow G \longrightarrow 1$, $n \in \mathbb{N}$. which are essential. Then (Gruenberg) there is an order preserving bijection between $\binom{G}{S}^e$ and the projective $\mathrm{End}_{\mathbb{Z}G}(S)$ space of finitely generated $\mathrm{End}_{\mathbb{Z}G}(S)$-submodules of $H^2(G,S)$.

G. ROSENBERGER: Anwendungen der Nielsenschen Kürzungsmethode bei Gruppen mit einer definierenden Relation

Satz 1: Sei $G = \langle a_1, b_1, \ldots, a_q, b_q \mid (W_1(a_1, b_1) \ldots (W_q(a_q, b_q))^{\gamma} = 1 \rangle$ $q \geq 1$, $\gamma \geq 2$, $W_j(a_j, b_j) \neq 1$ für $j = 1, \ldots, q$. Dann können wir in endlich vielen Schritten entscheiden, ob eine beliebige Gruppe mit einer definierenden Relation zu G isomorph ist oder nicht. Weiterhin gilt:

a) G ist Hopfsche Gruppe.

b) Die Automorphismengruppe von G ist endlich erzeugt.

Für $q = 1$ wurde dieser Satz von S. J. Pride bewiesen. Für $q \geq 2$ beruht der Beweis auf der Einführung eines geeigneten Längenbegriffs und der Anwendung der Nielsenschen Kürzungsmethode. Mit Hilfe dieser Methode wird gezeigt, daß G genau ein T-System besitzt. Die Nielsensche Kürzungsmethode erlaubt bei Gruppen mit einer definierenden Relation auch Aussagen zu Untergruppenproblemen und zum Problem der Zerlegung in ein freies Produkt mit Amalgam; z.B. gilt

Satz 2: Sei $G = \langle a, b \mid (R(a,b))^{\gamma} = 1 \rangle$, $\gamma \geq 2$, $R(a,b) \neq 1$. Genau dann ist $G = H_1 \underset{A}{*} H_2$ mit $H_1 \neq A \neq H_2$, $A \neq 1$ und A zyklisch, wenn $G = \langle x, y \mid ((x^m)^{\alpha_1} y^{\beta_1} \ldots (x^m)^{\alpha_r} y^{\beta_r})^{\gamma} = 1 \rangle$, $r \geq 1$, $m \geq 2$, $\alpha_i \neq 0$, $\beta_i \neq 0$ für $i = 1, \ldots, r$.

L. L. SCOTT: Matrices and cohomology

A theorem of Ree-Riemann-Hurwitz on permutations is generalized to matrices, and the main ingredient of the proof has several consequences of a homological nature. Specifically:

Theorem: Let G be a group acting linearly on a finite dimensional vector space V. For x an element or subgroup of G set $v(x) = \dim(V/C_V(x))$. Then if $G = \langle x_1, \dots, x_m \rangle$ with $x_1 \dots x_m = 1$, we have

$$\sum_{i=1}^{m} v(x_i) \geq v(G) + v(G^*) \qquad (G^* = G \text{ acting on } V^* \text{(dual)})$$

The proof has many ramifications for the theory of cohomology and relation modules. For example, the cohomology of polyhedral groups with arbitrary coefficients (trivial or nontrivial action) is completely determined in all dimensions, and a theory of polyhedral relation modules is developed.

D. SEGAL: The residual simplicity of certain modules

We use Krull dimension techniques to prove the

Theorem: If G is a finitely generated nilpotent group and M is a finitely generated $\mathbb{Z}G$-module, then N is poly-(residually simple) as a $\mathbb{Z}G$-module.

The motivation is that one can deduce a number of group-theoretic results about f. g. $\mathfrak{A}\mathfrak{N}$ groups, for example that if H is any subgroup of such a group, then the Frattini subgroup of H is nilpotent.

D. SEGAL: Irreducible representations of finitely generated nilpotent groups

Let G be a f. g. nilpotent group and k a field.

1) Then every primitive ideal of kG is the kernel of an irreducible representation of G which is induced from an irreducible representation of an abelian-by-finite section of G. So if k is algebraically closed, then every primitive ideal of kG is the kernel of a monomial irreducible representation.

2) We give an example of a primitive irreducible representation of G of infinite dimension (counter-example to a suggestion of Zalesski).

J. TAPPE: Isoclinic groups and projective representations

The following theorem is proved:

Two finite groups with isomorphic central factor groups are isoclinic in the sense of P. Hall, if and only if the irreducible complex representations of both groups induce the same irreducibleprojective representations on the central factor groups

This yields a unified proof of results of P. Hall, J. C. Bioch, and M. R. Jones and J. Wiegold.

M. TYRER JONES: Direct decomposition of certain non-Hopfian groups

I am constructing a f. g. group G such that $G \cong G \times G \times G$, but $G \not\cong G \times G$. The only difficulty is the non-isomorphism, and I have done enough of the proof to be confident that I can finish it. I am also confident that the construction easily generalizes to construct a group such that, given r, $G^m \cong G^n$ if and only if $r \mid (m-n)$.

B. WEHRFRITZ: Nilpotence in groups of semilinear maps

We discuss nilpotency and centrality in groups of semilinear maps of finitely generated modules over finitely generated commutative rings and perhaps explain the connection with groups of automorphisms of certain metabelian groups.

H. WIELANDT: Frame's Quotient

1941 hat Frame gezeigt, daß für eine Permutationsgruppe G des Rangs r vom Grad n mit den Untergraden $n_1, \dots n_r$ der Quotient $q = n^{r-2} \prod n_\varsigma / \prod f_\lambda^{e_\lambda^2}$ stets eine ganze Zahl ist; dabei bezeichnet e_λ die Vielfachheit, f_λ den Grad der im Permutationscharakter von G auftretenden irreduziblen Bestandteile χ_λ. Frame hat vermutet, daß q ein Quadrat ist, falls die Werte aller dieser χ_λ rationale Zahlen sind. Im Vortrag wird bewiesen:
Genau dann ist q ein Quadrat, wenn für jede zur Gruppenordnung teilerfremde Zahl a der Form $4x+1$ die Abbildung $\alpha: G \to G$, $g \mapsto g^a$ eine gerade Permutation derjenigen Charaktere χ_λ induziert, für welche e_λ ungerade ist. In ähnlicher Weise werden diejenigen Primteiler von $|G|$ gekennzeichnet, welche q in gerader Potenz teilen.

J. S. WILSON: Groups with many serial subgroups

A group G is called an $\widetilde{N}$-group if and only if each of its subgroups is a serial subgroup (equivalently, if and only if $L \lhd M$ whenever $L < M \leq G$ and L is a maximal subgroup of M). The construction of a periodic $\widetilde{N}$-group which is not locally nilpotent is described; the construction uses the Golod-Šafarevič argument.

On the other hand, locally finite or locally soluble groups with 'enough' serial subgroups are relatively well behaved. Groups which are locally finite or locally soluble and satisfy the minimal condition for non-serial non-locally-nilpotent subgroups can be classified: they either are Černikov groups (and so satisfy the minimal condition on all subgroups) or are locally nilpotent by finite cyclic and in fact have no non-serial non-locally-nilpotent subgroups. Some corollaries of this latter theorem (the result of joint work with R. E. Phillips) are the Šunkov-Kegel-Wehrfritz theorem and some results of Černikov concerning groups with the minimal condition on non-normal subgroups.

Th. Schmid-Leißler
Tübingen