MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 24 /1976

Differentialgeometrie im Großen

6.6 bis 12.6.1976

Die Tagung wurde von S.S. Chern (Berkeley) und W. Klingenberg (Bonn) geleitet. Die Anzahl der Teilnehmer war dieses Mal so hoch, daß einige von ihnen im Dorf übernachten mußten.

Hauptthemen dieser Tagung waren der Laplace Operator und harmonische Abbildungen. Neben den angekündigten Vorträgen nutzten viele die Möglichkeit, im kleineren Kreis über neuere Ergebnisse zu berichten.

Die traditionelle Mittwochswanderung fand dieses Mal bei strahlender Sonne am Donnerstagnachmittag statt und führte zu einem nahegelenen Waldcafé.

Teilnehmer

W. Ballmann, Bonn
P. Bérard, Paris
L. Bérard-Bergery, Paris
J.P. Bourguignon, Palaiseau
J. Brüning, Marburg
E. Calabi, Philadelphia
J. Cheeger, Stony Brook
S.S. Chern, Berkeley
Y. Colin de Verdière, Paris
J. Dodziuk, Philadelphia
P. Dombrowski, Köln
J.J. Duistermaat, Utrecht
P. Eberlein, Chapel Hill

J. Eells, Coventry

P. Ehrlich, Bonn

J.-H. Eschenburg, Bonn

D. Ferus, Berlin

S. Gallot, Paris

P. Gauduchon, Paris

P.B. Gilkey, Princeton

A. Gray, College Park

K. Grove, Kopenhagen

E. Heintze, Bonn

W. Henke, Köln

H. Karcher, Bonn

W. Katz, Köln

H. Kaul, Bonn

J. Kern, Bonn

P. Klein, Bonn

W. Klingenberg, Bonn

J.A.C. Kolk, Utrecht

N.H. Kuiper, Bures-Sur-Yvette

J. Lelong-Ferrand, Paris

L. Lemaire, Coventry

P. Marry, Bonn

K. Maurin, Warschau

Min-Oo, Bonn

J.S. Mitteau, Toulouse

J.D. Moore, Bonn

J.J. O'Sullivan, Bonn

W.A. Poor, Bonn

H. Reckziegel, Köln

E. Ruh, Bonn

E. Seidel, Graz

U. Simon, Berlin

B. Smyth, Bonn

M. Takeuchi, Bonn

S. Tanno, Sendai, z.Zt. Berlin

G. Thorbergsson, Bonn

K.K. Uhlenbeck, Evanston

W. Ziller, Bonn

Vortragsauszüge

E. Calabi: Contributions to the Existence Problem for Complete Affine Hyperspheres

A negative valued, strongly convex function $u : D \to \mathbb{R}$ of class C^3, where D is a bounded, convex, open domain in $\mathbb{R}^n$, determines two embeddings of D into $\mathbb{R}^{n+1}$, resp. $(\mathbb{R}^{n+1})^*$, that are mutually dual, defined by

$$X(x) = \frac{-1}{u(x)}(x_1,\dots,x_n,1)$$

$$X^*(x) = -\left(\frac{\partial u}{\partial x_1},\dots,\frac{\partial u}{\partial x_n},u(x) - \Sigma x_i\frac{\partial u}{\partial x_i}\right)$$

The hypersurfaces $X(D)$ and $X^*(D)$ are called affine hyperspheres, mutually dual, if $u(x)$ satisfies the differential equation

$$(*)\quad (-u(x))^{n+2}\det\left(\partial^2 u(x)/\partial x_i \partial x_j\right) = \text{const}>0$$

Question. Given any bounded, convex $D\subset\mathbb{R}^n$, does there exist a complete affine hypersphere?

Cheng and Yau recently claimed to have a positive answer to this question, using former results of Calabi-Pogorelov-Yau and Calabi-Nirenberg.

J. Cheeger: The Ray-Singer Conjecture on Reidemeister Torsion

Let M be a smooth riemannian manifold. Then a certain invariant $\tau_{riem}(M,\varepsilon)$ can be defined for coefficients in a flat orthogonal bundle ε using a natural volume element in the space of harmonic forms. Ray and Singer wrote down a formula for an analytic invariant $T(M)$ in terms of the spectrum of the Laplacian of M acting on i-forms. They conjectured that $\tau_{riem} = T$. A proof of this conjecture was dicussed.

S.S. Chern: Abel's Theorem and Web Geometry

Theorem. Given a d-web of hypersurfaces in R^n, $n\geq 3$, $d\geq 2n+1$, of rank $\pi(n,d)$ (in the sense of Blaschke), where

$$\pi(n,d) = \frac{1}{2(n-1)}\{(d-1)(d-n) + s(n-1-s)\}$$

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$s \equiv -d+1, \bmod n-1$
$0 \le s \le n-2$.
There exists a coordinate system in which all the hypersurfaces are hyperplanes.

The Theorem was proved by G. Bol for $n = 3$ and by P. Griffiths and Chern for all $n \ge 3$.

Remark. According to Castelnuovo, $\pi(n,d)$ is the max genus of a non-degenerate curve of degree d in $\mathbb{P}^n$.

Y. Colin de Verdière: The spectrum of the Laplacian on a compact Riemannian manifold with integrable geodesic flo[w]

If a d-dimensional compact Riemannian manifold has an integrable geodesic flow (for example a surface of revolution or an ellipsoid), we can construct on it by using a Fourier Integral Operator some "quasi-mode", which give asymptotic estimates for a great part of the eigenvalues of the Laplacian. More precisely, let K be the homogeneous function on the open cone C of $\mathbb{R}^d \backslash 0$, which is the pull-back of the Riemannian norm on $T^*M \backslash 0$ by an "action-angles" mapping $\chi : \mathbb{R}^d/\mathbb{Z}^d \times C \to V \subset T^*M \backslash 0$. There exists a μ_0 in $\mathbb{Z}^d$ such that we have $\lambda_{n_\nu} = 4\pi^2 K^2(\frac{1}{4}\mu_0 + \nu) + O(1)$, where $\nu \to n_\nu$ is an injective map of $C \cap \mathbb{Z}^d$ into $\mathbb{N}$ and $(\lambda_n)_{n \in \mathbb{N}}$ is the spectrum of the Laplacian. As a corollary, we obtain that in the non-degenerate case (i.e. $K^n(\xi)$ has maximal rank almost everywhere, then for each $\varepsilon > 0$, we can split the spectrum of the Laplacion into 2 parts A_ε and B_ε, such that

$$\mathrm{Card}\{\lambda_n \le \lambda \mid \lambda_n \in A_\varepsilon\} = C_d(\mathrm{vol}(M) \backslash \varepsilon)\lambda^{d/2} + O(\lambda^{d/2-1+1/d+1})$$

$$\mathrm{Card}\{\lambda_n \le \lambda \mid \lambda_n \in B_\varepsilon\} \sim C_d . \varepsilon \lambda^{d/2}.$$

J. Dodziuk: L^2-cohomology of infinite coverings and harmonic forms

Let $\tilde{M}$ be an oriented Riemannian manifold acted on freely by a discrete group of orientation preserving isometries, so that the quotient $M = \tilde{M}/\Gamma$ is compact. The group Γ is represented on the Hilbert space $\mathcal{H}^p(\tilde{M})$ of L^2 harmonic forms.
Theorem. The representation of Γ on $\mathcal{H}^p(\tilde{M})$ is a homotopy invariant of the pair (M,Γ).
The theorem is proved by comparing $\mathcal{H}^*(M)$ with simplicially

defined L^2-cohomology.

Corollary. The numbers $\beta^p_\Gamma(M) = \dim_\Gamma \mathcal{H}^p(\tilde{M})$ are homotopy invariants.

Open Problem. To give an example of a pair $(\tilde{M},\Gamma)$ so that one of the numbers $\beta^p_\Gamma(M)$ is irrational.

P. Dombrowski: Jacobi fields and totally geodesic foliations

Let L be a k-dim totally geodesic foliation of an m-dim. riem. manifold M, J an open interval, $0\in J$. If $c : J \longrightarrow M$ is a normal geodesic into a leaf of L, then the Jacobi fields along c parallel to L satisfy a certain first order linear differential equation. Therefore $Y(0) \neq 0$ implies $Y(t) \neq 0$ for all $t\in J$. With these Jacobi fields as a tool, one gets:

(A) Completeness results for totally geodesic nullity distributions of certain tensorfields on complete riemannian manifolds.

(B) Restrictions on the dimensions of totally geodesic folitations with at least one complete leaf in a positively curved manifold.

As an application one gets e.g. the following

Prop. If M is a complete surface in E^3, $K\geq 0$ and $K\not\equiv 0$, which has one constant principal curvature, then M is a sphere.

S. Gallot: Obata's Theorem and Generalizations

Let (M,g) be a connected n-dim. riem. manifold.

$$\delta(M) = \sup_{p\in\mathbb{R}} \left[\sup\{r\in\mathbb{R}\,|\,\exp_p \text{ is defined on } B(0_p,r)\subset T_pM\}\right]$$

Proposition 1. If $\delta(M)>\frac{\pi}{2}$, $\pi_1(M) = 0$ and there exists a non-trivial solution of (E_p), then $(M,g) = (S^n,\text{can})$. Here (E_p) denotes a system of differential equations, which is on (S^n,can) characteristic for the eigenfunctions corresponding to the p-th eigenvalue λ of Δ. For $p=1$ this is Obata's theorem and for $p=2$ a conjecture of Obata.

Now let $\lambda_1(\Lambda^qM)$ be the first eigenvalue of Δ on closed q-forms.

Proposition 2. If M is compact, $\pi_1(M) = 0$ and if the curvature operator ρ is such that $\rho\geq g$ (where g is the metric on 2-forms) and there exists one q such that $\lambda_1(\Lambda^qM) = \lambda_1(\Lambda^qS^n)$.

Then (1) If n even or q odd: $(M,g) = (S^n,\text{can})$.

(2) If n odd and q even and if multiplicity of $\lambda_1(\Lambda^qM)\geq 2$,

then $(M,g) = (S^n, can)$.

Remark. A counter-example of (2) with mult. = 1 is given by Berger's spheres in $\mathbb{P}^n\mathbb{C}$.

P. Gilkey: The Lefschetz fixed point formula and the heat equation

Let M be a compact Riemannian manifold and $T : M \to M$ be a smooth map. We assume the fixed point set consists of the disjoint union of smooth submanifolds N_i. dT induces a map on the normal bundle over the fixed point set - we assume $\det(I-dT_v) \neq 0$. Let $L(T)$ denote the Lefschetz number of T, then $L(T) = \sum_i \text{sign}(I-dT_v)\chi(N_i)$ where the sum ranges over the components of the fixed point set and $\chi(N_i)$ denotes the Euler-Poincaré characteristic.
By using heat equation methods one obtains a local formula $L(T) = \sum_i \int_{N_i} P_i(X,G,T)$ where P_i is a functorial expression in the derivatives of the metric and of T. By using techniques of invariance theory one shows $P_i = \text{sign}(I-dT_v)E_i$, where E_i is the integrand of the Chern-Gauss-Bonnet theorem. This gives an analytic proof of the Lefschetz theorem; there are similar applications to the G-signature theorem and to the Dolbeault complex for a holomorphic isometry and Kaehler metric. These results have also been derived by Donnely for the Dolbeault complex and Kawasaki for the signature complex.

A. Gray: Compact Kähler manifolds with nonnegative sectional curvature

Theorem A. Let M be a compact Kähler manifold with nonnegative sectional curvature and constant scalar curvature. Then M is locally symmetric.
The same method yields a well known theorem of Berger:
Theorem B. Let M be a compact Einstein Kähler manifold with positive sectional curvature. Then M is isometric to a complex projective space.
Moreover, theorem B can be generalized to noncompact manifolds:
Theorem C. Let M be an Einstein Kähler manifold with positive sectional curvature. Then the holomorphic sectional curvature has no local maximum or minimum.

K. Grove: A Pinching theorem for the diameter.

The following generalization of the classical sphere-theorem by Klingenberg is proved:
Theorem. (Grove-Shiohama). Let M be a complete riemannian manifold with sec. curv. $k \geq \delta > 0$ and diameter $d(M) > \pi/2\sqrt{\delta}$. Then M is a topological sphere. (There are several manifolds M with $k \geq \delta > 0$ and $d(M) = \pi/2\sqrt{\delta}$, in particular the projective spaces $P^n(\mathbb{R})$, $P^n(\mathbb{C})$, $P^n(\mathbb{Q})$ and P^n (Cayley) with their standard metrics).
The idea of the proof is to exhibit M as the union of two embedded discs (centered at two points $p, \bar{p}$ at maximal distance) and one cylinder ($M \setminus$ the discs) joined along their common boundaries.

E. Heintze: Horospheres in manifolds of negative curvature and some applications

The geometry of horospheres is used essentially in the proofs of the following theorems:
Theorem 1. $\exists$ a lower bound for the volume of complete n-manifolds with curvature between $-b^2$ and $-a^2$.
Theorem 2. The number of homeomorphism types of complete n-manifolds with curvature between $-b^2$ and $-a^2$ and diameter bounded from above is finite.
Theorem 3. Let M/Γ be a complete manifold of finite volume and curvature between two negative bounds. Then any canonical fundamental domain of Γ has only finitely many cusps and these are fixed points of parabolic isometries of Γ.

H. Karcher: Riemann Center of Mass and Mollifier Smoothing

For a mass distribution $f : A \to B_\rho$ (vol $A = 1$) on a convex Riemannian ball $B_\rho \subset M$ the center of mass C_f can be defined as the minimum point of the function $x \to \frac{1}{2} \int_A d(x,f(a))^2 da =: P_f(x)$. Properties of C_f follow from estimates of grad $P_f(x)$ via $\| D \operatorname{grad} P_f - id \| \leq \mathrm{const} \cdot \rho^2$. A continuous map $F : M^n \to \hat{M}^N$ can be smoothed (at m) by first concentrating the volume with a bump function near $m : \Phi_\rho(m,x) \cdot dx$ and then taking the center of mass of $F : (M, \Phi_\rho(m,x) \cdot dx) \to \hat{M}$ to be $F_\rho(m)$. This smoothing behaves as in the euclidean case: F_ρ approximates F well, Lipschitz estimates are deteriorated at most by $(1 + \mathrm{const} \cdot \rho^2)$ and if F is locally bi-Lipschitz with $L \leq 1 + (\frac{8}{\pi}(n-1))^{-\frac{1}{2}}$, then F_ρ is an immersion for sufficiently small ρ.

H. Kaul: Dirichlet's problem for harmonic mappings of Riemannian manifolds

Let M, N be Riemannian manifolds, M compact, $\partial M \neq \emptyset$, N complete, $\partial N = \emptyset$ and let $f_0 : \partial M \to N$ be of class C^1.

Theorem. (S. Hildebrandt, H. Kaul, K.O. Widman). Let $f_0(\partial M)$ be contained in a closed ball $K_\rho \subset N$ of radius ρ satisfying $K_\rho \cap C(K_\rho) = \emptyset$ and let $\rho < \pi/2\sqrt{\varkappa}$, where $C(K_\rho) = \bigcup_{y \in K_\rho} C(y)$ is the cut locus of K_ρ. and where $\varkappa \geq 0$ is an upper bound for the sectional curvature of N. Then there exists a harmonic mapping $f : M \to N$ such that $f|\partial M = f_0$ and $f(M) \subset K_\rho$.

The proof is based on direct methods in the calculus of variations and depends essentially on the existence of certain convex functions on the target space N.

Conjecture: The result is optimal.

J.A.C. Kolk: A Poisson formula for compact locally symmetric spaces of negative curvature and applications

Using harmonic analysis on semisimple Lie groups, a Poisson formula for compact locally symmetric spaces of negative curvature and of arbitrary rank, is obtained.

This formula contains more refined information than the formula that are obtained using wave operator techniques.

N. Kuiper: Tight C^0-embeddings of $\mathbb{R}P(2)$ substantially into E^5 (Work with William Pohl)

The Veronese surface in E^5 and Banchoff's PL-embedding of the six vertex triangulated projective plane in E^5, have the following properties:

a. The image is not in any hyperplane

b. The image is a C^0-embedded $\mathbb{R}P(2)$

c. Every hyperplane divides it in at most two parts.

Theorem: There are no other examples.

J. Lelong Ferrand: Regularity of Conformal Mappings

We have the two following theorems, which involve the regularity of conformal maps of C^∞ manifolds, even if we take this term in the general meaning of 1-quasi-regular mapping.

Theorem A. For any integer $n \geq 3$, there exists a constant $K_n > 1$ such that any K-quasi-regular mapping $\phi : M^n \longrightarrow \bar{M}^n$, with $K < K_n$, is a local homeomorphism (conformal mappings are therefore local homeomorphisms).

Theorem B. If $M^n, \bar{M}^n$ are C^∞ [resp. C^ω] manifolds, with $n \geq 2$, any 1-conformal homeomorphisms $\phi : M^n \longrightarrow \bar{M}^n$ is C^∞ [resp. C^ω].

L. Lemaire: Harmonic mappings of surfaces

Let M,M' be connected, compact, oriented, 2-dim. C^∞-Riemannian manifolds, $\partial M = \partial M' = \emptyset$, with genus p resp. p'.

Question: Does there exist a harmonic map in a given homotopy class of maps from M to M'. When $p' \neq 0$, the answer is yes. Difficulties arise, when $p' = 0$. The homotopy classes are then parametrized by the degree d of the maps. J. Eells and J. Wood showed, that if $d \geq p$, every harmonic map of degree d is holomorphic with respect to complex structures associated to the metrics. Such maps don't exist, when $d = p$ and M hyperelliptic. When $d > p$, or $d = p$ and M not hyperelliptic, or p even, then a harmonic map of degree d exists. When $0 < d < p$ partial existence results have been obtained. For instance, there always exist metrics on M and M' with respect to which a harmonic map of degree d exists.

J.C. Mitteau: Harmonic mappings - The positive curvature case

Considering the elliptic polynomial differential operator τ associated to the variational problem of energy $E(f) = \frac{1}{2} \int_M |T_x f|^2 d\,\mathrm{vol}(x)$ where $f : M \longrightarrow M'$ is a C^∞-mapping of riemannian manifolds and where $|T_x f|$ is the norm of tangent mapping in vector bundle $T^* M \otimes f^* TM'$ herited from metrics on M and M', we obtain a formula giving τ in the exponential chart for manifold $C^2(M,M')$ (M is compact). Then we can prove majorations and by various methods (geometry and linear operator theory) attain two stability results on isometries among harmonics and a technical lemma on existence and convergence.

for solutions of the parabolic equation $\frac{\partial f}{\partial t} - \tau(f_t) = 0$.

E. Ruh: Compact manifolds with small curvature variation.

Let M be a compact manifold $X_1,\ldots,X_n$ a trivialisation of the tangent bundle TM. Let $c = (c_{ij}^k)$ denote the structure functions of the Liebracket: $[X_i,X_j] = \Sigma_k c_{ij}^k X_k$. Define the riemannian metric on M by the property $\langle X_i,X_j\rangle = \delta_{ij}$, and let $\bar{c}$ denote the average of c over M. We normalize the vector fields such that the first eigenvalue of the Laplace operator takes value one. The following conjecture would give pinching theorems for any symmetric space as model space.

Conjecture. There exists $A_n>0$ such that with the definitions and normalisation above $\|c-\bar{c}\|_{2\mu}<A_n$ implies the existence of vector fields $Z_1,\ldots,Z_n$ with properties

(1) $\|X_i - Z_i\|$ small

(2) $\{Z_i\}$ spann on n-dimensional Lie algebra of vector fields:

Theorem. The conjecture holds in case $|\bar{c}|$ is small.

U. Simon: Certain Differential Equations on Riemannian Manifolds and Applications to Submanifolds of Higher Codimension

Theorem. Let M be a closed, connected, n-dim. $(n\geq 2)$ Einstein manifold with constant scalar curvature and sectional curvature $K\geq\frac{1}{2}c$. M admits a diff. function $f: M\to\mathbb{R}$ with $\Delta f + c^2 nf = 0$ iff there exists an isometric diffeo. $\Phi: M\to S^n(c)$:

Remark. For arbitrary dimension n the lower bound $\frac{1}{2}c$ is the best possible.

Theorem. M as above, but $K\geq\frac{2}{3}$. For $f: M\to\mathbb{R}$ define $A(f) := \text{Hess } f + f\cdot g$, g metrictensor, $nA_{(1)} = \text{trace } A(f)$, $n(n-1)A_{(2)} =$ second elementary symmetric function of the eigenvalues of Hess f. M admits a diff. function $f: M\to\mathbb{R}$ with $A_{(2)}(f)\nabla(A_{(1)}(f))\geq A_{(1)}(f)\nabla(A_{(1)}(f), A_{(2)}(f))$ iff there exists $\Phi: M\to S^n(1)$ isom. diffeo.

Theorem. Let $\bar{x}: M\to S^{n+m}(1)$ be a minimal isometric immersion of M into $S^{n+m}(1)$, M closed, connected, $\dim M = n\geq 2$, $K\geq\frac{1}{2}$. Then $\bar{x}(M)$ is totally geod.

Theorem. $x: M\to E^N$ isom. immersion, M closed, conn., $K\geq 2$. If the mean curvature vector ξ is parallel in the normal bundle and

$K \geq \frac{1}{2}\frac{\langle\xi,\xi\rangle}{n^2}$, then $x(M)$ is a sphere $S^n(R) \subset E^{n+1}$ with radius $R = n\langle\xi,\xi\rangle^{-\frac{1}{2}}$.

B. Smyth: Complex submanifolds of tori

Let M^n be a compact complex manifold of dimension n, admitting a holomorphic immersion in a complex torus. Bochner (1950) showed $(-1)^n\chi(M) \geq 0$ and attempted to describe the structure of M when $\chi(M) = 0$, but without success. This was first done, for $n=2$, by Howard-Matsushima (1974). We prove the result for all n.

Theorem. If M^n is holomorphically immersible in a complex torus, and $\chi(M) = 0$, then $\mathrm{Aut}_o(M)$ is a complex torus acting freely on M. The quotient $M_1 = M/\mathrm{Aut}_o(M)$ is also holomorphically immersible in a complex torus and $\chi(M_1) = 0$.

M. Takeuchi: Γ-foliations and semisimple flat homegenous spaces

Let Γ be a pseudogroup acting on a smooth manifold B of dimension q and F a Γ-foliation of codimension q on a smooth manifold M. Denote by $\mathrm{Pont}^*(\nu(F))$ the subalgebra of $H^*(M.\mathbb{R})$ generated by the real Pontrjagin classes of the normal bundle $\nu(F)$ of F.

Theorem. Let L/L_o be a semisimple flat homogeneous space associated with a semisimple graded Lie algebra $\underline{l} = \underline{g}_{-1} + \underline{g}_o + \underline{g}_1$ and $\underline{k}_o$ a maximal compact subalgebra of $\underline{g}_o$. Let Γ be the pseudogroup of local automorphisms of a second order L_o-structure associated to L/L_o. Then for a Γ-foliation F of codimension q, the strong vanishing theorem:

$$\mathrm{Pont}^k(\nu(F)) = 0 \quad \text{for} \quad k>q$$

holds if

(1) Spencer cohomology $H^{2,1}(\underline{l})$ vanishes and

(2) $\mathrm{Pont}(\underline{k}_o) \subset I_L(\underline{k}_o)$.

These conditions (1),(2) are satisfied for "almost all" semisimple flat homogeneous spaces L/L_o.

S. Tanno: Differential equations of order 3 in Riemannian manifolds

In 1965 Obata announced a theorem: If a complete Riemannian manifold (M,g) admits a non-constant function f satisfying

$\nabla_k \nabla_j \nabla_i f + K(2\nabla_k f g_{ji} + \nabla_j f g_{ik} + \nabla_i f g_{kj}) = 0$, then (M,g) is of constant curvature K. However, the outline of Obata's proof is not complete. In this talk I explain the gaps in his proof and give a complete proof of this theorem by keeping some parts of Obata's ideas. The most essential part of my proof is the construction of the field of frames which is invariant under the 1-parameter group generated by grad f. Applying the K-nullity theory to these we can finish the proof.

K.K. Uhlenbeck: Harmonic maps from Surfaces into Higher Dimensional Riemannian Manifolds

Theorem. Let M be a surface and N a compact Riemannian manifold and assumed the based maps $\Omega(M,N)$ is not a contractible space. Then either there exists a harmonic map $s : M \longrightarrow N$ which is not trivial or there exists a harmonic map $s : S^2 \longrightarrow N$. Harmonic maps $s : S^2 \longrightarrow N$ are branched minimal immersions.

The method of proof for finding critical points of $E(s) = \int_M |ds|^2 d\mu$ is to find polyharmonic maps which are critical maps of $E_\alpha(s) = \int_M (1+|ds|^2)^\alpha d\mu$. Convergence properties of the critical maps of E_α as $\alpha \to 1$ are examined in detail to get the result.

W. Ballmann (Bonn)

G. Thorbergsson (Bonn)