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Reelle algebraische Geometrie

5. – 11. April 1987

Lou van den Dries : P-adic and real subanalytic sets
(Report on joint work with Jan Denef.)

Question (Serre, ± 1980) Let $f \in \mathbb{Z}_p\{X_1, \ldots, X_m\}$ be a power series over $\mathbb{Z}_p$ converging on $\mathbb{Z}_p^m$, let $Z(f) \subset \mathbb{Z}_p^m$ be its zeroset, put $N_k = \# \{ \text{image of } Z(f) \text{ under } \mathbb{Z}_p^m \to (\mathbb{Z}/(p^k))^m \}$, $k = 0, 1, 2, \ldots$.
Is then the Poincaré series $\sum_{k=0}^{\infty} N_k t^k$ a rational function in t?

Denef (Invent. 1984) : Yes, if f is a polynomial.
Denef & vdD (1986, to appear in Annals of Math.) : Yes.
For polynomials Denef used Macintyre's p-adic analogue of Tarski-Seidenberg. The general problem was solved by constructing a p-adic analogue of the theory of subanalytic sets. Since the proof of the real "fiber cutting lemma" has no p-adic analogue we invented a new technique that bypasses fibercutting completely, and that also works in the classical real case. We prove our central result (both in the p-adic and in the real case) by an elimination of quantifiers, which adds to the flexibility in working with subanalytic sets. In the real case our theorem can be stated as follows. Equip $I = [-1, 1]$ with operations $f : I^m \to I$, one for each power series $f \in \mathbb{R}[[X_1, \ldots, X_m]]$ that converges on a neighborhood of I^m and maps I^m into I, and also with the operation $D : I^2 \to I$ given by $D(x,y) = \begin{cases} x/y & \text{if } |x| \le |y|, \ y \neq 0 \\ 0 & \text{otherwise}, \end{cases}$

and finally with the binary relation $<$.
Formulas in the language with these basic operations and relation are called D-analytic formulas.
We now have :

Theorem. Each D-analytic formula is equivalent to one without quantifiers.

As immediate consequences one obtains that the definable subsets of I^m are exactly the subsets of I^m that are subanalytic in $\mathbb{R}^m$, hence that the complement and the interior of a subanalytic set (in any manifold) are subanalytic, etc.

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Jacek Bochnak (Amsterdam)

Working jointly with W. Kucharz (Albuquerque, U.S.A) we got several results concerning the structure of real algebraic (i.e. regular) mappings between real algebraic sets. Here is a sample of results (all spaces are suppose to be compact and connected). Let $S^n = \{x \in \mathbb{R}^{n+1} \mid \sum_1^{n+1} x_i^2 = 1\}$ be the unit sphere.

Th.1. Let M be a C^∞ surface. Then the following conditions are equivalent:

(i) M is homeomorphic to S^1, $\mathbb{P}^2(\mathbb{R})$ or the Klein bottle.

(ii) For each nonsingular real algebraic set X diffeomorphic to M, the set $\mathcal{R}(X, S^1)$ is dense in $C^\infty(X, S^1)$ (in C^∞ topology). ∎

Notation. Given a nonsingular real algebraic set, $\mathcal{R}(X, S^n)$ (resp. $C^\infty(X, S^n)$) denotes the set of regular (resp. C^∞) mappings from X into S^n. ∎

Th.2. Let M be a C^∞ surface. Then the following conditions are equivalent:

(i) M is nonorientable of odd genus.

(ii) For each nonsingular real algebraic set X diffeomorphic to M, the set $\mathcal{R}(X, S^2)$ is dense in $C^\infty(X, S^2)$. ∎

Th.3. Let M be a C^∞ surface. Assume that M is either orientable, or nonorientable of even genus. Then there is a nonsingular real alg. set X diffeomorphic to M, such that each $f \in \mathcal{R}(X, S^2)$ is homotopic to a constant. ∎

Th.4. Let M be an orientable 4-dimensional C^∞ manifold. Then the following conditions are equivalent:

(i) The signature $\sigma(M)$ of M is 0.

(ii) There is a nonsingular real algebraic set X diffeomorphic to M, such that each $f \in \mathcal{R}(X, S^4)$ is homotopic to a constant. ∎

Th5. Let M be an orientable 4-dimensional C^∞ manifold. Then for each nonsingular real alg. set X diffeomorphic to M, each $f \in C^\infty(X, S^4)$ of topological degree divisible by 6×(signature of M) can be approximated by regular mappings. ∎

Th.6. Let M be a C^∞ hypersurface in $\mathbb{R}^{2m+1}$. Then there is a nonsingular real algebraic set X diffeomorphic to M, such that each $f \in \mathcal{R}(X, S^{2m})$ is homotopic to a constant. ∎

Th.7. Let $\Sigma_n = \{(x,y,z) \in \mathbb{R}^3 \mid x^{2n} + y^{2n} + z^{2n} = 1\}$. Then $\mathcal{R}(\Sigma_n, S^2)$ is dense in $C^\infty(\Sigma_n, S^2)$. (Remark: of course Σ_n is diffeomorphic to S^2).

Th.8, 9, 10.... etc...

Foundations of Analysis over Surcomplex Number Fields

Norman L. Alling (Rochester, NY)

Over the field of complex numbers $\mathbb{C}$ the following hold.

(A) Locally, the simple roots of $\varphi^{\wedge}(X) \in \mathbb{C}[X]$ are analytic functions of its coefficients.

(B) Let $\varphi^{\wedge}(X,Y) \in \mathbb{C}[X,Y]$ and let $(x_0, y_0) \in \mathbb{C}^2$ with $(\partial\varphi^{\wedge}/\partial Y)(x_0, y_0) \neq 0$. Let $\varphi^{\wedge}(x,y) = 0$. Locally about (x_0, y_0), y is an analytic function of x.

(C) Let $\varphi_1^{\wedge}, \dots, \varphi_n^{\wedge} \in \mathbb{C}[X_1, \dots, X_n]$ define a map from $\mathbb{C}^n$ to $\mathbb{C}^n$, taking $\vec{0}$ to $\vec{0}$, that is non-singular at $\vec{0} \equiv (0, \dots, 0) \in \mathbb{C}^n$. Then it has an analytic inverse, defined in some neighborhood of $\vec{0}$, in the range space.

All of these classical theorems admit generalizations over the surcomplex numbers fields, and thus – a bit further restricted – over the surreal number fields.

Zofia Denkowska Kraków

As one can get some nice results applying subanalytic thms to other fields, I feel a search for explicit bounds for subanalytic sets should be made.
For instance, the following (already classical, with the proofs of Gabrielov, Hironaka, Hardt, Teissier and Denkowska / Łojasiewicz / Stasica) subanalytic thm. 1

Thm. 1.
Let E be a subanalytic (semi-an) rel. compact subset of $M \times N$, M, N real analytic manifolds, $\pi : M \times N \to N$ the natural projection.
Then there is a uniform bound C for the # of connected components of E_y, $y \in N$.

permits to obtain the following result ~~related~~ to Hilbert's XVI problem:

Thm (Françoise and Pugh, "Keeping track of limit cycles" 1986 J. of Diff. Equ)

Fix $d \in \mathbb{N}$, $T > 0$. Then the # of limit cycles of the dynamical system $\dot{x} = f(x, y)$ $\dot{y} = g(x, y)$ with ~~deg~~ f, g polynomials of deg $\leq d$, having the period $\leq T$, is uniformely bounded by a constant $B(d, T)$.

We obtain this result by applying thm 1 to the set

$$A = \{(t, \xi, v) \in [0,T] \times \mathbb{D} \times \mathcal{S} \; ; \; \varphi_v(t, \xi) = \xi\},$$

where $\mathbb{D}$ is ~~the~~ Poincaré compactification of $\mathbb{R}^2$,
$\mathcal{S}$ is the unit sphere in the space of polynomial vector fields of deg $\leq d$
φ_v is the flow of v.

The set A is semi-analytic $\overbrace{}^{\text{rel. compact}}$ (φ_v globally analytic!) and we take the projection on $\mathbb{R} \times \mathcal{S}$.
Then by thm. 1.

$$\#\{\text{connected components of } A_{(t,v)}\} \leq C \quad \text{for all } t, v$$
$$\rotatebox{90}{$\leq$}$$
$$\#\{\text{limit cycles of } v \text{ with the period } t\}$$

But the ~~bound~~ estimate here isn't explicit.

Another subanalytic thm. that is likely to apply (length of orbits?) is the following

Thm. 2 Let E, M, N be as in thm 1.
Then there exists a uniform bound C for the length of arcs in the ~~ef~~ fibers E_y.

Gilbert Stengle   Lehigh University.

A measure of complexity for complex polynomials with applications to the angular distribution of zeros.

Let $P(z)$ be a complex polynomial of degree n. Let $M(P) = \{m_1, \dots m_k\}$ be the set of nonzero exponents actually appearing in P. The following gives an internal measure of the additive complexity of the set $M(P)$.

Definition. Let M, G be subsets of a commutative semigroup $\mathfrak{M}$. Let the diameter of M with respect to G, $d(M,G)$ be ∞ if for no k is $M \subset G \cup G+G \dots \cup G+\dots+G$ (k summands) and otherwise the minimum such k. Let $\gamma_M(s) = \min\{d(M,G) \mid |G| = s\}$.

The function γ_M measures the disposition of M to be generated additively by sets of smaller cardinality. In terms of the function $\gamma_{M(P)}$ we can formulate estimates for the angular distribution of zeros of P.

Theorem. There exists a constant C such that if

$$k(P) = \min_{j>0} \left[(j+1)\log_2 \gamma_{M(P)}(j) + j^2\right]$$

then any open sector of aperture $\pi/\deg P$ contains no more that $C^{k(P)}$ zeros of P.

The point of this estimate is its independence of the coefficients and degree of P. The proof is a simple application of Khovansky estimates. As corollaries

Cor 1. The number of zeros of P in any sector of aperture π/n is no more than C^{k^2}.

Cor 2. If the set $M(P)$ is an arithmetic progression then the bound of the previous Corollary is polynomial in k.

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COMPLEX CURVES WITH REAL MODULI

The moduli space M^g of smooth complex algebraic curves of genus g, $g \geq 2$, is a quasiprojective variety defined over $\mathbb{Q}$. M^g can be embedded in a projective space $\mathbb{P}^N(\mathbb{C})$ in such a way that the complex conjugation in $\mathbb{P}^N(\mathbb{C})$, restricted to M^g, is the mapping $M^g \to M^g$, $[C] \mapsto [\bar{C}]$. Here $\bar{C}$ is the complex conjugate of the curve C.

We show, applying the above observation, that the real part of M^g, $M^g(\mathbb{R})$ is <u>the moduli space of genus g coverings of real algebraic curves</u>.

Using Teichmüller theory we can analyse the topological, analytic and algebraic structures of $M^g(\mathbb{R})$ and its various parts.

Use the notation

$$M_{\mathbb{R}}^{p,n} = \left\{ \begin{array}{l} \text{complex isomorphism classes of real algebraic} \\ \text{curves of genus } g \text{ with } n \text{ distinguished points} \end{array} \right\}.$$

<u>Thm.</u> $M_{\mathbb{R}}^{g,0}$ is the closure of the regular part of $M^g(\mathbb{R})$ provided that $g \geq 4$.

<u>Cor.</u> $M_{\mathbb{R}}^{g,0}$ is semialgebraic, $g \geq 4$.

These results hold also for the moduli spaces

$M_{\mathbb{R}}^{p,m}$ provided that $n \geq 4$ or $n = 3$ and $m > 9$ or $n = 2$ and $m > 6$.

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ON THE STABILITY INDEX OF EXCELLENT RINGS.

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We develope a general theory of stability index for excellent rings. Let A be an excellent ring, X_A its real spectrum and $U \subset X_A$ a basic open constructible subset (see [Bochnak, Coste & Roy, Ergeb. Math. vol. 12, 1987, Springer-Verlag for the definitions). We define

$$st\,U = \min\{k \in \mathbb{N} \mid U = \{x \in X_A : f_1(x)>0, \dots, f_k(x)>0\}\} \text{ and}$$
$$st\,A = \sup\{st\,U\}.$$

Our approach follows closely [Bröcker, Geometriae Dedicata 16 (1984)] and [Mahé, Inv. Math. 1986].

(A) We first show: (i) Łojasiewicz's inequality holds in X_A (ii) closures of constructible subsets of X_A are constructibles, (iii) the dimension of a constructible subset equals the dimension of its closure. Using these devices we obtain some pasting lemmas as in [Bröcker & Mahé, op. cit.]. Then we show

Theorem 1. The following are equivalent: (1) $st\,A < +\infty$ (2) $st\,k(\mathfrak{p}) < +\infty$ for all $\mathfrak{p} \in \operatorname{Spec} A$.

In fact only a finite number of $\mathfrak{p}$'s are required in (2): the zero-divisors of A, then the zero-divisors of the singular locus of A, and so forth. Then we get:

Corollary 2. (a) Let B a finitely generated A-algebra. If $st\,A < +\infty$ then $st\,B < +\infty$.
(b) Let B an excellent algebraic extension of A. If $st\,A < +\infty$ then $st\,B < +\infty$.

(B) Next assume that A is henselian, let k be its residue field and K its total ring of fractions. Then:

Theorem 3. $\dim A \leq st\,K \leq \dim A + st\,k + 1$.

Therefore from theorem 1 we get $st\,A < +\infty$ iff $st\,k < +\infty$. Theorem 3 is reduced, by means of Artin's ~~lang theorem~~ approximation property [Rotthaus, Inv. Math. 88 pp. 39-63] to the case of A be complete, and then proved by induction on $\dim A$ and some technical devices on formal power series.

(C) Finally we apply the above results to global semianalytic subsets of a compact analytic real manifold M of dimension d.

Theorem 4. Any basic open global semianalytic subset of M can be written as:
$S = \{x \in M : f_1(x) > 0, \dots f_s(x) > 0\}$ where $f_1 \dots f_s$ are analytic on M and
$$s \leq \begin{cases} d(d+2)/4 & \text{if } d \text{ is even} \\ (d+1)^2/4 & \text{if } d \text{ is odd.} \end{cases}$$

The proof uses the results above to show the property for basic open constructible subsets of the ring of analytic functions on M. Then we use Artin-Lang theorem [Ruiz, Mathematische Z. 190] to rephrase geometrically this information

N-GONAL CYCLIC REAL ALGEBRAIC CURVES

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Let C be a projective irreducible non-singular algebraic curve defined over $\mathbb{R}$, whose real part $C(\mathbb{R})$ is Zariski-dense in C. We denote by p the genus of C and by k the number of connected components of $C(\mathbb{R})$.

C is called N-gonal cyclic if there exists a birational isomorphism φ of C, of order N, such that C/φ is rational, and $\varphi(C(\mathbb{R})) \subset C(\mathbb{R})$ and $(C/\varphi)(\mathbb{R}) = C(\mathbb{R}) / \varphi|_{C(\mathbb{R})}$.

We study here the existence of such a curve according to the values of the couple (p,k) distinguishing whether $C \setminus C(\mathbb{R})$ is connected or not. Moreover, the characterization we obtain allows us to decide in an effective way the branching orders of the projection $C(\mathbb{R}) \to C(\mathbb{R})/\varphi$.

In a second step we calculate, given the number N, the minimum genus of an N-gonal cyclic curve; and determine the topological types of the curves achieving this bound.

The technique we use involves Klein surfaces and NEC groups.

Claus Scheiderer: QUOTIENTS OF SEMIALGEBRAIC SPACES

We consider affine semialgebraic spaces over a real closed base field R and semialgebraic maps between them. For M such a space and $E \subseteq M \times M$ a closed semialgebraic equivalence relation on it we say that the (geometrical) quotient M/E exists if there is an identifying map $f: M \to N$ to some space N such that $E = M \times_N M$ (that is, the set of equivalence classes, equipped with the quotient

topology, carries a (unique) semialgebraic structure.) This notion coincides with that of effective epimorphisms (in the affine category) provided that M is locally complete.
G. W. Brumfiel has shown M/E to exist if $p_1, p_2 : E \rightrightarrows M$ are proper. Assuming M to be locally complete we show that M/E exists if and only if there is some subspace K of M such that $p_1 | p_2^{-1}(K) : p_2^{-1}(K) \to M$ is proper and onto. (In fact, $E_K := E \cap (K \times K)$ is proper over K in this case, and M/E just "is" K/E_K.) On the other hand, M/E does always exist if $E \rightrightarrows M$ are open (again M locally complete). We present examples showing that these theorems do not hold without the local completeness hypotheses.
The proof of these theorems are given within the set-up of real spectrum. They make essential use of the fact that the theory of real closed fields with compatible non-trivial valuation admits elimination of quantifiers.

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KLAUS REICHARD: Some Remarks on Real Polynomial Mappings with Constant Determinant.

A famous conjecture sais: If $F: \mathbb{R}^n \to \mathbb{R}^n$ is a polynomial mapping with $\det\left(\frac{\partial F}{\partial x}\right) \equiv 1$, then there exists a polynomial invers mapping. This conjecture is not yet proven.
But the following, weaker special case is true:
If $n = 2$, $F: \mathbb{R}^2 \to \mathbb{R}^2$ polynomial with $\det \frac{\partial F}{\partial x} \equiv 1$, then F ist injectiv. With a theorem from Borel ~~...~~ it follows, that F ist bijektiv and so has a real-analytic invers mapping.

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TOMAS RECIO : The width of a semialgebraic set and the cost of an algebraic decision tree.

(joint work with L. M. PARDO).

Lower bounds for the complexity of the membership problem (to a semialgebraic set $S \subseteq \mathbb{R}^n$) in the model of computation of algebraic decision trees have been sistematically obtained either by considering the number of connected components of S or throught the notion of "width of a complete proof" as introduced for semilinear sets in Rabin (J. Computer Syst. Sc. 6 - 1972). Several attempts to extend this notion for the non-linear case (cf. Jaromczyk, Lect. N. Comp. Sc. # 118, 1981) have failed to produce non-linear lower bounds, as remarked by Ben-Or (Proc. 15th. ACM. Ann. Symp. Th. Comp. 1983). On the other hand it has been showed that some problems arising in computational geometry can not be formulated with semilinear tasks. Therefore we have presented here a general definition of the width of a semialgebraic closed set $S \subseteq \mathbb{R}^n$, $w(S) = \min\{r \in \mathbb{N} \mid S = \bigcup_{i \in I} \{\underline{x} \in \mathbb{R}^n \mid f_{i1}(\underline{x}) \geq 0, \ldots, f_{ir}(\underline{x}) \geq 0\}, f_{ij} \in \mathbb{R}[\underline{X}]\}$ (a similar definition holds for open sets). In the general case the width of the congruence class of a semialgebraic set S is defined as $w_{con}(S) = \min \{w(A) \mid A$ closed, s.a. set, $A \triangle S$ of cod. $\geq 1\}$, $\triangle$ = symmetric difference.

Then we have the following results:

i) The width of the congruence class of a s.a. set S is a lower bound for the cost of any algebraic decision tree solving the membership problem for S.

ii) $\forall S \subseteq \mathbb{R}^n$, s.a., $w_{con}(S) \leq n$. Therefore, taking n as a parameter, at best linear lower bounds can be obtained.

iii) The width of a basic closed s.a. set is in general smaller (and not always equal) to the t-invariant of its complement in $\mathbb{R}^n$, and smaller than the s-invariant of the given set (cf. Bröcker, CMS Conf. Proc. 4, 1984).

iv) Under certain conditions on $p_1, \ldots, p_m \in \mathbb{R}[\underline{X}]$, it can be shown that, $w(p_1 \geq 0, \ldots, p_m \geq 0) = m$; and that $w_{con}(p_1 \geq 0 \ldots p_m \geq 0) = m$. In particular these conditions hold when $\{p_1 \ldots p_n\}$ are part of a regular system of parameters of an algebraic variety (generalizing our definitions to a notion of width with respect to a subset of $\mathbb{R}^n$).

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Homologically and algebraically trivial cycles.

Let X be a projective, nonsingular, n-dimensional variety over $\mathbb{R}$, and let $Z_k(X)$ be the free abelian group generated by the k-dimensional subvarieties of X. There is a canonical homomorphism $cl = cl_k : Z_k(X) \to H_k(X(\mathbb{R}), \mathbb{Z}/2)$. We want to compute the kernel of cl_k. Define

$Z_k^{\phi}(X) = \{z \in Z_k(X),\ z$ rational equivalent to a cycle $\sum n_i z_i,\ z_i(\mathbb{R}) = \emptyset\}$

$Z_k^{*k}(X) = \{z \in Z_k(X),\ z$ rational equivalent to a cycle $\sum n_i z_i,\ \dim z_i(\mathbb{R}) < k\}$

1) The situation is well known in the cases $k=0$, $k=n-1$:

$$\ker(cl_k) = Z_k^{\phi}(X) \quad \text{for } k=0,\ k=n-1.$$

2) The following example is constructed:

X: proj $\mathbb{R}[X_0, \ldots, X_5]/(X_1^2 + \ldots + X_5^2 - X_0^2)$, hence $X(\mathbb{R}) = S^4$

there exists a cycle z with: $n \cdot z \notin Z_k^{\phi}(X)\ \forall_{n \in \mathbb{N}\setminus\{0\}}$ (but $cl(z)=0$)

IA will follow from 4) that $z \in Z_2(X)$.

3) Using a theorem of Ischebeck we can prove: $\ker(cl_k) = Z_k^{*k}(X)$ for every k. The main tool is the classification of nonoriented bordism

4) Using Hironaka's methods on "smoothing" of cycles, we prove $\ker(cl_k) = Z_k^{\phi}$ if $k \in \min(3, \frac{n-1}{2})$

Question: Can the 3 in the above bound be dropped?

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On the topological invariants of germs of analytic function - Zbigniew Szafraniec (Gdańsk)

There is given a definition of germs with property A_d (for example, each homogeneous polynomial of degree d has property A_d). Let $f:(\mathbb{R}^n,0)\longrightarrow(\mathbb{R},0)$ be a germ with property A_d, and let $f_{\mathbb{C}}$ be complexification of f. Let us denote by $e(f_{\mathbb{C}})$ the Euler characteristic of the Milnor's fiber of $f_{\mathbb{C}}$. The main results is:

$$\frac{\chi(L)}{2}\equiv\frac{\chi(S^{n-1})}{2}+\frac{e(f_{\mathbb{C}})}{d}\mod 2$$

, where $L=f^{-1}(0)\cap S_r$, $0<r\ll r$. This theorem is a generalization of a theorem which was proved by C.T.C. Wall (Topology, 1983).

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Nash manifolds and nonsingular real algebraic varieties

Masahiro SHIOTA

A C^ω affine Nash manifold is by definition a C^ω submanifold of $\mathbb{R}^n$ which is also semialgebraic. Let r be a natural number or ω. Then a C^r Nash map between affine C^ω Nash manifolds is by definition a C^r map with semialgebraic graph.

Theorem. An affine C^ω Nash manifold is C^ω Nash diffeomorphic to some nonsingular affine algebraic variety.

The compact case is equivalent to the well-known theorem of Tognoli. The noncompact case is partly proved by Akbulut-King. Their famous theorem is that ~~a compact~~ the interior of a compact C^ω manifold is C^ω diffeomorphic to some nonsingular affine algebraic variety. We see easily that this is equivalent to the statement that an affine C^ω Nash manifold which is noncompact is C^ω diffeomorphic to some nonsingular affine algebraic variety. However Theorem does not follow automatically from this because there exist two affine C^ω Nash manifolds which are C^ω diffeomorphic but not C^ω Nash diffeomorphic.

For the proof of Theorem we need some facts of Nash manifolds and some topological methods. The most important fact is as follows. We can construct a topology on the set of C^r Nash maps between affine C^ω Nash manifolds so that a close approximation of a C^r Nash diffeomorphism is a diffeomorphism and that we can approximate a C^r Nash map by a C^ω one.

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Uniformization and rectilinearization of subanalytic sets

Edward Bierstone

Hironaka has used his desingularization and local flattening theorems to prove the following fundamental results: Let M be a real analytic manifold and let X be a subanalytic subset of M.

Uniformization theorem. Suppose X is closed. Then there is a real analytic manifold N ($\dim N = \dim X$) and a proper real analytic mapping $\varphi: N \to M$ such that $\varphi(N) = X$.

Rectilinearization theorem. Let $K \subset M$ be compact. Then there are finitely many real analytic mappings $\varphi_i: \mathbb{R}^m \to M$ ($m = \dim M$) such that: (1) There are compact subsets K_i of $\mathbb{R}^m$, such that $\cup \varphi_i(K_i)$ is a neighborhood of K in M. (2) For each i, $\varphi_i^{-1}(X)$ is a union of quadrants.

Elementary proofs of these results, using neither resolution of singularities nor local flattening are presented in this talk and the following one by P. Milman. Our approach stands in the same relation to local resolution of singularities of real or complex analytic spaces as Zariski's uniformization theorem does to desingularization of algebraic varieties.

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Transforming an analytic function to normal crossings by blowings-up

Pierre Milman

In this talk we presented an elementary proof of the following theorem (a variant of desingularization).

Theorem Let M be an analytic manifold (over

$\mathbb{K}=\mathbb{R}$ or $\mathbb{C}$). Let $f \in \mathcal{O}(M)$. (Assume that f does not vanish identically on any component of M.) Then there is a countable collection of analytic mappings $\pi_j : W_j \to M$ such that:

(1) Each π_j is the composition of a finite sequence of local blowings-up (with smooth centers).

(2) There is a locally finite open covering $\{U_j\}$ of M such that $\pi_j(W_j) \subset U_j$, for all j.

(3) If K is a compact subset of M, then there are compact subsets $L_j \subset W_j$ such that $K = \bigcup_j \pi_j(L_j)$. (The union is finite, by (2).)

(4) For each j, $f \circ \pi_j$ is locally normal crossings on W_j.

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COMPUTATION OF THE ANALYTIC STRUCTURE OF A REAL ALGEBRAIC CURVE.

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Given a polynomial $F(X,Y) \in \mathbb{Z}[X,Y]$ and it zero set $\mathcal{C} \subseteq \mathbb{R}^2$, some algorithms have been recently given which computes the topology of $\mathcal{C}$, i.e. they produce a planar graph homeomorphic to $\mathcal{C}$.

In our work an improvement of this algorithm is given, which outputs the same graph with the edges numbered in such a way that they share the number iff they belong to the same global analytic component.

To do so we calculate the discriminant locus, the points of the curve lying over this set and for each one of this points the half-branches of the curve centered at it. Once we have this information edges of the same global components are collected togheter by just a poursuite of the component in the graph: at each critical point we follow a half-branch in the half-branch that gives us the whole branch.

The computation of the branches (and half-branches) are performed using: a) Duval's algorithm for rational Puiseux expansions (see. D. Duval "Développements de Puiseux rationnels", Thèse d'Etat, Univ. de Grenoble) and

b) Coste & Roy's codification of real algebraic numbers (see. M. Coste and M.-F. Roy "Thom's lemma, the coding of real alg. numbers and the topology of semi-algebraic sets" (submitted to the J. of Symb. Comp)).

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Real torii and Jacobians associated to A.C.I Systems.

J. P. Francoise
(Orsay)

M. Adler and P. van Moerbeke introduced the notion of algebraically completely integrable Hamiltonian systems. For these systems one can compute the Actions and relate the singularities of these actions to singularities of period mappings of families of Riemann surfaces. Some problems of real algebraic geometry nature arise to find the real invariant torii inside the complex torii (Jacobians or Prym variety) or to understand the monodromy of the Actions. We gave a complete description of the three cases of integrability of the motion of a solid body about a fixed point as an illustration.

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FIRST ORDER AXIOMS FOR CHAIN CLOSED FIELDS and 17th Hilbert's problem at level n.

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For orderings of exact higher level,
chains of orderings,
chain closure and chain-closed fields,
See Becker (Impa 1978) and Harman (G. Math. 1982) —

I First order axioms for chain closed fields (to appear CRAS 1987)

$\mathcal{L}$ language for fields. $\mathcal{L}(\alpha)$ adding a constant symbol.

thm I-1 K is chain closed iff $\exists \alpha \in K$ s.t. (K, α) satisfies (in $\mathcal{L}(\alpha)$)

(a) axioms for commutative fields

(b) $\forall n \geq 1$: $\bigwedge x_1 \dots \bigwedge x_n \ \neg(-1 = x_1^2 + \dots + x_n^2)$

(c) $\forall n \geq 0$: $\bigwedge x_1 \dots \bigwedge x_n \ \neg(\alpha^2 = x_1^4 + \dots + x_n^4)$

(d) $\bigwedge x \bigwedge y \bigvee z \ (x^2 + y^2 = z^2)$

(e) $\bigwedge x \bigvee y \ (x = y^2 \vee x = -y^2 \vee x = \alpha y^2 \vee x = -\alpha y^2)$.

(f) $\forall n \geq 1$: $\bigwedge x_0 \dots \bigwedge x_{2n+1} \ (x_{2n+1} = 0 \vee \bigvee y \ (x_0 + \dots + x_{2n+1} y^{2n+1} = 0))$

Note that 1) models of $(a) \cup (b) \cup (c)$ are fields with chains.
2) models of $(a) \cup (b) \cup (c) \cup (d) \cup (e)$ are fields with only 2 orders, only one chain of orderings (up to the exchange of the two orders) and are pythagorean.

thm I-2 $K \subset L$ chain closed fields satisfying axioms of th I-1 for the same α: Then $K \prec L$.

thm I-3 An axiomatization of the theory of chain closed fields K in $\mathcal{L}$ is

(1) axioms for commutative fields

(2) $K^2 + K^2 = K^2$

(3) $K^4 + K^4 = K^4$

(4) $\bigvee \alpha \bigwedge x \ (\neg(\alpha^2 = x^4) \wedge \bigvee y \ (x = y^2 \vee x = -y^2 \vee x = \alpha y^2 \vee x = -\alpha y^2))$

(5) Every polynomial of odd degree has a root in K.

– other results on the model theory of chain closed fields are to appear in JSL:

" Model theory of chain closed fields " Max Dickmann. (University of Paris 7)

One of these is the following theorem:

thm I – 4 (Dickmann)

The theory of chain closed fields is identical with the theory of Rolle fields with exactly two orders.

II **17^{th} Hilbert's problem at level n for chain closed fields.**

(Joint work with Françoise Delon (University of Paris 7))

Let v_0 be the coarsest henselian valuation with real closed residue field (see Jacob (Crelle 1981)) and v_1 be the finest one with real closed residue field (which will now be archimedean) (see Becker Jnger 1978).

thm II . 1 Let K be chain closed such that $v_0 = v_1$.

Let $f \in K(X)$ then the following are equivalent.

(i) $f \in \Sigma(K(X))^{2^n}$

(ii) For every real algebraic extension L of K f satisfies:

$\forall x \in L,\ f(x) \in L^{2^n}$.

There exist counter examples showing that $v_0 = v_1$ is necessary and also that if f is such that $f(a) \in K^{2^n}$ only on K the result does not hold.

thm II _ 2 $K \subset L$ chain closed fields then the following are equivalent:

(1) $K \cap L^2 = K^2$

(2) K is relatively algebraically closed in L

(3) $K \prec L$.

thm II _ 3 Let $K \subset L$ L chain closed, K relatively algebraically closed in L then K is real closed or K is chain closed.

To show Th II-1 we use these result more Becker's caracterization of sums of 2^n powers in a real field: $\Sigma K^{2^n} = (\cap P_n) \cap \Sigma K^2$ where.

P_n is any ordering of exact higher level 2^n. First step is a particular case of:

thm II – 4 (K chainclosed field (any) – $f \in K(\overline{X})$ such that for every real algebraic extension L of K satisfies $\forall \overline{x} \in L$, $f(\overline{x}) \in L^2$; then $f(\overline{X}) \in \Sigma K(\overline{X})^2$

Some remarks on the real spectrum under real closed field extensions

Let R be a real closed field and $V|R$ an affine real variety. Then $V(R)$ can be naturally identified with a dense subspace of the compact space $\operatorname{Max}\operatorname{Sper} R[V] := \widehat{V(R)}$. In order to understand this compactification one may first study varieties over $\mathbb{R}$ and investigate thereafter the behaviour of $\widehat{V(R)}$ under real closed field extensions.

Prop. 1. i) If $V|\mathbb{R}$ is an integral curve then $\#(\widehat{V(\mathbb{R})} \setminus V(\mathbb{R})) = 2 \cdot \#(Y(\mathbb{R}) - \tilde{V}(\mathbb{R}))$ where Y is the smooth proj. model of $\mathbb{R}(V)$ and $\tilde{V}$ is the normalization of V.

ii) if $K \subset \mathbb{R}$, K real closed, then $\widehat{V(K)}$ and $\widehat{V(\mathbb{R})}$ are homeomorphic (V still an integral curve).

For a more general study let $K \subset R$ be an extension of real closed field, V be a real variety $/K$, $A := K[V]$, $\tilde{V}$ the base extension of V to R, $B = R[\tilde{V}] = K[V] \otimes_K R$.

Prop. 2. i) B is also a real algebra,

ii) every $\alpha \in \operatorname{Sper} A$ extends to $\beta \in \operatorname{Sper} B$ with $\dim \alpha = \dim \beta$,

iii) if $R|K$ is Archimedean then $\operatorname{res}: \operatorname{Sper} B \to \operatorname{Sper} A$ induces a surjective map $\operatorname{res}: \widehat{\tilde{V}(R)} \to \widehat{V(K)}$, if $R|K$ is even dense, then $\operatorname{res}_{|V(R)}: \tilde{V}(R) \to \widehat{V(K)}$ is injective,

iv) if $K \subset \mathbb{R} = R$ then we have a surjective map $\operatorname{res}: \widehat{\tilde{V}(\mathbb{R})} \to \widehat{V(K)}$, injective on $\tilde{V}(\mathbb{R})$. If additionally V is a curve or $\tilde{V}(\mathbb{R})$ is compact then this map is a homeomorphism,

v) if $K \subsetneq \mathbb{R}$ then $\operatorname{res}: \operatorname{Max}\operatorname{Sper} \mathbb{R}[X,Y] \twoheadrightarrow \operatorname{Max}\operatorname{Sper} K[X,Y]$ is not injective.

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"Sets with polynomial cusps in approximation theory"
(a work elaborated in common with W. Pleśniak;
Math. Ann. (1986))

A subset E of $\mathbb{R}^n$ is said to be uniformly polynomially cuspidal (UPC), if there are three positive constants M, m, d such that for each point $x \in \overline{E}$ there exists a polynomial map $h_x : \mathbb{R} \to \mathbb{R}^n$ of deg. $\leq d$, and such that:

(i) $h_x((0,1]) \subset E$, $h_x(0) = x$, and

(ii) $\operatorname{dist}(h_x(t), \mathbb{R}^n \setminus E) \geq Mt^m$, for each $x \in \overline{E}$ and $t \in [0,1]$.

Every open bounded subanalytic subset of $\mathbb{R}^n$ is UPC; in particular every open bounded semi-algebraic subset is UPC. Let E be a UPC subset of $\mathbb{R}^n$.
We have the following version of Markov's inequality:

$\|D^\alpha p\|_E \leq C k^{r|\alpha|} \|p\|_E$, where $p : \mathbb{R}^n \to \mathbb{R}$ is a polynomial of deg $\leq k$, $\alpha \in \mathbb{N}^n$, $\|p\|_E = \sup\{|p(x)| : x \in E\}$ and positive constants C, r depend only on E. Markov's inequality is used in the proof of the following version of Bernstein's theorem: Assume that E is compact. Let $f : E \to \mathbb{R}$. Let P_k denote the space of polynomials of deg. $\leq k$, on $\mathbb{R}^n$. Then the following conditions are equivalent:

(1) there exists a C^∞ extension $\tilde{f} : \mathbb{R}^n \to \mathbb{R}$ of f,

(2) for every $r > 0$: $\lim_{k \to \infty} k^r \operatorname{dist}_E(f, P_k) = 0$ (where $\operatorname{dist}_E(f, P_k) \overset{df}{=}$
$= \inf\{\|f - p\|_E : p \in P_k\}$).

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Real Cubic Surfaces (revisited?) R. SILHOL (Angers)

The study of real cubic surfaces has a long history and one would think that almost everything is known on the subject. In the real case a classification of such surfaces can be made in terms of the number of lines and the number of connected components. It goes as follows:

type:	F_1	F_2	F_3	F_4	F_5
# of real lines:	27	15	7	3	3
# components:	1	1	1	1	2

It is well known and not hard to prove that surfaces of type F_1 to F_4 are all birationally isomorphic (over $\mathbb{R}$) to $\mathbb{P}^2_{\mathbb{R}}$. But the fact that the surfaces of type F_5 are all birationally isomorphic seems to be unknown or at least forgotten. This is what we prove.

The idea of the proof is to relate such surfaces to surface defined by $x^2+y^2=g(t)$ where g is a function regular on $\mathbb{P}^1(\mathbb{R})$ with 4 real simple zeros. Modulo an elementary transformation such a surface is isomorphic (over $\mathbb{C}$) to $\mathbb{P}^2_{\mathbb{C}}$ blown up in 5 points. The real part has 2 components and blowing up one more real point transforms the surface into a real cubic surface of type F_5. All surfaces of type F_5 are obtained in this way. To see this let W be a surface of this type and D a real line in it. If P is a plane passing through D, $P\cap(W\setminus D)$ is a conic. Using classical theory of cubic surfaces one can prove that the fibration of $(W\setminus D)$ thus obtained is of the preceding type.

A priori the birational isomorphism class of the surface depends on the cross-ratio of the 4 zeros of g. On the other hand we have 3 real lines, and hence 3 fibrations.

The proof of the assertion then reduces to building a cubic surface such that 2 of the fibrations correspond to 2 given cross-ratios. Such a surface is obtained by considering the following real structure on $\mathbb{P}^2$. Choose 5 real points $P_1,\dots,P_4$ and t and for any point x ($\neq P_i, t$) consider the conic C_x through the P_i's and x. Let y be the second point of intersection of C_x and tx. $\varphi: x \mapsto y$ is an involution of $\mathbb{P}^2(\mathbb{C})$ defined over $\mathbb{R}$. Hence φ composed with complex conjugation defines an anti-holomorphic involution of $\mathbb{P}^2(\mathbb{C})\setminus\{tP_i\}$. By blowing up the 5 points we get a surface of the ~~#~~ type above. The cross ratio of the fibration is just the cross-ratio of the 4 lines $\{tP_i\}$. It is then easy to choose another real point for this new structure such that the cross ratio of the $\{xP_i\}$ be arbitrary. This proves the assertion.

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~~[illegible]~~

If $p(x_1, \dots, x_n)$ is a psd form, when is $p(x_1^k, \dots, x_n^k)$ sos? B. Reznick (Urbana)

Suppose $p = p(x_1, \dots, x_n)$ is a real psd form of degree m. Let $Y(p) = \{k : p(x_1^k, \dots, x_n^k)$ is a sum of squares of forms (sos).$\}$ Clearly $k \in Y(p)$ implies $rk \in Y(p)$. The only other information comes from explicit examples. An <u>agiform</u> is a psd form with shape $c\{\lambda_1 \underline{x}^{\underline{u}_1} + \dots + \lambda_n \underline{x}^{\underline{u}_n} - \underline{x}^{\underline{w}}\}$ where $c > 0$, $\lambda_i \geq 0$, $\Sigma \lambda_i = 1$ and $\underline{w} = \Sigma \lambda_i \underline{u}_i$ is a lattice point. These occur in the literature from Hurwitz $(x_1^m + \dots + x_m^m - m x_1 \cdots x_m)$, Motzkin $(M(x,y,z) = x^4y^2 + y^4z^2 + z^4x^2 - 3x^2y^2z^2)$ Choi, Lam and Reznick. There is a purely geometric criterion to determine whether an agiform is sos. Using this, one can show that $Y(M) = \{2, 3, \dots\} = \mathbb{N} - \{1\}$. More generally, there exist agiforms p_s and q_s so that $Y(p_s) = \{s, s+1, \dots\}$ and $Y(q_s) = \{2, 4, \dots, 2s-2, 2s, 2s+1, 2s+2, \dots\}$. By contrast, let $H(\underline{x}) = \Sigma x_i^4 - 2\Sigma x_i^2 x_{i+1}^2 + 2\Sigma x_i^2 x_{i+2}^2$ $(x_{i+5} = x_i)$ denote the psd (non-agiform) Horn form: $Y(H) = \emptyset$. Questions: Does there exist a form p in ≤ 4 variables for which $Y(p) = \emptyset$. If $Y(p) \neq \emptyset$ does $Y(p)$ contain all but finitely many k? How do the degree and number of variables restrict $Y(p)$? Is there an <u>even</u> form p with $2 \in Y(p)$, $3 \notin Y(p)$. <u>Conjecture</u>: If p is psd then there exists an invertible linear transformation T such that $q(\underline{x}) = p(T\underline{x})$ with an odd $k \in Y(q)$. If the conjecture is true then every psd p can be written as a sum of squares of forms in the "fractional" variables $(\Sigma a_{ij} x_j)^{1/k}$ $1 \leq i \leq n$.

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Arcwise symmetric semi-algebraic sets K. Kurdyka (Kraków, St. Martin d'Hères)

A semi-algebraic set E in $\mathbb{R}^n$ is said to be arcwise symmetric iff for each $\gamma:(-1,1)\to\mathbb{R}^n$ analytic arc, $\gamma(-1,0)\subset E$ implies $\gamma(-1,1)\subset E$. The class of all arcwise symmetric sets in $\mathbb{R}^n$ forms a class of closed sets for a noetherian topology, We call it AR-topology. This topology is between Zariski topology and strong topology. We claim that

- if V is algebraic, $\dim V = k$, then there is (1-1) correspondence between AR-irreducible components of V of dim. k and connected components of a resolution of singularities of V
- AR-irreducible immerged components correspond to Nash sheets (J. Nash. Real algebraic manifold. Ann. of. Math. 1952) of V
- if E is AR-closed, $f: E\to\mathbb{R}^m$ regular injective and proper then $f(E)$ is AR-closed
- we define a ring of semi-algebraic functions on $\mathbb{R}^n$ which satisfies the following condition
  if $\gamma:(0,1)\to\mathbb{R}$ is an analytic arc then $f\circ\gamma$ is analytic
  We call this ring $A_a(\mathbb{R}^n)$
- for each E AR-closed in $\mathbb{R}^n$ there exists $f\in A_a(\mathbb{R}^n)$ s.t
  $$f^{-1}(0) = E$$
- $\operatorname{codim} \operatorname{Sing}(f) \geq 2$
- ring $A_a(\mathbb{R}^n)$ is an integral domain, not noetherian, nor factorial, but $\operatorname{Spec} A_a$ is a noetherian space.
- Nullstellensatz holds true for $A_a(\mathbb{R}^n)$ (as in complexe case)

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Part I. A fixed point theorem for semi-algebraic sets.
G. Brumfiel, Stanford.

Theorem Let $X \subset \mathbb{R}^n$ be any semi-algebraic set, $f: X \to X$ a continuous semi-algebraic map, $tr(f_*) = \Sigma(-1)^i \operatorname{trace}(H_i(X) \to H_i(X))$. Let $\tilde{X} \subset A^n = \operatorname{Spec} \mathbb{R}[x_1, \dots x_n]$ be the associated constructible to X. Then^R: If $Tr(f_*) \neq 0$, then $\tilde{f}: \tilde{X} \to \tilde{X}$ has a fixed point, where $\tilde{f}: \tilde{X} \to \tilde{X}$ is the extension of $f: X \to X$.

Part II. The Tree of a non-Archimedean hyperbolic plane.

Def $\Lambda \subset (\mathbb{R}, +)$ is an ordered group, a Λ-tree [more precisely, the vertices of a Λ-tree] is a metric space $d: T \times T \to \Lambda$ which satisfies certain axioms such as (i) Each pair of points of T are endpoints of a unique segment (subspace isometric to an interval in Λ) (ii) if S_1, S_2 are segments and $S_1 \cap S_2$ = point then $S_1 \cup S_2$ is a segment. (iii) If two segments S_i have an endpoint of one in common then $S_1 \cap S_2$ is a segment (maybe trivial)

These axioms are approximate. The point is to generalize $\mathbb{Z}$-trees = ordinary contractible graphs, with integral distance function between vertices.

Let F non. Arch. ordered field, HF^2 the hyperbolic plane with cross ratio distance $D(A, B) \in F^{\geq 1}$ between points of HF^2. Define $d(A, B) = \log D(A, B) \in \mathbb{R}$ where $\log: F^* \to \mathbb{R}$ is log with base a big element $b > 0$. (b big means $\forall a \in F$, $a \leq b^m$ some m.) Then $-\log |\ |: F^* \to \mathbb{R}$ is a valuation, say Λ-valued, $\Lambda \subset \mathbb{R}$.

Theorem $\underset{\sim}{HF^2}$ with metric d is a Λ-tree, where $A \sim B$ means $d(A, B) = 0$

On transversals to semialgebraic/subanalytic sets

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We report on recent joint work with Tzee-Char KUO (Univ Sydney). The notion of transversal submanifold to an embedded singular space is in terms of transversality to the strata of a C^1 stratification.

<u>Thm 1.</u> In $C^1(\mathbb{R}^n, \mathbb{R}^p)$ the set of transversal maps to a C^1 stratification Σ of a closed subset Z of $\mathbb{R}^p$ is open if and only if Σ is Whitney A regular.

It is thus interesting to study transversals to A-regular stratifications.

<u>Thm 2 (Kuo-Trotman).</u> The topological type of the germ at 0 of the intersection $X \cap T$ for T a C^1 transversal at 0 to Y, where (X,Y) are C^1 A regular strata, is independent of T.

We describe how various blowingup constructions provide new proofs of this Thm 2.

For example the map $\underset{\mathbb{R}^m \times \mathbb{R}^k}{(x,\xi)} \xrightarrow{\Phi} \underset{\mathbb{R}^m \times \mathbb{R}^k}{(x, |x|\xi)}$ defines a "blowingup" such that if X is A regular over $Y = 0 \times \mathbb{R}^k$ then $\Phi^{-1}(X)$ is strongly Verdier W-regular over $0 \times \mathbb{R}^k$.

Of more general interest we consider

$$\begin{array}{ccc} \mathbb{R}^n \times \mathbb{P}^{n-1} & \xrightarrow{\pi_1} & \mathbb{R}^n \\ (x,\ \xi) & & \\ \cup & \nearrow_{\gamma} & \\ \{(x,\xi) : x \in \xi\} = E & & \end{array}$$

where $\mathbb{P}^{n-1}$ parametrises <u>hyperplanes</u> through 0.

Then if (X,Y) is Whitney A regular in $\mathbb{R}^n$, $\gamma^{-1}(X)$ is Verdier W-regular over $\mathbb{P}^{n-1}_0$ = space of hyperplanes ~~[illegible]~~ transverse to Y. Thm 2 follows.

Transversals to $\mathbb{P}^{n-1}_0$ of class C^k are sent by γ to transversals to Y of class C^k_+. ~~[illegible]~~ We say a fn. $f : \mathbb{R}^n \to \mathbb{R}$ ~~is~~ is of class C^k_+ if $f(x) = \sum_{i=1}^{n} x_i g_i(x)$ where g_i are of class C^k.

We deduce by stratification arguments and by a 1985 (Topology) theorem ~~that~~
"transverse C^1 transversals $\rightarrow$ homeomorphic C^1 transversals",
(t^1) (h^1)
that "transverse C^1_+ transversals $\Rightarrow$ finitely many homeomorphic C^1_+ transversals"
for subanalytic sets. (t^1_+) (fin^1_+)
This provides a partial confirmation of a conjecture of the author ("$t^k \rightarrow (fin^k)$") concerning subanalytic sets.
We are also led to consider Lipschitz transversals, Hölderian C^α transversals, in this semialgebraic context.

Separation of semialgebraic sets by polynomials and Nash-functions.

Let V be a real algebraic affine variety of dimension n. Let $A, B \subset V(R)$ be disjoint basic closed semialgebraic subsets.

Theorem a) If $n \leq 2$, then A and B can be separated by a polynomial.
b) If $n \geq 3$ there exist semialgebraic disjoint closed basic sets, which cannot be separated by a polynomial.

Part b) is proved by an explicite counterexample. Part a) follows from the following result on abstract spaces of orderings.

Theorem Let (X, G) be a space of orderings and let A, B be open and closed disjoint constructible subsets of X. Then A and B can be separated by an element $g \in G$ iff this holds for the restriction to all finite subspaces of X.

More general, the Mostowski-number $m(A, B)$ is considered for any pair of disjoint semialgebraic subsets $A, B \subset V(R)$, A, B closed. This is the minimal number m such that $f \in R[V][\sqrt{q_1}, \ldots, \sqrt{q_m}]$ separates A and B, where the $q_i \in R[V]$ are strictly positive. There are estimates given for the Mostowski number $m(n) := \sup\{m(V) \mid \dim(V) = n\}$ where $m(V) = \sup\{m(A, B) \mid A, B \subset V(R)\}$ namely
$n - 1 \leq m(n) \leq \bar{s}(n)\,\bar{t}(n)$ for $n \geq 2$. Here $\bar{s}(n)$ $\bar{t}(n)$ is the minimal number of polynomials which describes an arbitrary closed semialgebraic set as a union of basic closed sets.

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Arbeitsgemeinschaft Geyer-Harder:
Integrable Hamiltonsche Systeme

Leitung: Horst Knörrer, Eugene Trubowitz

12.4. - 18.4. 1987

1. Wirkungs-Winkel-Variable bei vollständig integrablen Hamilton-systemen

Zunächst wurde an die Grundbegriffe des symplektischen Formalismus erinnert: Hamiltonsche Vektorfelder, Poisson-Klammern und Impulsabbildung. Damit wurden dann integrable Systeme nach dem Vorbild von Guillemin/Sternberg wie folgt behandelt.

Auf einer symplektischen Mannigfaltigkeit M, $\dim M = 2n$, seien n Funktionen $f_1, \dots, f_n : M \to \mathbb{R}$ in Involution gegeben, d.h. mit $\{f_i, f_j\} = 0$ stets. Man arbeitet lokal und nimmt an, daß $f = (f_1, \dots, f_n) : M \to \mathbb{R}^n$ eine Submersion mit kompakten zusammenhängenden Fasern ist.

Für $h : B = f(M) \to \mathbb{R}$ ist das Hamiltonsche Vektorfeld zu f^*h tangential an die Fasern von f. Damit erhält man eine faserweise Aktion von T^*B auf M, wodurch $M \cong T^*B/L$ als Torusbündel zu einem Gitterbündel $L \subset T^*B$ erkannt wird.

Lokal (bez. B) läßt sich nun eine Gitterbasis durch geschlossene 1-Formen $\beta_j = 2\pi\, dI_j$ beschreiben. Damit hat man Wirkungsvariable $I_1, \dots, I_n$ und erhält aus den zugehörigen Faserkoordinaten θ_j für T^*B durch Übergang nach M Winkelvariable $\theta_1 \bmod 2\pi, \dots, \theta_n \bmod 2\pi$ auf M. Dabei gilt für die symplektische Form ω auf M: $\omega = d\sigma$ mit $\sigma = \sum I_j\, d\theta_j$.
I_j und $\theta_j \bmod 2\pi$ sind also kanonische Koordinaten für M.

~~Ist daß~~ Die ursprünglich gegebenen Funktionen f_i sind jetzt beschrieben durch Funktionen $h_i(I_1,\dots,I_n)$ der Wirkungsvariablen. Für die zugehörigen Hamiltonschen Differentialgleichungen hat man

$$\dot\theta_j = \frac{\partial h_i}{\partial I_j} \quad \& \quad \dot I_j = 0 ,$$

d.h. die betreffenden Hamiltonschen Vektorfelder integrieren sich zu einem faserweise linearen Fluß.

Als Beispiel wurde der Lagrange-Kreisel genannt: das ist der vollständig integrable Spezialfall der Kreiselbewegung, wo der Kreisel eine Symmetrieachse hat und sich unter dem Einfluß der Schwerkraft bewegt. Die Gesamtenergie $H: TSO_3 \to \mathbb{R}$ wird hier durch die Impulsfunktionen zur Gruppe der Drehungen um die Symmetrieachse und zu derjenigen um die z-Achse zu einem System von 3 Funktionen in Involution ergänzt.

J. Gamst, Bremen.

Die Birkhoffsche Normalform.

A useful way to analyze the structure of a dynamical system in the neighbourhood of a stationary point is to use coordinate transformations to transform the system $\dot x = F(x)$, $x \in \mathbb{R}^n$ in as simple as possible normal form. Writing $F(x) = A.x + h.o.t$ one can find a formal coordinate transformation bringing F into normal form (assuming A to be semisimple). When the eigenvalues λ_i of A satisfy an estimate of the form $|\lambda_s - \langle\lambda, m\rangle| \geq \frac{C}{|m|^\nu}$, $\forall m \in \mathbb{N}^n$ Siegel showed that the formal coordinate change actually is convergent. In the case of Hamilton systems $\dot w = J.\partial_w H$, $w = (q,p)$, $J = \begin{pmatrix} 0 & E \\ -E & 0 \end{pmatrix}$ one has to use symplectic transformations and transform H in normal form. When we write

$H = H_2 + H_3 + \dots$; H_i homogeneous of degree i then H is said to be in Birkhoff(-Gustavson) normal form if $\{H_2, H_i\} = 0$ $i = 2, \dots$.
Again it is trivial that H can be brought into normalform by a formal coordinate change (in case that H_2 is semisimple). When we write $H_2 = \sum \omega_i x_i y_i$; $L(\omega) = \ker(\mathbb{Z}^n \to \mathbb{C}; m \mapsto \langle \omega, m \rangle)$ then if $\mathrm{rk}_{\mathbb{Z}} L(\omega) = 0$ (no resonance) or $= 1$ (simple resonance) then the system becomes formally complete integrable (with integrals $I_\gamma = \sum \gamma_i x_i y_i$ with $\gamma \perp (L(\omega) \otimes \mathbb{C})$). The question of convergence of the normal form transformation is very subtle ~~and exceptional~~. A theorem of Siegel states that convergence and hence integrability cannot be decided by looking at a finite part of the Taylor expansion of H.

Dr. Straten, Leiden.

Die Methode K-A-M für das eingeschränkte 3-Körper-Problem.

Ich habe die Differentialgleichung und den Physikern geläufige Hamiltonfunktionen für das 2- und das 3-Körper-Problem referiert. Für eine elegante Lösung für alle Zeiten muß muß man die Hamiltonfkt. nicht infinitesimal isometrisch sondern infinitesimal symplektisch, d.h. "kanonisch" transformieren. Solche kanonischen Transformationen erhält man klassisch aus jeder fkt. $S: \mathbb{R}^{2n}_{(\underline{x},\underline{\eta})} \to \mathbb{R}$ mit $\underline{y} := S_{\underline{x}}$ und $\underline{\xi} := S_{\underline{\eta}}$ mit dem Satz über implizite Funktionen. Wir suchen deshalb zu $H = H(\underline{x}, \underline{y})$ die fkt's $S = S(\underline{x}, \underline{\eta})$ und $K = K(\underline{\eta})$ so, daß $H(\underline{x}, S_{\underline{x}}(\underline{x}, \underline{\eta})) = K(\underline{\eta})$ unabhängig von den kanonischen Ortsvariablen ist.

Um diese Differentialgleichung von Hamilton-Jacobi zu lösen, brauchen wir eine Theorie für partielle Differentialgleichungen auf dem Torus, wie $U_\xi(\xi)\cdot\underline{\omega} = f(\xi)$ für U. Die Lösbarkeit und Periodizität von U erzwingt, daß $\underline{\omega}$ sehr irrational sein muß, wegen dem berühmten Problem der "kleinen Nenner". Aber mit diesem Verfahren erhält man eines der Haupt-Stücke der K-A-M-Theorie: Sind die Winkelgeschwindigkeiten des ungestörten Problems hinreichend irrational, so gibt es eine Konvergente Folge von kanonischen Transformationen, die im Limes auch die Hamiltonfunktionen des gestörten Problems von den Ortsvariablen unabhängig macht. Deshalb hat auch das gestörte Problem dann eine quasiperiodische Bahn. Das vollendet eine weltberühmte Preisschrift von H. Poincaré.

R. Böhme Bochum

Geodätische auf dem Ellipsoid

Geodätische werden als Bahnen eines kräftefreien Massenpunkts auf einem Ellipsoid des $\mathbb{R}^n$ beschrieben. Um geeignete Koordinaten zu erhalten, betrachtet man, Jacobi (Vorlesungen über Dynamik) folgend, die konfokalen Quadriken $\sum x_i^2(a_i-z)^{-1}=1$ und zeigt:

a) Jeder Punkt des $\mathbb{R}^n$ liegt i.a. auf n Quadriken verschiedenen Typs ($n=3$: Ellipsoid, ein- und zweischaliges Rotationsellipsoid)

b) Die Parameter $z=u_i$ dieser Quadriken liefern ein "elliptisches" Koordinatensystem.

c) Die konfokalen Quadriken schneiden sich orthogonal, es gilt sogar der Satz (Chasles) daß die Normalen in den Berührpunkten einer gemeinsamen Tangente zweier Quadriken orthogonal sind.

d) Im allgemeinen berührt eine Gerade $n-1$ konfokale Quadriken.

Die Hamilton-Jacobi-Gleichung $H(\frac{\partial S}{\partial u}, u)=E$ kann bei dieser Koordinatenwahl durch Trennung der Variablen gelöst werden. Dies liefert eine kanonische Transformation auf Wirkungs-Winkel-Variablen. Die Integrale der Bewegung, die zunächst als willkürliche Integrationskonstanten auftreten, erhalten noch eine Deutung als elementarsymmetrische Funktionen der Parameter derjenigen $n-2$ Quadriken, die von der Tangente an eine Geodätische im Lauf der Bewegung tangiert werden.

W. Ingrisch, Erlangen

Toda - Gitter

Im Vortrag wurde der folgende Artikel von Moser referiert: Three integrable hamiltonian systems connected with isospectral deformations, Adv. Math. (1975). Die Notation des Lax-Paares ($\dot{L} = [B, L]$) und der isospektralen Deformation wurde erklärt. Als erstes Beispiel wurde das System

$$\dot{a}_k = a_k(a_{k+1}^2 - a_{k-1}^2), \quad k = 0, \ldots, n, \quad a_0 = a_n \equiv 0$$

in Form eines Lax-Paares geschrieben. Als zweites Beispiel wurde das Toda-Gitter in eine solche Form gebracht. Das konstante Spektrum der Matrix L hängt in diesem Fall zusammen mit den Streuzuständen des Systems.

W. Barth, Erlangen

Isospektralmengen für Sturm-Liouville-Operatoren auf $[0, 1]$ mit Dirichlet-Bedingung

Im Vortrag wurde ein großer Teil des Buches "Inverse Spectral Problems" von J. Pöschel und E. Trubowitz (Academic Press, 1987) im "Erzählerton" vorgestellt:

1. Gerade L^2-Potentiale q werden durch das Dirichlet-Spektrum $\mu_1, \mu_2, \ldots$ der Gleichung $-y'' + q(x)y = \lambda y$ eindeutig bestimmt.
2. Beliebige L^2-Potentiale q werden durch das Dirichlet-Spektrum $\mu_1, \mu_2, \ldots$ und die "Normierungs-Konstanten" $\kappa_1, \kappa_2, \ldots$ (= "Endgeschwindigkeiten"

der Lösungen des Dirichlet-Problems, die Anfangsgeschwindigkeit 1 haben) eindeutig bestimmt.

3. Jede Zahlenfolge mit Wachstum $\mu_n = n^2\pi^2 + \text{const} + \ell^2$-Rest tritt als Dirichlet-Spektrum eines L^2-Potentials auf.

4. Die "Isospektralmengen" $M(p) = \{q \mid q \in L^2,\ \mu_n(p) = \mu_n(q),\ n = 1, 2, \dots\}$ sind reell-analytische Untermannigfaltigkeiten in L^2 und diffeomorph zum linearen Raum h^1 aller Folgen ξ_n mit $\sum_n n^2 \xi_n^2 < \infty$.

5. Die Normierungs-Konstanten $\kappa_1, \kappa_2, \dots$ bilden ein globales Koordinatensystem auf $M(p)$.

6. Die Abbildung $q \mapsto (\kappa_1(q), \kappa_2(q), \dots; \mu_1(q), \mu_2(q), \dots)$ ist ein globales Koordinatensystem auf L^2, welches sogar symplektisch bzgl. der Poisson-Klammer $\{F, G\} = \langle \frac{\partial F}{\partial q}, \frac{d}{dx} \frac{\partial G}{\partial q} \rangle$ ist.

Jürgen Appell, Augsburg

Das Neumann Problem

Carl Neumann untersuchte in 1859 eine reibungsfreie Bewegung auf der $(n-1)$-Sphäre unter dem Einfluss einer linearen Kraft $-Aq$, also die Differentialgleichung ~~unter dem Ei~~

$$\ddot{q} = -Aq + \nu q \qquad \text{mit} \quad A = \operatorname{diag}(\alpha_1, \dots, \alpha_n)$$

wobei $\nu(q)$ so bestimmt wird, das der Punkt q auf der Kugel bleibt.
Man bekommt diese Differentialgleichung auch wenn man die Hamiltonfunktion

$$H = \tfrac{1}{2} \langle Aq, q \rangle + \tfrac{1}{2} \left(|q|^2 |p|^2 - \langle q, p \rangle^2 \right)$$

einschränkt auf S^{n-1}. Das Hamiltonsystem ist volständig integrabel.
Man nimmt sich eine Schar von konfokalen Quadriken,

$$Q_z(x, x) + 1 = 0 \qquad \text{mit} \quad Q_z(x, y) = \langle (zI - A)^{-1} x, y \rangle .$$

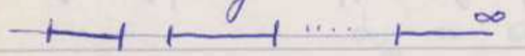
Die Funktionen $\Phi_z(x, y) = (1 + Q_z(x, x))\, Q_z(y, y) - Q_z(x, y)^2$ sind Integralen für die Geodätischen auf dem Ellipsoid $Q_0(x, x) + 1 = 0$ und $\Phi_z(p, q)$ sind Integralen für das Neumannproblem.
$\Phi_z(x, y) = 0$ definiert die Tangentialkegel an $Q_z(y, y) + 1 = 0$.
Nach Knörrer bildet die Gauß-abbildung die Geodätischen auf dem Ellipsoid (in geeignete Parametrisierung) ab auf de Lösungen von das Neumannproblem mit matrix $-A^{-1}$, und in gewissene Sinne is die Umkehrung auch wahr.

D. Siersma, Utrecht

Beziehungen zwischen dem Schrödinger Operator $L = -d^2/dx^2 + q(x)$, der Korteweg-de Vries Gleichung und dem Neumannschen Problem. (nach Jürgen Moser)

Der Operator L wird für quasiperiodische q auf ganz $\mathbb{R}$ mit Werten in $L^2(\mathbb{R})$ betrachtet. Es sei $G(x,y;\lambda)$ der Integralkern der Resolvente $(L-\lambda)^{-1}$. Der Mittelwert $w(\lambda) = M\left(\frac{1}{2G}\right) = \lim\limits_{x\to\infty} \frac{1}{x}\int_0^x \frac{dt}{2G(t,t,\lambda)}$ besitzt eine asymptotische Entwicklung $w(\lambda) = i\sqrt{\lambda}\left(1 + \frac{w_1}{\lambda} + \frac{w_2}{\lambda^2} + \dots\right)$,

wobei z. B. $w_3 = M(2q^3 + q'^2)$ ist. Auf dem Raum aller q's läßt sich ein Fréchetscher Differentialkalkül entwickeln, der jedem differenzierbaren Funktional H einen Gradienten ∇H zuordnet. Wenn man den Operator $\begin{pmatrix} 0 & I_n \\ -I_n & 0 \end{pmatrix}$ auf $\mathbb{R}^{2n}$ durch d/dx auf den Raum der q's ersetzt, kann man in Analogie die Hamiltonsche Differentialgleichung $\partial q/\partial t = d/dx\, \nabla H$ bilden. Dies ist für $H = w_3$ die KdV-Gleichung. Sie besitzt unendlich viele erste Integrale $w_1, w_2, w_3, w_4 \dots$

Über die Lösung des Neumannschen Problems gelingt es, zu einem Bandspektrum

endlich vieler Intervalle ein L zu finden, so daß $G(x,x;\lambda)$ als Funktion von λ auf die zweiblättrige, in den Endpunkten des Spektrums verzweigte Überlagerung der λ-Ebene meromorph fortgesetzt werden kann.

K. Lamotke (Köln).

Das Dirichlet-Spektrum eines eindimensionalen Schrödinger Operators.

In diesem Vortrag wurde der zweite Teil des Buches "Inverse Spectral Theory" von Pöschel und Trubowitz behandelt.

Es wurde gezeigt, dass die Isospektralmengen des Schrödingeroperators mit Dirichlet-Randwerten unendlich dimensionale, unbeschränkte Mannigfaltigkeiten sind. Diese können als invariante Flächen eines unendlichdimensionalen Hamilton'schen Systems aufgefasst werden. Die Eigenwerte des Operators (aufgefasst als Funktionen des Potentials) bleiben konstant auf den Isospektralmengen, und sind die Integrale des Systems. Die Lösungen des Hamilton'schen Flusses sind in geeigneten Koordinaten Geraden, d.h. das System ist vollständig integrierbar. Ausserdem wurde gezeigt, dass der Hamilton'sche Fluss entlang den Isospektralmengen explizit integriert werden kann, d.h. die Lösungen können in geschlossener Form angegeben werden. Dies erlaubt dann die vollständige Charakterisierung der Spektren des Schrödinger-Operators.

Bernhard Ruf (Köln)

Isospektralmengen des Hill's Operator

Im Vortrag wurde der Artikel „Hill's Operator and Hyperelliptic Function Theory in the Presence of Infinitely Many Branch Points" von H.P. McKean und E. Trubowitz referiert.

Die isospektrale Menge des Hill's Operators $Q = -D^2 + q$, $q \in C^\infty(S^1)$, kann mit einem reellen Torus $(S^1)^g$ identifiziert werden und dieser mit einer „Jacobi"-Abbildung in einen komplexen Torus $\mathbb{C}^g/\Gamma$ eingebettet werden. $\mathbb{C}^g/\Gamma$ ist durch eine Riemannsche Fläche $S \sim \sqrt{-(\lambda-\lambda_0)\dots(\lambda-\lambda_{2g})}$ gegeben.

Peter [illegible] (Bochum)

Eine algebro-geometrische Methode zur Lösung der Korteweg-de Vries und ähnlicher Gleichungen

Nach Burchnall/Chaundy (1922-1928) und Krichever (1976) gibt es eine eineindeutige Entsprechung zwischen Quadrupeln $(X, P, \mathcal{F}, \varphi)$ aus einer vollständigen irreduziblen reduzierten Kurve über dem Grundkörper k der Charakteristik Null, einem nichtsingulären k-rationalen Punkt $P \in X$, einer torsionsfreien Garbe $\mathcal{F}$ vom Rang Eins mit $h^0(\mathcal{F}) = h^1(\mathcal{F}) = 0$, und einem Isomorphismus $\varphi: T_{X,P} \to k$ und Äquivalenzklassen modulo Konjugation mit Elementen von $k[[t]]^*$ von kommutativen Ringen $R \subseteq k[[t]][\frac{d}{dt}]$, die k enthalten sowie zwei Operatoren A, B mit höchstem Koeffizienten Eins und coprimen Ordnungen. Auf den Quadrupeln (X, P, F, φ) ~~operiert~~ definiert eine Umgebung von $\mathcal{O}_X$ in $\mathrm{Jac}\, X$ einen Fluß $(X, P, F \otimes L, \varphi)_L$, dem eine Familie $D_s : R \to k[[t]][\frac{d}{dt}]$ von Einbettungen entspricht. Für festes $a \in R$ definiert dies eine Familie $D_s(a)$ von Operatoren; ist b eine Funktion aus $R = \Gamma(X \setminus P, \mathcal{O}_X)$ mit einem Pol der Ordnung k in b, so erhält man für eine geeignete einparametrige Familie von $L \in \mathrm{Jac}\, X$ eine Deformation $D_s(a)$ zu einem Lax Paar $\frac{d}{ds} D_s(a) = [D_s(a), D(f)^{f/k}_+]$.

<u>Beispiel:</u> X hyperelliptisch, P Weierstraßpunkt, F mit $h^0(F) = h^1(F) = 0$, beliebig, φ beliebig, $a \in \Gamma(X, \mathcal{O}_X(2P))$, $D_0(a) = \left(\frac{d}{dt}\right)^2 + \alpha(s,t)$,
$\frac{d}{ds} D_s(a) = [D_s(a), D_s(a)^{3/2}_+] = -\frac{1}{4}\left(\frac{\partial^3 a}{\partial t^3} + 6\alpha(t)\alpha'(t)\right)$, d.h. $\alpha(s,t)$ genügt der Korteweg-de Vries Gleichung

$$\frac{\partial a(s,t)}{\partial s} = -\frac{1}{4}\frac{\partial^3 a}{\partial t^3} + 6a\frac{\partial a}{\partial t},$$

W. K. Seiler (Mannheim)

Die Methode von S. Kowalewski zum Auffinden integrabler Hamiltonscher Systeme.

In dem Vortrag wurde ein Kriterium von S. Kowalewski vorgestellt, das besagt, daß sich ein $2n$-dimensionales Hamiltonsches System, dessen Definition sich über die komplexen Zahlen ausweiten läßt, nur dann mit abelschen Funktionen vollständig integrieren läßt, wenn es $(2n-1)$-dimensionale Familien von komplexen Lösungen mit Polen in beliebigen Zustandsvariablen gibt.
Die Anwendung dieses Kriteriums auf den Fall der Bewegung eines starren Körpers um einen festen Punkt zeigt, daß neben den Fällen des kräftefreien Kreisels und des Lagrange-Kreisels höchstens noch der heute so genannte Kowalewski-Kreisel mit zwei gleichen und einem halb so großen Hauptträgheitsmoment und der Aufhängung in der Ebene der beiden gleichen Trägheitsmomente mit abelschen Funktionen integriert werden kann.

C. Kahn (Bonn)

Das Schottky-Problem und die KP-Gleichung (I)

In diesem Vortrag wurde über Arbeiten von Shiota, Arbarello-de Concini und Welters referiert. Bekanntlich lautet das Schottky-Problem wie folgt: Wann kommt eine Matrix

$\tau \in \mathcal{H}_g = \{\tau \in M(g \times g, \mathbb{C}),\ \tau = {}^t\tau,\ \operatorname{Im} \tau > 0\}$ (Siegelsche ob. Halbebene)

von einer Jacobischen Varietät? Ausgehend von der Fay'schen Trisekantenformel wurde gezeigt, daß die Kummer-Varietät $\mathrm{Km}(X) \subseteq \mathbb{P}^{2^g-1}$ einer Jacobischen eine nicht-triviale Familie von Trisekanten besitzt. Das Kriterium von Gunnings besagt andererseits, daß jede prinz. pol. abelsche Varietät (irreduzibel), deren ~~Jacobische~~ ~~genügend~~ Kummerfläche genügend viele Trisekanten besitzt, bereits eine Jacobische ist. Dieses Kriterium wurde von Welters auf Wendetangenten verallgemeinert. Schließlich wurde gezeigt, wie das Weltersche Kriterium in die Sprache der Differentialgleichungen übersetzt werden kann.

Klaus Hulek (Bayreuth)

Das Schottky-Problem (II)

Zur Lösung ist noch zu zeigen, daß alle Gleichungen bis auf eine überflüssig sind: Die Jacobi-Varietäten sind im Modulraum aller prinzipal polarisierten abelschen Varietäten dadurch ausgezeichnet, daß ihre Thetafunktion die Kodomcev-Petviašvili-Gleichung erfüllt. Das Problem wurde mit etwas algebraischer Geometrie auf die Lösung einer Differentialgleichung zurückgeführt, die man dann auch tatsächlich integrieren kann.

Thomas Höfer (Bonn)

Mathematical Methods in the Study of
Natural and ~~Programming~~ computer Languages,

Organizers: Jon Barwise, Jens-Erik Fenstad, Hans Kamp, Michael Richter

19 – 25 april 1987

* Mathematical Logic
19.-25.04.1987

Mathematics in the Study of Natural Language.

This talk was a survey of existing, as well as desirable future uses of mathematics in the study of natural language (as well as related topics, such as computer science or artificial intelligence). Examples of existing uses included applications to

Syntax: Formal Languages and Automata, Categorial Grammar and Type Theory, 'Logical Syntax'.

Semantics: Universal Algebra in Montague Grammar, Situation Theory. Logicality and Permutation Invariance, Finite Model Theory, Type-free Theories. Special topics: generalized quantifiers, temporal representation.

Inference: Natural Logics (large decidable fragments), Non-standard Consequence Relations (Non-monotonic reasoning), Interaction of Computation and Representation.

These applications often call for fine-structure of, or variants of existing parts of mathematics/mathematical logic. Moreover, in almost all cases, quite similar problems arise both in natural languages and programming languages.

Further areas which call for new mathematics are the study of larger structures (texts, discourses) as well as the dynamic aspects of interpretation (context change, side effects). There are already some promising attempts.

*Johan van Benthem (Amsterdam)

Syntactical Characterization of Theories which admit Initial Structures

The question 'Why universal Horn formulas matter in Computer Science?' (cf. Makowsky, LNCS 115) has a model theoretic and a proof theoretic answer. The model theoretic answer characterizes universal Horn theories as theories T which admit uniformly (i.e. for all consistent extensions by new facts (= insertions)) term models M which are generic for T (i.e. every element of M is denoted by a term and any fact is satisfiable in M iff its existential closure is derivable from T (≈ Closed World Assumptions)). Note that a term model M is generic iff it is initial for T (i.e. there is a unique homomorphism into any other model of T. Moreover, the existence of a generic model for T is equivalent to an irreducibility property of the theory.

We obtain the known result for universal Horn theories (cf. Malcev, Algebraic Systems) from the more general characterization of pseudo-universal Horn theories (= limit theories in Volger, Math. Zeitschr. 166 (1979)) as theories T which uniformly admit initial structures. Note that a structure is initial iff it is a pseudoterm structure which is generic for T. The missing link in Makowsky is the closure under equalizers of homomorphisms. This result helps to understand Malcev's result and it shows that up to a definitional extension by partial operations nothing can be gained using initial structures rather than term structures which are initial. – A related result characterizes the generic Horn theories which are axiomatized by formulas of the form $\forall \bar{x} (\alpha(\bar{x}) \rightarrow \exists \bar{y} \beta(\bar{x}, \bar{y}))$ with $\alpha, \beta \in \Lambda At$. – Adding some restrictions we obtain a syntactical characterization which yields Prolog-Programs i.e. universal Horn theories which are strict and non-identifying. In this context Herbrand structures, where each element is denoted by a unique term, are used instead of term structures.

H. Volger (Passau)

Resolution on Formula-Trees

We introduce a nonclausal resolution calculus on formula-trees which comprises classical resolution as a special case. The resolvents produced in this calculus are more structure preserving than in nonclausal resolution by Murray and Manna & Waldinger and simpler than in nested resolution by Traugott. Proofs of correctness and completeness are sketched. In some examples, first experiences made when implementing the calculus are discussed.

Ulf R. Schmerl,
München

*

Decision problems for PROLOG-programs

We give a general method for showing recursive unsolvability of various decision problems for programs in (extensions of) PROLOG. In particular we give a positive answer to the conjecture (made by Flannagan at the ASL meeting, Stanford '85) that the floundering property for queries in MU-PROLOG programs is undecidable.

Egon Börger
(Università di Pisa)

Semantics and Paradox

The semantical paradoxes pose a challenge to any mathematical account of language that contain negation and their own truth predicate, a property of all human languages. In this talk I presented such a language $\mathcal{L}$ and two semantical accounts of the language, one ~~with~~ a Russellian semantics, the other an Austinian semantics. The Russellian semantics is very similar to Kripke's account of truth. I then discussed various theorems that related the semantic behaviour of sentences on the two accounts. For example, for any sentence $\varphi \in \mathcal{L}$, φ is intrinsically paradoxical in the Russellian semantics iff for any actual situation s, the propositions expressed by φ about s and the proposition expressed by $\neg\varphi$ about s are both false. The talk was based on joint work with John Etchemendy, which has just appeared in <u>The Liar</u>, O.U.P.

Jon Barwise
Stanford University

Honest Polynomial-Time Reducibilities

An honest polynomial-time (hp-) reduction is a reduction which can be performed in polynomial time and in which the lengths of the queries are polynomially related to the length of the input. Extending work of S. Homer, we show that there are recursively enumerable sets of minimal degree w.r.t. to the standard hp-reducibilities provided that $P = NP$.

Klaus Ambos-Spies
Universität Dortmund

* Über Robinson-Charakterisierungen der einstelligen rekursiven Wortfunktionen.

Es werden drei interne induktive Charakterisierungen der Klasse Prim^1_A der einstelligen primitiv rekursiven Wortfunktionen über einem Alphabet $A = \{a_1, \ldots, a_r\}$ $(r \geq 2)$ gegeben. Insbesondere wird gezeigt, daß Prim^1_A die kleinste Klasse von einstelligen Wortfunktionen über A ist, die die Nachfolgerfunktionen $\sigma_i(w) := wa_i$ $(i = 1, \ldots, r)$ und entweder π oder ϱ enthält und abgeschlossen ist in bezug auf Superposition $\circ$, Verkettung $\hat{}$ und Wortiteration it_A. Dabei ist π bzw. ϱ die zu einer geeigneten primitiv rekursiven Paarbildungsfunktion $\gamma : A^* \times A^* \to A^*$ gehörige Links- bzw. Rechtsfunktion und $\circ$, $\hat{}$, it_A sind definiert durch

$$(\varphi \circ \psi)(w) := \varphi(\psi(w)), \quad (\varphi \hat{} \psi)(w) := \varphi(w)\psi(w)$$

$$it_A(\varphi_1, \ldots, \varphi_r) = \chi :\Leftrightarrow \chi(\Lambda) = \Lambda \ \& \ \chi(wa_i) = \varphi_i(\chi(w)).$$

Günter Asser
(Universität Greifswald)

* Subrecursive Hierarchies

The Slow-Growing (G), "Hardy" (H) and Fast-Growing (F) hierarchies were defined and their most significant properties outlined:

(1) $G(\alpha)$ represents α as a direct limit
(2) $H(\alpha)$ witnesses the (combinatorial) well-foundedness of α
(3) $F(\alpha) = H(\omega^\alpha)$.

These suggest the construction (from below) of a large class of so-called "accessible recursive functions". The definition follows standard recursion theoretic principles, using $G(\alpha)$ to code α:

i.e. generate $F(\alpha)$ only if $\exists \beta < \alpha \, (G(\alpha) = F(\beta))$.

Theorem For appropriate Bachmann-style collections $\overset{n}{\varphi} : \Omega_{n+1} \times \Omega_n \longrightarrow \Omega_n$ we have $G(\overset{n+1}{\varphi}) = \overset{n}{\varphi}$, where $\overset{0}{\varphi} = F$.

Corollary $G(\text{"}id_{n+1}\text{"}) = F(\text{"}id_n\text{"})$, so the hierarchy of accessible recursive functions closes off at "$id_{<\omega}$".

S. S. Wainer
(Leeds UK).

Quantification in Automatic
Theorem Proving
by
Robert S. Boyer
University of Texas at Austin

We described a theorem-proving program which has been used to check such results as quadratic reciprocity and Gödel's incompleteness theorem. We asked for help in finding or creating a logic with (a) the power of set theory, (b) the convenience of quantifier notation, (c) no bound variables, and (b) ease of hand proof. The apparent contradiction between (b) and (c) may be resolved by consideration of the support that the von Neumann-Bernays-Gödel set theory provides for the $\{x : p(x)\}$ notation with a finite axiomatization; however, this avenue violates condition (d). A technique was described to achieve (b), (c), and (b) but not (a).

Inductive definitions in type theory

We construct a formal type theory where we include some type-operators given in e.g. Per Martin-Löf type theory together with an operator forming the least fix-point of certain strictly positive type operators.
We may also define a family of types indexed over a type using simultanous induction.
The theory contains a general rule for proofs by induction and construction of types by recursion over the natural ordering of the inductively defined types.
Finally it is shown that the strictly positive type-operators are in a sense categorical and that this is provable within the theory

Dag Normann, Oslo, Norwegen

Shifting types in natural language semantics

Montague's strong form of compositionality requires a homomorphism from the syntactic algebra to the semantic algebra (Montague, "Universal Grammar", 1970), including the assignment of a unique semantic type to each syntactic category. But perhaps natural languages are better described with a framework which allows the assignment of a family of types to each syntactic category. I described some empirical linguistic data supporting such a view (including facts from cross-categorial conjunction and evidence of noun phrase type multiplicity) and presented some ideas for formal and empirical considerations in identifying "basic" types to assign to expressions lexically and "natural" type-shifting operations to provide constraints on the resulting system and predictions about preferred interpretations. The need for collaborative work on such problems by mathematicians and linguists was emphasized in the conclusions.

References: (1) B. Partee (1987) "Noun Phrase Interpretation and Type-Shifting Principles", in Groenendijk et al, eds. ... (Foris)
(2) B. Partee & M. Rooth (1983?), "Generalized Conjunction and Type Ambiguity", in Bäuerle et al, ... (de Gruyter)

Barbara H Partee April 23, 1987
Univ. of Massachusetts, Amherst

Annotations on the consistency of the closed world assumptions

In recent years the general theme of "Negative Information" has attracted a certain amount of attention, especially in the context of logic programming, logical data bases and the like. One of the most important concepts in this connection is Reiter's closed world assumption. In this talk we introduce the notion of "closed world of a theory T w.r.t. a sequence of predicates" and give natural conditions for its consistency.

Gerhard Jäger
ETH Zürich

Formal properties of parallel and sequential algorithms.

About half of the talk was devoted to a brief exposition of the theory of (pure, concurrent, no-side-effects) algorithms. After the basic definitions of the appropriate structures and the algorithms on them, this theory studies a language of terms, FLR = the formal language of recursion: the terms of FLR define the algorithms of the structure. The first and basic result of the theory is an axiomatization of the relation $s \sim t \iff$ the terms s and t define the same algorithm on all structures.

The second half of the lecture outlined the basic ideas in a recent extension of the theory to cover communicating algorithms, with side effects which change the state. The basic result is that the terms of FLR may be used to name such algorithms in a new modelling, and that the axiomatization of pure algorithm equality still stands: thus the formal properties of communicating algorithms are the same as those of pure algorithms.

Yiannis N. Moschovakis UCLA
April 24, 1987

The outcome of LEX

LEX (linguistics and logic based legal expert system) is a project currently being carried out (and nearly finished) at IBM's Heidelberg Scientific Center. Its aim is to develop a natural language understanding system which helps the lawyer to deal with cases of §142 StGB (German Criminal Law). Presented a description of a road traffic accident, it checks (with user interaction) whether the case is subsumed by §142. This check is carried out by searching for a proof (in the sense of Mathematical Logic) that the facts are fulfilled.
Though we made progress in some parts of the system (lexicon, grammar, resolution of referents, inference), the system as a whole seems, at the moment, to be unrealizable. Main reasons are:
- Some parts are underdeveloped, both in theory and in implementation.
- There is no unifying overall theory.

Wolfgang Schönfeld
23. April 1987

Generating Natural Language Descriptions of Image Sequences

The aim of the VITRA project (Visual TRAnslator) is the development of a computational theory of the relation between natural language (NL) and vision. We investigate two different conversational settings:

1) VITRA-CITYTOUR answers NL queries about spatial relations and recognized events. It is assumed that both dialog partners are located in the scene.

2) VITRA-SOCCER generates a report on a soccer game, which it is watching. The hearer does not see the scene.

In the talk we show how the semantics of path prepositions like 'past' and 'along' can be formalized and implemented. The geometrical representation we use for these discourse domains are closed polygons and trajectories in 2D space. Then, we present a formal analysis of the deictic and intrinsic use of spatial predicates. Finally, we discuss the role of a spatial imagination component for generating anaphoric expressions.

Wolfgang Wahlster, Univ. Saarbrücken
24. April 1987

NATURAL LANGUAGE SYSTEMS AND COMPUTATIONAL SEMANTICS.

A system for natural language analysis shall provide a framework for relating the linguistic form of utterances and their semantic interpretation. This calls for an extension of computational linguistics with its traditional emphasis on syntax and morphology to include a theory of computational semantics

Basic to the approach which we presented in this talk is an algorithm for converting linguistic form to a format which we call a situation schema. The algorithm is in the spirit of current unification based approaches to grammar and exploit the idea of constraint propagation.

A situation schema has a well-defined (algebraic) structure, suggestive of "logical form", but it is a structure different from the standard model-theoretic one. We will argue that it is a structure better adapted for the analysis of the meaning relation in natural langues and that it provides a format useful for further processing.

In the first part of the paper we provided the necessary background from the model-theory of partial information (situation semantics); in particular, we reported briefly on some mathematical investigations into situational logic.

Jens Erik Fenstad.

Background to Situation Theory

We motivate the development of a theory of the structures required by an account of information-content. The theory involves a variety of sorts, including the postulation of a subdomain of relations, taken as primitive and as intensional, and of propositions, construed as structured complexes. Various local + global relations were described. A crucial requirement is that to each of the axioms of the theory there corresponds an object of the theory

David Israel
24/04/87

Discourse Representation Theory describes in detail how sentences are interpreted in the light of the preceding part of the discourse or texts to which they belong. Central to the theory are Discourse Representation Structures, or DRS's. A DRS acts simultaneously as a representation of the joint content of the sentences that have been interpreted already and as the context for the sentence that comes next.

When the next sentence is processed relative to the DRS this results in a new DRS which incorporates the content of the old DRS as well as the contribution which the new sentence makes to that content. Thus sentence interpretation can be described as a function from DRS's and (syntactic parses of) sentences to DRS's. Sentence meaning should no longer be thought of in terms of the proposition a sentence expresses, but rather as the capacity of the sentence to modify given DRS's into new ones, normally with stronger content. (Cf the file change potential of I. Heim).
Discourse Representation Theory seems particularly well-equipped to handle intersentential connections such as pronominal or temporal anaphora. The talk presented a number of sample texts and showed how the processing algorithm converts into DRS's with intuitively correct truth conditions.

Hans Kamp, April 23, 1987

Noun Phrases and Quantified Terms
To resolve anaphoric references (see H. Kamps Summary above) or to identify scopes it would be nice to have a formal language with quantifiers within terms, and marking pronouns by the quantified variables. We argued against that kind of modelling in favor of a theory which satisfies some properties needed in parsing.
A formal language was presented with the aim of keeping syntax as close to the syntax of German as possible. The fragment we gave included various kinds of noun phrases, locative prepositions and relative clauses. A translation into 1.st order predicate logic, with additional generalized

quantifiers (2.nd order relations) and a means of modifying predicates by prepositions and terms, was presented. Essentially, noun phrases translate to quantifier blocks with ~~q~~ relativized quantifiers, prepositions translating to predicate modifiers.
On the semantical side, we tried to strongly make plausible that the notion of consistency property familiar from logic (Henkin's construction) has to be refined to give a mathematically precise notion of 'coherence' (or formal understandability) of texts. At the very least, the analog of the model construction has to make essential use of the linear ordering of texts – as opposed to sets of sentences in logic – to be able to resolve anaphora during the construction. The 'admissible' steps in the construction have to be defined in terms of consistency plus a series of linguistic restrictions, which partly seem to have been made explicit in Kamps DRS theory.

Hans Leiß, 24. 4. 87

* Incompleteness and proof complexity

Gödel's second incompleteness theorem is equivalent to non-trivial lower bounds (LB) on proof complexity. Therefore usual formal theories T of mathematics cannot prove any non-trivial LB_T on their proof complexity. We set up wide-spread examples of provable formulas F_n (some consisting only of Boolean ones) assuming such LB for elementary or NP≠P-reasons thus constituting a new type of formal proof limit which can only be overcome by making almost all (proved) F_n axioms. So a smaller ideal on the adequacy of formalization is also destroyed, namely that every recognized set of new true statements [like $LB_S(F_n)$ for all consistent $S \supseteq T$] can be formally derived by adding suitable new (true) formulae as axioms. – Our Boolean examples also give indirect hints in favour of Cook's $NP \neq P$-thesis and point out some proof-theoretic difficulties a proof has to overcome. – Finally we obtain formal undecidability of the following well-known theorems: recursive undecidability, proof-theoretic Π_1^0-uniformity and proof speed-ups.

H. Zuckhardt (Frankfurt a. M.)

* Syntax of E-logic

Stimulated by the observation that the class of Kripke structures mentioned by D. Scott in LNM 753 is not complete for E-logic, M. Unterhalt from Münster gave a complete class in a study of the semantics of E-logic (1986), and the following is joint work with him.

We develop a general concept of rule in natural deduction. A theory is then given by its language and its specific rules (instead of axioms only). Besides standard redexes and simplifications, we consider as permutation redex any formula occurrence, that is conclusion of $\vee\exists E$ and major premiss of an E-rule. By standard reduction techniques, any natural deduction reduces to a normal (redex-free) deduction. We give an inductive definition of these normal deduction. After applying the crude discharge convention, this leads to:

Proposition. There is an isomorphism between normal deductions and cut-free sequent derivations.

For the large class of prime theories, the hour-glass theorem holds with the usual proof-theoretic consequences, e.g. disjunction property and existential definability.

Whereas in theories with global existence, non-logical rules may be replaced by axiom-schemata, M. Unterhalt could show by a semantic argument that in proper E-logic there are non-axiomatizable rules, e.g. the Görnemann-rule characterizing Kripke-structures with constant domains.

J. Diller (Münster)

Correspondences between DRT, Situation Schema Theory and Situation Semantics.

Over the past 10 years several new grammatical and semantical theories in the study of natural language have emerged: the Discourse Representation Theory (DRT) developed by Hans Kamp [1], the Situation Semantics by Jon Barwise and John Perry [2], and the Situation Schema Theory by Jens Erik Fenstad et. al [3].

In the first part of the lecture we discussed the relationship between DRS (Discourse Representation Structures) and Situation Schemata. Given a lexicon and a set of phrase structure rules we can for each sentence φ of the language construct an associated DRS which consists of a structured set of conditions, atomic conditions ($\alpha = u, \alpha(u), \alpha(u,v)$) and complex conditions ($m_1 \to m_2$). The DRS represents a unique reading of the sentence with respect to scope order and coreference. From the corresponding situation schema of φ we can extract exactly the same atomic fact schemata, such that given a suitable Q-mode (quantifier scope reading) and coreferential conditions we get the same first order transcription of the DRS(φ) and SIT.φ.

In the second part of the talk we discussed a Situation Semantics interpretation of a DRS. In Situation Semantics the meaning of a sentence φ is a conventional constraint between the utterance situation and the situation described: $[\![\varphi]\!]$: involves, DC_φ', S_φ; 1

From a DRS we can systematically construct the event-type of S_φ, the described situation, giving an interpretation of the DRS into situation semantics.

[1] Kamp, H, 1981, "A Theory of Truth and Semantic Representation", in Groenendijk, J. et. al. (eds) "Formal Methods in the Study of Language", Amsterdam.

[2] Barwise, J., and J. Perry, 1983, "Situations and Attitudes", The MIT Press

[3] Fenstad, J. E., P. K. Halvorsen, T. Langholm and J. van Benthem, 1987, "Situations, Language and Logic", Reidel

Helle Frisak Sem

Generalized Quantifiers and Anaphora

Different kinds of noun phrase in natural languages express different sorts of semantic content. For example, the following English noun phrases have the syntactic and semantic types shown.

Noun phrase	Syntactic variety	Content sort
John	proper name	individual
she	pronoun	
the lecturer	definite description	
a logician	indefinite description	
no linguist	quantified phrase	generalized quantifier
most tables		
two theorems	numeral phrase	

The content of any phrase varies systematically with the circumstances in which the phrase is used; it is determined jointly by the meaning of the phrase and the context of utterance.

A generalized quantifier can be regarded, in extension, as a set of subsets of the domain of quantification. For instance, the content of "most tables" is $\{A \subseteq D \mid R(T, A)\}$ in a context where the determiner "most" expresses the relation R between sets, the noun "tables" corresponds to the set T, and D is the domain of quantification. (R might be the relation such that $R(X, Y)$ iff there is no function from $X \setminus Y$ onto $X \cap Y$.)

A pronoun can function either deictically (indexically) or anaphorically, depending on context. Deictic uses are, in effect, parameters whose values are fixed by context. Anaphoric uses of pronouns subdivide into two kinds.

(1) A co-parametric use of "his" in "John saw his watch" gives the sentence the content that $j \in \{x \mid x \text{ saw } j\text{'s watch}\}$.

(2) A role-linking use of "his" gives the same sentence the content that $j \in \{x \mid x \text{ saw } x\text{'s watch}\}$.

The difference between co-parametric and role-linking uses becomes clear in considering sentences like "John saw his watch and so did Bill".

(1) $j \in \{x \mid x \text{ saw } j\text{'s watch}\}$ & $b \in \{x \mid x \text{ saw } j\text{'s watch}\}$.

(2) $j \in \{x \mid x \text{ saw } x\text{'s watch}\}$ & $b \in \{x \mid x \text{ saw } x\text{'s watch}\}$.

Very different properties are ascribed to Bill.

An anaphorically used pronoun with a quantifying noun phrase such as "no logician" as its antecedent can only serve for role-linking. E.g., "No logician saw his watch" can express $\{x \mid x \text{ saw } x\text{'s watch}\} \in \{A \subseteq D \mid \text{logicians} \cap A = \emptyset\}$. Accordingly, "No logician saw his watch, but John did" cannot express that John saw every logician's watch but no logician saw his own.

Stanley Peters

Locality and Linguistic Structure

Different grammatical formalisms are characterized by different domains of locality. For example, each rewriting rule in a context-free grammar (CFG) constitutes a domain of locality for CFG. Head Grammars (HG), Tree Adjoining Grammars (TAG), Categorial Grammars (CG), Indexed Grammars (IG), etc. all have different domains of locality. The particular domains of locality for a given grammatical formalism has implications for the specification of constituency, constraints (e.g. agreement), function-argument relationships, word-order variations, and characterization of unification constraints. The elementary trees of a TAG provide a larger domain of locality as compared to CFG. This particular domain of locality enables one to <u>localize</u> all the so-called long-distance dependencies in natural languages. The long distance nature of these dependencies then comes out as a byproduct of the operation of composition, called <u>adjunction</u>, This locality constrains the flow of information with respect to feature compatibility checking in unification framework. ~~This embedding~~ I described how TAG's can be embedded in the unification framework. This embedding results in a constrained unification based framework. The precise nature of the complexity results due to these constraints is still being worked on. An exact semantics for this formalism can be given via a recursive transition network, as contrasted with the finite state automaton which has been used for specifying the semantics for feature-structures by Aït-Kaci and Rounds and Kasper.

I also described results which show how a variety of systems such as HG, IG (restricted), and others are all equivalent to TAG.

Aravind Joshi.

An unusual truth definition for Δ_0-formulas.

We define a Δ_1 universal relation for Δ_0-formulas in the standard model $\mathbb{N}$. The construction of the relation uses König's lemma on finitely branching trees instead of using the existence of Skolem functions for Δ_0-formulas.

Zofia Adamowicz

MacLane Set Theory

Saunders MacLane has emphasised the rôle in mathematics of the two axiom systems M = Extensionality, Null set, Pair set, Union, Power set, Foundation, Infinity and Δ_0 comprehension scheme, and MAC = M + "Every set has a wellordering."

The following comparisons with KP (= Kripke-Platek, formulated as M less Power set plus Δ_0 collection and Π_1-foundation schemata) and with Quine's ~~system~~ concepts of stratified formulae were made:

Thm 1 $\mathrm{Con}(M) \Rightarrow \mathrm{Con}(MAC + KP)$

Thm 2 (following Boffa & Coret) MAC + KP is conservative over MAC for stratified wffs; ~~[illegible]~~ every stratified instance of replacement is provable in MAC.

Thm 3 There is an instance of stratified collection not provable in MAC, since the following (stratifiable) statement is unprovable in that system: "there is an infinite set of infinite sets, no two with the same cardinal."

ARD Mathias

Extensional partial combinatory algebras

We formulate a general method for the construction of non-total λ-models.

Thm. 1 (following Scott) Let D be a CPO and $[D \rightarrow_s D] = \{f \in [D \to D] \mid f(\perp_D) = \perp_D\}$. Let $F: D \to [D \rightarrow_s D]$, $G: [D \rightarrow_s D] \to D$ be continuous maps such that

- range $G \subseteq D \setminus \{\perp_D\}$
- $F \circ G = id_{[D \rightarrow_s D]}$.

Define for $d, d' \in D \setminus \{\perp_D\}$

$$d \cdot d' = \begin{cases} F(d)(d') & \text{if } F(d)(d') \neq \perp_D \\ \text{undefined} & \text{otherwise.} \end{cases}$$

(i) $\mathcal{M} = (D \setminus \{\perp_D\}, \cdot)$ can be considered as a non-total λ-model such that the representable functions on $\mathcal{M}$ are exactly the partial continuous functions on $D \setminus \{\perp_D\}$.

(ii) Moreover, $\mathcal{M}$ is extensional, i.e.

$\forall d, d' \in D \setminus \{\bot_D\} \ (\forall d'' \in D \setminus \{\bot_D\} \ d \cdot d'' \simeq d' \cdot d'' \rightarrow d = d')$
iff $G \circ F = id_{D \setminus \{\bot_D\}}$. ∎

As an example of how a CPO satisfying the domain equation $D \setminus \{\bot_D\} \cong [D \underset{S}{\rightarrow} D]$ can be constructed we describe a modification of the free PSE-algebra (Plotkin-Scott-Engeler) generated by an arbitrary poset A with bottom $\bot_A$: let $G(A)$ be the closure of A under ordered pairs of non-empty, finite subsets and elements. Define the extensional collapse of $\mathcal{P}(G(A))$ by

- $X \equiv Y$ iff $X \sqsubseteq Y$ and $Y \sqsubseteq X$
- $X \sqsubseteq Y$ iff $\forall x \in X \ \exists y \in Y \ x \sqsubseteq y$
- $x \sqsubseteq y$ holds if either
  (i) $x = y$ or
  (ii) $x = \bot_A$ or
  (iii) $x \in A$ & $y = (\{\bot_A\}, b)$ & $x \sqsubseteq b$ or
  (iv) $x = (B, b)$ & $y \in A \setminus \{\bot_A\}$ & $b \sqsubseteq y$ or
  (v) $x \sqsubseteq_A y$ or
  (vi) $x = (B, b)$ & $y = (C, c)$ & $C \sqsubseteq B$ & $b \sqsubseteq c$.
- $\mathcal{M} = (\mathcal{P}(G(A)) \setminus \{\emptyset\} / \equiv, \cdot)$, where
  $[X] \cdot [Y] = [\{b \mid \exists B \sqsubseteq Y \ \{(B, b)\} \sqsubseteq X\}]$.

The resulting non-total extensional PCA $\mathcal{M}$ has the following properties:

Thm. 2 $\mathcal{M}$ has neither a completion nor a total submodel.

Thm. 3 There are unsolvable λ-terms $T_0, T_1, \dots T_\infty$ such that
$[\![T_0]\!]^{\mathcal{M}} \subsetneq [\![T_1]\!]^{\mathcal{M}} \subsetneq \dots \subsetneq [\![T_\infty]\!]^{\mathcal{M}} = \sup\{[\![T_n]\!]^{\mathcal{M}} \mid n \in \omega\}$.

Ingeborg Bethke

An open problem concerning intensional continuous functionals.

Let APP be the "applicative part" of Feferman's theory of operators and classes, i.e. the part without the ~~open~~ classes, but with operators, combinatorial constants $\underline{k}$, $\underline{s}$, etc., a predicate N for the natural numbers, induction.

Within any model of APP we can define an intensional type structure $IT_{\mathcal{M}} = \langle IT_{\mathcal{M},\sigma} \rangle_{\sigma \in \mathcal{T}}$, where $\mathcal{T}$ is the collection of finite types containing 0 $(= \mathbb{N})$, closed under $\rightarrow$ and $\wedge$ (cartesian closed):

$$IT_0 = N,$$
$$IT_{\sigma\to\tau} = \{x : \forall y \overset{\in I_\sigma}{} (xy \in I_\tau)\}$$
$$IT_{\sigma\times\tau} = \text{pairs of } I_\tau, I_\sigma$$

Question: is there a model $\mathcal{M}$ for APP such that

$[\![IT_0]\!]_{\mathcal{M}} = IT_{\mathcal{M},0} = N \cong \mathbb{N}$,

$[\![IT_1]\!]_{\mathcal{M}} = IT_{\mathcal{M},1} \cong \mathbb{N} \to \mathbb{N}$,

$[\![IT_2]\!]_{\mathcal{M}}$ consists of objects continuous on $\mathbb{N} \to \mathbb{N}$, i.e.

$$\mathcal{M} \models \forall z^2 \forall \alpha^1 \exists x^0 \forall \beta^1 \in \bar{\alpha}^1 x^0 (z^2\alpha = z^2\beta)$$

$[\![IT]\!]_{\mathcal{M}}$ ~~consists~~ contains a modulus of continuity functional for the type two objects.

The question may be given a more specific term: let ICF be the model of the intensional continuous functionals:

$$ICF_0 = \mathbb{N}, \quad ICF_{0\to 0} = \mathbb{N} \to \mathbb{N},$$
$$ICF_{0\to\sigma} = \text{sequences of objects of } ICF_\sigma \quad (\sigma \neq 0)$$
$$ICF_{\sigma\to 0} = \{\alpha : \forall \beta \in ICF_\sigma (\alpha(\beta)\downarrow)\} \quad (\sigma \neq 0)$$
$$ICF_{\sigma\to\tau} = \{\alpha : \forall \beta \in ICF_\sigma \exists \gamma \in ICF_\tau (\alpha|\beta = \gamma)\} \quad (\sigma, \tau \neq 0)$$

where
$$\alpha(\beta) = x \leftrightarrow \alpha(\min_z [\alpha(\bar{\beta} z) > 0]) \dot{-} 1 = x$$
$$\alpha|\beta = \gamma \leftrightarrow \forall x (\alpha_{\langle x \rangle}(\beta) = \gamma x) \text{ where}$$
$$\alpha_{\langle x \rangle} = \lambda n . \alpha(\langle x \rangle * n).$$

Can ICF be isomorphically embedded as $[\![IT]\!]_{\mathcal{M}}$ in a model $\mathcal{M}$ of APP?

A.S. Troelstra.

Some Problems ~~between~~ on the Borderline between Natural Languages, Computer Languages and Logic Arising from Expert Systems.

In Expert Systems one faces the problem of representing different kinds of knowledge in an adequate manner. Natural languages have the most flexibility but their implementation leads to slow programs; computer languages are fast but have little "expressive power". Between those two levels we have various knowledge representation languages, which try to combine the positive aspects of both sides. Here we discuss the following problems:

(1) Amalgamation of logic, functional and object oriented languages together with (polymorphic) types
(2) Horizontal information flow in object oriented approaches
(3) Hypothetical and default reasoning
(4) The logic of question and answer
(5) The interval time logic.

Michael M. Richter
(Kaiserslautern)

Functions provably total in $I^-\Sigma_n$.

Let $I^-\Sigma_n$ denote the theory of Σ_n-induction without parameters, in the language of PA. The following theorem is a generalization to the case $n \geq 2$ of the theorem proved for $n=1$ by Z. Adamowicz and T. Bigorajska. Let $n \geq 2$.

Theorem. If in the proof of the totality of a recursive function $f: N \to N$ in the theory $I^-\Sigma_n$ we use only m-different axioms of Σ_n-induction then f can be bounded (almost everywhere) by a function $H_{\omega_{n-1}^{\omega^m k}}$ in Hardy's hierarchy for a certain $k \in \omega$.

The estimation of the rapidity of growth for $n=1$ which we obtain by [1] is $H_{\omega^{m+2} \cdot k}$ for a certain $k \in \omega$. In the proof we use only the usual semantical method and combinatorial consideration. We do not need to analyse ~~the~~ proofs but only provability. In comparison with the proof in [1] we need the notion of largeness and the notion of approximations. We use the properties of approximations proved in [2]

[1] Z. Adamowicz, T. Bigorajska, Functions provably total in $I^{-}\Sigma_1$, to appear in F.M.

[2] Z. Ratajczyk, A combinatorial analysis of functions provably recursive in $I\Sigma_n$, to appear in F.M.

Zygmunt Ratajczyk

Strong Existential Types and Normalization in the Second Order Typed λ-calculus

The second order typed λ-calculus of Girard is extended with Martin-Löf's dependent types and existential types with the strong introduction and elimination rules of Howard. It is shown that there are non-normalizable terms in the resulting calculus. This would seem to support the view that impredicativity is incompatible with the identification of types and propositions.

Edward Griffor

On modified Kruskal–Friedman combinatorics.

In comparison to the talk given last time in Oberwolfach here I present a more precise estimation of the proof theoretic strength of the modified combinatorial statements
M^n_α (α – countable ordinal) : „ for every infinite sequence $\langle T_1, l_1\rangle, \langle T_2, l_2\rangle, \ldots, \langle T_n, l_n\rangle, \ldots$ of $(\leq n)$–branching trees T_n with labeling functions $l_n : T_n \to \alpha \ (=\{\beta : \beta < \alpha\})$ there are $i<j$ and a homeomorphism $h : T_i \to T_j$ satisfying :
(1) $l_i(x) \leq l_j(h(x))$, for every node $x \in T_i$,
(2) if x and x' are neighbours in T_i and y a node between $h(x)$ and $h(x')$ in T_j, then $l_j(y) \geq \min\{l_i(x), l_i(x')\}$.
It turns out that from the proof theoretical viewpoint the hierarchy $\{M^1_\alpha\}_\alpha$ corresponds to the predicative hierarchy $\{(\Pi^1_0\text{-CA})_\alpha\}_\alpha$ while for $n>1$ each hierarchy $\{M^n_\alpha\}_\alpha$ is analogous to the impredicative hierarchy $\{ID_\alpha\}_\alpha$ (these theories are familiar in the literature).

28. Tagung zur Geschichte der Mathematik

2.

26.4. – 1.5. 1987

The Determination of tawassut (mediatio) in Arabic Astronomy.

The mediatio, or the co-culminating point of the ecliptic, was a recognized coordinate of a star in medieval Arabic astronomy. It is usually found in combination with declination or (especially in instrument-texts) with the distance between the co-culminating point and the star. In the Qānūn al-Bīrūnī (11c.) treated the determination of declination and mediation from longitude and latitude, and vice-versa, almost at the beginning of the section on spherical astronomy. The methods of the Qānūn & other works by al-Bīrūnī will be compared with those of the tabulae probatae (9c) & al-Battānī (c. 900) as well as Ptolemy (2c), to illustrate the development of spherical trigonometry up to al-Bīrūnī's time.

Richard Lorch.

Probable influences of Arabian algebra on the development of algebra in Italy in the fourteenth and fifteenth centuries – R. Franci –

The study of many manuscripts of the 14th and 15th centuries has shown that algebra, in Italy, greatly developed in these centuries (see. Toward a history of algebra from Leonardo of Pisa to Luca Pacioli. JANUS, 72, 1985, 17-82). In connection with this development an important problem is still open, if there were Arabian influences in addition to those well known on Leonardo. No quotation of Arab sources have been found in Italian manuscripts, nevertheless a positive answer may be possible. In order to give raise to an answer to the above mentioned question, some subjects on which an influence seems to be possible ~~ha~~ have been pointed out.

An introduction to Inca quipus – M. Ascher & R. Ascher

The Incas of ~~the~~ 16^{th} century western South America are commonly depicted as the exception to the rule that a civilization has writing. Yet, the Inca bureaucracy recorded and transmitted information over a vast empire with a device they called quipus. Quipus are multicolored spatial arrays of knotted cords embodying a logical-numerical system. We introduce the system, the logical structures of some quipu formats, and the means of encoding quantitative and nonquantitative data. Some specific examples of arithmetic and logical relationships on quipus are given.

On Samuel Klingenstierna's Contributions to the Solution of Riccati Equations. – O. Kurola

Samuel Klingenstierna (1698–1765), professor of geometry (and later physics) at Uppsala, left behind an extensive collection of manuscripts (largely unpublished), dealing with elementary algebra, classical geometry, calculus, differential equations, mechanics and optics. Among his manuscripts with title "Methodum Fluxionum" (volume A 9 c in the library of the university of Uppsala) his writings on Riccati differential equations are especially interesting. In problems dealing with equations $dy + ay^2dx + bx^n dx = 0$, $dy + y^2dx = ax^{\frac{1}{n}-2}dx$ and $ax^m dx + bx^{m+1} y dx + cy^2 dx + dy = 0$ he shows to master the solution of these not only by infinite series but also by continued fractions. He also finds relationships between certain second order equations and first order Riccati equations. Although having been influenced by Euler, Klingenstierna shows great originality in dealing with these problems.

The Western discovery of Indian mathematics: Henry Colebrooke and his contemporaries as a test case for Orientalism.

In his book Orientalism (1978) Edward Said argued that the Orient is a Western ideological creation, with Orientalist discourse the mechanism of European power over the East. The great flourishing of British Sanskrit scholarship in the early nineteenth century is central in what Said sees as the formative period of this hegemony. I examined the work of Henry Colebrooke and other British Orientalists of the period (Taylor, Strachey, Rosen, etc) — and the response of their critics such as Dugald Stewart and John Leslie — to ascertain whether the unprecedented early nineteenth-century uncovering of Indian mathematical works is helpfully seen from Said's perspective.

John Fauvel (Open University)

Dr. Alexander Volodarsky
(Institute of the history of science and technology
USSR Academy of Sciences, Moscow)
The study of ancient and medieval Indian mathematics: its achievements and prospects.
The talk deals with the different problems of the mathematics which are essential not only for India but also for other ancient civilizations: Babylonia, Egypt, China. These problems are: time and place of the origin of mathematics as a science: a unity of mathematical knowledge or plurality of mathematical sciences; a single source of origin of mathematics or a plurality of independent births; the problem of interpretation of ancient texts; mathematizing

of natural science in antiquity; the place of origin of the decimal place-value system of numeration.

Über die Einwirkung der Muhammedaner auf die mittelalterliche abendländische Mathematik.

Die Moslems sorgten nach der Eroberung ihres Weltreiches dafür, daß die ihnen zugänglichen Werke der Griechen bald in arabischen Übersetzungen vorlagen. Hinzu kamen noch mit der Jahrhundertwende beachtliche eigene wissenschaftliche Leistungen. Der Einfluß auf Westeuropa machte sich erstmals nachweislich schon zur Zeit Karl d. Gr. bemerkbar, während der Arbeiten mittelalterlicher Gelehrter, die wie Gerbert von Aurillac, Leonardo von Pisa, Albertus Magnus und andere den direkten Zugang zum arabischen Kulturkreis suchten. Nach der Reconquista blühten in Spanien regelrechte Übersetzerschulen. Hauptsächlich von dort her gelangte ehemals griechisches, ferner mittelasiatisches etc. Wissen aus dem 8.–10. Jh. in den Westen, verbunden freilich auch mit irrigen und falschen Ansichten. Speziell traf dies für die Mathematik zu. Die abendländischen wissenschaftlichen Bemühungen in Westeuropa fußten zu nicht geringem Teil mit auf solchen Grundlagen, die mehrmals über den islamischen Bereich zu uns gelangt waren.

Wolfgang Kaunzner

Our Numeral System: A Chinese Legacy

The main evidence put forward to support my claim that our numeral system has its origins in the Chinese rod numeral system are briefly summarised as follows: The two systems are conceptually identical with respect to three central features, namely, (i) nine signs and the concept of zero, (ii) a decimal number base and (iii) a place value system. Among numeral systems of antiquity, the rod system is the only one which has all three features. There is every possibility of a diffusion of the concept of the rod system to India before the 6th century; this would then be transcribed to a written form with zero denoted by a dot or circle. It is possible for the rod numeral concept to be transmitted to more than one place and this would serve as a plausible explanation for the two different versions of the Arabic numerals, which have hitherto not been satisfactorily explained. The Chinese procedures for multiplication and division with counting rods as described in the Sun Zi suanjing (c. 400) are identical to those described in three Arabic texts, one of them being al-Khwarizmi's book on arithmetic. Furthermore, the identical procedures of division led to identical forms in expressing the concept of a fraction in both numeral systems.

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Mathematics in the Service of Islam

الرياضيات في خدمة الاسلام

The Islamic dimension of "Islamic science" is provided by five main topics: (1) the regulation of the lunar calendar; (2) the organization of the five daily prayers, the times of which are astronomically defined; (3) the determination of the sacred direction towards the Kaaba in Mecca; (4) the division of legacies according to Kuranic prescriptions; and (5) the generation of geometric patterns for religious and secular decoration. This paper addresses topics (1), (2) and (3).

Muslim astronomers developed sets of criteria for predicting the visibility of the lunar crescent. These were mainly expressed in terms of conditions on the apparent elongation of the sun and moon, the difference in setting times of the sun and moon, and the altitude of the moon above the horizon. From the 9th century onwards Muslim astronomers tabulated the ecliptic elongation of the sun and moon corresponding to various conditions, sometimes taking into consideration the lunar latitude, sometimes assuming that the moon was on the ecliptic. The purpose of such tables was to predict month by month whether the crescent would be seen on the first day of the civil month.

The times of Muslim prayer are defined in terms of twilight phenomena and by shadow lengths. Already in the ninth century tables were prepared showing how these prayer-times vary throughout the solar year for a specific latitude. These tables became increasingly more sophisticated over the centuries. This activity was just one part of Muslim interest in the mathematics underlying spherical astronomy; Muslim astronomers also tabulated the time of day and night as functions of solar or stellar altitude, developed sundial theory and devised instruments for converting from one set of orthogonal coordinates to another.

The earliest Muslim solutions to the problem of determining the direction of Mecca from any locality were cartographic and mathematically approximate. The first exact solution (ca 800 AD) was achieved by solid trigonometry. By ca 900 AD exact solutions had been formulated by means of projection methods (notably by the analemma) and spherical trigonometry. By ca 1000 AD all possible cases had been investigated. From the 9th century onwards, tables displaying the direction of Mecca as a function of terrestrial latitude and longitude were compiled, based either on approximate or exact formulae. The mathematics underlying these procedures and tables merits consideration as a new chapter in the history of mathematics.

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King Yūsuf al-Mu'taman ibn Hūd as a geometer

The recently discovered "Kitāb al-Istikmāl" (Book of the Perfection) of al-Mu'taman ibn Hūd (who reigned in Saragossa (Northern Spain) from 1081 to 1085 A.D.) is largely a compilation of mathematical materials from works by earlier authors. The way in which this material was organized and abridged shows that Al-Mu'taman was in complete command of these earlier works, and in some cases understood them better than the authors themselves (I have shown an example from Book V of Ibn al-Haytham's Optics and the simplification made by Al-Mu'taman). The extant fragments of Parts 2-5 of the Istikmāl also contain some theorems that are not found in other known sources from antiquity and the middle ages. Because Al-Mu'taman makes no reference to his sources, the origin of these theorems is obscure; they may or may not be his own contribution. ~~[illegible]~~ Examples are the (so-called) theorem of Ceva, and a proof of the invariance of cross-ratios under a perspectivity. (This is the earliest general proof of this theorem in the known literature; Pappus of Alexandria (ca. A.D. 300) only proved a particular case). There are indications that Al-Mu'taman is the author of at least some theorems in the Kitāb al-Istikmāl. Examples are 1) a theorem on segments of a conic sections that Al-Mu'taman probably discovered when he tried to turn Ibrāhīm ibn Sinān's quadrature of the parabola into a method for finding the area of a segment of an arbitrary conic section, and 2) a construction of two mean proportionals by means of a circle and a parabola.

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Iterative Algorisms in Medieval Science

Five Examples:

Mathematics

1. Solution of "Kepler's Equation," from the zīj of Ḥabash al-Ḥāsib al-Marwazī (fl. 830)
2. Calculation of sin 1° by Kāshī (fl. 1420)

Graphical-Mechanical

3. Time of day with the shakkāziya astrolabe of al-Zarqālluh (d. 1100)

Astrology

4. Instant of birth by the namūdār (نمودار) of Hermes, from Kāshī's Zīj-i Khāqānī.

Astronomy

5. Determination of the rising point, from Kāshī zīj.

General Remarks:

Fields in which the study of Muslim work continues: computational mathematics, trigonometry, analog computers, graphical methods (analemma)

the relation between the histories of astronomy and mathematics.

the place of Islamic contributions in the history of mathematics.

E. Kennedy

Fortschritte in der Herausgabe des Lexikons des Mittelalters

Das Lexikon des Mittelalters, von dem bis heute die ersten drei Bände erschienen sind, bezweckt eine enzyklopädische Darstellung des gesamten europäischen Mittelalters unter Einbeziehung der angrenzenden Bereiche (Byzantinisches Reich, Kreuzfahrerstaaten, arabischer und jüdischer Kulturraum). Es umfasst die Zeit von circa 300 n. Chr. (Spätantike) bis 1500 (Renaissance), welche es in knapp gehaltenen Personen- und Sachartikeln sowie in grossen, fächerübergreifenden "Dachartikeln" behandelt. Die einzelnen Fachgebiete, wozu auch die Naturwissenschaften und die Technik gehören, werden von gegen 100 verschiedenen Herausgebern betreut. Der Vortrag gab eine Würdigung der vom Redner geleiteten Fachbereiche Mathematik, Astronomie und Mechanik.

E. Neuenschwander

On the recognition of Arabic mathematics in Montucla and Delambre

This talk was a small contribution to the history of the history of Arabic mathematics. Two works were considered: the Histoire des mathematiques, vol. 1 (1799) of Montucla; and the Histoire de l'astronomie du moyen age (1819) of Delambre. In each case the author honestly admitted his ignorance of Arabic, but did not worry much over his forced reliance on Latin editions. Montucla gave an impression of Arabic mathematics close to that required by some of the participants at this meeting, in that he covered astronomy, optics & music

(& such topics) as well as arithmetic, algebra and geometry.

In addition to these two authors, various other contemporaries took an interest in Arabic (and/or oriental) mathematics. In particular, the Caussins de Perceval and the Sédillots (both père and fils) produced editions of and translations of some texts, especially the tables of 'Ebn Younis' (1804) drawing on a manuscript held in the library of ~~Leid~~ Leiden University.

The talk began with some preliminary remarks about the origins of Egyptology in the French campaign to Egypt (1798-1801), which took place at the time when Montucla's book appeared. The conscious ignorance of Egyptian mathematics, explicitly admitted by (especially) Montucla and Delambre (in his *Histoire de l'astronomie ancienne* (1817)), was noted.

I. Grattan-Guinness

Analysis and Synthesis in arabic Mathematics: The Book of Ibn al-Haytham

The aim of my lecture was just to present one of the three actually extant medieval books on the method of Analysis and Synthesis. This method of reasoning was very widespread in greek and especially in arabic Mathematics. But while we have many greek and arabic mathematical texts where this method is used, we have very few texts where this method is explained and commented: two pages only in Pappus *Collection* for almost all greek litterature! This emphasizes the importance of the three arabic extant books: the book of Ibrahim ibn Sinān (909-946); the book of as-Sizjī (X^{th} century) and the book of

Ibn al-Haytham (965-1041 A.D.). This latter is composed, roughly, of five parts and deals with analysis and synthesis in the four fundamental ancient and medieval sciences, namely: arithmetic, geometry, astronomy and music. In the first part, Ibn al-Haytham explains what is, following him, the nature of mathematical reasoning and shows that it cannot be reduced to pure syllogistic deduction as it has also recourse to some ingenious devices which necessitate the intervention of the intuition of the mathematician. After having given in the second part an interesting classification of the different kinds of analyses and syntheses, Ibn al-Haytham explains in the third part what is the logical nature of analysis. In the fourth part he has some comments on the Data of Euclid, which are, following him, an important "instrument" for carrying on analysis. In the fifth part, he gives actual examples of analysis and synthesis, terminating his book with this classical and somewhat difficult problem: to construct a circle tangent to three given circles.

K. Jaouiche

Chinese Mathematics and the Significance of the 勾股 Theorem (Gou-Ku Theorem) in Chapter Nine of the 九章算術 (Chiu-chang Shan-Shu)

Joseph W. Dauben, Dept. of History
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This paper presents some very general observations about the nature of early Chinese geometry—especially in terms of one specific example, namely the 勾股 (Gou-Ku) Theorem as it is represented in two of the early Chinese classics 周髀算經 (The Arithmetical Classic of the Gnomon and the Circular Paths of Heaven), and 九章算術 (Nine Chapters on the Mathematical Art). Comparisons are drawn between the Chinese demonstration of the theorem and comparable versions in Egyptian, Babylonian and Greek sources, including Euclid's proof of the Pythagorean Theorem. Concluding remarks are made about why Chinese mathematics developed as far as it did, but no further in antiquity. Here my interests are not so much sociological (Chinese society was more practical than theoretical in its orientation) or philosophical (Confucianism placed little value on theoretical knowledge), but instead focus on certain logical and linguistic factors in the development of Chinese mathematics. These include problems of "Entification" and "Counterfactual" reasoning within the general linguistic structure of the Chinese language. Above all, the difficulty in handling counter-factual situations in Chinese is considered, and the extent to which this may have affected the presentation and development of mathematical reasoning in general in ancient China.

Quelques aspects de Théorie des nombres à travers le 8e chapitre du Fiqh al-Ḥisāb d'Ibn Munᶜim (m. 1228)

A. DJEBBAR (ORSAY)

Ibn Munᶜim est un mathématicien d'origine andalouse qui a vécu au Maghreb à l'époque du grand empire almohade. Il a écrit de nombreux livres dans deux disciplines : la géométrie et la théorie des nombres. Un seul livre nous est parvenu : le Fiq al-Ḥisāb. Son chapitre VIII et ses prolongements constitués par les chapitres IX et X nous informent sur :

1. Le prolongement de la double tradition arithmétique d'Euclide et de Nicomaque, en Andalousie et au Maghreb, relative à l'étude de certaines classes de nombres entiers : pairs, impairs, pairement-pairs, pairement impairs, pairement-pair-impairs. Cette étude concerne : a) leur répartition dans la suite des entiers naturels
   b) leur dénombrement dans une suite donnée
   c) leurs propriétés en tant que suites arithmétiques ou géométriques.
   d) l'expression de chaque élément en fonction du nombre d'éléments de la suite.
2. Le prolongement de la double tradition arabe de l'étude des séries finies (arithmétiques, ~~et~~ géométriques, ou séries de puissance) et leurs sommations.
3. L'utilisation de l'Analyse et la Synthèse pour démontrer les propositions (lorsque cela lui est possible)

Le Fiqh al-Ḥisāb est important également pour les informations qu'il contient sur des écrits mathématiques perdus de l'Andalousie et sur des mathématiciens du Maghreb.

Il constitue enfin une illustration de la pratique combinatoire avancée, en Occident musulman. Son chapitre XI est en effet entièrement consacré à l'analyse combinatoire.

Spheres of Influence

Out of a collection of Greek treatises, written between the 4th c. B.C. and the 1st century A.D., there was formed, at least by the time of Pappus in Alexandria a collection of treatises for the purpose of introducing the student to the subject of mathematical astronomy. This collection was known as The Little Astronomy and it included treatises by Euclid, Autolycus, Hypsikles, Aristarchus and Theodosius. During the 9th century of our era this material was translated into Arabic, some of the treatises more than once, and it was incorporated into a larger teaching collection known in the Islamic World as The Middle Books. This collection included not only material introducing questions about the celestial sphere but treatises on a wide variety of mathematical topics. Our goal in this talk is to show how this material was altered and shaped both by its transformation into Arabic and its subsequent study in Islamic lands.

We have concentrated on three works: Hypsikles On Risings, Theodosius' Spherics and Autolycus On The Moving Sphere. What we have found are changes of the following sorts: (1) The "signposting" of proofs to emphasize the logical structure (2) The addition of material seemingly to fill gaps in the argument and (3) The alteration of proofs to provide a unified approach to several cases. In addition we have found material that suggests a closer study of the Arabic texts could shed considerable light on problems connected with the Greek texts.

This work is part of a study on the history of Greek texts on spherics from the 4th c. B.C. to the 13th c. A.D. in Greece and Islam.

A.P. Juschkevitch

D'abord je formule quelques questions ou problèmes concernant l'emploi des expressions qui restent souvent assez floues et qu'on utilise à tort et à travers, comme p.ex. „la science",

„la démonstration", la révolution scientifique", la rigueur mathématique" la "démonstration" et autres. Les "notions correspondantes évoluent avec" le temps et sont interprétées différemment par divers auteurs, p. ex. lorsque on parle de la naissance de la science mathématique".

" Puis suivent quelques réflexions au sujet de la soi-disante conception européocentrique" si répandue au XIX siècle et même" aujourd'hui et après cela sur la conception asiocentrique' qu'on a commencer à introduire dans les ouvrages historico-mathématique, soit ~~sous~~ la forme arabocentrique", soit „sinocentrique" et autres. Ces conceptions s'entrechoquent parfois dans les recherches sur les mathématiques médiévales.

Il est nécessaire de préciser chaque fois, surtout dans les ouvrages embrassant des longues périodes ou concernant des vastes territoires le sens qu'on donne aux expressions mentionnées là-dessus, p. ex lorsqu'on affirme que les mathématiques scientifiques sont nées en Grèce antique, ou que les mathématiques égyptiennes, babyloniennes, chinoises ne sont pas dignes d'être ~~appelées selon~~ qualifiées comme scientifiques. A mon avis l'esprit axiomatique n'est pas le seul symptome qui permet d'apprécier la valeur scientifique des mathématiques pratiquées par les peuples anciens. On ne peut pas juger de la mentalité des mathématiciens d'autrefois s'appuyant seulement les textes des manuels utilisés par les praticiens, les scribes, les constructeurs, les marchands. Plusieurs règles exprimées dans les papyrus ou dans les tablettes cunéiformes etc. ne pouvaient pas être trouvées par hasard ou de manière empirique; la réflexion mathématique y jouait certainement un certain rôle. Je propose l'expression (tiré du langage mathématique moderne) « mathématiques démonstratives par morceaux ».

Les expressions „mathématiques orientales" et „mathématiques occidentales" désignent une réalité historique pour tle mathématique moyenâgeuse, si l'on les interprète de manière correcte. Mais le plus important problème qui se pose, problème difficile à résoudre immédiatement, c'est le problème de l'interaction de diverses structures mathématiques dévelopées dans les pays différents. C'est la prémisse d'une étude impartiale et objective de la mathématique conçue comme une entité en soi, — entité dont la teneur variait en fonction du développement de l'humanité civilisée.

Je critique l'expression lancée autrefois par Ernest Renan, celle du „miracle grec". Il y avait beaucoup de miracle de cette sorte dans l'histoire de l'art, de la philosophie, de la littérature etc. Trop de miracle, plus de miracle. On peut enfin de compter utiliser le mot joli de Renan, mais ce qui importe, c'est ce de chercher les causes de tous ces „miracles", „révolutions" etc, d'analyser le progrès réel des mathématiques ou de la mathématique dans le plan historico-social et logico-psychologique.

Yvonne Dold - Samplonius

Quadratic Equations in Arabic Mathematics.

Although it was already known in Babylonian times how to solve quadratic equations, the first to write systematic treatises on the subject were Arabic mathematicians: Sind ibn 'Alī, Ibn Turk, and especially Muḥ. b. Mūsā al-Khwārizmī. After the latter's treatise Compendium of Algebra (Kitāb al-mukhtaṣar fī ḥisāb al-jabr wa'l-muqābala) a sequence of scientists continued through the centuries to work on quadratic equations. The simple algebraical solution remained the same and is even found, put into a neat scheme, in al-Kāshī's Key of Arithmetic (Miftaḥ al-ḥisāb), which appeared in 1427. But at the same time more specialized solutions were developped and al-Khwārizmī's geometrical demonstration, which was purely illustrative and explanatory, was very soon replaced by a more rigid demonstration.

Yvonne Dold.

G.P.MATVIEVSKAYA

THE INVESTIGATIONS IN THE HISTORY OF MEDIEVAL ORIENTAL MATHEMATICS IN UZBEKISTAN

The history of mathematics of Near and Middle East has been and is being studied in Uzbekistan (USSR). Many of the works of great scientists of Central Asia (al-Khwarizmi, al-Biruni, Ibn Sina, Ulug Beg etc.) have been published in Russian and in Uzbekian here. The main problem of present-day research is to study the mathematical and astronomical manuscripts, a considerable number of which is found in the Institute of Oriental studies of the Academy of Sciences in Uzbek SSR in Tashkent. In my ~~talk~~ talk I give information about this manuscript collection and its history.

Г. Матвиевская

M.M.Rozhanskaya.

Arabische Statik

Рассматриваются основные направления средневековой арабоязычной механики: теоретическое и практическое, геометрическое и кинематическое. Определен предмет статики как научной дисциплины средневековья. В теоретическом направлении основные проблемы – учение о весе и тяжести, центре тяжести, закон рычага, учение о равновесии, гидростатика. Основные проблемы практического направления – учение о простых машинах, теория весов и взвешивания, удельный вес, определение состава сплавов.

Дан обзор основного сочинения по статике на средневековом Востоке – "Книги весов мудрости" ал-Хазини.

М. Рожанская

Alireza DJAFARI NAINI

Muhammad Bāqir Yazdī, ein persischer Mathematiker aus dem 17. Jahrhundert

Zur Zeit der Ṣafaviden-Dynastie lebte (wahrscheinlich in Iṣfahān) Muḥammad Bāqir Yazdī (gest. etwa 1637). Sein wichtigstes Werk ist ʿUyūn al-ḥisāb. Es gibt Zeugnis über den damaligen Stand des mathematischen Wissens in Persien. Hieraus werden zwei zahlentheoretisch wichtige Abschnitte behandelt: die Gewinnung der befreundeten und der gleichgewichtigen Zahlen. Zu den befreundeten Zahlen hat Yazdī nach der Regel von Tābit ibn Qurra (830-901) ein neues Zahlenpaar, 9363584 und 9437056, gefunden. Ein Novum stellen die gleichgewichtigen Zahlen dar, für deren Auffinden er eine Bildungsregel aufgestellt hat und das Zahlenpaar 39 und 55 angibt. Nach Angaben seines Übersetzers Ḫātūnābādī jedoch reicht diese Regel nicht aus, um alle gleichgewichtigen Zahlen aufzufinden. Als Beweis dafür gibt er ein kleineres Paar an, das er nicht nach Yazdī's Anleitung gefunden hat.

Im Vortrag werden nun drei neue Bildungsregeln vorgestellt. Außerdem wird der Satz bewiesen, daß die Anzahl der gleichgewichtigen Zahlen, die ein gemeinsames Gewicht > 1 haben, endlich ist. Weiter werden in einer Aufstellung die ersten 101 Gewichte daraufhin untersucht, wieviel gleichgewichtige, leere und isolierte Zahlen sie enthalten.

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Alireza Djafari Naini

den 1. Mai 1987

Vortragender: Hermann Kogelschatz

Thema des Vortrages: Kritische Anmerkungen zur Bearbeitung des frühen mathematischen Schrifttums Chinas unter Kaiser Ch'ien-lung.

Kurze Zusammenfassung (höchstens 15 Zeilen):

Im Zentrum ddr textgeschichtlichen Untersuchung stehen zwei Druckausgaben des Chiu chang suan shu: (A) die Palastdruck-Ausgabe (1774/75) in der Bearbeitung von Tai Chen (1724-1777), eine Rekonstruktion des Textes anhand der Exzerpte in der handschriftlichen Ming-Enzyklopädie (1403 ff.), und (B) die Wei-po-hsieh-Ausgabe von K'ung Chi-han (1739-1784), die das Datum 1773 trägt und eine getreue Wiedergabe eines Sung-Druckes (1084 bzw. 1213) zu sein verspricht. Angesichts des schon von Ch'ien Pao-ts'ung aufgezeigten Widerspruchs zwischen der Kennzeichnung von Ausgabe (B) und ihrer faktischen Abhängigkeit von Ausgabe (A) stellt sich die Frage nach den Hintergründen und Motiven dieses bis in die Gegenwart hinein erfolgreichen Täuschungsmanövers. Zugleich stellt sich die Aufgabe einer Revision von Tai Chens mathematischen Interpretationen auf der Basis des 1980 vom Verlag Wen-wu (Peking) der Allgemeinheit zugänglich gemachten Sung-Fragments.

H. Kogelschatz
柯宝山

J. Sesiano: Der "Liber mahameleth"

Eines der umfangreichsten mathematischen Werke des frühen lateinischen Mittelalters ist der "Liber mahameleth", er wurde in der zweiten Hälfte des XII. Jahrhunderts in Spanien verfaßt, und zwar von einem unbekannten Autor, dem arabische Quellen zur Verfügung standen. Das Werk zerfällt in zwei Hauptteile. Im ersten werden die Grundlagen der Rechenkunst erklärt (einschließlich Operationen mit ganzen Zahlen und Brüchen, sowie Näherungsverfahren für das Ausziehen der Quadratwurzeln). Der zweite enthält eine Anwendungen

[illegible]

THE CHINESE TA-YEN 大衍 RULE

ULRICH LIBBRECHT, LEUVEN (BELGIUM)

In a first part I discussed some general characteristics of Chinese mathematics: 1) the rather high starting point in the Chiu-chang suan-shu hindered further development for a millenium, because this work became canonical, and mathematicians made only variations on its contents 2) Mathematical life was organized in the Suan-kuan (Office of mathematics), where officials were trained in practical mathematics, and in the Li-suan, where astronomical and calendrical problems were solved. However, in the 13th century, the "independent" mathematician appears, but he was never in the service of the Office of mathematics.

In a second part I dealt with one of the great achievements of Chinese mathematics: the solution of indeterminate equations of the first degree, having the general form:

$$N \equiv r_1 \pmod{m_1} \equiv r_2 \pmod{m_2} \equiv r_3 \pmod{m_3} \equiv \dots$$

In India a method, called kuttaka, has been developed, but this has nothing to do with the ta-yen rule. The oldest ta-yen problem appears in the Sun-tzu suan-ching 孫子算經 of the 5th century A.D. It was fully developed in the Shu-shu chiu-chang 數書九章 by Ch'in Chiu-shao 秦九韶, published in 1247. Ch'in Chiu-shao was even able to solve the problem when the moduli are relatively prime in pairs.

I made also a comparison with the same rule in Europe. We see that it was very underdeveloped in the Liber Abbaci and other mediaeval books. A first culminating point is found in a Göttingen manuscript of c. 1550. Full explanation was given by Beveridge in 1669, by Euler in 1734 and by Gauss in 1801. The general problem with moduli not relatively prime in pairs was solved by Lebesgue in 1859 and by Stieltjes in 1890.

李培德

GRUPPENTHEORIE

3.5.87 – 9.5.87

Matrix Groups over Finite-Dimensional Division Algebras.

Extending work of Alexander Lichtman we are able to partially describe the structure of subgroups of $GL(n, D)$ for D any finite-dimensional division algebra. As a consequence the derived length of a soluble such subgroup is bounded in terms of n and the exponent of the degree of D. If $n = 1$ the results are particularly complete. For example: let D be a division algebra of degree q^m for q a prime. Let G be a subgroup of $D \setminus \{0\} = D^*$ with no non-cyclic free subgroups. Let A be any maximal abelian normal subgroup of G and set $H = C_G(A)$.

Then either $(G : H)$ divides q^m, $H/A = \langle 1 \rangle$,

or $\operatorname{char} D = 0$, $q = 2$, $(G : H)$ divides 2^{m-1}, $H/A \cong Alt(4), Sym(4)$ or $Alt(5)$,

or $\operatorname{char} D = 0$, $q = 2$, $(G : H)$ divides 2^{m-2}, $H/A \cong Sym(5)$.

Bert Wehrfritz (London).

Groups with thin lattices of subgroups

The width of a lattice is defined to be the maximum possible cardinality of its antichains. For a group G, let $w(G)$ (resp. $w_n(G)$) denote the width of the lattice of all (normal) subgroups of G.

Theorem. Let n be a positive integer. Then there exist finitely many finite groups $H_1, \dots, H_t$ with the following property. If $w(G) = n$, then G is a split extension of a normal Hall subgroup $H \cong H_i$ (some i) by a locally cyclic torsion group Q.
If Q is infinite, then there exists exactly one subgroup of type p^∞ in Q, and it is a direct summand in G.

Corollary 1. For every given $n \neq 1$ there exist only finitely many p-groups of width n.

Corollary 2. The derived length of a soluble group (class of a nilpotent group) is bounded by a function of its width.

Finally, relations between finite p-groups G satisfying $w_n(G) = p+1$ are discussed.

Rolf Brandl (Würzburg)

The Tits alternative for one-relator quotients of free products of cyclics

For a finitely generated group H, the Tits alternative says that H either contains a free subgroup of rank 2 or a solvable subgroup of finite index. We ask wether the Tits alternative holds for the one-relator product of cyclics

$G = \langle a_1, \dots, a_n \mid a_1^{e_1} = \dots = a_n^{e_n} = R^m(a_1, \dots, a_n) = 1 \rangle$

with $n \geq 2$, $m \geq 2$, $e_i = 0$ or $e_i \geq 2$ for $i = 1, \dots, n$ and $R(a_1, \dots, a_n)$ a cyclically reduced word in the free product on $a_1, \dots, a_n$ which involves all $a_1, \dots, a_n$.

Theorem 1: (B. Fine, F. Levin, G. Rosenberger)
Suppose one of the following holds
(i) $n \geq 3$
(ii) $n = 2$ and $e_i = 0$ for $i = 1$ or $i = 2$
(iii) $n = 2$ and $m \geq 3$.
Then the Tits alternative holds for G. □

If $n = 2$ and both generators have finite order then G is called a generalised triangle group and can be written in the form $G = \langle a, b \mid a^p = b^q = R^m(a,b) = 1 \rangle$ with $2 \leq p \leq q$ and $R(a,b) = a^{p_1} b^{q_1} \dots a^{p_k} b^{q_k}$

with $1 \leq k$ and $1 \leq p_j < p$, $1 \leq q_j < q$ for $j = 1, \dots, k$. By a theorem of G. Baumslag, J. Morgan and P. Shalen [Preprint] G has a free subgroup of rank 2 if $\frac{1}{p} + \frac{1}{q} + \frac{1}{m} < 1$. If $m \geq 3$ then the Tits alternative holds for G by Theorem 1. If $m = 2$ then the corresponding question seems to be fairly difficult in general and we only have the following partial result.

Theorem 2:

If $n = m = 2$ and $1 \leq k \leq 2$ then the Tits alternative holds for G. $\square$

Conjecture: The Tits alternative holds for all $k \geq 1$.

Gerhard Rosenberger

Trifactorized Soluble Minimax Groups

Suppose that the soluble minimax group G has a triple factorization

$$G = AB = AC = BC$$

where A, B and C are subgroups.

The following holds:

(a) if A and B are nilpotent and C is locally nilpotent, then G is locally nilpotent and hence hypercentral

(b) if A and B are nilpotent and C is locally supersoluble, then G is locally supersoluble and hence hypercyclic.

(joint work with B. Amberg and F. de Giovanni)

Silvana Franciosi (Napoli, Italy)

The subnormal embedding of relatively complete groups

All groups in this abstract are assumed to be finite. The following generalisation of complete groups is considered: Definition: A is complete with respect to a Fitting class $\underline{F}$, if (i) $Z(A) = 1$ and (ii) for every U with $\mathrm{Inn}(A) \leq U \leq \mathrm{Aut}(A)$, $\mathrm{rad}_{\underline{F}}(U) = \mathrm{Inn}(A)$.

Theorem: If (i) A is not isomorphic to the $\{2,p\}$-Hall subgroup of some $\mathrm{Hol}(C_{p^n})$, $n \in \mathbb{N}$, $p \equiv 3 \bmod 4$, and (ii) A is complete with respect to a Fitting class $\underline{F}$ which is Q-closed, and (iii) A is directly indecomposable and subnormal in the group G,

then (a) $(A^G)^*$ is the direct product of all conjugates of A^*

and (b) $A^G/F(A^G)$ is the direct product of all conjugates of $AF(A^G)/F(A^G)$ in $G/F(A^G)$.

(joint work with P. Soules). Partial generalisation of the corresponding theorem on complete groups (H.H. and J.C. Lennox)

M. Menth (Würzburg)

Amalgamation of soluble groups

Let $\mathfrak{X}$ be either the class of all finite soluble π-groups, or the class of all such groups of derived length $\leq n$. B. Maier has shown that there are either $2^{\aleph_0}$ (isomorphism types of) countable existentially closed $L\mathfrak{X}$-groups, or one which is unique with respect to an additional requirement. All of this depends just on the question, whether or not amalgamation of $\mathfrak{X}$-groups over any $A \in \mathfrak{X}$ can be controlled by some $A \leq B \in \mathfrak{X}$ in the sense that any two $\mathfrak{X}$-supergroups of B can be amalgamated over A.

For this reason we study amalgamation of $\mathfrak{X}$-groups. We give a necessary and sufficient condition which shows that the heart of the problem actually is amalgamation of operator-groups ($\mathfrak{X}$-groups acting on abelian groups). Using tensor products for the amalgamation of operator groups, we obtain results about amalgamation of finite soluble π-groups over supersoluble groups, and about amalgamation of metabelian groups.

Felix Leinen (Mainz)

Modules and crossed homomorphisms of finite groups especially p-solvable groups.

Let G be a p-solvable group and P_1 the projective hull of the trivial representation of G over the prime field $\mathbb{F}_p$. Let $\mathcal{H}$ be a principal series of G and A the direct product of all splitting p-chief-factors of $\mathcal{H}$. A chief-factor L/K is a G-module by $(xK)^g = g^{-1}xgK$. So A is a G-module. We show the existence of a „canonical" crossed homomorphism ψ of G onto A, $\ker\psi = D$ is a p-prefrattini subgroup of G. The group-ring $\mathbb{F}_p[A]$ is made a G-module by: $a \circ g = a^g\,\psi(g)$. This module is called $\mathbb{F}_p[A]_\psi$. Then we get:

Theorem:

a) There is a G-epimorphism $\hat{\psi}: P_1 \longrightarrow \mathbb{F}_p[A]_\psi$
$\ker\hat{\psi} \cong P_1(D)\,J_D \otimes_D \mathbb{F}_p[G]$ (J_D is the radical of $\mathbb{F}_p[D]$)

b) $\hat{\varphi}: P_1 J_G \twoheadrightarrow J_{A,\varphi}$ the augmentation ideal of $\mathbb{F}_p[A]$ with the „o" structure.

c) there is a G-isomorphism $\pi: J_{A,\varphi} \to A$, given by $a-1 \xrightarrow{\pi} a$.

d) let τ be the composed mapping: $P_1 J_G \twoheadrightarrow A$, then $\ker \tau = P_1 J_G^2$.

So we get a new proof of Gaschütz's theorem: $P_1 J_G / P_1 J_G^2 \cong A$. Moreover: all composition factors of $\mathbb{F}_p[A]_\varphi$ are composition factors of $\mathbb{F}_p[A]$, so we get more and new information on P_1. Especially we get a very simple proof of the theorem of Green and Hill.

Alfred Brandis (Heidelberg)

Non Conjugate Fitting functors

A subgroup functor f on a class $\mathfrak{D}$ of finite groups assigns to each group $G \in \mathfrak{D}$ a set of subgroups $f(G)$ of G such that if $\alpha: G \to \bar{G}$ is an injective homomorphism, then $f(G^\alpha)^{\alpha} = \{X^\alpha \mid X \in f(G)\}$. In two recent papers Beidleman, Hauck and I (M.Z. 182 (1983) 359-384 and Proc. Cambr. Phil. Soc. (1987) 101, 37-55) studied a particular type subgroups functor. We were trying to analyze how $\mathfrak{F}$-injectors for a Fitting class $\mathfrak{F}$ behaved in soluble groups without dependence upon the class. We call a subgroup functor f a Fitting functor provided f satisfies both: [A] For $N \trianglelefteq G$ and $X \in f(G)$, $X \cap N \in f(N)$
[B] For $N \trianglelefteq G$ and $Y \in f(N)$, $\exists X \in f(G)$ such that $Y = X \cap N$.

The primary examples are injectors and radicals of Fitting classes but others were produced too. A Fitting functor f satisfies the Frattini argument provided for every group in $\mathfrak{D}$, $K \trianglelefteq G$ and $U \in f(G)$, $G = K \cdot N_G(U \cap K)$. Our experience indicates that if extra properties are imposed on f, then f tends to satisfy the Frattini when these extra properties are those of injectors.

We say the Fitting functor f satisfies the ~~Frattini/avoidance~~ cover-avoidance property, when each $U \in f(G)$ either covers or avoids each chief factor of G. Certainly a Fitting functor which satisfies the Frattini argument has the cover-avoidance property.

Unfortunately it is difficult to discern whether they are equivalent. We know of only two types of constructions which produce Fitting functors which do not satisfy the Frattini argument.

① If f, h are Fitting functors on $\mathfrak{D}$, then define for $G \in \mathfrak{D}$, $f \cap h(G) = \{U \cap V \mid U \in f(G), V \in h(G)\}$. $f \cap h$ ~~(G)~~ a Fitting functor on $\mathfrak{D}$ which in general does not satisfy the ~~Frattini~~ cover-avoidance property. In particular if $f(G) = Syl_2(G)$, $f \cap f$ does not have the cover-avoidance property.

② If $\bar{F}$ is a subgroup functor on an S_n-closed class $\mathfrak{B}$ such that $\bar{F}$ satisfies condition [A] in definition of Fitting functor, then $f(G) = \{U \mid \exists B \lhd\lhd G, B \in \mathfrak{B} \text{ with } U \in \bar{F}(B)\}$ is a Fitting functor (also $\lhd\lhd$ may be replaced by $\leq$).

Note f satisfies [B] because it satisfies (★):

(★) For $N \lhd G$, if $U \in f(N)$, then $U \in f(G)$.

③ If f is a Fitting functor on domain $\mathfrak{S}$ (= finite soluble groups) p is a prime and $U \in f(G)$ such that $p \mid |U|$, then $O_p(G) \leq U$.

Corollary. If f is a Fitting functor on $\mathfrak{S}$ which satisfies ★ and has the cover-avoidance property, then $f(G) = \{1\}$ for all $G \in \mathfrak{S}$.

Ben Brewster

Table algebras and applications to finite group theory.

For a better understanding of the analogy between theorems concerning theorems about product of irreducible characters and theorems concerning theorems about products of conjugacy classes, Arad and Blau introduced a concept which they call table algebras:

Definition: A table algebra $(A, \mathfrak{X})$ is a finite dimensional commutative algebra A with identity 1 over $\mathbb{C}$, which has a specified basis $\mathfrak{X} = \{1 = a_1, a_2, \dots, a_k\}$ such that the following properties hold:

(i) $a_i a_j = \sum_{m=1}^{k} \lambda_{ijm} a_m$ where $\lambda_{ijm} \in \mathbb{R}^+ \cup \{0\}$, $1 \le \forall\, i,j,m \le k$.

(ii) There is an algebra automorphism $-$ of A whose order divides 2, s.t. $-$ permutes $\mathfrak{X}$ (we denote the image of $a \in A$ under $-$ by $\bar{a}$.

(iii) There is a $\mathbb{C}$-algebra homomorphism $f: A \to \mathbb{C}$ s.t. $f(\bar{a}) = \overline{f(a)}$, $\forall a \in A$, and $f(a_i) \in \mathbb{R}^+$, $\forall a_i \in \mathfrak{X}$.

(iv) $\exists$ hermitian form $(\ ,\ ): A \times A \to \mathbb{C}$ s.t. $(a_i, a_j) = \delta_{ij}$, $\forall a_i, a_j \in \mathfrak{X}$.

(v) $\exists$ a function $g: \mathfrak{X} \times \mathfrak{X} \to \mathbb{R}^+$ s.t. $(a_i a_j, a_m) = g(a_i, a_m)(a_i, \bar{a}_j a_m)$ for all $1 \le i,j,m \le k$.

Examples: If G is a finite group the two algebras induced by the set of conj. classes and the set $Irr(G)$ are table algebras.

Therefor theorems about table algebras can be applied to theorems about irreducible characters and also to theorems about conjugacy classes in finite groups.

Recently, we developed a general theory on table algebras (Z. Arad, E. Fisman and H. Blau) with interesting applications to finite group theory.

Zvi Arad

The classification of finite simple Moufang loops.

A Moufang loop is a loop which satisfies the identity $(xy)(zx) = (x(yz))x$. These were introduced by Ruth Moufang in connection with geometry.

Some properties (1) Moufang's theorem: every Moufang loop is di-associative – that is, any two elements generate a subgroup.

(2) There is a Jordan-Hölder theorem – every finite Moufang loop is built from a unique set of simple Moufang loops (where a simple Moufang loop is one with no proper normal subloops or, equivalently, no proper homomorphic images).

Examples of simple Moufang loops. (a) Simple groups.

(b) Let $\mathcal{C}$ be the 8-dimensional split Cayley algebra over $GF(q)$ and let $n : \mathcal{C} \to GF(q)$ be the norm function on $\mathcal{C}$. Then

$$M(q) = \{x \in \mathcal{C} \mid n(x) = 1\} / \langle -1 \rangle$$

is a simple Moufang loop which is not associative (L.J. Paige, 1956).

Theorem Let M be a finite simple Moufang loop. Then either M is a simple group or $M \cong M(q)$ for some prime power q.

The proof is based on work of S. Doro, 1977. We say that a group G has triality if it has automorphisms ρ, σ of order 3, 2 respectively such that $\langle \rho, \sigma \rangle \cong S_3$, $[G, S_3] = G$, and for every $g \in G$,

$$[g,\sigma][g,\sigma]^{\rho}[g,\sigma]^{\rho^2} = 1.$$

Now let M be a Moufang loop. For $x \in M$ define the permutations

$$L_x : m \to xm, \qquad R_x : m \to mx \qquad (m \in M)$$

and let $T(M) = \langle L_x, R_x \mid x \in M \rangle \leq \mathrm{Sym}(M)$, the multiplication group of M.

Doro's theorem (1977) If M is finite, simple and not associative, then $T(M)$ is a simple group with triality.

Note that for $M = M(q)$, $T(M)$ is the 8-dimensional orthogonal group $P\Omega_8^+(q)$, which has triality with respect to a group S_3 of graph automorphisms.

Using Doro's theorem and many intricate calculations with simple groups, I have obtained the above classification theorem.

Martin Liebeck,
Imperial College, London.

The Diameter of the Finite Simple groups

Theorem For every finite simple non-abelian group G, there ~~is~~ is a set S of 7 generators for which $\text{diameter}(X(G;S)) = O(\log|G|)$ ($X(G;S)$ denotes the Cayley graph of G with respect to S).

The Theorem is proved by consider A_n (or S_n) seperately. Note that for $\tau = (1,2)$ and $\sigma = (1,2,\dots,n)$ $\text{diameter}(X(S_n;\{\tau,\sigma\})) = O(n^2)$ and not $O(n\log n) = O(\log n!)$. But other generators do it! All simple groups of Lie type can be reduced to the family $SL_2(p)$. For $SL_2(p)$ we have:

Theorem $U = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. Then $\text{diameter}\, X(SL_2(p);\{U,T\})$

$= O(\log p)$

The proof is based on a deep result of Selberg bounding the first non-zero eigenvalue of the laplacian operator acting on functions on $\mathbb{H}/\Gamma(p)$ where $\mathbb{H}$ is the upper half plane and $\cancel{\Gamma(p)}$
$\Gamma(p) = \text{Ker}(SL_2(\mathbb{Z}) \to SL_2(p))$. (Selberg Theorem is based on Weil's Riemann hypothesis for curves over finite fields).

(Still: using $U = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and $V = \begin{pmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$ one can show easily that $SL_2(p)$ has diameter $\log p$).

This work was done in collaboration with L. Babai (Budapest) and Bill Kantor (Oregon)

Alex Lubotzky
(Jerusalem)

Free subgroups of various unit groups

The following theorem was discussed :-

Theorem. Let G be a finite group and $U = U(\mathbb{Z}G)$ the unit group of the integral group ring $\mathbb{Z}G$. Then U contains a normal subgroup U_0 of finite index such that if $N \trianglelefteq U$, then either N contains a free non abelian subgroup, or $N \cap U_0$ is in the centre of U. Further, $U = U_0$ unless some quotient of G is isomorphic to a non abelian subgroup of the real quaternions $\mathbb{H}_{\mathbb{R}}$. In particular $U = U_0$ if G has odd order.

The theorem is a fairly straightforward consequence of the observation that U can be realized as ~~the~~ an arithmetic subgroup in a reductive algebraic $\mathbb{Q}$-group, and a density theorem of Borel on arithmetic subgroups of semisimple algebraic $\mathbb{Q}$-groups. In fact the ~~conclusion of~~ the first part of the theorem holds for arbitrary ~~reductive~~ arithmetic subgroups U of reductive algebraic $\mathbb{Q}$-groups.

B Hartley

Fitting classes of $\underline{\underline{S}}_1$-groups - J.C. Beidleman

An $\underline{\underline{S}}_1$-group is one possessing a finite normal series in which the factors are abelian groups of finite rank whose torsion subgroups are Černikov groups. A Fitting class of $\underline{\underline{S}}_1$-groups is a subclass $\underline{\underline{X}}$ of $\underline{\underline{S}}_1$ such that:

(F1) If $G \in \underline{\underline{X}}$ and A asc G, then $A \in \underline{\underline{X}}$.

(F2) If $G \in \underline{\underline{S}}_1$ is generated by ascendant $\underline{\underline{X}}$-subgroups, then $G \in \underline{\underline{X}}$.

Let $\underline{\underline{X}}$ be a Fitting class of $\underline{\underline{S}}_1$-groups. (F2) ensures that the join of ascendant $\underline{\underline{X}}$-subgroups of G is a normal $\underline{\underline{X}}$-subgroup $G_{\underline{\underline{X}}}$ of G. $G_{\underline{\underline{X}}}$ is called the $\underline{\underline{X}}$-radical of G. $\underline{\underline{N}}$ = class of locally nilpotent $\underline{\underline{S}}_1$-groups is a Fitting class of $\underline{\underline{S}}_1$-groups. Also $G_{\underline{\underline{N}}}$ is just the Hirsch-Plotkin radical of G. Let p be a prime. For $G \in \underline{\underline{S}}_1$, the p-socle of G is $Soc_p(G) = \langle M \mid M$ a minimal normal p-subgroup of $G \rangle$. If G has no minimal normal p-subgroups, then $Soc_p(G) = 1$. $\mathcal{C}(p) = \{G \in \underline{\underline{S}}_1 \mid Soc_p(G) \leq Z(G)\}$. $\mathcal{C}(p)$ is a Fitting class of $\underline{\underline{S}}_1$-groups and if $G \in \underline{\underline{S}}_1$, then $G_{\mathcal{C}(p)} = C_G(Soc_p(G))$.

A subgroup X of $G \in \underline{\underline{S}}_1$ is called an $\underline{\underline{X}}$-injector of G provided that $X \cap A$ is a maximal $\underline{\underline{X}}$-subgroup of A for each ascendant subgroup A of G. The following theorems are considered.

<u>Theorem 1</u> Let $\underline{\underline{X}}$ be a Fitting class of $\underline{\underline{S}}_1$-groups and let $G \in \underline{\underline{S}}_1$. If $G/G_{\underline{\underline{X}}}$ is finite, then G has $\underline{\underline{X}}$-injectors and any two $\underline{\underline{X}}$-injectors are conjugate.

<u>Theorem 2.</u> Let $\underline{\underline{X}}$ be a Fitting class $\underline{\underline{S}}_1$-groups such that $\underline{\underline{N}} \nsubseteq \underline{\underline{X}}$. Then there is a polycyclic group G which does not have $\underline{\underline{X}}$-injectors

Theorem 3. Let $\underline{\underline{X}}$ be a Fitting class of $\underline{\underline{S}}_1$-groups containing $\underline{\underline{N}}$ and let $G \in \underline{\underline{S}}_1$. Suppose that G has a normal subgroup M such that $M/G_{\underline{\underline{X}}}$ is finite and M contains all $\underline{\underline{X}}$-subgroups of G which contain $G_{\underline{\underline{X}}}$. Then

(i) G has $\underline{\underline{X}}$-injectors and any two such subgroups are conjugate.

(ii) $\mathrm{Inj}_{\underline{\underline{X}}}(M) = \mathrm{Inj}_{\underline{\underline{X}}}(G)$.

Theorem 4. Let $\underline{\underline{X}}$ be a Fitting class of $\underline{\underline{S}}_1$-groups such that every $\underline{\underline{S}}_1$-group G has a unique conjugacy class of $\underline{\underline{X}}$-injectors. If $X \in \mathrm{Inj}_{\underline{\underline{X}}}(G)$, $G \in \underline{\underline{S}}_1$, then

(i) $X^G/G_{\underline{\underline{X}}}$ is finite.

(ii) $\mathrm{Inj}_{\underline{\underline{X}}}(X^G) = \mathrm{Inj}_{\underline{\underline{X}}}(G)$.

Let $G \in \underline{\underline{S}}_1$. By Theorem 1 G has a unique conjugacy class of $\mathcal{C}(p)$-injectors. Also G has a normal subgroup N such that $N/G_{\underline{\underline{N}}}$ is finite and N contains all the $\underline{\underline{N}}$-subgroups of G which contain $G_{\underline{\underline{N}}}$. Hence G has a unique conjugacy class of $\underline{\underline{N}}$-injectors. This fact was first established by M. Tomkinson (see Proc. Edinburgh Math. Soc. 1979).

Structure of crossed group algebras of a class of infinite groups over the real field.

Let $\langle a \rangle$ be the infinite cyclic group, D the infinite dihedral group, G a group containing infinite cyclic subgroups of finite index, K a field.

S.D. Berman and K. Buzási described all the finitely generated KG-modules (with some restriction on the characteristic of K). They showed that the investigation of KG-modules can be reduced to the study of modules over some crossed group algebras of $\langle a \rangle$ and D. The authors described all the finitely generated modules over the real crossed group algebras of $\langle a \rangle$ and D, expect the finitely generated torsion-free modules over the crossed real group algebras

$$A = \langle R, a, b \mid \lambda a = a\lambda;\ \lambda b = b\lambda;\ b^{-1}ab = a^{-1};\ b^2 = -1;\ \lambda \in R \rangle$$

$$B = \langle C, a, b \mid \lambda a = a\lambda;\ \lambda b = b\bar{\lambda};\ b^{-1}ab = a^{-1};\ b^2 = -1;\ \lambda \in C \rangle$$

where R and C are the real and complex fields.

The main results of the present lecture are as follows:

1) Algebra A is not a principal left ideal ring.

2) For the non-principal left ideals $J_1, J_2 \subseteq A$ either $J_2 = J_1 \cdot d$; $d \in R(a)$ or J_1 and J_2 are not A-isomorphic.

3) Every finitely generated torsion-free A-module either is A-isomorphic to a left ideal of A or is a free A-module.

K. Buzási

Präfrattinigruppen und simpliziale Komplexe mit einer guten Strategie.

Wir sagen, daß ein simplizialer Komplex $K = K_\Omega$ (d.h. $K \subseteq 2^\Omega$) ein Komplex mit einer guten Strategie ist, wenn für eine beliebige Teilmenge $\Delta \subseteq \Omega$ es immer möglich ist, mit weniger als $|\Omega|$ vielen Fragen der Form "Ist $\omega \in \Omega$ Element von Δ?" zu entscheiden, ob Δ in K liegt.

Sei H eine Untergruppe der endlichen, auflösbaren Gruppe G. Wir ordnen dem Intervall $[G/H] = \{H \leq A \leq G\}$ einen Komplex $K = K(G:H)$ auf folgende Weise zu: Ω sei die Menge aller Nebenklassen $Hg \neq H$ von H in G; die maximalen Elemente von K seien die maximalen Elemente von $[G/H]$. Dann ~~ist~~ besitzt $K(G:H)$ genau dann eine gute Strategie, wenn die Eulercharakteristik $\chi(K(G:H)) = 1$ ist, oder genau dann, wenn die H-Praefrattinigruppen von G $\neq H$ sind. Dabei sind H-Praefrattinigruppen von G eine natürliche Verallgemeinerung der von Gaschütz eingeführten Praefrattinigruppen $(H = 1)$. Sie kann man z. B. – in unserem Kontext – definieren als die minimalen Elemente der Menge

$$\{ U \in [G/H] \mid \chi(K(G:U)) \neq 1 \}$$

Sie haben analoge Deck- und Meide-Eigenschaften, und sind insbesondere unter G konjugiert.

Hans Kurzweil (Erlangen)

Symmetric presentations and some finite simple groups

Let G be the group presentation $G = \langle a_1, a_2, \ldots, a_r \mid w_i(a_1, \ldots, a_r) = 1,\ 1 \le i \le s \rangle$
and for $n \geq r$ define $S_n(G) = \langle a_1, a_2, \ldots, a_n \mid w_i(a_{1\theta}, a_{2\theta}, \ldots, a_{r\theta}) = 1,\ 1 \le i \le s,\ \theta \in S_n \rangle$
We say that the n-generator group presentation G is symmetric if $S_n(G) = G$.

Suppose we start with a 2-generator symmetric group presentation G. What can we say about $S_n(G)$?

<u>Theorem</u> Let $G_m = \langle a, b \mid ab^2 = ba^2,\ a^m = b^m = 1 \rangle$ and let g_i be the ith Lucas number. Then suppose m is odd. If $(m,3) = 1$ then $S_n(G_m)$ is finite of order $m\, g_m^{n-1}$. If $(m,3) = 3$ then $S_2(G_m)$ order $m g_m$, $S_3(G_m)$ order $2 m g_m^2$, $S_n(G_m)$ infinite $n \geq 4$.

Define $G_1(p) = \langle a, b \mid a^p = b^p = (a^i b^i)^2 = 1,\ 1 \le i \le [p] \rangle$ and for p prime define $G_2(p) = \langle a, b \mid a^p = b^p = (a^i b^{1/i})^2 = 1 \rangle$. Now $G_1(p)$ is finite of order $p\, 2^{p-1}$ being an elementary abelian 2-group of order 2^{p-1} extended by C_p. For $p \geq 5$ $G_2(p) \cong PSL(2,p)$ (proved by Beetham 1971). Now $S_n(G_1(3)) = S_n(G_2(3)) = A_{n+2}$ (Coxeter)

We conjecture that $S_n(G_1(p))$ and $S_n(G_2(p))$ are series of 'essentially' orthogonal group over $GF(2^{\frac{p-1}{2}})$ and $GF(p)$ respectively. Sample results

$S_3(G_1(5)) = PO^-(4,4)$, $S_4(G_1(5)) = PO(5,4)$, $S_5(G_1(5)) = PO^-(6,4)$, $S_6(G_1(5)) = 2^{12}.PO^-(6,4)$
$S_7(G_1(5)) = PO^-(8,4)$, $S_8(G_1(5)) = PO(9,4)$, $S_9(G_1(5)) = PO^-(10,4)$, $S_{10}(G_1(5)) = 2^{20}.PO^-(10,4)$
$S_2(G_2(5)) = PO(3,5)$, $S_3(G_2(5)) = PO^-(4,5)$, $S_4(G_2(5)) = PO(5,5)$, $S_5(G_2(5)) = 2.PO^+(6,5)$
$S_6(G_2(5)) = 5^6.2.PO^+(6,5)$, $S_7(G_2(5)) = 2.PO^+(8,5)$, $S_8(G_2(5)) = PO(9,5)$.

<u>Theorem</u> $S_3(G_2(p)) = 2.PO^+(4,p)$ if $\sqrt{2} \in GF(p)$. If $\sqrt{2} \notin GF(p)$ $S_3(G_2(p))$ has $PO^-(4,p)$ as a homomorphic image and we conjecture that $S_3(G_2(p)) = PO^-(4,p)$.

We have other series of orthogonal groups starting with 2-generator symmetric presentations for $SL(2,8)$, $SL(2,16)$, $SL(2,32)$, $SL(2,64)$, $PSL(2,25)$, $PSL(2,27)$, $PSL(2,49)$, $PSL(2, \mathbb{Z}_{25})$ etc

This is joint work with C.M. Campbell (St Andrews).

Edmund F. Robertson (St Andrews)

Parabolic Systems and $\circ\overset{\sim}{=\!=}\circ$

Suppose G is a (not necessarily finite) group, and $I = \{1, \ldots, n\}$ an index set and that G contains finite subgroups $S, P_1, \ldots, P_n$ which satisfy :-

(i) $S \leq \bigcap_{i \in I} P_i$ and $S \in Syl_2 P_{ij}$ for all $i, j \in I$ (where $P_{ij} := \langle P_i, P_j \rangle$);

(ii) $G = \langle P_i \mid i \in I \rangle \neq \langle P_i \mid i \in J \subsetneq I \rangle$; and

(iii) for each $i \in I$, $P_i / O_2(P_i) \cong SL_2(2)$.

Setting $\overline{P_{ij}} = P_{ij} / O_2(P_{ij})$ we define a diagram for $\{P_1, \ldots, P_n\}$ whose nodes are I and for $i, j \in I$

$\underset{i}{\circ} \quad \underset{j}{\circ}$ iff $\overline{P_{ij}} \cong SL_2(2) \times SL_2(2)$

$\underset{i}{\circ}\!\!-\!\!-\!\!-\!\!\underset{j}{\circ}$ iff $\overline{P_{ij}} \cong SL_3(2)$

$\underset{i}{\circ}\overset{\sim}{=\!=\!=}\underset{j}{\circ}$ iff $\overline{P_{ij}} \cong \hat{S}_6$

An outline of a proof of the following result was given

Theorem Suppose $\{P_1, P_2, P_3, P_4, P_5\}$ has diagram $\underset{1}{\circ}\!-\!\underset{2}{\circ}\!-\!\underset{3}{\circ}\!-\!\underset{4}{\circ}\overset{\sim}{=}\underset{5}{\circ}$

Assume that

$$|S / S_{234}| \neq 2^9 \text{ and } |S / S_{2345}| \neq 2^{25}.$$

Then $|S / \mathrm{core}_G S| \leq 2^{46}$. (Here, $S_{234} := \mathrm{core}_{\langle P_2, P_3, P_4 \rangle} S$ and $S_{2345} := \mathrm{core}_{\langle P_2, P_3, P_4, P_5 \rangle} S$.)

Peter Rowley

Hermitian forms over $\mathbb{Z}[\omega]$ and Fischer groups.
Let M be a free ~~Rank~~ rank n module over $\mathbb{Z}[\omega]$ $(\omega = e^{2i\pi/3})$ and let $\varphi : M \times M \to \mathbb{Z}[\omega]$ be an hermitian form.
For $p \in \mathbb{Z}$, we put $M_p = \{ m \mid m \in M, \varphi(m,m) = p \}$.
We suppose that φ is positive non degenerate and that M_2 contains a basis B of M.
Theorem 1: The following conditions are equivalent
(i) $M_1 = \emptyset$
(ii) $\forall (a,b) \in B$, $a \neq b$, $\varphi(a,b) \in \{0\} \cup \mathbb{Z}[\omega]^*$.

If $a \in M_2$, we have the reflection $r_a : m \mapsto m - \varphi(m,a)a$: they are isometries of φ. We put $D = \{ r_a \mid a \in M_2 \}$ and $G = \langle D \rangle$.
Theorem 2. If the equivalent conditions of theorem 1 are satisfied then D is a set of 3-transpositions of G. Moreover the possibilities for G and φ are determined

François Zara (Amiens)

Supersoluble subgroups of symmetric groups

(Joint work with M. Bianchi, A. Gillio Berta Mauri)
All maximal supersoluble subgroups of symmetric groups of finite degree are classified.
Among the consequences of this result we have the following:
The symmetric group S_n contains exactly one conjugacy class of maximal supersoluble transitive subgroups exactly when $n \in M$ or $n \in 2M$ where $M = \{ m \in \mathbb{N} \mid \text{if } p, q \text{ are primes dividing } m, \text{ then } p \text{ does not divide } q-1 \}$.

Peter Hauck (Freiburg)

GROUPS OF FINITARY PERMUTATIONS

Let V be a vector space over $\mathbb{K}$ and $g \in GL(V, \mathbb{K})$. The transformation g is finitary if $[V, g] = \{ v(g-1) \mid v \in V \}$ has finite dimension. Denote by $FGL(V, \mathbb{K})$ the groups of finitary transformations in $GL(V, \mathbb{K})$. We call $FGL(V, \mathbb{K})$ the group of $\mathbb{K}$-finitary transformations. Our main result is

THEOREM: Let G be a periodic subgroup of $FGL(V, \mathbb{K})$ and suppose that either $\mathrm{char}\,\mathbb{K} = 0$ or that G is a p'-group and $\mathrm{char}(\mathbb{K}) = p$. Then

i) G is a subdirect product of irreducible $\mathbb{K}$-finitary groups

ii) If G is irreducible, then G has a normal subgroup N such that N is a subdirect power of a finite dimensional $\mathbb{K}$-linear group and G/N is a transitive group of finitary permutations.

iii) If G is irreducible and V has infinite dimension then G' is the unique minimal normal irreducible subgroup of G.

A key ingredient in the proof is a recent result of J. I. Hall which asserts that if $\mathrm{char}\,\mathbb{K} = 0$ and G is an infinite, simple, periodic subgroup of $FGL(V, \mathbb{K})$, then G is alternating and V is the natural module for G.

R. E. Phillips

On induced automorphisms

(P. Gawron &) Olga Macedońska

In 1965. S. Bachmuth and S. Andreadakis proved independently that for the free group F_n of rank n

$$\mathrm{Aut}\, F_n \xrightarrow[\text{onto}]{\text{is not}} \mathrm{Aut}\, F_n/\gamma_4(F_n)$$

The same is not onto for γ_n, $n > 4$.
(γ_n is n-th member of the low central series)

We consider the similar map in case of F_∞ — the free group of countably infinite rank. It was proved, that

$$\mathrm{Aut}\, F_\infty \xrightarrow{\text{is onto}} \mathrm{Aut}\, F_\infty/\gamma_4(F_\infty)$$

Sufficient conditions are given for the map

$$\mathrm{Aut}\, F_\infty \longrightarrow \mathrm{Aut}\, F_\infty/V$$

to be onto, for V - a verbal subgroups of F_∞, such that $V \leq F_\infty'$.

OMacedońska

Automorphisms and normal subgroups

Let G be a group, and denote by $\mathrm{Aut}_n G$ the group of all automorphisms of G which leave every normal subgroup of G invariant. It is clear that $\mathrm{Aut}_n G$ is abelian if G is abelian, and one of the problems about this group is to decide when $\mathrm{Aut}_n G$ is nilpotent. Obviously the groups G for which $\mathrm{Aut}_n G$ is nilpotent are nilpotent, and the following results can be proved:

Theorem 1. Let G be a nilpotent p-group. Then:

(a) If G has finite exponent and $p>2$, the group $\mathrm{Aut}_n G$ is a semidirect product $\mathrm{Aut}_n G = \Phi \ltimes \Lambda$, where Λ is a nilpotent p-group of finite exponent and Φ is cyclic with order dividing $p-1$.

(b) If G has infinite exponent ~~then~~ or $p=2$, then $\mathrm{Aut}_n G$ is a nilpotent group, and its torsion subgroup is a p-group, provided that G is non-abelian.

Theorem 2. Let G be a torsion-free nilpotent group of class c. Then:

(a) There exists a torsion-free nilpotent normal subgroup Λ of $\mathrm{Aut}_n G$ which has class $\leq c-1$ and index ≤ 2.

(b) If G' is not radicable, then $\mathrm{Aut}_n G$ is a torsion-free nilpotent group with class $\leq c-1$.

For mixed nilpotent group a result similar to Theorem 2 holds.
(Joint work with S. Franciosi)

F. de Giovanni (Napoli)

On pairs of groups with periodic cohomology

(joint work with R. Bieri)

Let G be a group and $\underline{S} = \{S_i \leq G \ 1 \leq i \leq n\}$ and consider the short exact sequence of $\mathbb{Z}G$-modules : $0 \to \Delta_{\underline{S}} \to \bigoplus_i \mathbb{Z}(G/S_i) \xrightarrow{\varepsilon} \mathbb{Z} \to 0$

$xS_i \longrightarrow 1$

Def. $(G, \underline{S})$ is called periodic if the $\mathbb{Z}G$-module $\Delta_{\underline{S}}$ has periodic cohomology (i.e. $\mathrm{Ext}^i_{\mathbb{Z}G}(\Delta_{\underline{S}}, -) \simeq \mathrm{Ext}^{i+q}_{\mathbb{Z}G}(\Delta_{\underline{S}}, -)$ for $i \geq 1$)

I. G finite

Thm. A If $(G, \underline{S})$ is p-periodic then G is p-periodic iff S_i is p-periodic for all $1 \leq i \leq n$.

Thm B $(G, \underline{S})$ is p-periodic iff one of the following holds

(i) G is p-periodic

(ii) $S_i = G$ and S_j is p-periodic for all $j \neq i$

(iii) $S_i \neq G$ for all i and there is a p-Sylow subgroup P of G s.t.

α) $N_G(P) \leq S_i$

β) $P \cap S_i^x$ is periodic for all $x \notin S_i$

γ) $P \cap S_j^y$ is periodic for all $y \in G$ and $j \neq i$

II. G finitely generated accessible group

Thm. C If $(G, \underline{S})$ is periodic then G is the fundamental group of a graph $(\mathcal{G}, X)$ of groups where all the edge groups are finite and $\{G_v ; G_v \text{ infinite}\} = \{S_i^{x_i} ; S_i \in S_{inf}\}$.

(S_{inf} = all the infinite members of $\underline{S}$).

Olympia Talelli (Athens)

Fitting Classes in Infinite Groups

The class $\mathfrak{X}$ of periodic radical groups with min-p for all primes p is considered. A Fitting class $\mathfrak{Y}$ of $\mathfrak{X}$ is a subclass of $\mathfrak{X}$ satisfying

i) $H \text{ asc } G \in \mathfrak{Y} \Rightarrow H \in \mathfrak{Y}$

(ii) $G \in \mathfrak{X}$ and $G = \langle H_\lambda : H_\lambda \text{ asc } G,\ H_\lambda \in \mathfrak{Y},\ \lambda \in \Lambda \rangle$
$\Rightarrow G \in \mathfrak{Y}$.

(here H asc G means H is ascendant in G.)

<u>Question</u>: For which Fitting classes $\mathfrak{Y}$ are there $\mathfrak{Y}$-injectors? ($V \leq G \in \mathfrak{X}$ is an $\mathfrak{Y}$-injector if $V \cap A$ is a maximal $\mathfrak{Y}$-subgroup of A whenever A asc G.)

<u>Theorem 1</u>: a) Every $\mathfrak{X}$-group has $L\mathfrak{N}$-injectors.
b) The $L\mathfrak{N}$-injectors are finitely conjugate (that is if V, W are $L\mathfrak{N}$-injectors then the Sylow σ-subgroups of V and W are conjugate in G for all finite sets of primes σ.)
c) The $L\mathfrak{N}$-injectors are isomorphic.

<u>Theorem 2</u>: a) Every $\mathfrak{X}$-group has $\mathfrak{S}_\pi$-injectors for each set of primes π.
b) There is a soluble $\mathfrak{X}$-group which has non-isomorphic $\mathfrak{S}_\pi$-injectors.

Martyn Dixon (Alabama)

Universal nilpotent groups

If $\underline{K}$ is a class of groups then $G \in \underline{K}$ is called universal if any $H \in \underline{K}$ with $|H| \le |G|$ embeds into G. Assume $\underline{K} = L\underline{X}$ for a countable set $\underline{X}$ of isomorphism types of finitely generated groups. $G \in \underline{K}$ is called closed if $\forall A, B \in \underline{X}$ ($\exists H \in \underline{K}$

$$\begin{tikzcd} & B \arrow[dr] & \\ A \arrow[ur] \arrow[dr] & & H \\ & G \arrow[ur] & \end{tikzcd} \Longrightarrow \begin{tikzcd} & B \arrow[d] \\ A \arrow[ur] \arrow[r] & G \end{tikzcd}$$

) where arrows denote embeddings. The class of closed groups in $\underline{K}$ is cofinal in $\underline{K}$ in each power.

Theorem. If $\underline{K}$ is first order axiomatizable and has a countable universal group, then there exists a unique countable closed universal homogeneous group G in $\underline{K}$. Here homogeneous means that any two copies of a ~~[illegible]~~ group from $\underline{X}$ in G are conjugate by an automorphism of G provided they have the same supergroups from $\underline{X}$ in G.

Proposition 1. There exists no countable universal group in $\underline{N}_c$

Proposition 2. " " " " " " $\underline{N}_{c,p}$

Proposition 3. If G is closed in $\underline{N}_c^n$ and the prime divisors of n are $> c$ then $G = T \times G^n$ and T is closed in $\underline{N}_c \cap \underline{B}^n$ and G^n is closed in $\underline{N}_c^+$. If G^n is countable then it is unique up to isomorphism.

Corollary. There is a unique countable closed group in $\underline{N}_2^n$ if $2 \nmid n$.

Proposition 4. The number of countable closed groups in $\underline{N}_2^2$ is $2^{\aleph_0}$.

Remark: There is a unique countable closed group in the classes $L\underline{F}$, $\underline{N}_2 \cap L\underline{F}$, $\underline{N}_2 \cap \underline{B}^n$, $\underline{N}_c^+$, $L\underline{F}_p$, $L\underline{N}^+$ and L(finitely presented)

Notation $\underline{N}_c$: group of nilpotency class $\le c$. $\underline{N}_c^+$: torsion-free groups in $\underline{N}_c^{\#}$. $G \in \underline{N}_{c,p}$: $T(G)$ is a p-group & $G \in \underline{N}_c$. $G \in \underline{N}_c^n$: $T(G) \in \underline{B}^n$ & $G \in \underline{N}_c$
$\underline{B}^n$: Groups of exponent n. $\underline{F}$ finite groups $\underline{F}_p$ finite p-groups.

B. Maier

Alexander Lichtman, On linear groups over division fields generated by enveloping algebras.

Let L be a Lie algebra over a field K, $U(L)$ be its universal ~~[illegible]~~ envelope. It was proven by P.M. Cohn that $U(L)$ can be embedded in a division ring. We denote this division ring by D and prove the following theorems.

Theorem 1. Let D^* be the multiplicative group of D. Then

$$D^* \simeq K^* \times D_1,$$

where the group D_1 is residually torsion free nilpotent if $\operatorname{char} K = 0$ and D_1 is a residually nilpotent p-group of bounded exponent if $\operatorname{char} K = p$.

Theorem 2. Let R be a finite dimensional (over its center Z) skew subfield of D. Then

i) If $\operatorname{char} K = 0$ then R must be commutative

ii) If $\operatorname{char} K = p$ then $(R:Z)$ is a power of p.

D.J. Collins: <u>The automorphism of a free product of finite groups</u>

If F_n is the free group of rank n, then its automorphism group $\mathrm{Aut}\, F_n$ is well-known to be virtually torsion-free. Recently it has been shown that the virtual cohomological dimension $\mathrm{vcd}(\mathrm{Aut}\, F_n)$ is $2n-2$. (Gersten, Culler + Vogtmann) We investigate the corresponding situation for a free product $G = \ast_{i=1}^{n} G_i$ of n finite groups.

Define $C(\ast_{i=1}^{n} G_i)$ to be the kernel of the natural map from $\ast_{i=1}^{n} G_i$ to $\prod_{i=1}^{n} G_i$. Then it is well known that $C(G)$ is free and is of finite rank when all G_i are finite.

<u>Proposition</u> $C(\ast_{i=1}^{n} G_i)$ is characteristic in $\ast_{i=1}^{n} G_i$ and the ~~natural~~ restriction map $\mu : \mathrm{Aut}(\ast G_i) \longrightarrow \mathrm{Aut}(C(\ast_{i=1}^{n} G_i))$ given by $\mu : \alpha \longmapsto \alpha|_{C(\ast G_i)}$ is an embedding.

<u>Corollary</u> $\mathrm{Aut}(\ast_{i=1}^{n} G_i)$ is virtually torsion-free and of finite virtual cohomological dimension.

There is good evidence to support the conjecture:

<u>Conjecture</u> $\mathrm{vcd}(\mathrm{Aut}(\ast_{i=1}^{n} G_i)) = n-1$, ~~for~~ if the groups G_i are finite.

Donald J Collins.

The conjugacy problem for free centre-by-metabelian groups
(with C. K. Gupta and W. Herfort)

The solution of the conjugacy problem for free metabelian groups (Matthews, 1966) reduces the problem for free centre-by-metabelian groups to that of solving $[u,g] \equiv v \pmod{[F'',F]}$, F free, for given $u \in F'$, $v \in F''$ and $g \in F'$ unknown. We first solve the problem modulo $[F'',F]\cdot K$, where K is the torsion subgroup of F''. This uses the faithful representation of $F/[F'',F]K$ in $\mathbb{Z}F/\mathfrak{f}\mathfrak{a}\mathfrak{f}$, where $\mathfrak{f}$ is the augmentation ideal of $\mathbb{Z}F$ and $\mathfrak{a}$ of $\mathbb{Z}F'$. The solution is completed by showing that any solution ~~of~~ for $F/[F'',F]$ comes from one in $F/[F'',F]K$.

F. Levin (Bochum)

Quasi-injective groups

A group G is said to be quasi-injective if every homomorphism θ from a subgroup H of G to G can be extended to an endomorphism $\bar{\theta}$ of G. Abelian quasi-injective groups are either divisible (injective) or are periodic and each p-component is homocyclic (ie $\cong C_{p^n} \times C_{p^n} \times \cdots$ for some $n \in \mathbb{N} \cup \{\infty\}$).

Theorem 1 If G is a nonperiodic soluble quasi-injective group, then G is abelian.

The remaining soluble quasi-injective groups are included in:

Theorem 2 A locally finite quasi-injective group is one of the following

I) abelian

II) G is a split extension of $K = G'$ by H where (a) K is an abelian q.i. π'-group and H an abelian q.i. π-group, (b) for $h \in H$ $\exists\, m = m(h,p,r)$ such that for each $a \in K$ of order dividing p^r, $h^{-1}ah = a^m$, (c) if $\sigma \subseteq \pi'$ then $C_H(K_\sigma)$ is a direct factor of H, (d) every maximal π-subgroup of G is a complement to K in G.

The condition (d) appears to be very strong and may imply that $H/C_H(K)$ is countable (??). It is unlikely that there is a general characterization of q.i. groups as ~~the~~ Ol'šanskii's constructions indicate the possible existence of periodic non-locally finite examples

Mike Tomkinson

Commutator laws

B. H. NEUMANN (Canberra & Bielefeld)

To try to answer a question posed by Luise-Charlotte KAPPE, Ian D. MACDONALD and I have collaborated in a study of the mutual interdependences or independeces of various laws that the commutator in groups either always satisfies, or satisfies in some interesting classes of groups. To do this, we add to the group operations $x \cdot y$, x^{-1}, e, with the usual derived operations $[x,y] := x^{-1} \cdot y^{-1} \cdot x \cdot y$, $[x,y,z] := [[x,y],z]$, $x^y := y^{-1} \cdot x \cdot y = x \cdot [x,y]$, a further binary operation κ, written as a right-hand operator, $xy\kappa$, the "κ-tator", which is to satisfy some but not all of the laws that $[x,y]$ satisfies. Central to the investigation are the laws of JACOBI-WITT-HALL type in 3 variables and their extensions to 4 and more variables. Although much more remains to be done, we have some results of sufficient interest to have collected some of them in a joint paper, submitted for publication to the Journal of the Australian Mathematical Society, Series A.

B. H. Neumann. 1987-05-08

Regular Elements in Galois Extensions

In a finite Galois Extension L/K an element x will be called 'completely regular' if x serves as a generator for the MU-module L for all $U \leq G := Gal(L/K)$ and M the field of fixed elements of U. One can show that completely regular elements always exist. The proof of this theorem (joined work with K. Johnsen in Kiel) for infinite K is a variation of a proof given by Emil Artin for the existence of a normal basis for L/K. If K is a finite field we can restrict ourselves to the case that G is a cyclic q-group for some prime q. The case $q = \mathrm{char}\, K$ is easy. If $q \neq \mathrm{char}\, K$ one has to look closer on the structure of L as a K_iG_i-module where $G = G_0 > G_1 > \dots > G_{n-1} > G_n = 1$ and $K = K_0 < K_1 < \dots < K_{n-1} < K_n = L$ are the chains of all subgroups of G respectively of all subfields of L containing K. By a comparison of the direct decompositions of $T = \mathrm{Ker}\, \mathrm{Tr}_{L/K_{n-1}}$ into the irreducible K_iG_i-submodules for $0 \leq i \leq n-1$ and induction for K_{n-1} the theorem is proved.

Dieter Blessenohl (Kiel)

Chief factors and projective indecomposable modules

The abelian p-crowns C/D of a finite group G are canonically embedded in the second term PJ/PJ^2 of the lower Loewy series of the principal indecomposable projective module P over the group algebra $GF(p)\, G$, and there is a canonical bijective correspondence between the set of conjugacy classes of supplements of C/D in G and the set of supplements of the image of C/D in PJ/PJ^2. This permits us to clarify the relation of this term PJ/PJ^2 of P with the normal structure of G.

Julio Lafuente (Zaragoza)

Automorphisms of free nilpotent groups

Let $F_{n,c}$ be the free nilpotent group of class c on n generators $x_1, \ldots, x_n$ $(n \geq 2)$. Thus $F_{n,c} \cong F_n / \gamma_{c+1}(F_n)$ where F_n is free. Let T be the subgroup of $\mathrm{Aut}(F_{n,c})$ consisting of those automorphisms which are induced from $\mathrm{Aut}(F_n)$. Let δ be the automorphism of $F_{n,c}$ satisfying $x_1\delta = x_1[x_1, x_2, x_1]$ and $x_i\delta = x_i$ $(i \geq 2)$. Then, for $n \geq \frac{c}{2}+1$, $\mathrm{Aut}(F_{n,c}) = \langle T, \delta \rangle$. Hence, for $n \geq \frac{c}{2}+1$ and $n \geq 4$, $\mathrm{Aut}(F_{n,c})$ is a 3-generator group. (Joint work with C. K. Gupta.)

R. M. Bryant.

Primitive subgroups of wreath products in product action

Given a primitive group G on a finite set Ω such that $\mathrm{soc}\, G$ is not regular, one may wish to account for the W with $G \leq W \leq \mathrm{Sym}\,\Omega$ such that $\mathrm{soc}\, W = \mathrm{soc}\, G$ and W is a wreath product in product action. To this end, let H be a point stabilizer in G, and K a maximal normal subgroup of $\mathrm{soc}\, G$. Let P denote the intersection of those maximal normal subgroups K_i of $\mathrm{soc}\, G$ for which $H \cap K_i = H \cap K$. For each X with $(\mathrm{soc}\, G)N_H(H \cap K) \leq X < G$, do:-

(1) Form the orbit space $\Omega_X := \Omega / \mathrm{core}_X P$; denote by $\bar{X}$ and M_X the restrictions of X and $\mathrm{soc}\, G$ to Ω_X; & let a_X be the number of those A_X with $\bar{X} \leq A_X \leq N_{\mathrm{Sym}\,\Omega_X}(M_X)$ which are not wreath products in product action;

(2) Form the group $G \div X$ induced by G on the coset space G/X, & let b_X be the number of those B_X with $G \div X \leq B_X \leq \mathrm{Sym}\, G/X$.

There are precisely $\sum_X a_X b_X$ such W, namely the $A_X \mathrm{wr} B_X$ in product action.

L G Kovács

Groups given by presentations in which each defining relator involves exactly two generators

Stephen J. Pride

Let $G = \langle \underline{x} ; \underline{r} \rangle$ where each element of $\underline{r}$ is cyclically reduced and involves exactly two generators $\in \underline{x}$. Let Γ be the graph with vertex set $\underline{x}$ and edge set

$$E = \{\{x, y\} : \text{some element of } \underline{r} \text{ involves both } x \text{ and } y\}.$$

For $\{x, y\} \in E$ let $G\{x, y\} = \langle x, y ; \underline{r}\{x, y\} \rangle$ where $\underline{r}\{x, y\}$ consist of all elements of $\underline{r}$ involving x and y. We call $G\{x, y\}$ an "edge group". Under rather mild restrictions on Γ and the edge groups we obtain a series of results concerning the structure of G, namely: the embeddability of the edge groups into G; the diagrammatic asphericity of G; the structure of the relation module of G; the (co)homology of G in dimensions ≥ 3; torsion in G. Some of these results were obtained jointly with Ralph Stöhr. Our theorems have been generalized in two different directions, by M. Edjvet, and by E. Fennessey. In particular, Fennessey's work concerns the case when $G = \langle A_i \ (i \in I) ; \underline{r} \rangle$ where the A's are non-trivial groups and each $R \in \underline{r}$ is a cyclically reduced element of $\underset{i \in I}{*} A_i$ involving terms from exactly two factors.

Stephen J Pride Glasgow.

The groups $G^{k,l,m}$

Marston Conder (Auckland, NZ.)

8 May 1987

For positive integers k,l,m, the group $G^{k,l,m}$ may be defined by

$$G^{k,l,m} = \langle A,B,C \mid A^k = B^l = C^m = (AB)^2 = (BC)^2 = (CA)^2 = (ABC)^2 = 1 \rangle$$

or by $\langle x,y,t \mid x^2 = y^k = (xy)^l = t^2 = (xt)^2 = (yt)^2 = (xyt)^m = 1 \rangle$

or by $\langle X,Y,Z \mid X^2 = Y^2 = Z^2 = (XY)^2 = (YZ)^k = (ZX)^l = (XYZ)^m = 1 \rangle$

Such groups were dealt with extensively by Coxeter (Trans AMS, 45 (1939))
In particular, when $\cos\frac{2\pi}{k} + \cos\frac{2\pi}{l} + \cos\frac{2\pi}{m} < 1$ he shows $G^{k,l,m}$ is finite, eg $G^{4,5,6} \cong S_5 \times C_2$, $G^{5,5,5} \cong PSL(2,11)$, $G^{3,7,12} \cong PGL(2,13)$.
Supposing $k \le l \le m$, for $G^{k,l,m}$ to be infinite we require $k \geq 3$, and if $k=3$ then $l \geq 7$, and so on. The groups $G^{3,7,m}$ are known to be finite for all $m \le 17$, infinite for $m = 18, 20$, and $G^{3,7,19}$ has J_1 and $PSL(2,113)$ as quotients, but it is not known whether $G^{3,7,19}$ is finite or infinite.
For some larger m, $G^{3,7,m}$ is not just infinite but has all but finitely many A_n and S_n as quotients (eg $m = 720720$ M.C.; $m = 3960$ Graham Higman; recently $m = 168$ M.C.). We conjecture $G^{3,7,24}$ has this property too.
On the other hand, Graham Higman has shown $G^{6,8,8}$ has all but finitely many S_n as quotients. This can be improved: for all $n \geq 250$, both A_n and S_n are factor groups of the group $G^{6,6,6}$. The proof uses coset diagrams for the latter group, together with a method of composition for obtaining transitive permutation representations of arbitrarily large degree. [It may be possible that $G^{5,6,6}$ will also do, in which case that would be the "best" result of this type]

MDEC.

Dynamische Systeme 1987

10. V – 16. V.

Flows on surfaces.

D. V. Anosov.

Notations: M = closed surface of Euler characteristic ≤ 0, $\tilde{M}$ = its universal covering, A = absolute, $\{\varphi_t\}$ = flow on M, $\{\tilde{\varphi}_t\}$ = its lift to $\tilde{M}$.

General idea is that under rather general conditions (but not always) each semitrajectory $\tilde{L} = \{\tilde{\varphi}_t \tilde{z}; t \geq 0\}$ either remains bounded or tends to some point a of A (this a plays a role similar to "rotation number"). In the latter case sometimes (but not always) $\tilde{L}$ remains a bounded distance from geodesic going to a. In exceptional case $\tilde{L}$ can even have all points of A as its "limit points at infinity". Related questions concern behaviour of lifts to $\tilde{M}$ of leaves of foliations on M or, more generally, nonselfintersecting infinite curves on M.

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Other publications will appear in the same J. and in the "Proc. of Steklov Math. Inst.", v. 185.

Аносов

Minimal laminations and commuting circle homeomorphisms
(V. Bangert)

Let $F: \mathbb{R}^n \times \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$ be an integrand as considered by J. Moser (Ann. Inst. Fourier – Analyse non linéaire 1986): F is periodic in the first $n+1$ variables, has positive second derivative with respect to the last n variables and satisfies certain natural growth conditions. A function $u: \mathbb{R}^n \to \mathbb{R}$ is an (F-) minimal solution if $\int F(x, u(x), u_x(x))\,dx$ is minimal with respect to all compactly supported variations of u. We say that u has no selfintersections if the projection of graph$(u) \subseteq \mathbb{R}^{n+1}$ into the torus $\mathbb{R}^{n+1}/\mathbb{Z}^{n+1}$ does not have proper selfintersections. Extending results by J. Moser (l.c.) we give a complete description of the set of all minimal solutions without selfintersections. Roughly it says the following: For every possible "type" of non selfintersecting graphs there exists a nonempty set of minimal solutions of this type unless the graphs of the minimal solutions of some "more periodic type" foliate $\mathbb{R}^{n+1}$. The graphs of the minimal solutions of the same "type" do not intersect and form a foliation or a lamination (≈ foliation with gaps). This is a generalization of Aubry-Mather theory to this higher dimensional setting.

V. Bangert

SMOOTH CONJUGACY OF ANOSOV SYSTEMS (R. MORIYON)

The results on smooth conjugacy of Anosov systems that have been proved up to now, and the technics that are involved, are discribed. The situation is completely understood for low dimensional systems (maps in dimension two and flows in dimension three): given two topologically conjugate Anosov systems, of this sort, a necessary and sufficient condition for the smoothness of the diffeomorphism that conjugates one to the other is that corresponding periodic points have the same Lyapunov exponents. Nothing is known in the higher dimensional case, except in the case of symplectic (canonical or hamiltonian) systems, where there are partial results.

R M

An infinite dimensional KAM - theorem
j. pöschel

A perturbation theory of KAM - type is presented concerning certain weak couplings of harmonic oscillators on a lattice $\Lambda = \mathbb{Z}^d$, $d \geq 1$. The system is described by the Hamiltonian

$$H = \sum_{i \in \Lambda} \omega_i I_i + \varepsilon \sum_{A \in \mathcal{A}} P_A(I, \varphi)$$

where $\mathcal{A}$ is a system of finite subsets of Λ, and P_A "lives on A". The frequencies ω_i are considered as parameters in a set $\Omega \subset \mathbb{R}^\Lambda$. Given a measure μ for subsets of Λ, for example

$$(*) \qquad \mu(A) = \sum_{i \in A} |i| ,$$

it is shown that if (roughly speaking!)

$$|P_A| \sim e^{-\mu(A)}$$

then for sufficiently small ε the perturbed system possesses a large family of ∞-dim invariant tori; their frequencies are strongly nonresonant:

$$|(k, \omega)| \geq \gamma \cdot f(|k|) \cdot f(\mu(\operatorname{supp} k)) , \quad \forall k \in \mathbb{Z}^\Lambda \setminus \{0\}$$

for $f(t) = e^{-t / \log^{1+\varepsilon} t}$, say. The point (among other) is to show μ so that this can be satisfied. For (*) this is the case. — The above represents an improvement of a recent result by Fröhlich, Spencer and Wayne.

Pöschel

Algebraic differential equations in the plane have a finite number of limit cycles.

J. Martinet.

Theorem. An analytic differential equation on an analytic, compact, real surface, has a finite number of limit cycles.

A classical argument (DULAC) reduces this theorem to proving that the Poincaré first return mapping $G : (\mathbb{R}^+, 0) \to (\mathbb{R}^+, 0)$ of a polycycle (made up of hyperbolic or $1/2$-hyp. singular points and separatrices) is either the identity or has no fix points close to 0. This map G is the composition of local maps G_i corresponding to the singular points. At a hyperbolic saddle, G_i belongs to a quasi-analytic ring $\mathcal{I}$ (essentially identified by Il'yashenko). At a saddle-node, G_i is essentially the exponential of a resurgent function (in the sense of Ecalle) $R_i \in \mathcal{R}$; $\mathcal{R}$ is another quasi analytic ring such that $\mathcal{I} \cap \mathcal{R} =$ Convergent series. The theorem is proved by means of further similar quasi analyticity and independence arguments. This proof is due to J. Ecalle, J.M, R. Moussu and J.P. Ramis; an independent proof is announced by Il'yashenko

J. Martinet

PRACTICAL STABILITY FOR A HAMILTONIAN SYSTEM NEAR AN ELLIPTIC EQUILIBRIUM POINT

Consider an n-degrees of freedom Hamiltonian system near an elliptic equilibrium point, with Hamiltonian $H = \frac{1}{2}\sum_{l=1}^{n} \omega_l (q_l^2 + p_l^2) + H_3 + \dots$, where $\omega \in \mathbb{R}^n$ and H_s, $s \geq 3$, is a homogeneous polynomial of degree s.

It is a classical result that, for a fixed $r \geq 2$, one can find a canonical transformation $(q,p) \to (q^{(r)}, p^{(r)})$ such that the transformed Hamiltonian takes the form $H^{(r)} = Z^{(r)} + \mathcal{R}^{(r)}$, where $Z^{(r)} = H_2 + Z_3 + \dots + Z_r$ is in Birkhoff normal form, and $\mathcal{R}^{(r)} = H^{(r)}_{r+1} + \dots$ is the unnormalized remainder. The normal form depends on the resonance relations which exist between the ω's.

Such result is made rigorous by giving an explicit lower bound on the convergence radius of the Hamiltonian $H^{(r)}$, and an upper bound for the size of the remainder $\mathcal{R}^{(r)}$. More precisely, one proves that $H^{(r)}$ is convergent in any polydisc $\mathcal{D}_R$ with $R < R_r^* \approx c/r^\tau$, and that in such polydisc one has $|\mathcal{R}^{(r)}| \leq d\, r^{\tau r} R^r$, where $\tau \geq 1$ and $c, d > 0$. The integrals of motion of $Z^{(r)}$ can then be used to prove that orbits starting in the polydisc $\mathcal{D}_{R_0}$ remain bounded in $\mathcal{D}_R$ with $R = \sigma R_0$, $\sigma > 1$, for times $|t| < T \approx \frac{\eta}{r^{\tau r} R^r}$, $\eta > 0$.

The apparently poor dependence on r in these bounds can be used in a powerful way by adapting the normalization order r to the radius R_0 of the polydisc containing the initial data. In particular, given R_0 one looks for the optimal value r_{opt} of r for which the bound above for the remainder is near to a minimum, and T is maximized. This gives $r_{opt} = \left[\frac{1}{e}\left(\frac{\hat{R}}{\sigma R_0}\right)^{1/\tau}\right]$, and

$T = \tau \exp \frac{I}{e}\left(\frac{\hat{R}}{R_0}\right)^{1/\tau}$, $\hat{R}$ and τ being constants. Thus the result is obtained that the stability time exponentially increases with $1/R$.

For example, for the Lagrangian point L_4 of the circular restricted three body problem in the spatial case one obtains a stability radius of some kilometers for a time interval $T = 10^{10}$ years, i.e. the estimated age of the universe.

Antonio Giorgilli

(ANTONIO GIORGILLI)

Critical points on the boundaries of Siegel singular disks.

Michael R. HERMAN

We described the recent open questions on Siegel's linearization theorem for analytic maps of 1 complex variable near an elliptic fixed point (the center problem). We showed (E. Ghys 83) the relation between the study of the boundary behaviour of Siegel's linearization map for an elliptic fixed point of a polynomial and the existence analytic conjugacies of diffeomorphisms of the circle to rotations. Using a construction of E. Ghys 83 we obtained the following theorem:

Theorem There exists $\alpha \in \mathbb{R} - \mathbb{Q}$ such that $P_\alpha(z) = e^{2\pi i \alpha}(z + z^2)$ is linearizable at 0 and the Siegel singular disk S, $0 \in S$ is quasi disk but the critical point $-\frac{1}{2}$ of P_α does not belong to the boundary of S.

Herman

Solution of the Stability Conjecture

A diffeomorphism is called C^r structurally stable if any C^r nearby diffeomorphism is conjugate to it via a

homeomorphism of the ambient manifold. After a series of contributions (specially by Anosov, Palis-Smale) these last two authors conjectured in 1967 that "f is structurally stable iff $\Omega(f)$ is hyperbolic and for any pair of points in $\Omega(f)$ their stable and unstable manifolds are in general position. A similar conjecture is: "$f/\Omega(f)$ is stable iff $\Omega(f)$ is hyperbolic and there are no cycles on $\Omega(f)$". The "if" part of these conjectures were proven by Robin-Robinson and Smale, resp.
Theorem (Mañé): The C^1 stability conjecture is true.
Theorem (Palis): The C^1 Ω-stability conjecture is true.

J Palis

Refinement of Shannon-McMillan-Breiman Theorem for some maps of an interval
Krystyna Ziemian (WARSAW UNIVERSITY)

Let f be a piecewise monotone map of an interval I into itself, $\mathcal{A}$ - the natural partition of I into the pieces of monotonicity of f, $\mathcal{A}_n = \bigvee_{i=0}^{n-1} f^{-i}\mathcal{A}$, $A_n(x)$ - this atom of $\mathcal{A}_n$ which contains x. We assume that f has non-positive Schwarzian derivative, trajetory of critical points stays far from the set of critical points, there are no attracting periodic orbits (all periodic orbits are repelling) and near critical points f looks like x^u, $u \geq 0$. For some iterate f^k we have a probabilistic measure μ, f-invariant, absolutely continuous with respect to the Lebesgue measure, and such that the system (f^k, μ) is weakly mixing. Assume for simplicity (f, μ) is weakly mixing. (M. Misiurewicz)

We prove that the sequence $\log(\mu(A_m(x))) + nh_\mu(f)$ satisfies an almost sure invariance principle for a large class of f (in particular all unimodal maps except fully chaotic with $h_\mu(f) = \log 2$).

This theorem (Refinement of Ph.-B.-McM.) implies the speed of convergence of $-\frac{1}{n}\log \mu(A_m(x))$ to $h_\mu(f)$ is not faster than $\sqrt{\frac{\log\log n}{n}}$ for a large class of f.

Global bifurcation of periodic solutions with symmetry

Bernold Fiedler

For differential equations with a compact symmetry group Γ we present some results on global Hopf bifurcation. In particular we investigate, how the spatial and temporal action of the group Γ on a periodic solution may vary along global bifurcation branches, e.g. via period doubling bifurcations. The results are obtained geometrically by generic but equivariant approximation, rather than by topological techniques. We discuss applications to reaction diffusion systems.

Ergodic Geodesic Flow on S^2 (V. J. Donnay)

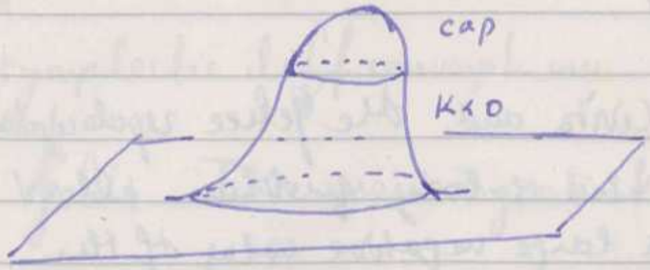
We show that any compact orientable surface can be given a C^∞ Riemannian metric so that the geodesic flow is ergodic with positive measure entropy.

Start by deleting 3 or more points from the sphere (1 or more for genus $g \geq 1$). The universal cover of this punctured surface is the disk and the Poincaré metric projects onto the surface. In ~~the~~ a neighborhood of the deleted point, this metric produces a cusp going to infinity. We cut this cusp off and replace it with a cap made from a surface of revolution. Using the Clairaut Integral, we can calculate explicitly the behaviour of geodesics in the cap. We choose the cap so that a diverging family of geodesics will focus when they go thru the cap and then again be diverging when they leave the cap.

We then construct an invariant cone family in $T(SM)$ which by Wojtkowski's work implies non-zero Lyapunov exponent almost everywhere. For ergodicity, we construct stable and unstable foliations and show these foliations are global. We then use a Hopf-type argument.

A second type of example is given by the flat torus with cap. Here the curvature is predominately $K=0$ outside the cap rather than $K=-1$ as in the previous construction. Considering the cap as a perturbation of the flat metric on the torus we have a one-parameter family of dynamical systems starting with ~~the~~

integrable case and ending with the ergodic case.



Flat torus with cap.

Victor J. Donnay

Periodic and Quasi-periodic solutions for non-linear ~~way~~ Wave equations. (C. E. Wayne)

Recently some progress has been made in understanding the existence of ~~peri~~ periodic and quasi-periodic orbits in hamiltonian systems with infinitely many degrees of freedom via classical mechanical perturbation theory. A natural class of systems to consider from this viewpoint is the class of perturbations of completely integrable partial differential equations. Probably the simplest example of such an equation is

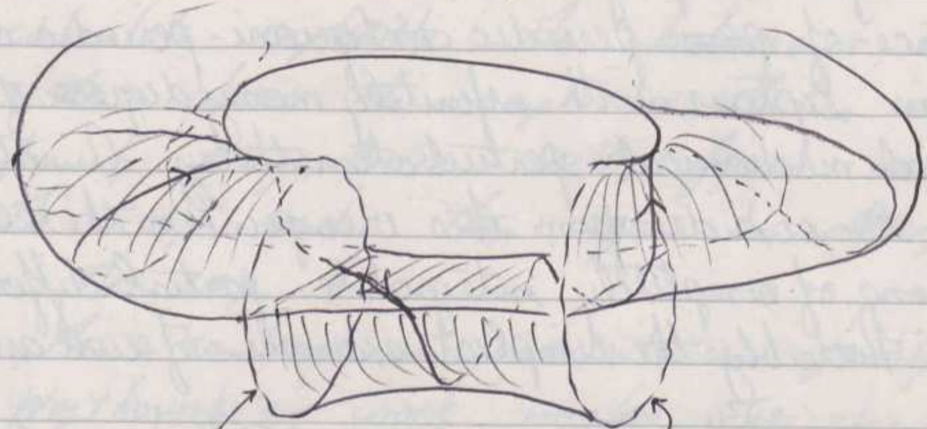
$$\frac{\partial^2 u}{\partial t^2}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) + v(x)u(x,t) + \varepsilon u^3(x,t) \quad ; \quad 0 \le x \le 1, \ t \in \mathbb{R}$$
$$u(0,t) = u(1,t) = 0 .$$

Expanding $u(x,t)$ in terms of the eigenfunctions of $(\partial_x^2 + v)$ (plus Dirichlet boundary conditions, one rewrites this equation as an infinite system of non-linearly coupled oscillators. One can prove the existence of periodic solutions for this system by means of KAM theory for a large class of potentials $v(x)$ (and more general non-linearities.) It is hoped that these methods will ~~also~~ yield a proof of the existence of quasi-periodic trajectories too. These methods are somewhat complementary to the bifurcation theory and variational techniques previously applied to this equation.

C. E. Wayne

Invariant punctured tori in the restricted 3-body problem (A. Chenciner).
(Joint work with J. Llibre).

A comparison of the Levi-Civita and McGehee regularizations of the (planar circular) restricted 3-body problem allows us to prove the existence, for large negative values of the Jacobi constant, of invariant punctured tori in the phase space of the problem (diffeomorphic to an open solid torus).
When the collisions (ejections) are added, these tori become honest KAM tori. Even if complete integrability occured, such tori would lead to complicated motions in the moving frame. After a linear change of coordinates, these tori appear like in the following picture:



When this circles are removed, one gets a torus minus two points.

Th

Invariant Cones and Ergodicity

Anatole Katok

We establish verifiable criteria for ergodicity and for stronger stochastic properties including Bernoulli property for several classes of classical dynamical systems including symplectic diffeomorphisms and contact flows on compact manifolds. The main ingredient is the existence of a continuous family of symplectic cones, defined for all points of a sufficiently large open subset U of the phase space, which is mapped inside itself and eventually strictly inside itself by the linearization of the dynamical system induced on U. A result of Wojtkowski insures that Lyapunov characteristic exponents are different from zero almost everywhere. Thus, Pesin theory can be applied to deduce that ergodic components have positive measure. Our main technical advance is an observation based upon a simple lemma about symplectic linear maps, which allows us to extend almost every stable and unstable manifold to uniform size without too much wiggling. Then one can show that ergodic components are essentially open sets and with an extra condition, which is easy to satisfy in most cases, that a single ergodic component encompasses the set U. In the case of flow Bernoulli property follows from non-integrability of contact structure.

These results allow us to unify and simplify the proofs for the number of previously known cases of ergodicity as well as to obtain some new ones. The primary new application is ~~the~~ a construction of a C^∞ Riemannian metric with ergodic and Bernoulli geodesic flow on every three-dimensional manifold

Alex

Applications of Nekhoroshev-like exponential estimates
(Giancarlo Benettin)

Nekhoroshev-like exponential estimates are worked out for a class of Hamiltonian dynamical systems, which are interesting for physical applications. In the simplest case the Hamiltonian is $H = \omega \underline{I} + h(p,q) + \omega^{-1} f(I, \varphi, p, q)$, $(p,q) \in \mathbb{R}^{2n}$, and one wants to estimate the rate of the energy exchange between ωI and h, in the limit $\omega \to \infty$, but with a finite total energy. Two physical applications are suggested:
(a) the introduction of a constraint in a dynamical system,

by means of a physical device having a large elastic constant; (b) Jeans' problem (1903) of the collision of an atom with a vibrating molecule. The result is that the energy exchange per unit time is exponentially small, in ω, so that: (a) the constrained system does not exchange an appreciable amount of energy with the transversal vibrational motion, for an exponentially long time; (b) an exponentially large number of collisions are needed, in order the vibrating molecule changes appreciably its internal energy. This means that, as conjectured by Jeans, there exists a completely classical mechanism for the so-called "freezing" of the high-frequency degrees of freedom, which is usually considered to be a purely quantum effect.

[illegible]

Small divisors & Siegel's method.

L. H. Eliasson

An idea behind a new proof of the existence of quasi-periodic solutions of a perturbation of an analytic (non-deg.) integrable Hamiltonian system is explained.

Such a system has a unique formal quasi-periodic solution for any fixed frequency.

The main difficulty in proving the convergence of the formal solution is that the "standard" series representation – the Lindstedt series – of this solution is not absolutely convergent. But the formal solution has another series representation which is absolutely convergent. This can be proved by direct estimates of the coefficients using a generalization of Siegel's method.

Niles Ch[illegible]

Dynamical properties of monotone twist map of the annulus.

(Patrice le Calvez)

J. Mather proved, using a variational method, that if f is an area-preserving monotone twist map of the annulus which has a region of instability there is a point whose orbit goes from a border of the region to the other one. Using topological arguments, introduced by Birkhoff in the study of instable elliptic fixed points, we can give a simple proof of this fact. These topological techniques permit to give a more precise description of the dynamic of f inside the region of instability. Adding a very weak property we show the existence of a certain ~~set~~ compact connected invariant set which has all the "non trivial" properties of the region and which is the common closure of the stable and unstable sets of each border. They also permit to show the following result.

If we perturb our given map, the new one, which is not necessarily conservative, will still have ~~[illegible]~~ a large interval of rotation of Aubry-Mather sets.

[illegible]

Central Configurations in the 1+n body problem

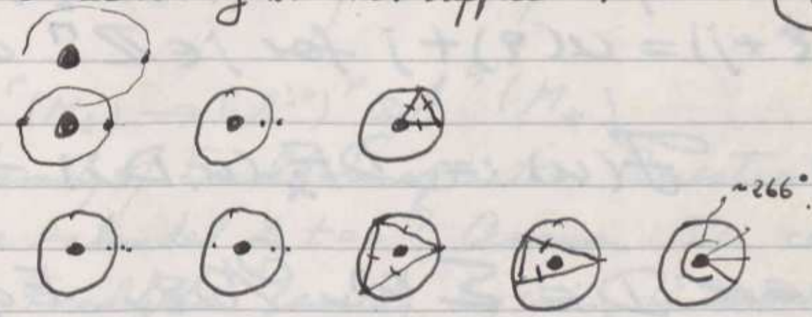
G.R. Hall.

Let $q_0, \dots q_n \in \mathbb{R}^2$ be positions, $m_0 = 1$, $m_1 = \dots = m_n = \varepsilon$ be masses of n+1 bodies. We consider 1+n body central configurations which are (def.) limits of central configurations as $\varepsilon \to 0$. (By Conley-Moeckel $\perp$ bisector theorem, $q_0 = 0$, $|q_1| = \dots = |q_n|$ is necessary but not sufficient.)

∃ Results: 1+1 body

1+2 bodies

1+3 bodies



1+n bodies regular n-gon about 0.

Prop. When $n \gg 1$ there is only one 1+n body central configuration with $q_i \neq q_j$, $i \neq j$. (it is the regular n-gon).

The best n known is $n > e^{14\cdot 16\cdot (2\pi)^3}$.

Glen R. Hall

Boston.

KAM Theory in Configuration Space

(D. Salamon & E. Zehnder)

A new approach to KAM theory is presented adopting the Lagrangian rather than the Hamiltonian point of view. Let $F(x, \dot{x})$ be a Lagrangian which is periodic in the x-variables. An invariant torus for the

Euler equations

(1) $\frac{d}{dt} F_{\dot{x}} = F_x$

with frequency vector $\omega \in \mathbb{R}^n$ is given by a transformation $x = u(\xi)$ of T^n satisfying $u(\xi + j) = u(\xi) + j$ for $j \in \mathbb{Z}^n$ and

(2) $\mathcal{F}(u) := DF_{\dot{x}}(u, Du) - F_x(u, Du) = 0$

where $D = \sum \omega_\nu \, \partial/\partial\xi_\nu$. Equation (2) can be solved with the usual small divisor techniques provided that ω satisfies Diophantine estimates, $F_{\dot{x}\dot{x}}$ is nonsingular, F is sufficiently smooth and $\mathcal{F}(id)$ is sufficiently small. The proof is based on Newton's method and the key point is to overcome the nondegeneracy in the equation

$$\mathcal{F}'(u) v = -\mathcal{F}(u),$$

which has to be solved approximately in each step of the iteration.

This approach to KAM theory has been motivated by Moser's perturbation theorem of minimal foliations for variational problems on a torus.

J. Mann

Elliptische Operatoren auf singulären und nichtkompakten Mannigfaltigkeiten (17.5. – 23.5.1987)

Extending $\bar\partial$ over limit points in moduli space

R. Seeley + I. Singer

One can form a family M_t of Riemann surfaces of genus g, varying with a modular parameter t, by "operating" on a surface M^{g-1} of genus $g-1$: Take two disjoint disks $\{|z|<1\}$ and $\{|w|<1\}$ in M^{g-1} and, for $|t|<1$, identify $\{|t|<|z|<2\}$ with $\{|t|<|w|<1\}$, by the map $zw=t$. Then for $t=0$,

$$\bar\partial_p : (\Lambda^{1,0})^p(M_t) \longrightarrow (\Lambda^{1,0})^p \otimes \Lambda^{0,1}(M_t)$$

is a holomorphic family of Fredholm operators. Question: Can this family be extended to $t=0$? Answer: It can, as a continuous Fredholm family, by using L^2 norms defined in the two annuli by

$$\left\| u\left(\frac{dz}{z}\right)^p \right\|^2 = \int |u|^2 r^{-1}\,dx\,dy, \quad \left\| f\,d\bar z \left(\frac{dz}{z}\right)^p \right\|^2 = \int |f|^2 r^{-1}\,dx\,dy.$$

Similar results hold for half-integer p. As a consequence, one recovers $\operatorname{index}(\bar\partial_p) = (2p-1)(g-1)$.

R. Seeley

Singular Hilbert boundary value problem

Consider an n-dimensional subvariety $R \subset \mathbb{C}^n$ which is almost everywhere totally real. The problem is to describe the space of compact holomorphic curves (Riemann surfaces) $C \subset \mathbb{C}^n$ with boundary in R. The local structure of this space is effected by singularities of R. If these are mild enough and $(\mathbb{C}^2, R)$ is tamed by a symplectic form ω, then one can study global properties as well.

M. G.

An n-dimensional Borg-Levinson theorem

Gunther Uhlmann

A classical result of Borg and Levinson says that we can determine a real-valued potential $q \in L^\infty[0,1]$ from the Dirichlet eigenvalues of $-\frac{d^2}{dx^2} + q(x)$ and their normal derivative at the boundary ~~boundary value~~ of normalized Dirichlet eigenfunctions.

We generalize this theorem to higher dimensions in the following fashion:

Let $\Omega \subseteq \mathbb{R}^n$, $n \geq 2$, be a bounded, smooth domain and $q_1, q_2 \in C^\infty(\bar{\Omega})$ are real-valued potentials. Let us consider the Dirichlet eigenvalues $\{\mu_j(q_i)\}_{j=1}^{\infty}$ associated to the Schrödinger equation $-\Delta + q_i$ $i=1,2$.

Let $\{\varphi_j(q_i)\}$ be a corresponding complete system of orthonormal eigenfunctions. We proved

Theorem Suppose $\lambda_j(q_1) = \lambda_j(q_2) \ \forall j$

$$\frac{\partial}{\partial \nu}\left(\varphi_j(q_1)\right)\Big|_{\partial\Omega} = \frac{\partial}{\partial \nu}\left(\varphi_j(q_2)\right)\Big|_{\partial\Omega}$$

where $\frac{\partial}{\partial \nu}$ denotes outer normal derivative. Then

$$q_1 = q_2 \quad \text{in } \overline{\Omega}.$$

The proof consists of showing that the information provided in the theorem ~~allows to determine the~~ shows that the Neumann map for $q_1 - \lambda$ and $q_2 - \lambda$ coincides $\forall \lambda \in \mathbb{C}$.

If we have a solution of $(*)$ $(-\Delta + q - \lambda)u = 0$

$$u\big|_{\partial\Omega} = f$$

then the Neumann map, $\Lambda_{q-\lambda}$, associated to $q - \lambda$ is defined by

$$\Lambda_{q-\lambda}(f) = \frac{\partial u}{\partial \nu}\Big|_{\partial\Omega} \quad \text{with } u \text{ solution of } (*).$$

This is joint work with A. Nachman and J. Sylvester.

G. Uhlmann

COMPLEX POWERS OF OPERATORS ON NON-COMPACT MANIFOLDS

In order to analyze complex powers of pseudodifferential operators (pdo) on non-compact manifolds, a class of weighted symbols and Sobolev spaces, first investigated by H-O. Cordes on $\mathbb{R}^n$ is transferred to manifolds with a compatible structure. These include manifolds consisting of a compact center and finitely many ends. In particular, some cases of manifolds

with singularities can be treated via coordinate transforms.
For certain elliptic pdo with positive differentiation and multiplication order, a family of complex powers $\{A_s : s \in \mathbb{C}\}$ is constructed.
The kernel function $k_s(x,y)$ as well as the zeta and eta functions of A then show an analytic behavior very similar to that in case of a compact manifold.

Elmar Schrohe

Branch points of the resolvent, standing waves and resonances in some classes of unbounded domains

Let A be the self-adjoint extension of $-\Delta$ in a domain $\Omega \subset \mathbb{R}^n$ with respect to the boundary condition $u = 0$ (or $(\partial/\partial n)u = 0$) on $\partial\Omega$. If Ω is bounded, then the spectrum of A is discrete, the resolvent $(A-z)^{-1}$ is meromorphic with simple poles at the eigenvalues $\lambda_1, \lambda_2, \ldots$ of A, and the solution of the initial and boundary value problem $\partial_t^2 u - \Delta u = f e^{-i\omega t}$ in Ω, $u = 0$ on $\partial\Omega$, $u(x,0) = \partial_t u(x,0) = 0$ with $f \in C_0^\infty(\Omega)$ has a resonance of order t if $\omega = \sqrt{\lambda_i}$ $(i = 1, 2, \ldots)$. New phenomena arise if Ω is unbounded. Certain unbounded domains admit resonances of order $t^{1/2}$ and $\ln t$ which are not related to eigenvalues of A. These resonances are connected with branch points of the resolvent on the real axis and discontinuities of the derivative $(d/d\lambda)P_\lambda f$ of the spectral family P_λ of A. As examples, we discuss the asymptotics of $u(x,t)$ as $t \to \infty$ in $\mathbb{R}^2$, in two-dimensional exterior domains, in $D_0 := \mathbb{R}^n \times (0,1)$, and in local perturbations Ω of D_0. Resonances occur in these domains if and only if the corresponding homogeneous boundary value problem for $\Delta u + \omega^2 u = 0$ has certain non-trivial solutions ("admissible standing waves"), which can be characterized by suitable conditions at infinity. In contrast to the bounded case, these resonances are extremely unstable and can be simultaneously removed by small perturbations of the domain. Peter Werner (Stuttgart)

Fredholm and Liouville Properties of Laplaceans on Non-compact Manifolds: Suppose M is a non-compact manifold with ends. Thus outside a compact set M_0 we have $M - M_0 = \partial M_0 \times \mathbb{R}^+$. On such a manifold it is sensible to talk about asymptotically translation invariant metrics. Suppose h is such a metric and $g = e^{2\rho} h$ for $\rho \in C^\infty$ and all $D_h^t \rho$ converging to a translation invariant tensor at ∞. Then the associated Laplacean Δ_g is a bounded operator from $W^p_{s+2,\delta,a}(\Lambda^q M)$ to $W^p_{s,\delta,a+2}(\Lambda^q M)$ where the norm for the weighted Sobolev space is $\|\sigma\| = \left(\sum_{t=0}^{s} \int_M \|e^{\delta z + (t+a)\rho} D_g^t \sigma\|_g^p \, dV_g\right)^{1/p}$. Furthermore there is a discrete set $\mathcal{D}_\Delta \subset \mathbb{R}$ $\ni$ if $\delta \notin \mathcal{D}_\Delta$ then Δ_g is Fredholm. As a consequence the space of harmonic forms satisfying $\|e^{\delta z}\sigma\|_g < \infty$ is finite dimensional. Another consequence is that $\exists$ an interpolation space $\tilde{W}^p$ $\ni$ $\Delta_g : \tilde{W}^p \to L^p$ is always Fredholm. Similar results hold for the operator $d + d_g^*$. In particular $L^2(\Lambda^q) = d\tilde{\tilde{W}}^p(\Lambda^{q-1}) \oplus d_g^* \tilde{\tilde{W}}^p(\Lambda^{q+1}) \oplus h^q$. Often much can be said about h^q. For instance if g is q-bdd. above then h^q provides unique representation for the de Rham cohomology classes that have representatives in L^2

R. Lockhart

Global Invariants of Strongly Pseudo Convex CR manifolds D. Burns & C.L. Epstein

We define a global $\mathbb{R}$-valued invariant of a compact, strictly, pseudo convex, 3-dim'l C.R. manifold, M. The invariant arises as the evaluation of a deRham cohomology class on the fundamental class of the manifold. To construct the relevant form we start with the Chern structure bundle, Y over M. The form is a secondary characteristic class of this structure. If the 2-plane field underlying the holomorphic tangent space of M is trivial then the form can be pulled down to M. Surprisingly, this form is defined up to an exact term and thus is well defined in $H^3(M;\mathbb{R})$. By examples one sees that the invariant actually depends on the complex structure. E.g. for the unit circle in the canonical bundle $\mathcal{F}_1 \to \Sigma$ of a Riemann surface

$$\mu(\mathcal{F}_1) = -\frac{|\chi(\Sigma)|}{4} + \frac{1}{12\pi}\int_\Sigma \frac{|\Delta \log K|^2 dA}{K}.$$

C L Epstein

Conformal Deformations of Riemannian Metrics on Noncompact Manifolds

– R McOwen

I consider the problem of conformally deforming a noncompact Riemannian manifold (M,g) to a complete metric $\tilde{g}$ with constant scalar curvature. One quantity which is useful is the 1^{st} eigenvalue (for Dirichlet conditions) $\lambda_0(L)$ of the "conformal Laplacian" $L = -\Delta + S$ where $S = (\text{scalar curvature})\frac{(n-2)}{4(n-1)}$: If (M,g) has a conformal metric $\tilde{g}$ with $\tilde{S} \geq 0$ then $\lambda_0(L) \geq 0$. On the other hand:

<u>Theorem</u> (Aviles & McOwen) If $\lambda_0(L) < 0$ then there is a conformal metric $\tilde{g}$ with $\tilde{S} = -1$. If, moreover, g is complete and $S \leq -\varepsilon < 0$ outside of a compact set $M_0 \subset M$ then $\tilde{g}$ is complete.

We can use $\lambda_0(L) \geq 0$ to achieve $\tilde{S} = 0$ provided more information is known about ∞: for example if (M,g) has cylindrical ends with $g = dz^2 + h$ at each end where $S_h > 0$, or if (M,g) is asymptotically Euclidean.

We can also consider a compact manifold (M,g) and a submanifold $\Gamma \subset M$ of dimension d, and ask for a complete conformal metric $\hat{g}$ on $\hat{M} = M \setminus \Gamma$ with constant $\hat{S}$. Toward that end we have:

<u>Theorem</u> (Aviles & McOwen). For any compact (M,g) there is a complete $\hat{g}$ on $\hat{M}$ with $\hat{S} \equiv -1 \iff d > \frac{n-2}{2}$ $(n = \dim M)$.

Crack singularities in three dimensions

(Joint work with E. Stephan)

Motivated by error estimates for numerical approximation schemes, one is interested in the precise form (in terms of a-priori estimates in Sobolev norms) of the singularities of the solutions of the Lamé equations in the exterior of a two-dimensional surface (crack) in $\mathbb{R}^3$. The requirement that the singularities are sought in the form of a tensor product $u^{sing}(\rho,\varphi) = \rho^{1/2}\alpha(\varphi)$ leads to a loss of one derivative in the a-priori estimate.

Martin Costabel

On the heat equation for pseudo-differential boundary problems. By Gerd Grubb

(University of Copenhagen, Denmark)

Pseudo-differential boundary problems appear in many applications, often arizing from manipulations with differential operator problems. We mentioned two new time-evolution cases:

1° In control theory: The "taming" of unbounded solutions (for $t \to \infty$) of the heat equation for a strongly elliptic Dirichlet problem with some negative eigenvalues, by introduction of a (non-local) feedback in the boundary condition, $\gamma u = \sum_{j \le N} (u, w_j)_\Omega \, g_j$.

2° In hydrodynamics: Removal of the degeneracy in the parabolicity of the Navier-Stokes initial-boundary value problems, by transformation to problems containing pseudo-differential boundary terms.

The tools for the ps.d.o. heat semigroup construction are given in a recent book [Progress in Mathematics #65, Birkhäuser Boston 1986], and we explained some basic ingredients, comparing with the resolvent construction of R. Seeley for differential problems. Finally, we described the finite asymptotic expansion of the trace for $t \to 0$.

Zoll-Stein manifolds by Victor Guillemin

This talk consisted of three parts. I. A discussion of the I. E. Segal approach to scattering theory on $\mathbb{R}^{n-1+1}$ as Floquet theory on the universal cover of the conformal compactification of $\mathbb{R}^{n-1+1}$ II. A brief overview of the theory of Zoll-Stein manifolds: These are compact Lorentzian manifolds all of whose null-geodesics are closed. III. A construction of conformal spectral invariants of these manifolds using the Floquet theory described in I.

Harmonic Maps between Noncompact Manifolds

Jürgen Jost

Firstly, the problem of finding harmonic maps between noncompact Riemannian manifolds and some approaches together with ~~the~~ the assumptions required for them are discussed. In particular, the initial step of finding a finite energy map in the homotopy class under investigation is stressed. Then, a special case of certain Kähler manifolds is studied in detail (locally Hermitian symmetric spaces, the Poincaré metric on the punctured unit disk and its generalizations in Kähler geometry). Finally, applications are given to rigidity questions in Kähler geometry (joint work with S.T. Yau).

The Laplacian on Asymptotically Hyperbolic Manifolds - Rafe Mazzeo.

The Hodge Laplacian (= Laplacian on k-forms) on hyperbolic space H^n is degenerate elliptic. A class of operators is introduced modelling this type of degeneracy, and a further class of degenerate pseudodifferential operators is defined containing parametrices for the 'elliptic' differential operators above. In particular, it is then proved that for the class of complete metrics (M,g) where M is a compact manifold with boundary, $g=\rho^{-2}h$ where h is a smooth metric, ρ a defining function for ∂M, that the Hodge Laplacians are of the above type, and their essential spectrum is thereby understood completely (and mimics that of H^n). The dimension of the space of L^2 harmonic forms is also identified and interpreted in terms of the topology of the compact manifold M. This pseudodifferential calculus, or closely related ones, has proved useful in various geometric problems of this type.

Spectral invariants of elliptic operators on noncompact manifolds.

M.A. Shubin

Continuous spectrum of selfadjoint elliptic operators on noncompact manifolds can be studied by means of spectral invariants which are constructed either by some limiting procedures from bounded domains in these manifolds or by von Neumann traces. An example of such an invariant is the integrated density of states $N(\lambda)$ which can be ~~defined~~ defined e.g. for selfadjoint elliptic operators with almost periodic or random homogeneous coefficients on $\mathbb{R}^n$ and for periodic operators on the universal covering $\widetilde{M}$ of a compact manifold M (periodicity means that the operator commutes with deck transformations by the fundamental group $\Gamma=\pi_1(M)$). In the latter case let the operator be the Laplacian Δ_p on p-forms on $\widetilde{M}$ constructed by means of some Riemannian metric on M pulled back to $\widetilde{M}$, so $N(\lambda)$ becomes

$$N_p(\lambda) = \int_F tr\, e_p(\lambda, x, x)\, dx$$

where $e_p(\lambda,x,y)$ is the spectral function of $(-\Delta_p)$ and F is the fundamental domain of Γ on $\widetilde{M}$. It was noticed in a joint paper by S. P. Novikov and M.A. Shubin that if $0 \in \mathrm{spec}(\Delta_p)$ and $N_p(\lambda) \sim c\lambda^{\alpha}$ as $\lambda \to +0$ then $\alpha = \alpha_p$ does not depend on the metric on M and so it has to be a topological invariant of M itself. The same invariant can be ~~const~~ computed by considering asymptotic behaviour of heat kernel on p-forms (the Schwartz kernel of $\exp(t\Delta_p)$ as $t \to +\infty$).

The Dirichlet problem at infinity for manifolds of nonpositive curvature

W. Ballmann

A simply connected, complete Riemannian manifold M with sectional curvatures nonpositive is in a natural way diffeomorphic to the interior of the unit ball B. If M is irreducible and admits a compact quotient, then either M is a symmetric space of higher rank or the Dirichlet problem at $M(\infty) = \partial B$ is solvable on M. This result is related to recent work of Anderson and Sullivan.

L^2-cohomology

Steven Zucker

L^2-cohomology is the homological object associated to the L^2 harmonic forms on a Riemannian manifold. Given M, there is an intrinsic notion of L^2 forms, and the L^2-cohomology is defined by $H^i_{(2)}(M) = Z^i/B^i$, where Z^i is the space of closed L^2 i-forms, and B^i is the subspace consisting of exterior derivatives of L^2 $(i-1)$-forms. Here, one can use either C^∞ forms, or measurable forms with weak derivatives. Taking the latter choice, one sees that $H^i_{(2)}(M) \simeq h^i \oplus R^i$ (general Hodge theorem) where $h^i = Z^i \cap (B^i)^\perp$ is the space of L^2 harmonic forms satisfying the requisite domain condition (Neumann problem), and $R^i = \overline{B^i}/B^i$ is either trivial or of infinite algebraic dimension. As simple examples, we considered a) $M = D^2$ (to show the role of domain conditions) and b) $M = \mathbb{R}^+$ (Euclidean, where $R^1 \neq 0$). We continued with c) $X \subset \mathbb{CP}^N$ a projective variety (with singularities) and $M = X^{reg}$, with metric induced by the embedding; d) M an arithmetic quotient of a non-compact symmetric space (when M is Hermitian, it has the Baily-Borel Satake compactification X); e) X has metrically conical singularities, M the regular locus. The L^2-cohomology is known to have the topological interpretation on X as the middle intersection homology of Goresky-MacPherson in (d) and (e). The same is conjectured for (c) by Cheeger-Goresky-MacPherson. As is well-known, it is a local issue on X.

Zucker's Conjecture

Leslie Saper and Mark Stern

Let D be an hermitian symmetric domain and Γ a neat arithmetic subgroup of the group G of automorphisms of D. With respect to the metric induced from the Bergman metric on D, $\Gamma\backslash D$ is a complete Kähler manifold with finite volume. Let $\Gamma\backslash D^*$ denote the Baily-Borel-Satake compactification; this is a normal projective variety which in general is highly singular. Let $\mathcal{E}$ be a local coefficient system on $\Gamma\backslash D$ defined by a finite-dimensional representation of G. We prove the following theorem, conjectured by Zucker.

<u>Theorem</u>: $H^{\cdot}_{(2)}(\Gamma\backslash D;\mathcal{E}) \cong IH^{\cdot}(\Gamma\backslash D^*;\mathcal{E})$, where $H_{(2)}$ denotes L^2-cohomology and IH denotes (middle) intersection cohomology.

Let x belong to the complex codimension k singular stratum of $\Gamma\backslash D^*$. In order to prove the theorem one needs to show the following local vanishing condition:

$$H^i_{(2)}(U\cap\Gamma\backslash D;\mathcal{E})=0 \qquad \text{for } i\geq k,$$

for U belonging to a nice fundamental system of neighborhoods of x in $\Gamma\backslash D^*$. To show this we establish the estimate:

$$\|d\varphi\|^2+\|d^*\varphi\|^2 \geq C\,\|\varphi\|^2 \quad \text{for } \varphi\in \operatorname{dom} d^*\cap A^i_c(\bar{U}\cap\Gamma\backslash D;\mathcal{E}).$$

The proof requires extensive use of the structure of hermitian symmetric spaces.

Existence of ground states for semilinear equations in $\mathbb{R}^n$ — Alice Chaljub-Simon.

Let us consider in $\mathbb{R}^n$ $(n>2)$ the following equation:

(1) $\Delta u - c^2 u + g(x,u) = 0$

c is a constant, and g a positive function, such that the growth of $u \mapsto g(x,u)$ is less than u^k with $k < \frac{n+2}{n-2}$. We want to prove the existence of positive solutions of (1) tending to zero at infinity (ground states). For this we introduce some functions with an exponential weight.

First, we prove that: $L = \Delta - c^2$ is an isomorphism in the weighted spaces. Then transforming (1) into a non-linear integral equation:

(2) $u = \underline{G}[\tilde{g}(u)]$

we prove that $\Phi : u \mapsto \underline{G}[\tilde{g}(u)]$ is compact under convenient assumptions on g; we prove, then, the existence of an a priori estimate for the positive, C^2, bounded solutions of (1). By use of a fixed point theorem in the cone of positive functions in the weighted space, we get a solution with the required properties

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Elliptic Operators on ramified spaces –

Serge Nicaise

We introduce a class of boundary value problems on t -dimensional polygonal topological networks which are ramified spaces (G. Lumer, C. R. Acad. Sc. Paris, t. 291, série A, 1980, p. 627-630) such that each face is homeomorphic to a polygon. This class contains mixed boundary value problems (Kondratier 1967, Maz'ya and Plamenevskii 1978, Grisvard 1985) or interface problems (Kellogg 1971, Lemrabet 1977) in polygonal domains of the plane. Following Grisvard (1985), we show that, in a neighbourhood of a vertex, the singularities can be calculated using a Laplace operator defined on an associated topological network (one-dimensional ramified space, cf. G. Lumer)

These considerations are related to Felix Ali Mehmeti's results on nonlinear wave equations on ramified spaces.

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Interior elliptic singular problems

Gerardo Mendoza

A problem posed on a smooth compact manifold together with a closed submanifold X, both without boundary, is used as a means to describe part of work done jointly with R. Melrose. The problem leads to the following situation. Let N be a compact manifold with boundary Γ, $\rho: \Gamma \to X$ a fibration (with compact fibers). The space $\mathcal{V}$ of vector fields on N which are tangent to the fibers of ρ generates a ring of degenerate operators, $\mathrm{Diff}_{\mathcal{V}}(N)$. Let $\dot{H}^s$ be the space of distributions on N which extended as 0 to a neighborhood of N belong to H^s. Let $I_{\mathcal{V}}\dot{H} = \{ u \in \dot{H}^s \mid Au \in \dot{H}^s \ \forall A \in \mathrm{Diff}_{\mathcal{V}} \}$. If $P \in \mathrm{Diff}_{\mathcal{V}}(N)$ is elliptic in the appropriate sense and $u \in (C^\infty(N))'$ then $Pu \in C^\infty(N) \Rightarrow u = v + w$ with $v \in C^\infty(N)$ and $w \in I_{\mathcal{V}}\dot{H}^s$ for some s. If furthermore $Pu = 0$ to ∞ order at Γ then u has an asymptotic expansion at the boundary resembling a Taylor series, a series which in certain significant cases arising from regular elliptic boundary value problems, where $\rho: \Gamma \to X$ is just the identity, is the usual Taylor series. Conditions imposed on the number of terms that vanish together with additional assumptions on P lead to Fredholm properties of the operator.

Interaction-problems

The physical problem of vibrations of certain complicated systems leads to nonlinear wave-equations on ramified spaces (notion introduced by G. Lumer, developed further by S. Nicaise). We indicate a possibility to construct a selfadjoint operator from the elliptic spatial part of the equation by Friedrichs-extension.
The information of the connexion of the media is

contained in the choice of a certain closed subspace of the product of function spaces on these media.
Much of the formalism works, when we take any closed subspace. This leads to interaction-problems (e.g. identifications in the interior of the media, integral conditions, mixing of the dimension etc.)

Finally we give an existence-theorem for global solutions of certain nonlinear abstract wave-equations with damping and indicate the applicability to mixed initial-value-interaction-problems. Also the theory of T. Kato and results of J. Shatah can be applied!

Concerning the question of regularity, there are relations to the research on abstract C^∞-notions of B. Gramsch.

F. Ali Mehmeti (Mainz)

On spectral theory of some non compact complete manifolds

Laurent Guillopé F_Grenoble

Let be M one manifold with cylindrical ends and cusps of symmetric space of rank one. We introduce Eisenstein functions (generalized eigenfunctions of the laplacian), which depend meromorphically from one complex variable on some infinite ramified cover over $\mathbb{C}$. The action of the group of this cover is explicit determined by some transfer coefficients. We examine the particular situation of Eisenstein series in the case of locally symmetric spaces. We give too the differential of the transfer coefficients with respect to conformal deformation of metric.

Guillopé

Spectral properties of a Mixed Laplacian

Santiago R. Simanca

We construct a suitable inverse of an unbounded Laplacian with mixed Dirichlet and Neumann conditions at the boundary. Using it, we prove there exist a complete set of eigenfunctions which allow us to invert the operator in the orthogonal complement of its kernel. For polyhedral domains in the plane, and via direct computation we prove that

$$N(\lambda) = \frac{\text{Area } \Omega}{4\pi}\lambda + \frac{L_N - L_D}{4\pi}\lambda^{1/2} + O(\lambda^{1/2})$$

where $N(\lambda) = \#$ eigenvalues $\leq \lambda$, L_N = length of part of $\partial\Omega$ where Neumann conditions are imposed and L_D = length $\partial\Omega - L_N$. This shows that for large λ

$$N_N(\lambda) \geqslant N(\lambda) \geqslant N_D(\lambda)$$

where N_N (resp N_D) is the counting function with purely Neumann (resp. Dirichlet) conditions.

Some Laplace comparison algebras on noncompact spaces

We study C^*-algebras $\mathfrak{A}$ of singular integral operators on Ω, a noncompact Riemannian manifold. Specifically, with conical ends: $\tilde{\Omega} = [0,\infty) \times \Theta$, $ds^2 = dr^2 + r^2 d\theta^2$, Θ compact, or cylindrical ends $\bar{\Omega} = [0,\infty) \times B$, $ds^2 = dr^2 + dz^2$. Or, a combination of both: $\tilde{\Omega} = [0,\infty) \times \Theta \times B$, $ds^2 = dr^2 + r^2 d\theta^2 + dz^2$. In each case $\mathfrak{A} \subset \mathcal{L}(\mathfrak{H})$, $\mathfrak{H} = L^2(\Omega, dS)$, is generated by a class $\mathcal{A}^\#$ of multiplications (by $a \in C^\infty(\Omega)$ $a = O(1)$) and a class of "Riesz-Operators" $D(1-\Delta)^{-1/2}$, with $D = b^j \partial_{x^j} + p \in \mathfrak{D}^\#$, a class of bounded first order operators.

With suitable choice of $\mathcal{A}^\#, \mathfrak{D}^\#$ one obtains an ideal chain $\mathfrak{A} \supset \mathfrak{E} \supset K(\mathfrak{H})$, $\mathfrak{E} = [\mathfrak{A}, \mathfrak{A}]$, where $\mathfrak{A}/\mathfrak{E} = C(M)$, $\mathfrak{A}/K(\mathfrak{H}) \oplus C(E_j, \mathfrak{S}\mathfrak{I}(B_j))$ with (locally) compact spaces M, E_j (one E_j for each cylinder end) and with the algebra $\mathcal{SI}(B)$ of singular integral operators over $\underline{B}$.

A necessary and sufficient condition for $A \in \mathfrak{A}$ to be Fredholm is that both induced symbols σ_A and j_A are invertible.

Details about $\mathbb{M}$, $\mathbb{E}_j$, and criteria for differential operators "within reach" are discussed.

Heinz Otto Cordes

Necessary and sufficient conditions for differential operators on $\mathbb{R}$, with continuous matrix-valued coefficients that approach 2π-periodic functions at $+\infty$ and $-\infty$, to be Fredholm are given. If $A_j = A_j^+(x)\,\chi^+(x) + A_j^-(x)\chi^-(x) + A_j^0(x)$, $A_j^\pm$ periodic, $0 \le \chi^\pm \le 1$, $\chi^+ + \chi^- = 1$, $\chi^\pm(x) = 0 \ \mp x > 1$, $\lim_{x\to\pm\infty} A_j^0(x) = 0$, are $N\times N$ matrices, we can prove:

(Cordes-Melo) $L = \sum_{j=0}^{m} A_j(x)\left(\frac{1}{i}\frac{d}{dx}\right)^j : H^m(\mathbb{R})^N \to L^2(\mathbb{R})^N$ is Fredholm iff it is uniformly elliptic and $L^\pm(t) = \sum_{j=0}^{m} A_j^\pm(\theta)\left(\frac{1}{i}\frac{d}{d\theta} - t\right)^j : H^m(S^1)^N \to (L^2(S^1))^N$ is invertible $\forall t \in \mathbb{R}$. In the case $N=1$, we get that $L = \frac{1}{i}\frac{d}{dx} + A(x) : H^1(\mathbb{R})^N \to L^2(\mathbb{R})^N$ is Fredholm iff $L^\pm = \frac{1}{i}\frac{d}{d\theta} + A(\theta) : H^1(S^1)^N \to L^2(S^1)^N$ have no real eigenvalues. Denoting $N^\pm = \{\operatorname{Im}\zeta ; \zeta$ is an eigenvalue of $L^\pm\}$ we get N^+ and N^- finite and index $L = \#\, N^+ \cap \{t; t<0\} - \#\, N^- \cap \{t; t<0\}$.

Severino T. Melo.

Regularity for a class of degenerate operators
by Stephan Rempel

We consider solutions to an elliptic equation which degenerates at the boundary in a special way (treated earlier by Baouendi / Goulaouic, Višik / Gruškin, Shimakura, Bolley / Camus / Hetivier). Even for smooth right hand sides which have an infinite order zero at the boundary ~~the~~ any solution (possibly not satisfying boundary conditions!) will contain certain singularities at the boundary. Functionspaces and the optimal form of the singular terms are described including the case of branching asymptotics. A possible extension of the result to degenerate operators arising from the resolution of a wedge singularity is indicated

St. Rempel

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Double structures on Bordiga surfaces

A Bordiga surface S is a rational surface of degree 6 in $\mathbb{P}^4$. As an abstract surface, S is isomorphic to a blowing up $\tilde{\mathbb{P}}^2(x_1, \dots, x_{10})$ and embedded in $\mathbb{P}^4$ by the complete linear system $|4L - \sum_{i=1}^{10} E_i|$.
Some of them admit double structures $\tilde{S}$ with $\omega_{\tilde{S}} \cong \mathcal{O}_{\tilde{S}}(-2)$ which implies that there exists (by a result of Ferrand) a rank 2 vector bundle E on $\mathbb{P}^4$ and a section $s \in H^0E$ with $\tilde{S} = \{s = 0\}$. For Bordiga surfaces, E splits as $E \cong \mathcal{O}(3) \oplus \mathcal{O}(4)$, i.e., $\tilde{S}$ is a complete intersection.
These special Bordiga surfaces can be characterized in two different ways:

1. They contain a certain nondegenerate curve C and lie on its secant variety where C is one of the following
   a) a rational normal curve of degree 4 in $\mathbb{P}^4$
   b) a union of two conics which intersect in one point
2. The 10 points $x_1, \dots, x_{10}$ are in special position, namely
   in a) Fix a smooth quadric surface $F \subset \mathbb{P}^3$ and a 2:1-projection $F \to \mathbb{P}^2$. Then $x_1 \dots x_{10}$ are the 10 double points of a rational sextic C which is projection of a curve of bidegree (1,5) on F.
   in b) $x_1 \dots x_{10}$ are the two ordinary double points x_1, x_2 and 8 of the 9 intersection points of two rational cubics. Furthermore, the ninth intersection point lies on the line through x_1 and x_2.

Jürgen Rathmann (Göttingen)

Homology and rational equivalence on real varieties.

Let X be smooth, projective over $\mathbb{R}$ and

$$cl_k : Zyc_k X \longrightarrow H_k(X(\mathbb{R}), \mathbb{Z}/2)$$

the canonical homomorphism.

Theorem: $\ker cl_k = Zyc_k^{th}(X) + P_k(X)$,

where $Zyc_k^{th}(X)$ is the group of "thin" cycles, which is generated by k-dimensional subvarieties $V \subset X$ with $\dim_{top} V(\mathbb{R}) < \dim V$, and P_k is the group of cycles, which are rational equivalent to 0.

F. Ischebeck, Münster (FR Germany).

ASYMPTOTIC DEPTH AND CONNECTEDNESS OF FIBERS

Let (R, m) be a local noetherian ring, and let $I \subseteq m$ be an ideal. Let t denote the asymptotic depth of the higher conormal modules I^n/I^{n+1}: $t = \operatorname{depth}(I^n/I^{n+1})$, $\forall n >> 0$.

Moreover consider the blowing up-morphism $\pi : \operatorname{Proj}(\bigoplus_{n\geq 0} I^n) \longrightarrow \operatorname{Spec}(R)$ and the exceptional fiber $E := \operatorname{Proj}(\bigoplus_{n\geq 0} I^n/I^{n+1})$ of π as well as the special fiber $S := \operatorname{Proj}(R/m \otimes (\bigoplus_{n\geq 0} I^n))$ of π.

One knows the inequality $\dim S < \dim R - t$. We improve this estimate by the following result:

Assume that $t > 1$ and assume that at least one of the following conditions holds:

(i) R is excellent and normal, and I is $\neq 0$

(ii) $\mathrm{depth}_I(R) > 1$

(iii) $\mathrm{Spec}(\hat{R}) - V(I\hat{R})$ is $\neq \emptyset$ and connected.

Then $E-S$ is connected and satisfies the inequality

(*) $\underline{c}(\mathcal{O}_E | E-S) \geq t-2$.

Thereby $\hat{\cdot}$ stands for the m-adic completion, and – for an arbitrary coherent sheaf $\mathcal{F}$ over a noetherian scheme X – $\underline{c}(\mathcal{F})$ is defined by
$\underline{c}(\mathcal{F}) = \min\{\dim_x(\overline{T}_1 \cap \overline{T}_2) \mid x \in \overline{T}_1 \cap \overline{T}_2 \text{ closed}; T_1, T_2 \subseteq \mathrm{Ass}(\mathcal{F}), T_i \neq \emptyset, T_1 \cup T_2 = \mathrm{Ass}(\mathcal{F})\}$
So (*) implies in particular that $E-S$ is connected in dimension $t-2$.

M. Brodmann, Universität Zürich

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Some Problems on dimension of fibres of ring homomorphisms

Let $f: A \to B$ be a morphism of noetherian rings and assume that the going-down theorem holds for f (e.g. f is flat). For a prime ideal $\mathfrak{p}$ of A, define $\alpha(f, \mathfrak{p}) := \dim B \otimes_A k(\mathfrak{p})$, the dimension of the fibre at $\mathfrak{p}$ ($k(\mathfrak{p}) := A_\mathfrak{p} / \mathfrak{p} A_\mathfrak{p}$).

Theorem. If $\mathfrak{p} \subset \mathfrak{q}$ then $\alpha(f, \mathfrak{p}) \geq \alpha(f, \mathfrak{q})$.

Problem 1. Suppose that A, B, f are local. If $\mathfrak{p} \subset \mathfrak{q} \neq \mathfrak{m}_A$ and $\alpha(f, \mathfrak{p}) = \dim(B/\mathfrak{p}B) - 1$, does it follow that $\alpha(f, \mathfrak{q}) = \dim(B/\mathfrak{q}B) - 1$?

When A is local and f is the natural map $A \to \hat{A}$, set $\alpha(A) := \max\{\alpha(f, \mathfrak{p}) : \mathfrak{p} \in \mathrm{Spec}\, A\}$. By the above theorem we have $\alpha(A/\mathfrak{p}) = \alpha(f, \mathfrak{p})$. If A is essentially of finite type over a field then $\alpha(A) = \dim A - 1$ (if $\dim A > 0$). If A dominates a non-trivial complete local ring or if A is I-adically complete for some ideal I with $\mathrm{ht}\, I > 0$, then $\alpha(A) < \dim A - 1$, and usually we get $\alpha(A) = \dim A - 2$. Put $N(A) := \{\mathfrak{p} \in \mathrm{Spec}\, A : \alpha(A/\mathfrak{p}) = \dim A/\mathfrak{p} - 1 \text{ and } \dim A/\mathfrak{p} \geq 2\}$

Problem 1'. If $\mathfrak{p}, \mathfrak{q} \in \mathrm{Spec}\, A$ with $\dim A/\mathfrak{p} \geq 2$, $\dim A/\mathfrak{q} \geq 2$, and $\mathfrak{p} \supset \mathfrak{q}$, $\mathfrak{q} \in N(A)$, does it follow that $\mathfrak{p} \in N(A)$?

Problem 2 Are there local rings A with $0 < \alpha(A) < \dim A - 2$?

Remark Huneke says that Heinzer and he constructed examples of A with $\alpha(A) = 1$, $\dim A$ arbitrary. Someone says that Abhyankar constructed A with $\alpha(A) = m$, where m is any integer between 0 and $\dim A - 1$.

H. Matsumura (Nagoya)

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Automorphisms of order p of semi-stable curves.

Let R be a complete discrete valuation ring of mixed characteristic, K its quotient field, k its residual field of char $p > 0$. Suppose the maximal ideal is generated by p.

Let X be a smooth, proper R curve and u an automorphism of X of order p.

Then, for $p \geq 5$, u acts freely on X.

Question: Suppose X is smooth and proper over R of dimension d. Let u be an automorphism of X of order p. Does u acts freely on X when $d < p-2$?

M. Raynaud. Orsay. France.

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Fibres of morphisms of local rings, work with Avramov (Sofia).

Let A and B denote local rings, and let $\varphi: A \to B$ be a morphism ($\varphi(m_A) \subseteq m_B$). Assume that the flat dimension $fd_A B$ is finite. The fibre of φ is $F(\varphi) = k_A \otimes_A G$, where $k_A = A/m_A$ and G is a bounded DG-A-algebra resolution of B. (DG = differential graded.)

Let D be a DG-ring with $H(D)$ bounded and Noetherian, and let M be a DG-D-module with $H(M)$ bounded and Noetherian. The Bass series is $I_D^M(t) = \sum_{i \geq 0} [\mathrm{Ext}^i(k_D, M) : k_D]\, t^i \in \mathbb{Z}[[t]]$, when D maps onto the field k_D.

Thm. 1. $I_B^{M \overset{L}{\otimes}_A B}(t) = I_A^M(t)\, I_{F(\varphi)}^{F(\varphi)}(t)$, when M is a f.g. A-mod.

Thm. 2. A and $F(\varphi)$ are Gorenstein $\Leftrightarrow$ B Gorenstein.

Here $F(\varphi)$ is said to be Gorenstein, if $I_{F(\varphi)}^{F(\varphi)}(t)$ is a monomial.

Let D_1, D_2, D_3 be DG-rings like D above and $D_1 \to D_2$ and $D_2 \to D_3$. Assume $fd_{D_1} D_2 < \infty$ and $fd_{D_2} D_3 < \infty$. Fibres of morphism (augmented) of DG-rings are defined as for local rings.

Thm. 3. $F(D_1 \to D_2)$ and $F(D_2 \to D_3)$ are Gorenstein $\Leftrightarrow$ $F(D_1 \to D_3)$ is Gorenstein.

Thm. 4. If $F(A \to B)$ is Gorenstein and A has Gorenstein formal fibres, then $F(\hat{A} \to \hat{B})$ is Gorenstein and B has Gorenstein formal fibres.

Hans-Bjørn Foxby (København)

Algebraic equivalence of vector bundles   G. Horrocks (Newcastle upon Tyne)

A is a regular local ring with coefficient field k. Bundles over the punctured spectrum of A are said to be algebraically equivalent if they can be joined by a sequence of local algebraic deformations. The main result is that algebraically equivalent bundles determine isomorphic bundles over $\bar{k} \otimes_A S$, $S = A[y_1, \dots, y_n]/(\sum x_i y_i - 1)$ $y_1, \dots, y_n$ indeterminates $x_1, \dots, x_n$ a base for the maximal ideal. Thus ~~the~~ a class of algebraically equivalent bundles corresponds to a set of descent data on a projective module up to equivalence of data. This gives a standard model with which to approach the construction of moduli spaces

G.H.

The Ideal of an Arithmetically Buchsbaum Curve in $\mathbb{P}^3$

(joint work with Juan Migliore).

Let $\mathcal{C}$ be a curve in $\mathbb{P}^3$, (closed, pure 1-dim'l, locally C-M) and $I_{\mathcal{C}} \subseteq S = k[x_0, x_1, x_2, x_3]$ its defining ideal, $\mathcal{I}_{\mathcal{C}}$ its ideal sheaf. The Hartshorne-Rao module of $\mathcal{C}$ is the graded S-module, $M(\mathcal{C}) = \oplus M(\mathcal{C})_n = \oplus H^1(\mathcal{I}_{\mathcal{C}}(n))$. $M(\mathcal{C})$ is an S-module of finite length.

Def: $\mathcal{C}$ is arithmetically Buchsbaum (a.B) if S_1 acts trivially on $M(\mathcal{C})$.

If $m(\mathcal{C})_n = \dim M(\mathcal{C})_n$, then $N = \sum m(\mathcal{C})_n$ is called the Buchsbaum invariant of $\mathcal{C}$.

Let $\alpha(\mathcal{C})$ = least integer $t \ni: (I_{\mathcal{C}})_t \neq 0$;

$\beta(\mathcal{C})$ = " " " " $(I_{\mathcal{C}})_t$ contains a regular sequence of length 2, then

Prop 1: If $\mathcal{C}$ is a. B., $\alpha = \alpha(\mathcal{C})$ and $\mathcal{H}$ is a hyperplane not containing a component of $\mathcal{C}$. Then

i) $\alpha - 1 \leq \alpha(\mathcal{C} \cap \mathcal{H}) \leq \alpha$

ii) If $\alpha(\mathcal{C} \cap \mathcal{H}) = \alpha - 1$ then $h^0(\mathcal{I}_{\mathcal{C} \cap \mathcal{H}}(\alpha - 1)) = m(\mathcal{C})_{\alpha - 2}$.

iii). $M(\mathcal{C})_i = 0$ for all $i \leq \alpha - 3$.

Prop. 2: $\mathcal{C}$ a. B, $\mathcal{H}$ a hyperplane not containing a component of $\mathcal{C}$, t = least integer $\ni$: $h^1(\mathcal{I}_{\mathcal{C} \cap \mathcal{H}}(t)) = 0$. Then $\mathcal{I}_{\mathcal{C}}$ is generated in degrees $\leq t+1$.

Prop. 3: $\mathcal{C}$ reduced and irreducible a. B curve, $\alpha = \alpha(\mathcal{C})$, $\beta = \beta(\mathcal{C})$, N = Buchsbaum invariant of $\mathcal{C}$. Then $I_{\mathcal{C}}$ can be generated in degrees $\leq \alpha + \beta - 2N$. Moreover, if $\alpha(\mathcal{C} \cap \mathcal{H}) = \alpha - 1$ then $\alpha = \beta$ and so $I_{\mathcal{C}}$ can be generated in degrees $\leq 2(\alpha - N)$.

A.V. Geramita (Kingston, Ontario (Canada))

Introduction to Almost Split Sequences in the Category of Maximal Cohen-Macaulay Modules.

The purpose of this talk was to give the basic definitions and existence theorems for almost split sequences as developed by I. Reiten and myself. Also applications to studying Cohen-Macaulay rings of finite Cohen-Macaulay type were given including the fact that such rings are isolated singularities, that their Grothendieck group can be described using almost split sequences and Cohen Macaulay module as well giving a criterion for deciding when a Cohen-Macaulay isolated sing. is of finite Cohen-Macaulay type.

M. Auslander (Bladesburg, Va.)

Almost factorial singularities

Let X be a projective manifold over the complex numbers and $NS(X)$ its Neron-Severi group. It is well known from classical Hodge theory that the map induced by logarithmic derivation $d\log: NS(X)_{\mathbb{C}} \longrightarrow H^1(\Omega^1_X)$ is injective. In this lecture we gave a generalization to singularities. Let $A = \mathbb{C}[[X]]_n/\alpha$ be of pure dimension ≥ 3 satisfying (S_2). By a result of Boutot, the Picard group of the punctured spectrum $\mathcal{U}$ of A has an algebraic structure and so $NS\,\mathcal{U} = Pic\,\mathcal{U}/Pic^0\mathcal{U}$ makes sense. Again there is a map induced by logarithmic derivation $d\log: NS(\mathcal{U})_{\mathbb{C}} \longrightarrow H^1(\Omega^1_{\mathcal{U}/\mathbb{C}})/dH^1(\mathcal{O}_{\mathcal{U}})$, and the main result is, that for algebraic singularities this map is injective. The proof heavily depends on the vanishing theorems of Grauert-Riemenschneider and Steenbrink. As an application one gets under suitable depth assumptions for A and Ω^1_A criteria for A to be almost factorial.

Hubert Flenner (Göttingen)

Set-theoretic Complete Intersections on Cones

This is a report of the work of David Jaffe in his PhD thesis (Berkeley 1987).

It has been known for some time that certain rational curves in $\mathbb{P}^3$, such as the nonsingular quartic curve given by $x=u^4$, $y=tu^3$, $z=t^3u$, $w=t^4$ are set-theoretic complete intersections in characteristic $p>0$. For example, if $p=7$ one may use the equations

$$y^4 - x^3w = 0 \quad \text{and} \quad z^7 - xyw^5 = 0 .$$

On the other hand, it is not known whether this curve is a complete intersection in characteristic 0, and some authors had verified already that at least on that cone $y^4 - x^3w = 0$, it is not a set-theoretic complete intersection in characteristic 0.

Hence the objective of Jaffe's thesis is to study curves on cones, and to decide when they are set-theoretically the intersection of that cone with some other surface.

The general situation is this. Let $D \subseteq \mathbb{P}^2$ be an irreducible plane curve. Let $S \subseteq \mathbb{P}^3$ be the cone over D. Let $C \subseteq S$ be an irreducible nonsingular curve lying on S. Let $v \in S$ be the vertex.

When D is nonsingular the situation is easily understood:

Proposition. Suppose D is nonsingular. If $C \not\ni v$, then C is a (strict) complete intersection on S. If $C \ni v$, the tangent line to C at v determines a point $P \in D$. Then C is a set-theoretic complete intersection on S if and only if the class of P in $\operatorname{Pic} D / \mathbb{Z} \cdot \mathcal{O}_D(1)$ is torsion.

To state the main result, we need some definitions. Let Pic D be the Picard scheme of D, and let $\operatorname{Pic}^\circ D$ be the connected component. There is an exact sequence of group schemes

$$0 \to (\operatorname{Pic}^\circ D)_{mult} \times (\operatorname{Pic}^\circ D)_{unip} \to \operatorname{Pic}^\circ D \to (\operatorname{Pic}^\circ D)_{ab} \to 0$$

where ab denotes the abelian variety which is the Jacobian of the normalization of D; mult denotes the multiplicative part, which is a product of $\mathbb{G}_m$'s, and unip denotes the unipotent part, which is a successive extension of $\mathbb{G}_a$'s. We say that D is of cuspidal type if $(\operatorname{Pic}^\circ D)_{mult} = 0$ and $(\operatorname{Pic}^\circ D)_{unip} \neq 0$. We say D is of nodal type if $(\operatorname{Pic}^\circ D)_{mult} \neq 0$ and $(\operatorname{Pic}^\circ D)_{unip} = 0$.

Theorem (Jaffe). Assume that D is singular.

a) If C is a set-theoretic-complete intersection on S, then

1) $\mathrm{char}.k = p > 0$, and

2) D is of cuspidal type.

b) Conversely, suppose that a1) and a2) are satisfied, and assume furthermore either (i) $C \not\ni v$, or (ii) D is rational. Then C is a set-theoretic complete intersection on S.

Cor 1 [Hartshorne]. For each $d \geq 4$ and for each $\mathrm{char}\, k = p > 0$, the rational curve $x = u^d, y = u^{d-1}t, z = ut^{d-1}, w = t^d$ is a set-theoretic complete intersection in $\mathbb{P}^3_k$.

Cor 2 [Ferrand]. If C is a nonsingular curve in $\mathbb{P}^3$ over a field of char. $p > 0$, and if $\exists\, 0 \notin C$ such that the projection from 0 sends C birationally onto a plane curve $\bar{C} \subseteq \mathbb{P}^2$ having only cusps for singularities, then C is a set-theoretic complete intersection.

Ex. The rational quartic curve mentioned above is not a complete intersection of a cone with any other surface in characteristic 0.

The proof depends on a careful study of the groups $\mathrm{Pic\,loc}(s) = \mathrm{Pic}(\mathrm{Spec}\, \mathcal{O}_{S,s} - \{s\})$ for the singular points $s \in S$.

Robin Hartshorne (Berkeley)

Algorithms for finding roots of a polynome

Newton gives an algorithm for finding the roots of a polynome $P(X) = a_d X^d + \cdots + a_0$, $a_i \in \mathbb{C}$. Namely, the algorithm is: $N(z) = z - \dfrac{P(z)}{P'(z)}$, it is really working when all the roots are real and for z in a neighbourhood of a complex simple root.

Euler generalises Newton algorithm: $N_h(z) = z - h\frac{P(z)}{P'(z)}$ $0 < h \le 1$. For $h=1$, $N_h = N$. This algorithm works for z in a neighbourhood of any complex or real root. But in general, this algorithm is not genericly convergent. We prove (joint work with A. Douady and P. Sentenac) that there exist a lot of $u = he^{i\alpha}$, with $|1-u|<1$ such that, if we define $N_u(z) = z - u\frac{P(z)}{P'(z)}$, $z \in \mathbb{C}$, N_u is an algorithm genericly convergent (here $\alpha \in]-\pi/2, \pi/2[$)

Marguerite Flexor (Orsay)

Some new results about surfaces of general type.

Using a new invariant of Donaldson recently R. Friedman and J. Morgan proved that under some conditions the canonical class of an algebraic surface is a diffeomorphism invariant.

This allows to construct first examples of surfaces of general type which are homeomorphic and not diffeomorphic.

Let $X = \mathbb{C}P^1 \times \mathbb{C}P^1$, $\{x_1, \dots, x_m\}$ be a sequence of positive numbers. Define inductively finite morphisms $g_k(x_1, \dots, x_k): X(x_1, \dots, x_k) \to X$ as follows. Assume $g_{k-1}(x_1, \dots, x_{k-1})$ is constructed. Let $f_k: X(x_1, \dots, x_k) \to X(x_1, \dots, x_{k-1})$ be a triple cyclic covering ramified at a non-singular curve $B_k \in |3x_k(g_{k-1}(x_1, \dots, x_{k-1}))^* E|$, where $E = l_1 + l_2 \subset \mathbb{C}P^1 \times \mathbb{C}P^1$,

$\ell_1 = \mathbb{C}P^1 \times pt$, $\ell_2 = pt \times \mathbb{C}P^1$. Then define
$g_k(x_1, \ldots, x_k) = g_{k-1}(x_1, \ldots, x_{k-1}) \circ f_k$.

Examples of homeomorphic not diffeomorpic simply-connected minimal surfaces of general type are the following pairs of surfaces:

$$\{X(1,1,1,1,6,z_1,\ldots,z_m),\quad X(2,10,16,3z_1,\ldots,3z_m)\}$$

where $\{z_1,\ldots,z_m\}$ is an empty set or any sequance of positive numbers with $\sum_{i=1}^{m} z_i \equiv 0 \pmod 2$.

Boris Moishezon (Columbia University, New York)

INTERACTION BETWEEN COMPUTER AND COMMUTATIVE ALGEBRA: SOME NEW ASPECTS

I report on a joint paper of myself and Teo Mora, which deals with the description of the maximal monomial ideals associated to an ideal I in the polynomial ring $A = k[x_1 .. x_n]$ over a field k.

First we associate to every ordering of the monoid T of terms of A its "half line of first vectors".

This enables to associate to every set of orderings a suitable cone and a first result is that for every ideal I in A there is a partition of $(\mathbb{R}^n)^+$ into a ~~[illegible]~~ fan of polyhedral cones; over each of them the reduced Gröbner basis and the maximal monomial ideals are constant. The fan can be got constructively.

Another aspect of our work is to show that these cones may extend outside $(\mathbb{R}^4)^+$, giving rise to the so called Gröbner region of I.
The main feature of it is that every ordering "inside" it behaves, with respect to I, like a term-ordering.

LORENZO ROBBIANO
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The New Intersection Theorem

Let A be a local ring. The Intersection Theorem (new) states that if $F_\cdot = 0 \to F_n \to \ldots \to F_0 \to 0$ is a complex of free A-modules such that the homology is of finite length, and if $F_\cdot$ is not exact, then $n \geq$ the dimension of A. This theorem was proven by Peskine and Szpiro in positive characteristic and for rings of finite type over a field and extended to all rings containing a field by Hochster. We present here a proof in mixed characteristic. The proof uses the theory of local Chern characters of Fulton-MacPherson - the idea is to reduce modulo p to a complex $\bar{F}_\cdot$ of length exactly equal to the dimension of A/pA and show that if $F_\cdot$ were a counterexample to the theorem, we would have, first, that the d^{th} Chern character, where $d = \dim A$, is zero since $\bar{F}_\cdot$ is the reduction of a complex over A. Secondly, we show that the fact that length $(\bar{F}_\cdot) = \dim A/pA$ implies that this number is positive, so that a complex violating the theorem could not have existed.

Paul Roberts
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Set-theoretic Complete intersections.

We prove the following "addition" theorem about set-theoretic complete intersections of curves.

Theorem (Local version.)

Let R be a (Cohen-Macaulay) local ring. $I, J \subset R$ two set-theoretic complete intersection 'curves' such that $I+J$ is primary to the maximal ideal. Then $I \cap J$ is also a set-theoretic complete intersection.

Theorem (Projective version.)

Let $C \hookrightarrow \mathbb{P}^3$ be a ~~local~~ set-theoretic complete intersection curve; $L \subset \mathbb{P}^3$ any line such that $C \cup L$ is connected. Then $C \cup L$ is a set-theoretic complete intersection. In particular any connected union of lines is a set-theoretic complete intersection.

N. Mohan Kumar,
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We report on a work done jointly with Mohankumar and Amit Roy on Cancellation of projective modules over finitely generated commutative rings over $\mathbb{Z}$.

Th 1. Let A be a finitely generated ring of dimension

$d \geq 2$ over $\mathbb{Z}$. Let P, P' be projective A-modules of rank $\geq d$. Then $P \oplus A \simeq P' \oplus A \Rightarrow P \simeq P'$.

– : We say that a ring A has projective stable range $\leq r$ ($psr(A) \leq r$) if for for any projective A-module P of rank $\geq r$ and $(x, a) \in P \oplus A$ unimodular, $\exists\, y \in P$ s.t. $x + ay$ is unimodular.

Th 2. Let A be a finitely generated ring of dimension $d \geq 2$. Suppose all projectives of rank d have unimodular elements. Then ~~for~~

1) for $d \geq 3$, $psr(A) \leq d$

2) If $d = 2$ and $\exists\, n > 1$, $n \in \mathbb{Z}$, s.t. $1/n \in A$, $psr(A) \leq 2$.

The above theorem generalize work of Vaserstein.

Th 3. – Let A be a finitely generated ring of dimension $d \geq 2$. over $\overline{\mathbb{F}}_p$. If $d \geq 3$ $psr(A) \leq d$. Further if $d = 2$ and A is regular, then $psr(A) = 2$.

M. P. Murthy
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On the Canonical element Conjecture.

We mainly study the three equivalent Conjectures: Direct Summand, canonical element and improved new intersection Conjecture. (We prove ~~that~~ the equivalence of the last two in the Course of this talk). These conjectures are open in the mixed characteristics. First we study the effect of frobenius map on free complexes with finite length

homologies in ch $p>0$. We prove the following theorem:
<u>Th.</u> Let A be a complete local equidimensional ring without any embedded components in ch $p>0$. Let $F_\cdot$ be a free complex with finite length homologies and let N be a finitely generated module. Let $W_{j,n}$ be the jth homology of $\mathrm{Hom}(F_\cdot, {}^{f^n}N)$ where $f^n: A \to A$ is given by $f^n(x) = x^{p^n}$. Then
i) if $\dim N < \dim A$, $\lim l(W_{j,n})/p^{nd} = 0$; ii) if $\dim N = \dim A$,
a) ~~and~~ $j < \dim A$, $\lim l(W_{j,n})/p^{nd} = 0$; b) $j \geq \dim A$,

$\underline{\lim}\, l(W_{j,n})/p^{nd} = \underline{\lim}\, l(\cancel{\oplus} H_{j-d}(F^n(F_\cdot)) \otimes N^*)/p^{nd}$, here
$d = \dim A$, $N^* = \mathrm{Hom}(H^d_m(N), E)$.
We deduce the improved new intersection conjecture in ch $p>0$ and a special case of positivity of Serre's Conjecture on intersection multiplicity from the above theorem.
We then discuss ~~several~~ some cases of the canonical element Conjecture in the mixed characteristic and reduce it to a question of understanding ~~lengths of~~ the limits of lengths of some special cyclic module of finite length under the frobenius map.

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Tight Closures of Ideals I, II

We introduce the notion of tight closure for ideals and submodules in certain cases, and then use this notion to give new proofs that rings which are direct summands of regular rings are Cohen-Macaulay and of the Briançon-Skoda Theorems, as well as to obtain new constraints on the behavior of systems of parameters. In characteristic $p>0$, we define the tight closure I^* of $I \subseteq R$ as follows: $x \in I^*$ if there exists $c \in R - \bigcup\{\text{minimal primes of } R\}$ such that for all $e \geq 0$ $cx^{p^e} \in I^{[p^e]}$, where $I^{[p^e]} = (i^{p^e} : i \in I)$. A key point is that under mild conditions, $(x_1, \dots, x_n) : x_{n+1} \subseteq (x_1, \dots, x_n)^*$ when the x_i are parameters. In a regular ring, every ideal is tightly closed, and rings with this property are called F-regular. If $I \subseteq R$, an algebra finitely generated over a field of characteristic 0, we say that $x \in I^*$ if there exists a finitely generated $\mathbb{Z}$-algebra $D \subseteq R$, a finitely generated D-subalgebra R_D of R, an ideal $I_D \subseteq R_D$ and an element $c \in R_D$, not in any minimal prime of R, such that $R = K \otimes_D R_D$, $I = K \otimes_D I_D$ and ~~that~~ for all $x \in U$ in a certain open subset U of Max Spec D, if m is the corresponding maximal ideal, $K = D/m$, and $p = \operatorname{char} K$ then for all $e \geq 0$, $1 \otimes cx^{p^e} \in I_K^{[p^e]} \subseteq R_K$ where $R_K = K \otimes_D R_D$ and $I_K = IR_K$.

In general $I \subseteq I^* \subseteq \bar{I}$, the integral closure; I^* is usually much smaller than $\bar{I}$. F-regularity implies rational singularities if the ring has isolated singularities or is graded and has rational singularities except possibly at the irrelevant ideal.

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Unimodular elements and cancellation

Let $A = R[\underline{X}, \underline{Y}^{\pm 1}]$ be a Laurent extension over a noetherian ring R, $\dim R = d < \infty$, and let P be a projective A-module of $\mathrm{rank}(P) \geq d+1$. Generalizing a well known conclusion from a theorem of Eisenbud and Evans in case $A = R$, we show that for every ideal $\mathfrak{a} \subset R$ the canonical map $\mathrm{Um}\, P \longrightarrow \mathrm{Um}\, P/\mathfrak{a}P$ is surjective.

Moreover, we give a new proof of the following result of R. A. Rao:

Thm: Under the assumptions and notations above, then the elementary group of $A \oplus P$ acts transitively on $\mathrm{Um}(A \oplus P)$, if $\mathrm{rank}\, P + d \geq 2$.

This implies that projective A-modules P with $\mathrm{rank}\, P \geq d+1$ are cancellative, i.e. $P \oplus A \cong P' \oplus A$ implies $P \cong P'$.

— Hartmut Lindel

Elliptic curves and diophantine equations

We conjectured in 1982 the following: Let k be a number field and $\varepsilon > 0$ there exists a constant $C(k,\varepsilon)$ such that for any semistable elliptic curve on k its minimal discriminant satisfies $|\Delta| \leq C(k,\varepsilon) \left(\prod_{v \in S} N(v)\right)^{3+\varepsilon}$ where S is the set of places where the curve has bad reduction.

We gave in the talk evidence of the conjecture over function fields (any charact.), and we also explained how this is linked to

the Fermat conjecture via the work of G. Frey.
Thanks to him one can read the conjecture:
let $a+b=c$ natural numbers with no common factor then $\forall \varepsilon > 0$ $\exists C(\varepsilon)$ such that
$$|abc| \leq C(\varepsilon) N^{3+\varepsilon} \quad \text{where } N = \prod_{p|abc} p$$
We then explained the latest conjectures of Masser-Oesterlé, Vojta and Parshin wich implied this conjecture.

L. Szpiro
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Smooth surfaces in $\mathbb{P}_4$.

Which smooth surfaces can be imbedded in $\mathbb{P}_4(\mathbb{C})$? and how?
Implicit problem in Severi's theorem (except for Veronese surfaces, smooth surfaces in $\mathbb{P}_4$ are linearly complete).
Hartshorne conjectured that there is only a finite number of families of smooth rational surfaces (in $\mathbb{P}_4$).
With G. Ellingsrud we prove this fact, and the same about K_3, Abelian and birationally ruled surfaces. More precisely:
- Let $a < 6$. There is only a finite number of families of smooth surfaces in $\mathbb{P}_4$ verifying $K^2 \leq a\chi(\mathcal{O}_S)$.

By an easy numerical argument, the proof of the theorem is reduced to the proof of the following technical lemma:
- Let σ be a positive integer. There exists a polynome P_σ of $d^\circ 6$ with positive leading coefficient such that for every smooth surface S of $d^\circ d$ lying in a $d^\circ \sigma$ hypersurface of $\mathbb{P}_4$ one has $P_\sigma(\sqrt{d}) \leq \chi(\mathcal{O}_S)$.

Conjecture: $S \subset \mathbb{P}_4$, S smooth then $\chi(\mathcal{O}_S) \geq 0$?

Christian Peskine

On the Space of Plane Triangles
(with Alberto Collino)

Schubert's space X of (ordered) plane triangles is a closed subvariety of $\mathbb{P}^2 \times \mathbb{P}^2 \times \mathbb{P}^2 \times \check{\mathbb{P}}^2 \times \check{\mathbb{P}}^2 \times \check{\mathbb{P}}^2 \times G_2(\mathbb{P}^5)$. Using the natural torus action on X and known relations among divisors, one can compute the intersection ring $A^*(X) = H^*(X) = \mathbb{Z}[a,b,c,\alpha,\beta,\gamma,d]/I$, where I is generated by relations (1) $a^3, \dots, \gamma^3$; (2) $a\beta - a^2 - \beta^2, \dots$ (3) $(b-c)(b+c+\beta+\gamma-d)$, $(\beta-\gamma)(b+c+\beta+\gamma-d), \dots$ (4) $(d-a-b-c)(d-\alpha-\beta-\gamma)$ (The last relation does not follow from Schubert's equations) As an application one can calculate the number of triangles inscribed in a given curve, circumscribed about another. One must allow ordinary nodes and cusps in the first curve, so the dual argument applies to the second; this corrects ~~the~~ the formula given by Schubert.

William Fulton

The Jacobian Conjecture and Differential Operators

Let $A = \mathbb{C}[x_1, \dots, x_n] \subset B$ be an étale extension of polynomial algebras. The Jacobian Conjecture says that $A = B$. The $\partial_i = \partial/\partial x_i$ act on B, so $A \subset B$ are modules over the Weyl algebra $W = \mathbb{C}[x_1, \dots, x_n, \partial_1, \dots, \partial_n]$. In fact B is a holonomic W-module, hence cyclic of finite length. The linear derivations $\varepsilon_{ij} = x_i \partial_j$ span $\mathfrak{gl}_n \subset W$. Let $\mathfrak{g} \subset \mathfrak{gl}_n$ be a Lie subalgebra, and $U = U(\mathfrak{g})$. If $\dim_{\mathbb{C}}(\mathfrak{g}) > n$ then B must be a torsion U-module. One can thus try to prove the JC by choosing $\mathfrak{g}$ of $\dim > n$ and showing that B/A is a torsion free U-module.

We attempt this for $n=2$, $A = \mathbb{C}[x,y]$, with $\mathfrak{g}$ spanned by $\varepsilon_x = x\,\partial/\partial x$, $\varepsilon_y = y\,\partial/\partial y$, and $\delta = x\,\partial/\partial y$. It can be shown that B/A is torsion free over $\mathbb{C}[\varepsilon, \delta]$, $\varepsilon = \varepsilon_x + \varepsilon_y$, and $\mathbb{C}[\varepsilon_x, \varepsilon_y]$. Proof of the latter

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case invokes Siegel's Theorem on algebraic curves with infinitely many $\mathbb{Z}$-points, and Fabry's Theorem (1896) stating that a lacunary series $f = \sum_n a_n z^{r_n}$ with $r_n/n \to \infty$ is singular on the entire circle of convergence. Results for the full algebra $U(\mathfrak{g}) = \mathbb{C}[\varepsilon_x, \varepsilon_y, \delta]$ are still partial

Stephen Bose

Graded maximal Cohen-Macaulay modules (a survey)

This lecture is a report on joint papers with Eisenbud, Buchweitz, Backelin and Sanders. Jointly with Eisenbud we showed that the already known list of homogeneous Cohen-Macaulay rings of finite representation is complete, as long as the Cohen-Macaulay ring is defined over an algebraically closed field of char 0.

Next we discussed the representation theory of quadric hypersurface rings over an arbitrary field, which is completely understood and described in a joint paper with Buchweitz and Eisenbud.

Finally we reported on a paper with Backelin and Sanders in which we extend the methods and results obtained for quadratic forms to forms of higher degree.

Jürgen Herzog

Residual intersections

(with C. Huneke)

Let R be a local Gorenstein ring, let I be an R-ideal of grade g, and let $s \geq g$. An R-ideal J is called an s-residual intersection of I if grade $J \geq s$, and there exists an ideal $K \subset I$, $\mu(K) \leq s$, such that $J = K : I$. We prove:

<u>Theorem</u> Suppose I is in the even linkage class of an ideal which is strongly Cohen-Macaulay and (G_∞), and let $J = K : I$ be an s-residual intersection of I. Then

a) J is a Cohen-Macaulay ideal of grade s

b) depth $R/J = \dim R - s$

c) $\omega_{R/J} = S_{s-g+1}(I/K)$. In particular, J is Gorenstein if and only if I/K is cyclic.

Bernd Ulrich

Symmetric algebras and factoriality

Let R be a regular local ring - or a polynomial ring - and let E be a f.g. R-module. There are two puzzling questions regarding the symmetric algebra $S(E)$ of E.

i) Question 1: If $S(E)$ is factorial, must it be a complete intersection? This is equivalent to saying $\operatorname{pd}_R E \leq 1$. It is known that $\operatorname{pd}_R E \neq 2$.

ii) Denote by $B = \oplus S_t(E)^{**}$ = graded bi-dual of $S(E)$. B is a factorial domain.

Question 2: Is B Cohen-Macaulay?

A major aspect here is whether B is Noetherian, at least when R = polynomial ring.
We report on a computer-assisted approach to question 2.
Several classes of modules have B Cohen-Macaulay through the examination of the defining equations of the subalgebras B(r), generated by the forms of B of degree ≤ r.

Wolmer Vasconcelos

Principal generic space curves.

Generic (smooth connected) space curves are curves parametrized by generic points of irreducible components of the Hilbert scheme of $\mathbb{P}^3$. Principal ones are those with generic moduli, which exist and are unic for given degree d and genus g.
We prove that the family of plane sections of these curves enjoyes any general position property one would expect, for instance no flex (as known of Eisenbud-Harris), no quintisecant line, no quadritangent plane. All these properties flow from similar original properties of the Hilbert scheme of points in the plane thanks to the condition $H^1(N(-1))=0$, where N is the normal bundle of the curve. This condition is proven using reducible curves (unions of a lower degree generic curve and a cubic curve meeting in five points) and the smoothing result of Hartshorne-H., in the same way as other results on the normal bundle were obtained previously by Ellingsrud-H.

A. Hirschowitz

Darstellungstheorie endlichen Gruppen

31.5.87 – 6.6.87

Loewy series of permutation modules for p-groups

The theorems of Jennings and Hill on the Loewy series of the group algebra kP, k a field of characteristic p and P a finite p-group are generalized for kP-modules $k\Omega$ where Ω is a transitive P-set. In particular, in answer to a question of P. Neumann, it is proved that the upper and lower series of $k\Omega$ coincide.

J. L. Alperin

Prime factors of character degrees of solvable groups

Let $\sigma(G)$ be the maximum number of primes dividing any one character degree of G, and let $\rho(G)$ be the set of primes which divide some character degree of G. For G solvable, Huppert has conjectured that $|\rho(G)| \leq 2\sigma(G)$. In this joint work with O. Manz, we show that $|\rho| \leq 3\sigma + 32$ for every solvable group, considerably improving earlier results of Isaacs and of Gluck. We also obtain $|\rho| \leq 2\sigma + 32$ when G is solvable with no nonabelian normal Sylow subgroups.

David Gluck

Generalization of Brauer's 2nd M. Th. to virtual modules

Let G be a finite group, A an interior G-algebra, M an A-module and χ the character of M. For any pointed group H_β on A, denote by χ^β the character of the $\mathcal{O}H$-module $i\cdot M$ where $i\in\beta$ and if K_γ is a pointed subgroup of H_β denote by φ^β_γ the Brauer character of the $kC_H(K)$ module $s_\gamma(i)A(K_\gamma)s_\gamma(j)$ where $j\in\gamma$. In "Pointed groups and construction of characters" we prove $(*)$ $\chi^\alpha(us)=\sum_\varepsilon \varphi^\alpha_\varepsilon(s)\chi^\varepsilon(u)$ where $\alpha\in\mathcal{P}_A(G)$, $u\in G_p$, $s\in C_G(u)_{p'}$ and ε runs over $\mathcal{LP}_A(u)$. In particular, for any local pointed group Q_δ on A such that $Q_\delta\subset G_\alpha$, we get $(**)$ $\chi^\delta(u)=\sum_\varepsilon m^\delta_\varepsilon\chi^\varepsilon(u)$, where $u\in Q$, $m^\delta_\varepsilon=\varphi^\delta_\varepsilon(1)$ and ε runs over $\mathcal{LP}_A(u)$. Conversely, if for any local pointed subgroup Q_δ of G_α we choose a virtual character λ^δ of Q in such a way that $(**)$ holds, then $(*)$ defines a virtual character of G. Our main purpose here is to prove analogous statements replacing the ring of $\mathbb{C}$-valued central functions on G by the Green ring of G over $\mathcal{O}$, virtual characters by virtual modules, and values of virtual characters by "residues" of virtual modules on the subgroups H of G such that $H/O_p(H)$ is cyclic. The analogous of $(*)$ looks like

$$rd_H(M)=\sum_{\gamma\in\mathcal{LP}_A(P)^H} rd_{\hat{\bar H}^{\gamma\circ}}(V(P_\gamma))\otimes rdd^\gamma_K(M)$$

where $P=O_p(H)$, $\bar H=H/P$, $K=\mathrm{Im}(\bar H\to \widehat{\mathrm{Aut}}(P))$, $\mathrm{End}_k(k\otimes V(P_\gamma))\cong A(P_\gamma)$, and $d^\gamma_K(M)=\mathrm{Res}_{P\cdot\hat K^\gamma}(i\cdot M)$, $i\in\gamma$, having suitable "canonical" homomorphisms $\hat{\bar H}^{\gamma\circ}\longrightarrow A(P_\gamma)^*$ and $P\cdot\hat K^\gamma\longrightarrow A_\gamma^*$.

Ideals and codes in group algebras

Let $\mathfrak{a}$ be a right ideal in a group algebra $F[G]$, F of characteristic p, and define the divisor d of $\mathfrak{a}$ as the greatest common divisor prime to p of all the cardinalities of subsets of G which form the support of some non-zero element of $\mathfrak{a}$. Then there exists a subgroup H of G with d dividing the order of H such that furthermore $\mathfrak{a}$ is contained in the permutation ideal $(\sum_{h \in H} h)F[G]$. H is called the induction kernel of $\mathfrak{a}$ and the result is inspired from work by H. N. Ward in coding theory. This may also be used to explain how to determine the set of subgroups $\{X\}$ of G such that $S^H \neq 0$ for some simple module S. In particular we get an improved version of the Nakayama Relations for ideals.

Peter Landrock

Applying representation theory: a soluble quotient algorithm

To design feasible algorithms to compute the biggest finite soluble quotient group of a finitely presented group in case it exists has been an open problem in computational group theory ever after the spectacular success of the nilpotent quotient algorithm.
Given a finitely presented group G and an epimorphism ε of G onto a "known"

group H. To test whether ε lifts to an epimorphism onto an extension of a simple H-module by H amounts to solving linear equations. For H soluble one can find the simple modules and the cocycles describing the extensions algorithmically. This way one obtains a soluble π-quotient algorithm, where π is a finite set of primes. The proper choice of π can be made if the irreducible $\mathbb{Q}H$-modules are known for the various quotient groups H of G which occur.

Wilhelm Plesken, QMC London

Characters and solvable groups.

The machinery which has been developed for the study of the representations of solvable groups makes it possible often to answer questions about these groups which (as yet) seem impossibly difficult for general finite groups. (Examples are the McKay conjecture and Brauer's height conjecture which have been settled for solvable groups but are still open in general.) This talk presents a proof for solvable groups, using some of the now standard techniques, of the following general problem:

— Let $\chi \in \mathrm{Irr}(G)$ be primitive and suppose $\theta \in \mathrm{Char}(H)$ for $H \subseteq G$ and that $\theta^G = a\chi$ for some integer a. Show that $H = G$. —

The proof for solvable groups presented here uses factorization of characters.

I. Martin Isaacs, MADISON

~~Ieg~~ Eigenvalues of matrices of complex representations of finite Chevalley groups

Let $G = G(p^\alpha)$ be a Chevalley groups of normal or twisted type and $Z(G)$ its center. For $g \in G$ let $o(g)$ be the order of g modulo $Z(G)$. Let F be an algebraically closed field of characteristic $f \neq p$. For $\varphi \in \mathrm{Irr}_F G$ let $\deg \varphi(g)$ be the degree of the minimal polynomial of the matrix $\varphi(g)$. The following main problem is discussed.

Problem. To describe triples (G, g, φ) with $\varphi \in \mathrm{Irr}_F G$ such that $\deg \varphi(g) < o(g)$. and $\dim \varphi > 1$.

The main result is the following

Theorem. Let G, g, φ as above and $|g| = p$. If $\deg \varphi(g) < p$ then $p > 2$ and G is one of the groups: $A_1(p)$, $A_1(p^2)$, ${}^2A_2(p)$, $C_n(p)$ $(n > 1)$.

Morita equivalent blocks and Clifford theory

Burkhard Külshammer, Universität Dortmund

Clifford theory is concerned with the relationship between representations of groups K, H, G occuring in a finite group extension

$$1 \longrightarrow K \longrightarrow H \longrightarrow G \longrightarrow 1.$$

Let B be a (G-stable) block of K, and let A be a block of H covering B. J. Alperin has proved recently that A and B are isomorphic if and only if their Brauer correspondents are. We generalize his result, replacing isomorphism by a Morita equivalence satisfying a certain natural condition. When A and B are Morita equivalent, questions about A can often be reduced to questions about B. This reduction process complements other tools in Clifford theory such as the Fong-Reynolds correspondence.

Burkhard Külshammer, Dortmund

Compounding Clifford Theory

If G is a group, $N \trianglelefteq G$ and $\bar{G} = G/N$, then Clifford Theory is an equivalence of categories of modules over rings graded by the factor group $\bar{G}$. In particular, we use the natural $\bar{G}$-grading of $R = \mathcal{F}G$, over any coefficient ring $\mathcal{F}$. If V is a simple module over $R_1 = \mathcal{F}N$, then $V^R := V \otimes_{R_1} R = \sum_{\sigma \in \bar{G}} V^\sigma$ is a $\bar{G}$-graded R-module, where $V^\sigma = V \otimes_{R_1} R_\sigma$ is a conjugate simple R_1-module. For our category of R-modules we take the full additive subcategory $\mathrm{Mod}(R|V)$ of $\mathrm{Mod}(R)$ generated by V^R. It is easy to see that $\mathrm{Mod}(R|V)$ is closed under R-submodules, factor modules and direct sums, and that V^R is a finitely-generated, projective (in $\mathrm{Mod}(R|V)$) generator of $\mathrm{Mod}(R|V)$. A theorem of Mitchell (64) (and Gabriel (62)) gives us the Clifford equivalence $\mathrm{Mod}(R|V) \approx \mathrm{Mod}(R')$, where $R' = R'_V = \mathrm{End}_R(V^R)$. In fact the equivalence is given by:

$$\mathrm{Mod}(R|V) \underset{\cdot \otimes_{R'} (V^R)}{\overset{\mathrm{Hom}_R^{\#}(V^R, \cdot)}{\rightleftarrows}} \mathrm{Mod}(R').$$

Of course R' is also a $\bar{G}$-graded ring with

$$R'_\sigma = \{\varrho \in R' \mid \varrho(V^\tau) \subseteq V^{\sigma\tau}, \text{ for all } \tau \in \bar{G}\}, \text{ whenever } \sigma \in \bar{G}.$$
$$\xrightarrow[\text{res}]{\sim} \mathrm{Hom}_{R_1}(V, V^\sigma).$$

Since $V^1 = V$, V^σ are both R_1-simple we have $R'_\sigma = 0$ if $V \not\simeq V^\sigma$ and R'_σ contains a unit of R' if $V \simeq V^\sigma$, i.e., if σ lies in the stabilizer $\bar{G}\{V\}$ of V in $\bar{G}$. Thus R' is "really" a crossed product of $\bar{G}\{V\}$ over $R'_1 \simeq \mathrm{End}_{R_1}(V)$. This connects with the usual, two-step Clifford Theory.

Now let $N \leq M \trianglelefteq G$, $\bar{M} = M/N \trianglelefteq \bar{G}$, $\bar{\bar{G}} = \bar{G}/\bar{M} \simeq G/M$. Then $R'_{\bar{M}}$ denotes either of the naturally isomorphic rings $(R')_{\bar{M}} \simeq (R_{\bar{M}})' = \mathrm{End}_{R_{\bar{M}}}(V^{R_{\bar{M}}})$. Let W be a simple $R_{\bar{M}}$-module in $\mathrm{Mod}(R_{\bar{M}}|V)$, ~~then~~ and $W' \in \mathrm{Mod}(R'_{\bar{M}})$

be the corresponding simple $R'_{\bar m}$-module under Clifford theory for $\bar m$, $R_{\bar m}$ and $\bar v$. Then $\bar w^R = \bar w \otimes_{R_{\bar m}} R \in \mathrm{Mod}(R|\bar v)$ corresponds to $(\bar w')^{R'} \in \mathrm{Mod}(R')$. Since Clifford theory for $\bar G$, R and $\bar v$ is an equivalence of abelian categories, it gives an isomorphism:

$$(*) \qquad R'_{\bar w} := \mathrm{End}_R(\bar w^R) \xrightarrow{\sim} R''_{\bar w'} := \mathrm{End}_{R'}((\bar w')^{R'})$$

One can verify that the isomorphism $(*)$ preserves $\bar{\bar G}$-gradings. We have a diagram of equivalences of categories coming from the Clifford equivalence (C.E.) or from the isomorphism $(*)$:

$$\begin{tikzcd}
\mathrm{Mod}(R|\bar v) \arrow[d, "\wr"', "\text{C.E. for } \bar G,\ R \text{ and } \bar v"] & \supseteq \mathrm{Mod}(R|\bar w) \arrow[d, dotted, "\varphi"] \arrow[r, "\text{C.E. for } \bar{\bar G},\ R \text{ and } \bar w"] & \mathrm{Mod}(R'_{\bar w}) \arrow[d, "\wr"', "\text{from } (*)"] \\
\mathrm{Mod}(R') & \supseteq \mathrm{Mod}(R'|\bar w') \arrow[r, "\text{C.E. for } \bar{\bar G},\ R' \text{ and } \bar w'"'] & \mathrm{Mod}(R''_{\bar w'})
\end{tikzcd}$$

This induces (by restriction) an equivalence φ such that the diagram commutes to within natural equivalences of functors. The proof of this involves nothing deeper than associativity of tensor products.

Everett C. Dade

Die Charaktertafel von $E_6(2)$

$E_6(2)$ enthält als parabolische Untergruppe $N \cdot K$, wobei N elementar abelsch der Ordnung 2^{16} und $K \simeq D_5(2)$ ist. Zur Berechnung eine Charaktertafel einer Untergruppe von dem Monster wurde die Charaktertafel von NK berechnet. Da diese Tafel bekannt ist, konnte die Charakter-tafel von $E_6(2)$ mit Methoden, die man zur Berechnung von Charaktertafeln sporadischer Gruppen benutzte, berechnet werden. Ich bin S. Black und J. Janiszczak für ihre Mithilfe zu Dank verpflichtet.

B. Fischer, Bielefeld

Brauer characters of q'-degree

Take $N \trianglelefteq G$, $\alpha \in \mathrm{Irr}(N)$, suppose that G/N is q-solvable and that the prime q doesn't divide $\chi(1)/\alpha(1)$ for all $\chi \in \mathrm{Irr}(G|\alpha)$. By a fairly well-known theorem of D. Gluck and T. Wolf, under these circumstances a Sylow-q-subgroup of G/N is abelian. This extends the classical theorem of N. Ito, where $N=1$ and hence $\alpha = 1$ is taken. We consider p-modular analogues of the results above and henceforth denote by $\mathrm{IBr}(G)$ the Brauer characters of G w.r.t. a prime p.

Case 1. $q \neq p$: If $q \nmid \beta(1)$ for all $\beta \in \mathrm{IBr}(G)$ and G is p-solvable, then a Sylow-q-subgroup of G is at most metabelian, in particular, the q-length of G is at most 2. Furthermore, if $\alpha \in \mathrm{IBr}(N)$, G/N is solvable and $q \nmid \beta(1)/\alpha(1)$ for all $\beta \in \mathrm{IBr}(G|\alpha)$, then a q-Sylow of G/N has commutator length at most 3 and in case of $q \geq 5$ even at most 2.

Case 2. $q = p$: Although it's easy to see that in case of G p-solvable and $p \nmid \beta(1)$ for all $\beta \in \mathrm{IBr}(G)$ the group G has a normal p-Sylow-subgroup, a "local" version as above does definitely not hold.

(This report is about joint work with Tom Wolf (Athens, Ohio).)

Olaf Manz (MAINZ)

On the Loewy-structure of the Modular Group Algebra over a Finite p-Group

Let P be a finite p-group and K an arbitrary field of characteristic p, p a prime. In 1941, S. Jennings proved the following about the dimensions of the Loewy-factors $J(KP)^i/J(KP)^{i+1}$. Let $x_1 := P$ and $x_n := [x_{n-1}, P]\, x_m^p$, where m is the least integer with $pm \geq n$. Then $x_1 \geq x_2 \geq \ldots \geq x_\ell > x_{\ell+1} = 1$ is a central series with elementary abelian factors. If $|x_n/x_{n+1}| =: p^{d_n}$ and $\dim_K(J(KP)^i/J(KP)^{i+1}) =: c_i$, we have $\prod_{n=1}^{\ell} (1 + t^n + \ldots + t^{n(p-1)})^{d_n} = \sum_{i=0}^{s} c_i t^i$ (in $\mathbb{Z}[t]$). From this we easily have $c_i = c_{s-i}$, and for quite some time, the question circulated whether the Loewy-series is even monotonic, which means $c_{i-1} \leq c_i$ $(1 \leq i \leq \frac{s}{2})$. In 1986, Stammbach/Stricker and Manz/Staszewski independently found counterexamples, but in all of them, the prime p was either 2, or 3, or 5. Searching for counterexamples in a systematic manner, C. Leedham-Green and the author first proved some conditions which the sequence $d_1, d_2, \ldots$ has to fulfill whenever this sequence arises from a group and afterwards wrote a computer program ~~m~~ which constructs all d_i-sequences which might arise from a group and calculates the c_i's (up to a certain order of the group). By this method, it was shown that there is no counterexample P with $|P| \mid 7^{15}$, $|P| \mid 11^{17}$ or $|P| \mid 17^{19}$, whereas the smallest counterexamples are of order 2^5, 3^5 or 5^5, respectively.

Rainer Staszewski (Mainz)

Dixon's Character Table Algorithm Revisited

Let C_i, $1 \le i \le k$, denote the classes of a finite group G and χ_i the characters. The algorithm given by J. Dixon in 1967 for the automatic computation of group characters can ~~by~~ be significantly improved by using the equation

$$\frac{|C_r| \cdot \overline{\chi_i(x_r)}}{\chi_i(1)} \cdot \chi_i(x_t) = \sum_{s=1}^{k} \chi_i(x_s) \cdot c_{rst}$$

The c_{rst} are the class multiplication constants, i.e. the number of solutions in G to the equation $x_r \cdot x_s = x_t$, for $x_r \in C_r$, $x_s \in C_s$ and fixed $x_t \in C_t$. The characters are therefore row eigenvectors of the class matrix $M_r = (c_{rst})_{s,t}$ and can be obtained by successive computation of eigenspaces of various class matrices.

The new approach allows to predict whether a class matrix will split an existing space into smaller subspaces without having to determine the matrix. In addition not all columns of a matrix have to be determined to split a space. An algorithm was presented that finds the characters that span a 2-dimensional space without the need of a class matrix.

The performance of the new algorithm was demonstrated by giving the CPU-time requirements of various test cases. An implementation is available to the users of the CAYLEY system

Gerhard Schneider (Essen/Zürich)

Groups with only a few character degrees

Let G be a finite, solvable group and furthermore denote $\mathrm{Irr}_{\mathbb{C}}(G)$ the set of all irreducible complex characters of G. Define $c.d_{\mathbb{C}}(G) := \{\chi(1) \mid \chi \in \mathrm{Irr}_{\mathbb{C}}(G)\}$. Then it is a conjecture that $dl(G) \leq |c.d._{\mathbb{C}}(G)|$. This conjecture has been proved for $k := |c.d_{\mathbb{C}}(G)|$ equal 2 and 3 by Isaacs and Passman, for k equal to 4 by Garrison and for abitrary groups of odd order by Berger. For any solvable group G D. Gluck proved $dl(G) \leq 2 \cdot |c.d_{\mathbb{C}}(G)|$.
Now, if we look at the modular analog of those theorems what is to say about the structure of G. Since $\bigcap_{\beta \in \mathrm{IBr}_p(G)} \mathrm{Kern}\,\beta = O_p(G)$ if $\mathrm{IBr}_p(G)$ denotes the set of all irreducible Brauer characters for the prime p, it is natural to make the assumption that $O_p(G)$ is trivial. If you looking now at the case that G has only two p-modular irreducible Brauer caracters, then the derived length of G is less or equal to 3 if p is odd, and less or equal to 4 if p is equal to 2. Furthermore in any case the p-length of G is one and the groups of derived length four are "essentially" known.

Frank Bernhardt (Mainz)

Variations on McKay's Conjecture

Let P be a Sylow-p-subgroup of a finite group G. McKay's conjecture proposes that G and $N_G(P)$ have the same number of irreducible (ordinary) characters of p'-degree. Okuyama and Wajima have proven this (and the more refined Alperin-McKay conjecture) for p-solvable G. Let q be a prime (equal to or different from p) and let Q be a Sylow-q-subgroup of G. If G is $\{p,q\}$-separable, we show that G and $N_G(Q)$ have the same number of Brauer characters (for the prime p) of q'-degree. Furthermore, p and q may be replaced by sets of primes π and ρ, provided G is π-separable and ρ-separable. From this follows the Alperin-McKay conjecture for π-blocks of π-separable groups as well as an unpublished result of Isaacs regarding the number of π-special characters of a π-separable group.

Tom Wolf (Athens OH)

On Permutation Modules.

In this talk, G denotes a finite group, p a prime, k an algebraically closed field of characteristic p, S a Sylow p-subgroup of G. We discuss various connections ~~with~~ between the structure of permutation modules (and their endomorphism rings) of G and the group-theoretic structure of G.

In section 1, on Fusion, I ~~will~~ discuss the following result (and related ones).

THEOREM: The vertices of the non-projective summands (in the principal p-block) of $\mathrm{Ind}_S^G(k)$ constitute a conjugation family for S in G.

In section 2, on simplicial complexes and related topics, I will discuss (among other things) the following result related to a conjecture of Quillen.

THEOREM: Let Δ_p be the simplicial complex associated to the poset of non-trivial p-subgroups of G. Suppose that $p \geq 5$, that $O(G) = 1$, and that the components of G are of characteristic 2-type. Then Δ_p is not contractible.

I also discuss the following result (proved jointly with R. Knörr)

~~Let B~~. THEOREM: Let B be a block of kG, and for a simplex $C \in \Delta_p$,

let B_C be the Brauer correspondents (ie blocks of kG_C) (which are defined).

Let G_C act on B_C by conjugation. Then $\sum_{C \in \Delta_p/G} (-1)^{|C|} \mathrm{Ind}_{G_C}^G(B_C)$

is a virtual projective module.

This result relates to a conjecture of Alperin, and has applications to groups for which Alperin's conjecture has been verified.

Geoffrey R. Robinson (UMIST, Manchester).

An Application of prime characters.

If χ is a quasi-primitive irreducible character of G, let $Z(\chi)$, $F^*(\chi)$, and $M^*(\chi)$ be defined by $Z(\chi)/\ker(\chi) = Z(G/\ker(\chi))$, $\frac{F^*(\chi)}{\ker(\chi)} = F^*(G/\ker\chi)$, and

$M^*(\chi) = F^*(\chi)/Z(\chi)$. χ is a prime character if χ is a quasi-primitive irreducible character, $\chi_{F^*(\chi)}$ is irreducible and $M^*(\chi)$ is homogeneous.

I discuss the following:

Th 1 If χ is a quasi-primitive irreducible character of G, there is an extension $(\hat{G}, \pi)$ of G such that $\ker \pi \subseteq \hat{G}' \cap Z(\hat{G})$ and χ factors uniquely (up to associates) as $\chi = \prod_{i=1}^{n} \rho_i$ where $\{\rho_1, \rho_2, \ldots, \rho_n\}$ is an admissible set of prime characters.

As an application, I indicate a proof of the following theorem:

Theorem 2 Suppose χ is a quasi-primitive irreducible character of G of odd degree. If $\psi \in Irr(S)$ where S is a universal covering group of a non-Abelian composition factor of G and $\psi(1) \mid \chi(1)$, assume $\frac{\chi(1)}{\psi(1)}\psi$ is not induced from a proper subgroup of S and if $\psi(1) = p^r$ then $p \nmid |\frac{Z(S)}{\ker \psi}|$, then χ is a primitive character of G.

Pamela A. Ferguson

Maximal subgroups of classical groups

Let G be a finite classical group over a field of r^c elements. In view of a recent result of Aschbacher the problem of determining the maximal subgroups of G reduces to finding triples (X, Y, V), where X, Y are quasisimple groups, $X < Y$, and both X, Y are absolutely irreducible subgroups of G. Here V is the natural module for G.

Let $X = X(p^a)$ be of Lie type in characteristic p. For $p = r$ (the generic case) a great deal of work has been done and this case is now in reasonably good shape. Here we consider the cross-characteristic case, $p \neq r$.

The most likely candidate for Y is $Y = Y(p^b)$, also of Lie type in char. p, and we assume this to be the case. Hence, we have the embedding $X(p^a) < Y(p^b) < G = G(r^c)$. We have the following

Thm Assume $p^a > 3$ and Y is of classical type. Then there is a subgroup $C < Y$ such that $X \leq C$ and the pair $(C/Z(C), Y/Z(Y))$ is one of the following

1) $(PSp_{2m}(q), PSL^{\varepsilon}_{2m}(q))$ $\qquad (q = p^b)$
2) $(P\Omega_{m-1}(q), P\Omega_m(q))$
3) $(G_2(q), B_3(q))$
4) $(PSp_{2m}(q^a), PSp_{2ma}(q))$

In particular, if Y is maximal, then $(X/Z(X), Y/Z(Y))$ is one of the pairs 1) – 4).

For most of the types 1) – 4) a suitable V does exist, but we have been unable to determine all possible Vs.

Gary M. Seitz

Modular Representation of Chevalley Groups in Special Characteristics

Let G be a group with a split BN-pair, i.e.
$B, N \leq G = \langle B, N \rangle$; $B = UH$, $U \trianglelefteq B$, $U \cap H = 1$;
$N = N_G(H)$, $B \cap N = H$, $N/H = W$, the Weyl group of G.
Let r be a prime and D a Sylow r-subgroup of H. Suppose the following conditions are satisfied:

1. D is a Sylow subgroup of G.
2. $C_G(D) = H$.

In this situation we determine the decomposition matrice of the principal r-block of G. Two corollaries should be mentioned (k denotes a splitting field of characteristic r):

<u>Corollary 1.</u> The permutation module over k on the cosets of B is completely reducible. Its constituents are exactly the simple kG-modules in the principal block. The unipotent characters in the principal series are irreducible modulo r.

<u>Corollary 2.</u> The Green correspondents of the simple kG-modules in the principal block are exactly the simple kN-modules which have H in their kernel, i.e. the simple kW-modules.

The proof is a straightforward application of Green correspondence. L. Puig has obtained these results independently by using his theory of source algebras.

Gerhard Hiß

Resolutions as multiple complexes
(PhD thesis of R. Schmid)

Over a finite abelian group the minimal resolution of the field k can be described as the total complex of a multiple complex. Schmid has shown that this result generalizes to the case of a group $A \rtimes B$ where A is an abelian p-group and B is a p'-group such that for a large enough field k the B module $k \otimes A$ has 1-dimensional composition factors only. In principle the resulting complex can be used to calculate the cohomology of such a group.
Some time ago J. Alperin raised the question whether there always is a resolution that can be written as the total complex of a multiple complex. Recently Benson and Carlson have shown that this is indeed the case. Their proof however is not constructive. Regarding their multiple complex as iterated double complex, Schmid has proved that the associated spectral sequences have the property that $E_2 = E_\infty$. As a consequence the cohomology groups can be calculated as iterated homology. This implies in particular that the cohomology groups obey very strong periodicity rules.

Urs Stammbach
ETH-Zürich

Central extensions as Galois groups

Let G be a finite group and $\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3$ be conjugacy classes of G. Under certain conditions $(\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3)$ determines a regular extension E of $\mathbb{Q}(T)$ with Galois group G, if $Z(G) = 1$. For example, if $(\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3)$ is rigid, E exists. If K is an intermediate field $\mathbb{Q}(T) \subset K \subset E$, the Hasse-Witt invariant of $K/\mathbb{Q}(T)$ (with the quadratic form $Tr(X^2)$) is the obstruction of the existence of $\hat{E}$, with $\hat{G} \simeq Gal(\hat{E}/\mathbb{Q}(T))$, where $\hat{G}$ is the preimage in $\hat{A}_n$ of the image of G in A_n $(n = \dim(K))$, (when $G \subseteq A_n$).
A problem is to predict $w(K/\mathbb{Q}(T))$ from $(\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3)$.

To shed some light on this question we compute $w(K/\mathbb{Q}(T))$ for the rigid triples for S_n which contain the n-cycles and yield K of genus 0.

For n even, the answer only depends on n, and not on the other two parameters. Why?

A. Turull (Joint work with Núria Vila)

Reduced K-degrees of irreducible characters

Given a field K of characteristic 0 and a finite group, take a splitting field L ~~of characteristic zero~~ containing K and let $\chi \in \mathrm{Irr}_L(G)$. We define the reduced K-degree of χ by $t_K(\chi) = \chi(1)/m_K(\chi)$ where $m_K(\chi)$ is the Schur index of χ over K. We intend to see how numerical conditions on the $t_K(\chi)$ influence the structure of G.

Theorem 1 Suppose that there is a prime p such that $p \mid t_K(\chi)$ for all χ with $t_K(\chi) \neq 1$. Then G has a normal p-complement.

Corollary 2 Suppose that $t_K(\chi) = 1$ or k for all χ, $k > 1$. Then G is solvable and has derived length at most 4. If $k = 2$, G is even metabelian.

Theorem 2 Suppose that $t_K(\chi) = 1$ or a prime (not always the same prime). Then G is solvable of derived length at most 4 (this bound can be attained). Moreover if G has odd order, at most 2 primes can occur among the $t_K(\chi)$.

The proof of solvability requires the classification.

R. Gow (Joint work with B. Huppert)

On the finite linear groups with Frobenius section

Related to the results of Brauer, Leonard, Sibley and Ferguson on finite linear groups is the following theorem:

Theorem 1. Let a finite group G has a faithful complex character φ and $C_G(x) \subseteq P\,C_G(P)$ for each nontrivial element of some non-normal Sylow p-subgroup P of G. Then

1) If every irreducible constituent of character φ is less than $(|P|-1)/2$, then $G \simeq Sz(q)$

2) If $\varphi(1) \leq (|P|-1)/2$, then either $G \simeq Sz(q)$ or $G/Z(G) \simeq PSL(2,|P|)$

This result has been obtained together with Romanovskaya N. A. The proof is based on the theory of exceptional characters and not the classification of simple finite groups

Following results are obtained by me and Jadchenko A. A.

Theorem 2. Let G be a p-solvable group with an abelian Sylow 2-subgroup. Let G has a faithful representation of degree $n = 2p-2$ over the complex ~~$n = 2p-2$~~ number field. Then G has normal Sylow p-subgroup.

Theorem 3. Let p-solvable group G has a faithful complex irreducible imprimitive character of degree $2p-2$. Then either G is solvable or G has normal Sylow p-subgroup

A. V. Romanovskii (Gomel')

Braids and Galois Groups

Let $\mathcal{K}^s$ $(=(\mathbb{P}^1)^s(\mathbb{C}))$ be the s-fold product of the Riemann sphere and $\mathcal{D}^s$ the weak diagonal of $\mathcal{K}^s$. Then the fundamental group of $\mathcal{K}^s \setminus \mathcal{D}^s$ is the pure Hurwitz braid group $\mathcal{B}^s$. Therefore the Galois group of the maximal algebraic extension field $\hat{M}$ of $\mathbb{C}(\underline{t}) = \mathbb{C}(t_1,\dots,t_s)$ (geometric) unramified outside $\mathcal{D}^s$ is the profinite completion $\mathcal{B}^s := \hat{\mathcal{B}}^s$ of $\mathcal{B}^s$, called the profinite pure Hurwitz braid group. Now $\mathcal{B}^s$ is a split extension of a free normal subgroup Π of rank $s-2$ (for $s \geq 4$) with a group $\mathcal{B}$ of type $\mathcal{B}^{s-1}$. Let R be the subfield of $\hat{M}$ fixed by Π. Then we get rationality criteria for the intermediate fields of $\hat{M}/R$ with a given Galois group G by comparing the action of $\Delta := \mathrm{Aut}(R/\mathbb{Q}(\underline{t}))$ and of the braid group $\mathcal{B}$ on the set $\Sigma_s^i(G)$ of s-generator classes of G modulo $\mathrm{Inn}(G)$ (via the Hurwitz classification).

With these rationality criteria (braid orbit theorems) for example the Mathieu group M_{24} can be realized as a Galois group over the rational function field $\mathbb{Q}(t_1, t_2)$ and also over $\mathbb{Q}$ by Hilbert's irreducibility theorem.

The study of the braid orbits on $\Sigma_s^i(G)$ leads to interesting group theoretical questions hopefully to be solved in the future.

B. H. Matzat (TU Berlin)

News on the isomorphism problem.

Report on joint work with L. L. Scott.

Theorem: Let R be an unramified extension of the p-adic integers. G is a p-constrained group with $O_{p'}(G)=1$. Let $\alpha: RG \to RG$ be an augmented automorphism with $\alpha(I(O_p(G)) \to I(O_p(G))\uparrow^G$ — $I(X)$ is the augmentation ideal of the subgroup X of G. Then there exist a unit u in RG with $\alpha(G) = uGu^{-1}$!

Cor 1: Let G be as above. Then the Zassenhaus conjecture is true for $\mathbb{Z}G$.

Cor 2: Let G be a solvable group. Then for every prime p, the group $G/O_{p'}(G)$ is determined by $\mathbb{Z}G$.

K. W. Roggenkamp, Stuttgart

Periodic simple modules for Chevalley groups in the <u>defining</u> Characteristic.

I presented recent results of Janiszczak, Jantzen and myself.
If kG is a modular group algebra with char $k = p \mid |G| < \infty$, and M a kG-module, M is called periodic, iff $\Omega^i(M) = M$ for some $i > 1$. Here $\Omega(\cdot)$ denotes the Heller-operator, i.e. $\Omega^i(M) = \ker \partial_i$ where ∂_i = i-th differential of a minimal projective resolution $\to P_2 \xrightarrow{\partial_2} P_1 \xrightarrow{\partial_1} P_0 \xrightarrow{\partial_0} M \to 0$. Let k be algebraically closed, then the results are the following: $(q = p^n)$
ST denotes the Steinberg module

$A_1(q)$: There exist periodic simple modules $\neq$ ST which are classified (Jeyakumar 1979)

$A_n(q)$: No periodic simple modules $\neq$ ST exist ("Complexities" of irreducible $A_2(q)$ modules are $(n \geq 2)$ $0, 2, 4, \ldots, 2n$)

Same result for : $D_n(q)$, $E_6(q)$, $E_7(q)$, $E_8(q)$, $F_4(q)$
(all this Janiszczak 1985)

Same result for $B_2(q)$, $G_2(q)$ (and $B_n(q)$, $C_n(q)$ $n \geq 2$)
(Janiszczak + Jantzen 1986)

Twisted types:

$^2A_2(q^2)$: There exist simple periodic modules $\neq$ ST, which are classified.
(The complexities are $1, 2, 3, \ldots, 2n$) (Fleischmann 1986)

$^2A_n(q^2)$; $n \geq 3$: No periodic simple modules exist $(\neq ST)$ (Fleischmann + Jantzen 1986/87)

Same

$^2B_2$

$^2G_2$

$^2F_4$

Same result holds for: ${}^2D_n(q^2)$, $n \geq 4$, ${}^2E_6(q^2)$, ${}^3D_4(q^3)$, (Fl. + Jantzen)

${}^2B_2(2^{2m+1})$, (Suzuki groups): periodic simple modules exist and are classified. } (Fl. + Jauiszad)

${}^2G_2(3^{3m+1})$, (Ree groups): no periodic simple modules $\neq$ ST exist

${}^2F_4(2^{2m+1})$, not yet fully settled.

Peter Fleischmann, Essen.

The Grothendieck ring for $\mathbb{F}_q SL_3(q)$

Consider the homomorphism of the commutative polynomial ring $\mathbb{Z}[x,y]$ to the Grothendieck ring of $SL_3(q)$ over $\mathbb{F}_q$ which maps x to the natural module and y to its dual. This homomorphism is surjective, and its kernel is the ideal generated by the two polynomials

$$x^p - x + pR(x,y) \quad \text{and} \quad y^p - y + pR(y,x)$$

where

$$R(x,y) = \sum \frac{(i+j+k-1)!}{i!\,j!\,k!} x^i(-y)^j \in \mathbb{Z}[x,y]$$

with summation over all nonnegative i, j, k such that $i+2j+3k = p \neq i$.

A similar result holds for $\mathbb{F}_q SL_3(q)$ whenever q is a power of p.

L G Kovács (Canberra)

Specht modules and the cohomology of mapping class groups

Let $M^n_{g,k}$ ~~...~~ denote an oriented 2-manifold of genus g with n punctures and k boundary components, and $\Gamma^n_{g,k} = \pi_0 \, Top^+(M^n_{g,k})$ denote the group of connected components of the group of orientation preserving self homeomorphisms of $M^n_{g,k}$ (the mapping class group). Using recently developed diagrammatic methods in modular representation theory (due to myself and Jon Carlson), and the theory of Specht modules, I obtain the cohomology ring of $\Gamma^0_{2,0}$ with coefficients in any field. The interesting characteristics are 2, 3 and 5. For example, we have

$$H^*(\Gamma^0_{2,0}, \mathbb{F}_2) = k[\alpha, \beta, \delta, \xi] / (\beta\delta, \alpha\beta, \beta^2, \alpha^2 + \delta^4, \alpha\delta^3 + \delta^5).$$

As intermediate results we obtain information about the cohomology of $\Gamma^n_{0,0}$; the case of interest is $n=6$ because there is a short exact sequence

$$1 \to \mathbb{Z}/2 \to \Gamma^6_{0,0} \to \Gamma^0_{2,0} \to 1 .$$

Since the cohomology of $\Gamma^n_{0,0}$ is expressed in terms of Specht modules for Σ_n (the symmetric group), the diagrammatic methods are applied for modules for Σ_6.

Dave Benson, Oxford.

Sylow subgroups and isomorphic integral group rings

Report on joint work with R. Sandling

Thm. Let G be a finite group. Assume $\mathbb{Z}G \cong \mathbb{Z}G^*$.
Let $P \in Syl_p(G)$ and $P^* \in Syl_p(G^*)$.
Suppose that P is Hamiltonian. Then $P \cong P^*$.

The proof uses for $p > 2$ the classification of the finite simple groups. One crucial point of the proof is that the integral group ring of a finite group determines the chief series of the group. This result was obtained by R. Lyons and R. Sandling and independently by me. The result for the abelian Sylow subgroups should be seen in connection with R. Brauers question, whether the character table of a finite group determines the property of having abelian Sylow subgroups.

W. Kimmerle, Stuttgart

Intersections of maximal subgroups in simple groups of order less than 10^6 and associated amalgams

Report on joint work with S.V. Tsaranov

Let G be a finite group and H and K be subgroups of G.

Problem 1 Describe $H^g \cap K$ for all $g \in G$.

Result 1 List of intersections of all pairs of maximal subgroups in simple groups of order less than 10^6 excluding $PSL_2(q)$

The natural generalization of the Problem 1 is

Problem 2 Let $G_1, \ldots, G_n$, $n \geq 3$ be subgroups of a group G. Describe all residually connected amalgams $(G_1^{g_1}, \ldots, G_n^{g_n})$ for all g_i from G, such that G is generated by G_i, $1 \leq i \leq n$.

Result 2 List of all residually connected amalgams for

(1) $G \cong J_1$ and

(2) $G \cong U_4(2)$ with additional assumptions that all G_i are maximal subgroups of G and $\bigcap_1^n G_i$ is nontrivial.

E. Komissartschik, Moscow

Non rank 3 graph with 5-vertex condition.

Ordinary graph with t-vertex condition is defined. The graph with t-vertex condition is also the graph with t'-vertex condition for every $2 \leq t' \leq t$. The examples of such graphs are regular graphs ($t=2$), strongly regular graphs ($t=3$), rank 3 graphs ($\forall t$).

The graph G with 5-vertex condition

is constructed. Its parameters are $(v, k, \lambda, \mu, \alpha, \beta) = (256, 120, 56, 56, 784, 672)$ and $|Aut(G)| = 2^{20} \cdot 3^2 \cdot 5 \cdot 7$. Two subgraphs G_1 and G_2 of graph G are the graphs with 4-vertex condition. They parameters are $(120, 56, 28, 24, 216, 144)$, $(135, 64, 28, 32, 168, 192)$ respectively and $|Aut(G_1)| = |Aut(G_2)| = 2^{12} \cdot 3^2 \cdot 5 \cdot 7$. All these graphs are not rank 3 graphs.

Up to now all examples of non rank 3 graphs with t-vertex condition were known for $t \le 3$ only.

A. V. Ivanov, Moscow.

Table Algebras.

This is a report of joint work with Z. Arad. [For definitions, see his abstract on p. 95 of this book]. <u>Simple</u>, resp. <u>abelian</u>, table algebras are defined, to generalize the character ring and class algebra of a simple, resp. abelian, finite group. The <u>covering number</u>, $cn(A)$, is defined as the least positive integer m so that for every $a \ne 1 \in \mathfrak{A}$, $\mathfrak{A}$ = the support in $\mathfrak{A}$ of a^m.

<u>Prop.</u> $cn(A)$ exists $\Leftrightarrow$ $(A, \mathfrak{A})$ is simple.

<u>Thm.</u> Let $(A, \mathfrak{A})$ be nonabelian and simple. Then $cn(A) \le \frac{1}{2}(k^2 - (\lambda - 1))$, where $k = |\mathfrak{A}|$ and $\lambda = |\{a \in \mathfrak{A} \mid \bar{a} = a\}|$.

The structure of $(A, \mathfrak{A})$ which follows from assuming that there exist $b \ne c$ in $\mathfrak{A}$ with $bc = \lambda b + \gamma c$ or $\lambda \bar{b} + \gamma c$ (for $\lambda, \gamma > 0$) is given. Results of A. Mann on characters and classes are derived as corollaries.

Arad and Fisman have shown that if $(A, \mathfrak{A})$ is nonabelian and simple, then $\mathfrak{A} = (\prod_{a \in \mathfrak{A}} a) \cup \{1\}$. For classes, there is a better result due to

Brauer and Wielandt, but for characters the result seems new.

H. Blau (DeKalb, IL)

Exponents of Modules and Maps

Let G be a finite group and let R be a PID of characteristic zero. Let L and M be RG-lattices. If $\alpha : L \to M$, then $\exp(\alpha)$ is a generator for the ideal of all $r \in R$ such that $r\alpha$ factors through a projective. Also $\exp(M) = \exp(Id_M)$. Suppose that $\zeta_1, \dots, \zeta_r$ are homogeneous elements in $H^*(G,R)$ such that the ideal that is generated by their reductions mod P has the same radical as the annihilator of the cohomology of M/PM for every prime ideal $P \subseteq R$. Such a set can be found with $r = C_G(M)$. Then $\prod_{i=1}^{r} \exp(\zeta_i)$ is divisible by $\exp(M)$. In case $M = R$ then $\prod \exp(\zeta_i)$ is divisible by $|G|$ in R.

Suppose that R is a complete d.v.r. with prime element π. If $\exp(M) = \pi^a$ then we say M has property E provided $\pi^{a-1} \widehat{Ext}^0_{RG}(M,M) = \operatorname{Soc} \widehat{Ext}^0_{RG}(M,M)$. Joint work with A. Jones shows that property E is preserved by the Green correspondence for absolutely indecomposable modules. The work suggest that the height-0 conjecture may be provable by purely local methods.

Jon F. Carlson, Athens, Ga.

Diskrete Geometrie

7.6.87 — 13.6.87

Ein Kreisüberdeckungsproblem auf der Sphäre.

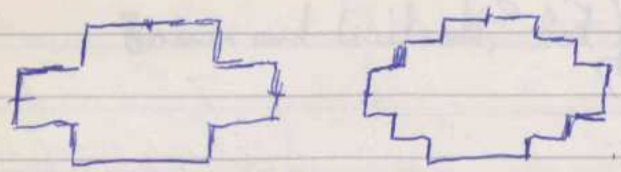
Untersucht wird, welcher Teil der Sphäre S^2 von n kongruenten Kreisen $\{K_1, \ldots, K_n\}$ einfach bedeckt werden kann; dabei sind bei gegebenem $n \in \mathbb{N}$ die Mittelpunkte der Kreise und ~~die~~ der Kreisradius variabel. Vorausgesetzt wird, daß kein Kreismittelpunkt im Innern eines anderen Kreises liegt.

Gezeigt wird: Ist M die von $\{K_1, \ldots, K_n\}$ einfach bedeckte Teilmenge der S^2, so gilt für ihren Flächeninhalt $|M| \leqq 12(n-2)\, F\left(\frac{\pi}{6}\frac{n}{n-2}\right)$ mit $F(\alpha) := \alpha - \pi + 2 \arccos \frac{1}{4\cos^{\alpha}/_2}$. Diese Schranke ist für $n \to \infty$ und für $n = 3, 4, 6, 12$ scharf; in den Fällen $n = 4, 6, 12$ liegen die Kreismittelpunkte in den Ecken eines regulären Tetraeders bzw. Oktaeders bzw. Ikosaeders, und jeder Kreis wird von den 3 bzw. 4 bzw. 5 benachbarten Kreisen in den Ecken eines regulären 6-Ecks bzw. 8-Ecks bzw. 10-Ecks geschnitten.

Gerd Blind, Stuttgart

Polymorphic Prototiles

A tile/prototile is a subset of the Euclidean plane E^2 that is homeomorphic to the closed unit disk. A tiling of the plane is a cover $\mathcal{T} = \{T_1, T_2, \ldots\}$ by tiles such that the T_i have pairwise disjoint interiors. A tile T admits a tiling if there is a tiling $\mathcal{T} = \{T_i\}$ such that all the T_i are congruent to (the prototile) T. If a prototile admits precisely k distinct (i.e. not congruent) tilings then it is called k-morphic (GRÜNBAUM-SHEPHARD [1977]; Tilings and Patterns 1986). k-morphic protiles are known for $k = 2, 3, \ldots, 10$, including many examples of dimorphic ($k=2$) prototiles. How can polymorphic prototiles be classified? Using the k-tuple of types of the corresponding tilings is not sufficient. Using (combined) incidence symbols does not satisfy, either. For instance, the two dimorphic prototiles ~~prototiles~~ shown below are not equivalent in this sense



The following definition is proposed: Two k-morphic prototiles T and T' are <u>topologically</u> equivalent iff the following is true:

(a) the tilings corresponding to T, namely $\mathcal{T}_i = \{T_{i1}, T_{i2}, \ldots\}$ $i = 1, \ldots k$ and T', namely $\mathcal{T}_i' = \{T_{i1}', T_{i2}', \ldots\}$, $i = 1, \ldots, k$, are topologically equivalent i.e. there are homeomorphisms $h_i : E^2 \to E^2$ mapping each T_{ik} on some T_{ik}'.

(b) there is a homeomorphism $h: T \to T'$

such that (for all i,k) there exist homeomorphisms φ, ψ such that the following diagram is commutative

$$\begin{array}{ccc} T_{ik} & \xrightarrow{h_i} & T_{ik}' \\ \varphi \downarrow & & \downarrow \psi \\ T & \xrightarrow[h]{} & T' \end{array}$$

Using halved Z-formed tiles as examples, it is shown how dimorphic prototiles can be generated and distinguished.

Peter Schmitt, Wien

<u>Exponential Sums + Volumes of Sum-sets.</u>

The classical inequalities (asymptotic formulae) of Vinogradov, Mordell + Tietäväinen are dependent upon upperbounds for

$$V(S+C) - V(S),$$

where V denotes the Jordan content of a convex set in $\mathbb{R}^N$ and $S+C$ denote the vector sum of S and C.

J H^2 Chalk

<u>Windschiefe Archimedische Polyeder höheren Geschlechtes</u>

Die elementare Theorie der klassischen Archimedischen Polyeder scheint nicht im Sinne der Topologisch-kombinatorischen Geometrie bis jetzt erweitert zu sein. Es wird versucht diese Theorie in dem Sinne zu verallgemeinern mit dem Hauptziel die hinreichende Existenzbedingung für das Polyeder $\{C;p\}$ zu finden. Um diese Idee zu realisieren wäre aber notwendig über etwas reicheres Anschauungsmaterial von mehreren verallgemeinerten Archimedischen Polyedern zu verfügen. Bis jetzt waren zu diesem Zweck schon einige besondere Polyederfamilien aus der Menge aller verallgemeinerten Archimedischen Polyeder untersucht. Jetzt wird die Familie der windschiefen Polyeder $\{(3,3,3,3,n); \{3\}\}$; $n = 4, 5, 6, \dots$ betrachtet, welche für $n = 4$ und $n = 5$ die zwei bekannten elementaren windschiefen Archimedischen Polyeder enthält.

Stanko Bilinski, Zagreb

Flexible Uniform Polytopes

The regular icosahedron $\{3,5\}$ is quite rigid in $\mathbb{R}^3$, but in $\mathbb{R}^4$ looses its rigidity and can easily be folded into the (starry) great icosahedron $\{3, 5/2\}$. In $\mathbb{R}^6$ it is even possible to perform this folding so that at each stage the (skew) icosahedron is regular. These and many other examples follow easily from a simple manipulation of Coxeter diagrams, which provides the orthogonal projections necessary to start the folding [c.f. "A Family of Uniform Polytopes with Symmetric Shadows", Geometriae Dedicata, 1987?].

Barry Monson, New Brunswick, KANADA.

NICE SETS IN EUCLIDEAN $\mathbb{R}^d$.

Measures of strength t in $V = \mathbb{R}^d$ generalize spherical t-designs and cubature formulae for the unit sphere S. In the case of finite support Y on p concentric spheres $M := \bigcup_{i=1}^{p} r_i S$, and $t = 2e$, the definition amounts to the isometry of the spaces $Pol_e(Y)$ and $Pol_e(M)$. This implies a lower bound for $|Y|$, as a consequence of

$$Pol_e(M) \cong \sum_{k=0}^{2p-1} Hom_{e-k}(V).$$

Joint work with A. Neumaier, and with P. Delsarte,

J.J. SEIDEL, EINDHOVEN.

Tightly packing a convex set with similar sets.

Let K be an d-dimensional convex body, $n \geq 2$ an integer, and $l > 0$. Let $K_n^{d,l} = \sup\{\sum_1^n d(K_i)^l / d(K)^l : K \supset \bigcup_1^n K_i,\ K_i^\circ \cap K_j^\circ = \emptyset,\ i \neq j,\ K_i \sim K\}$. Let $\alpha_n^{d,l} = \inf_K \{K_n^{d,l}\}$, $\beta_n^{d,l} = \sup_K \{K_n^{d,l}\}$. We wish to evaluate the α's and β's and analyze the cases of equality. In 1971, Beck and Bleicher, (Acta Math (Hung) 22) showed that $\alpha_2^{2,1} = 1$ and $\beta_2^{2,1} = \sqrt{2}$ with equality for α iff K is an isosceles right triangle or a parallelogram of side ratio $\sqrt{2}$ and equality for β iff K has constant width or is a regular k-gon. Here we show that, for the non-trivial case $0 < l < d$, $\beta_n^{d,l} = n^{\frac{d-l}{d}}$ with equality iff K is an n-rep-tile (n similarity tiling) and thus K is a polytope. For $d=2$, K has at most 5 edges and 5-edges is possible only for $n \geq 6$. It is conjectured that 5 edges never occurs. It is shown that $5/4 \leq \alpha_3^{2,1} \leq 6\sqrt{3} - 9$. It is conjectured that $\alpha_3^{2,1} = 6\sqrt{3} - 9$ with equality only for the circular disk.

Michael Bleicher

A lower bound on the number of sharp shadow boundaries of convex polytopes

Let P be a convex polytope in R^d $(d \geq 2)$ with n facets. We consider light sources x outside P, but on no facet hyperplane of P. By illumination from x some subset $I(x)$ of Bd P will be illuminated; its boundary (relative to Bd P) will be called the shadow boundary of P w.r.t. x. This is a $(d-2)$-complex $C(x)$ (a subcomplex of P). Denote $\sigma(P)$ the number of all such subcomplexes, as x varies arbitrarily (under the above restriction). We prove $\sigma(P) \geq 2^{d-2} \sum_{i=0}^{2} \binom{n-d+1}{i} - 1$, with equality if and only if P is a $(d-2)$-fold pyramid over a planar convex $(n-d+2)$-gon. If Q is an unbounded convex polytopal set with $n\ (\geq 3)$ facets we have for $\sigma(Q)$

defined in the same way $\delta(Q) \geq \sum_{i=0}^{2} \binom{n-1}{i} - 1$, while if the intersection of Q with the infinite hyperplane has dimension $t \,(\leq d-3)$ then $\delta(Q) \geq 2^{d-t-3} \sum_{i=0}^{2} \binom{n-d+t+2}{i} - 1$. The cases of equality are characterised.

E. Makai, Jr., Budapest - H. Martini, Dresden

Tiling $\mathbb{R}^3$ with circles and disks

A collection of circles or of disks gives a tiling of $\mathbb{R}^3$ if each point of $\mathbb{R}^3$ belongs to one and only one of the sets in question. We review a number of constructions and results selected to give some idea of the constraints that these tilings can satisfy. For example, while it is not possible to tile the plane with homeomorphs of the closed unit disk, it is possible to tile 3-space with hexagonal polyhedral tiles of this sort.

J. B. Wilker Toronto

REALIZATIONS OF REGULAR POLYTOPES

The automorphism group Γ of a (finite) regular incidence polytope $\mathcal{P}$ is generated by involutions $\rho_0, \dots, \rho_{n-1}$, and the v vertices of $\mathcal{P}$ are identified with the right cosets of the subgroup $\Gamma^* = \langle \rho_1, \dots, \rho_{n-1} \rangle$. A realization of $\mathcal{P}$ corresponds to some orthogonal representation G of Γ of degree (dimension) d say, with ρ_j going to a reflexion R_j (or the identity), which is identified with its mirror of fixed points; its Wythoff space is $W = R_1 \cap \dots \cap R_{n-1}$, whose dimension is denoted by w. The family of all realizations (up to congruence) of $\mathcal{P}$ forms a closed convex cone, of dimension r say. If $\bar{w}$ is the dimension of the Wythoff space of the realization whose vertices are those of a regular $(v-1)$-simplex, then the following relations hold:

$$\sum_G w d = v - 1 = \operatorname{card}\{\Gamma^*\sigma \mid \sigma \in \Gamma \setminus \Gamma^*\},$$
$$\sum_G \tfrac{1}{2}w(w+1) = r = \operatorname{card}\{\Gamma^*\sigma\Gamma^* \cup \Gamma^*\sigma^{-1}\Gamma^* \mid \sigma \in \Gamma \setminus \Gamma^*\},$$
$$\sum_G w^2 = \bar{w} = \operatorname{card}\{\Gamma^*\sigma\Gamma^* \mid \sigma \in \Gamma \setminus \Gamma^*\},$$

where the sums extend over the irreducible representations G of Γ. The realization cone of $\mathscr{P}$ is polyhedral exactly when $w \leq 1$ for each such G, or $r = \bar{w}$, and $\sigma^{-1} \in \Gamma^*\sigma\Gamma^*$ for each $\sigma \in \Gamma$.

Peter McMullen, London.

Rigid Plate Frameworks

This lecture gave a partial answer to a problem posed by A. Ehrenfeucht and J. Mycielski in 1981 (American Math. Monthly, unsolved problem #6367).

A plate framework in the plane is a collection of plates pivotted together that satisfies the following conditions:

(i) Each plate is a regular n-gon, and all plates are mutually congruent.

(ii) The number of plates is finite.

(iii) No two plates coincide

(iv) Every vertex of a plate is a pivot.

(v) Every pivot is a vertex of exactly two plates.

(vi) No two pivots coincide.

The problem is to find plate frameworks which are rigid. (In answer to the original problem, a construction was given for a plate framework with $2n$ n-gons that was not rigid.)

The following was proved.

Theorem There exist infinitely many rigid plate frameworks using n-gons, where $n = 3$ or 4.

It is not known if there exist rigid plate frameworks with n-gons for $n \geq 5$.

Some generalisations to three-dimensional analogues of the problem were also mentioned briefly.

G. C. SHEPHARD (NORWICH).

Some infinite families of finite incidence-polytopes

A type of partially ordered structures called incidence-polytopes generalize the notion of polytopes in a combinatorial sense. We discuss the possibility of constructing n-dimensional incidence-polytopes $\{\mathcal{P}_1, \mathcal{P}_2\}$ with preassigned facets $\mathcal{P}_1$ and vertex-figures $\mathcal{P}_2$. In particular when the facets are taken to be isomorphic to the maps $\{2q,4\}_4$ and $\{2q,3\}_6$ (on surfaces of genus $\binom{q-1}{2}$ and $(q-1)^2$ respectively) and their duals we obtain several infinite families of finite incidence-polytopes.

Asia Ivić Weiss, Toronto

A problem in discrete geometry

Let (R,G) be a general Kleinian space, that means, $R \neq \emptyset$ is a set with a group G of transformations of R onto R. In case that R is a metric space any collection $\mathfrak{M}$ of subsets of R is said to be discrete if $\mathfrak{M}$ is locally finite, that is, such that every bounded set $B \subseteq R$ meets only a finite number of $\mathfrak{M}$-elements. The group G is said to be discrete if each orbit Gx $(x \in R)$ consists of isolated points. Discrete geometry is the theory of discrete (or finite) systems $\mathfrak{M}$ of geometric objects, the theory of discrete (or finite) groups G of transformations, the theory of discrete (or finite) spaces R respectively.

A nice example for discrete geometry in this sense is the connection between the vertex-invariant embedding of a regular d-simplex in the d-cube and the geometry of the finite space $E_d = \{0,1\}^d$ with Hamming metric $h(e_i,e_k) = \sum_{\nu=1}^{d} |e_i^\nu - e_k^\nu|$. The open question whether each regular d-simplex with $d+1 \equiv 0\ (4)$ can be vertex-invariant embedded in the d-cube is equivalent to the problem of existence of Hadamard matrices of order $d+1$.

E. Hertel, Jena

On finite packing and covering

The basic problems of finite packing and covering in Euclidean d-space E^d are: Determine for a given $k \in \mathbb{N}$

A) the minimal volume of all convex bodies, into which k unit-balls can be packed,

B) " maximal " " " ", which can be covered by k unit-balls.

It turns out that the expected answer for $d \geq 5$ is completely different from that for $d \leq 4$.

For $d \geq 5$ it is conjectured (L. Fejes Toth, J. M. Wills) that linear arrangements of balls ("sausages") are best possible.

For $d \leq 4$ there is no such elegant statement. But the socalled sausage-catastrophes (1983) and the space-conjecture describe the expected behaviour in a satisfactoring way.

In the talk we give some new results, which support the above conjectures.

Jörg M. Wills (Siegen)

Inequalities involving points, lines and Planes

(1) Given n points in R^3 forming l lines and p points, we discuss the evidence for our conjecture that $n - l + p \geq 0$ if the points do not all lie on two skew lines or one plane. We can prove this in C^3 (and therefore R^3) if no 3 points are collinear and $n \geq n_0$ using an inequality due to F. HIRZEBRUCH.

(2) Given n points in R^3 and t arbitrary planes, we can show that the number of incidences I satifies

$$I \leq c\,(\log n)^{1/4}\, n^{3/4}\, t^{3/4}$$

if no three points are collinear.

We suspect however that $I \le c n^{3/4} t^{3/4}$, and ~~but~~ don't even know if this is sharp.

George B. Purdy (Cincinnati)

On the convex hulls of convex sets.

Let F be a family of ovals (compact, convex sets) in the plane. Three ovals are collinear if one is in the convex hull of the other two. Let $D = \mathrm{conv}(\bigcup_{A \in F} A)$. Then $X \in F$ is a vertex of D if $X \not\subset \mathrm{conv}(\bigcup_{A \in F \setminus \{X\}} A)$, and D is an n-gon if it has exactly n vertices.

Theorem: Let F be a family of mutually disjoint ovals, no three collinear and $n \ge 4$. Then there is a number $g(n)$ such that if $|F| > g(n)$, then F contains the vertices of an n-gon.

In this joint work with G. Fejes Tóth, we not only obtain the above generalization of the well know Erdős-Szekeres Problem but also that $g(4) = 4$ and $g(5) = 8$. Thus if $f(n)$ is the number where the ovals are points, then $g(n) \ge f(n) \ge 2^{n-2}$ with equality for $n=4$ and 5. We conjecture that $g(n) = f(n)$ for all $n \ge 4$.

T. Bisztriczky, Calgary.

Geometric Aspects of a New Linear Programming Algorithm

To solve LP: $\min c^T x$, $Bx = b$, $x \geq 0$ we consider the affine plane $E(t) = \{x \mid Bx = b, c^T x = t\}$ and the positive orthant $S = \{x \mid x \geq 0\}$. We define $d(t) = d(E(t), S)$ where $d(\,,\,)$ denotes euclidean distance. Apparently the smallest zero t_0 of $d(t)$ gives the solution of LP. Here we show that the exact t_0 can be computed in finitely many steps by Newton's algorithm and obtain thus an algorithm for LP.

Ulrich Betke, Siegen

On a metric for the class of convex sets

Let $\mathcal{K}_n$ be the class of all non-empty compact convex subsets of E^n. Define a distance function on $\mathcal{K}_2$ by setting

$$\varrho(K_1, K_2) = 2p(\mathrm{conv}(K_1 \cup K_2)) - p(K_1) - p(K_2),$$

where $K_1, K_2 \in \mathcal{K}_2$ and $p(M)$ denotes the perimeter of the set M. This definition can be extended to $\mathcal{K}_n$ by

$$\varrho^W(K_1, K_2) = 2W(\mathrm{conv}(K_1 \cup K_2)) - W(K_1) - W(K_2), \qquad (1)$$

where

$$W(K) = \frac{1}{|S^{n-1}|} \int_{S^{n-1}} w(K, u)\, d\sigma$$

is the average width of K. It is shown that (1) is in fact a metric on $\mathcal{K}_n$. For $n=2$, $\pi\varrho^W$ may be considered a modified perimeter deviation. Some properties of this metric are developed. E.g., $(\mathcal{K}_n, \varrho^W)$ is a complete and locally compact space, and a metric segment space as defined by K. Menger. In the terminology used by Shephard and Webster, ϱ^H and the Hausdorff metric are equivalent metrics on $\mathcal{K}_n$. Let C be a given plane convex disc, and let p_k be the minimal perimeter deviation (in the modified sense) from any convex k-gon. It is conjectured that p_k satisfies the Dowker-type inequality

$$p_{k-1} + p_{k+1} \geqq 2p_k, \quad \text{for } k = 4, 5, \ldots$$

A. Florian (Salzburg)

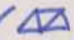
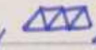
Packings with regular n-gons

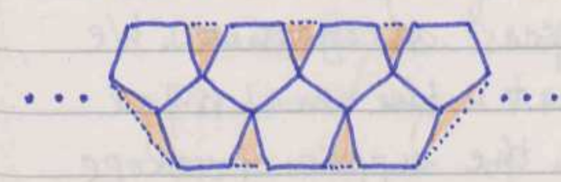
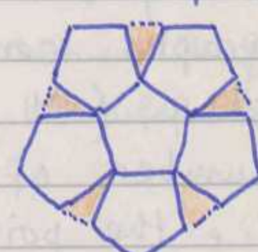
Packings of K regular n-gons in the plane are discussed, where every n-gon has an edge in common with one of the other n-gons, and the packing is connected by edges.

(1) The numbers of different shapes (polyominoes) are hopeless to determine.

(2) The strict Newtonian number $\bar{N}_n$ (maximum number of n-gons, which have an edge in common with a fixed n-gon): $\bar{N}_3 = \bar{N}_7 = 3$, $\bar{N}_4 = \bar{N}_8 = \bar{N}_9 = \bar{N}_{13} = \bar{N}_{14} = \bar{N}_{19} = 4$, $\bar{N}_n = 6$ for $n \equiv 0 \pmod 6$, $\bar{N}_n = 5$ otherwise.

(3) Maximum number $B_n(K)$ of common edges: $B_3(K) = 2K - \{\frac{1}{2}(K + \sqrt{6K})\}$, $B_4(K) = 2K - \{2\sqrt{K}\}$, $B_6(K) = 3K - \{\sqrt{12K-3}\} = B_n(K)$ for $n \equiv 0 \pmod 6$.
Conjecture: $B_n(K) = B_4(K)$ for $n \equiv 4, 8 \pmod{12}$, $B_n(K) = B_3(K)$ otherwise.

(4) Minimum area of the convex hull: For triangles only sausages (in linear sequence) are possible only for $K=3$ and $K=5$ (, ). For squares only sausages are possible only for K a prime number. For $K=3$ sausages exist only for $n \equiv 1 \pmod 2$ if $K = 7, 13, 19$, for $n \equiv 4 \pmod 6$ if $K = 4, 10, 16$, for $n \equiv 2 \pmod 6$ if $K = 8, 14, 20, 26, 32, 38$. — For 5-gons the minimum area is conjectured to occur as follows:

 with the only exception: 

Heiko Harborth (Braunschweig)

Packing problems

Let P be a packing of the space R^n with unit balls. Let $\{O_i\}$ and $\{DV_i\}$ be systems of the centres of balls and of Dirichlet-Voronoi cells, respectively. The centre of a supporting sphere is denoted by C^n.

A k-dimensional sphere $(1 \leq k \leq n-1)$ is called a k-supporting sphere if it lies on the boundary of a supporting sphere and contains k+1 points of $\{O_i\}$ ~~which~~ do not lie in a (k-2)-dimensional subspace. C^k denotes the centre of ~~a~~ k-~~dimensional~~ supporting sphere.
C^k affiliated to DV_j if O_j lies on the previous k-dimensional supporting sphere ($C^k \in DV_j$).
The k-dimensional closeness of the point-system $\{O_i\}$ is defined by

$$\sup_i \max_{C^k \in DV_i} O_i C^k = \varrho(n,k).$$

Theorem: The 2-dimensional ~~closes~~ closeness of a packing of unit balls in 3-dimensional Euclidean space is at least $\sqrt{\frac{3}{2}}$, and equality holds only for the space-centred cubic lattice, where the edge-length of the cubic is $\frac{4}{\sqrt{3}}$.

Károly Böröczky (Budapest)

Regions enclosed by plates

Let $f_1, \ldots, f_m$ be (partially defined) piece-wise linear functions of d variables whose graphs consist of n d-simplices altogether. We show (M. Sharir and me) that the maximal number of d-faces comprising the upper envelope (i.e. the pointwise maximum) of these functions is $O(n^d \alpha(n))$, where $\alpha(n)$ denotes the inverse of the

Ackermann function, and that this bound is tight in the worst case. If, instead of the upper envelope, we consider any single connected component C enclosed by n d-simplices in $\mathbb{R}^{d+1}$, then we show that the overall combinatorial complexity of the boundary of C is at most $O(n^{d+1-\alpha(d+1)})$ for some fixed constant $\alpha(d+1) > 0$.

Pach János.

Hamiltonian lattice graphs

This joint work with Cristina Zamfirescu describes sufficient conditions for a grid graph to be hamiltonian. A grid graph is a subgraph of the usual planar infinite lattice graph, which (i) is connected, (ii) has connected complement, (iii) has connected intersection with any vertical or horizontal infinite path. For one of the three main types of grid graphs our conditions are close to a characterization.

Tudor Zamfirescu

Zerlegungstheoretische Analoga zu Schläflischen Volumenformeln für Orthoscheme

Several geometrical dissections of regular simplexes and crosspolytopes in spherical, euclidean and hyperbolic n-space are considered. They exhibit in a purely combinatorial way volume relations previously deduced by L. Schläfli (Theorie der vielfachen Kontinuität, §29 + §31) and by H.S.M. Coxeter (On the Schläfli and Lobatschefski functions, 1935) with use of differential formulas.

Hans E. Debrunner

Hadwiger's Transversal Theorem in higher dimensions

Hadwiger's transversal theorem states that if n disjoint compact convex sets $B_1, \ldots, B_n$, in the plane, have the property that every 3 can be met by a directed line consistent with the order $1, \ldots, n$ then there is a line which meets all of them. We (J.E. Goodman + myself) prove that if n "separated" compact convex sets $B_1, \ldots, B_n \subset \mathbb{R}^d$ and a labelled configuration of points $C = \{P_1, \ldots, P_n\} \subset \mathbb{R}^{d-1}$ have the property that any $d+1$ of these sets can be met by an oriented hyperplane consistent with the "order type" of C, then there is a hyperplane which meets all of them. "Separated" means that no d of the sets are met by a $d-2$ flat and two configurations $\{P_1, \ldots, P_n\}$ and $\{q_1, \ldots, q_n\}$ have the same "order type" in $\mathbb{R}^{d-1}$ if every corresponding d of them $\langle P_{i_1}, \ldots, P_{i_d}\rangle$ $\langle q_{i_1}, \ldots, q_{i_d}\rangle$ have the same orientation. These are the natural generalizations of "disjointness" in the plane and "order" on the line. The key element in the proof is an exchange lemma for minimal Radon partitions.

Ricky Pollack (New York)

Convex bodies, coverings by caps, random polytopes

Motivated by a result of V. I. Arnold David Larman and I proved that for any convex compact body $K \subset \mathbb{R}^d$ $(d \geq 2)$ with $\operatorname{int} K \neq \emptyset$ and any $0 < \varepsilon < \varepsilon_0(d)$ $(= d^{-3d}$ say$)$ there are caps $C_1, \ldots, C_m$ with $m \leq \text{const} \cdot \varepsilon^{-\frac{d-1}{d+1}}$ such that these caps cover the boundary of K and $\varepsilon \leq \operatorname{vol} C_i \leq (18d^2)^d \varepsilon$ for $i = 1, \ldots, m$. Here "cap" means the intersection of a halfspace with K. We use then this result to show that for any convex polytope $P \subset \mathbb{R}^d$ with n vertices and $\operatorname{int} P \neq \emptyset$ there are $d+1$ vertices $v_0, \dots, v_d$, of P such that

$$\frac{\operatorname{vol}(\operatorname{conv}\{v_0, \dots, v_d\})}{\operatorname{vol} P} \leq \text{const.}\ n^{-\frac{d+1}{d-1}}.$$

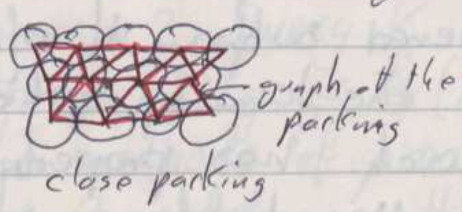
The cap-cover theorem can further be used to prove the following: Let again $K \subset \mathbb{R}^d$ $(d \geq 2)$ be a convex compact body with nonempty interior and chose n points randomly and independently from K according to the uniform distribution. Denote the convex hull of these points by K_n. Define further K_ε as the set of points of K that are contained in no cap of volume ε. Then there are constants c_1 and c_2 depending on K only such that with $\varepsilon = \frac{1}{n}$

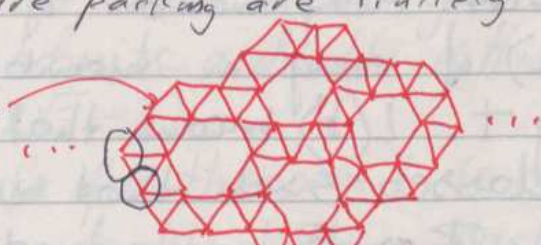
$$c_1 \operatorname{vol}(K - K_\varepsilon) \leq \operatorname{Exp} \operatorname{vol}(K - K_n) \leq c_2 \operatorname{vol}(K - K_\varepsilon)$$

Bárány Imre (London, Budapest)

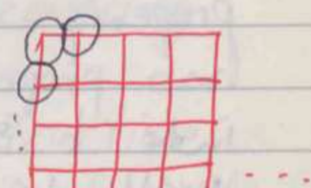
Uniformly Stable Circle Packings 6/9/87

A packing is finitely stable if every finite number of packing elements is held fixed by the rest. For example the close packing of equal circles, the close packing with holes and the square packing are finitely stable.



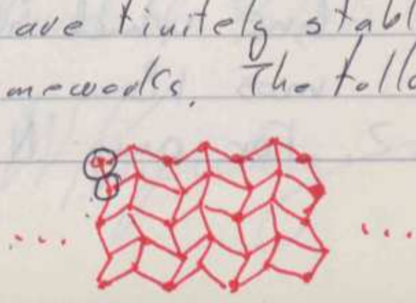


close packing with holes



square packing

The proofs ~~of these~~ that these are finitely stable follows from the theory of tensegrity frameworks. The following zigzag packing is not finitely stable.



Related to the above ideas is the following definition. A packing is ϵ-uniformly finitely stable if there is no other finite rearrangement of the members moving each member less than $\epsilon > 0$.

Theorem (Bárány, Dolbilin): The close packing with holes removed is uniformly finitely stable.

This brings up the question of whether every finitely stable packing is uniformly finitely stable. This is false, even for the square packing.

Example (with A. Bezdek): The square packing is not ~~b~~ uniformly finitely stable.

Take a large square region of the square packing, and replace it with the corresponding portion of the zigzag packing with the angles chosen just large enough to fit it in the same region.

R. Connelly

On common transversals

A finite family A of convex sets in the plane is said to have property T if the family admits a common transversal, that is, if there is a straight line which intersects every member of A. The family A has property T(m) if every m-membered subfamily of A has property T. L(k) means that the family splits into k subclasses such that each has property T.

With Károly Bezdek we proved the followings

Thm 1: If a finite family of congruent, disjoint circles satisfies T(3), then there exists a line which intersects every circles except at most 18.

Thm 2. For any $N \geq 6$ there is an arrangement of N congruent, disjoint circles having $T(3)$ but not $T(N-1)$.

Thm 3. In case of a finite family of homothetic convex sets $T(3)$ implies $L(4)$ and $T(4)$ implies $L(3)$.

Thm 4. If a finite family of translates of a convex sets has $T(3)$ then it has $L(2)$ too

András Bezdek

Double-lattice packings and coverings of the plane

Let K be a compact convex plane set with an interior point. A packing $\mathcal{P} = \{K_i\}$ with copies of K is a double-lattice packing, if $\mathcal{P} = \mathcal{P}_0 \cup \mathcal{P}_1$, where $\mathcal{P}_0$ consists of translates of K and $\mathcal{P}_1$ consists of translates of $-K$, and each $\mathcal{P}_0$ and $\mathcal{P}_1$ is a lattice ~~packing~~ arrangement with the same lattice of translations. Double-lattice coverings are defined analogously.

Theorem 1 (joint with Greg Kuperberg). Every K admits a double-lattice packing with density $\geq \sqrt{3}/2$.

This theorem generalizes a result of K. Mahler and L. Fejes Tóth about lattice packings of centrally symmetric convex domains with the same density bound.

Theorem 2. Every K admits a double-lattice covering with density $\leq \frac{8}{3}(2\sqrt{3}-3)$.

The method of proof for theorem 1 is constructive and it produces the double-lattice packing of greatest density. The bound of $\sqrt{3}/2$ however, is probably not the best possible, although it appears to be very close to it. The same can be said about the bound of $8/3(2\sqrt{3}-3)$ for the covering density.

W Kuperberg

Packing with rounded figures

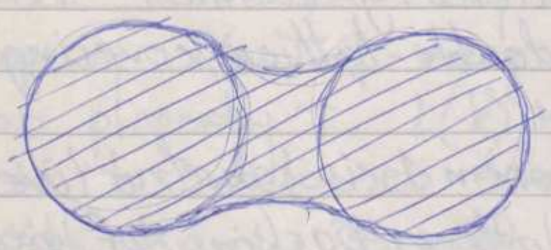
Definition of arc polygons on the sphere and in the Euclidean or hyperbolic plane:

* simply connected interior
* boundary consists of a sequence of finitely many circular arcs of radius r (inward or outward)
* the inner angles at vertices are $\geq \pi$ (concave), $= 0$ (smooth), $\leq \pi$ (convex).

Theorem. If k concave arc polygons (of radius r) are packed in a convex arc polygon, then the area of the uncovered part is $\geq (k-1) \cdot 2 \cdot \Delta_r$, where Δ_r is the area enclosed by 3 mutually tangent circles of radius r.

The theorem is sharp for all cases, when the above domains are smooth arc polygons, and the uncovered part consists of arc triangles.

Example: The two halfs of a tennis ball, the densest packing of the sphere with 12 circles (here the complement of one of the circles plays the role of the outer polygon), or the lattice packing of the figure below.



Aladár Heppes

Packing with translates of a special domain

Let S be an open connected bounded convex set in the Euclidean plane. $d(S)$ is the density of the densest packing of translates of S. $\bar{d}(S)$ is the density of the densest lattice-packing of translates of S. The domain S is called Rogers domain (R-domain) if $d(S) = \bar{d}(S)$

Theorem: If S is a connected union of two translates of a convex domain, then S is an R-domain

Gábor Fejes Tóth

Covering curves by translates of a convex set

In this joint paper with R. Connelly we investigated among others the following problem: What conditions will insure that one convex set can be translated into the other one? For instance Wetzel showed that for a given acute triangle, if a closed curve has length equal to or less than the pedal triangle, then the closed curve can be translated into the given acute triangle. Another example is when the covering set is a circular disk of diameter $1/2$. Then any closed (planar) curve of length one or less can be translated into the disk. With the help of the technique of the so called billiard triangles we proved the following.

Theorem Let X be any compact convex set of constant width $1/2$ in the plane. Then any closed curve of length one or less in the plane can be covered by a translate of X. Furthermore, if Y is any compact convex set such that any closed curve of length one or less can be covered by a translate of Y, then the length of the perimeter of Y is equal to or larger than $\frac{\pi}{2}$ with equality if and only if Y has constant width $1/2$.

We looked at other related problems and generalizations regarding translation covers, and mentioned several more results that we could obtain with our technique.

Károly Bezdek

Densest packing of translates of a domain

Let $d(s)$ be the density of the densest packing of translates of a domain s. Let $\bar{d}(s)$ be the density of the densest lattice-packing of s. It is known that if s is convex then $d(s) = \bar{d}(s)$. Some initial result have been presented about domains which share this property with convex domains.

László Dijustin

Packing and covering r-convex domains wit unit circles

Let $h(x)$ be the area of the intersection of a circle of unit area and a regular hexagon of area x concentric with the circle. It is shown that there is a function $r(d)$ (which can be given explicitly by a rather complicated formula and which satisfies $4 < r(d) < 17$) with the following property:

If R is the complement of the union of a set of circles of radius $r(d)$ and S is a system of unit circles such that the density of S with respect to R equals d and each component of R is met by at least two elements of S, then the part of R which is covered by the circles of S has density with respect to R

not exceeding $d\,f(1/d)$.

This implies several known results about packing and covering.

Gábor Fejes Tóth

Cyclic Homology

15.6.87 — 19.6.87

„Zyklische Kohomologie u. ihre Anwendungen" 14.-20.6.87

Introduction to K-homology and cyclic cohomology (A. CONNES)

The homology theory dual to ordinary K-cohomology is best described (by work of Atiyah - Brown Douglas Fillmore - Kasparov) by means of homotopy classes of K-cycles, which are called Fredholm modules. Given the algebra A (it is $C(X)$ for a compact space X) a Fredholm module over A is a $\mathbb{Z}/2$ graded Hilbert space $\mathfrak{h} = \mathfrak{h}^+ \oplus \mathfrak{h}^-$ which is a left A-module, together with an operator F in $\mathfrak{h}$ such that $F^2 = 1$, $F\varepsilon = -\varepsilon F$ where ε is the $\mathbb{Z}/2$ grading $\varepsilon = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and that every commutator $[F, a]$, $a \in A$, is a compact operator. We explain how first to introduce a dimension of a Fredholm module by the condition $[F, a] \in \mathcal{L}^p$ where $\mathcal{L}^p$ is the Schatten class $\{T \in \mathcal{L}^p \Leftrightarrow \sum \mu_n(T)^p < \infty\}$ where μ_n is the n^{th} characteristic value of T. Then we quantize the ordinary calculus of forms on a manifold by the following formulae:

$$df = i[F, f]$$

$$\Omega^k = \{\sum a^0 da^1 \ldots da^k\} \qquad (\subset \mathcal{L}^{p/k})$$

$$\int \omega = \mathrm{Trace}(\varepsilon \omega)$$

Given these laws one gets a differential graded algebra $\Omega = \oplus \Omega^k$ with $\Omega^0 = A$ and differential d, $d^2 = 0$, together with a closed graded trace $\int$ on Ω^n, n even large enough. We then explain how the character:

$$\tau(a^0, a^1, \ldots, a^n) = \int a^0 da^1 \ldots da^n = \mathrm{Trace}(\varepsilon a^0 i[F, a^1] \ldots i[F, a^n])$$

is a cocycle as follows:

1) $\tau^\lambda = \varepsilon(\lambda)\tau$ where λ is cyclic permutation
2) $b\tau = 0$ where b is the Hochschild cohomology

It is these properties which give the definition of cyclic cohomology (see this book for Oberwolfach meeting Sept. 1981), and the possibility of replacing n by $n+2$ which yield the operator S:

$$S: H^n_\lambda(A) \to H^{n+2}_\lambda(A)$$

We then explain how the long exact sequence with I, B, S comes out of the above considerations. We end up by discussing the meaning of the above construction when $A = C_0(\mathbb{R}^3)$ and F is the phase of the Dirac Hamiltonian of mass m, and relate it to the Dirac electron theory.

Cyclic homology of commutative algebras. Micheline VIGUÉ (C.N.R.S. France)

For any chain commutative differential graded algebra (A, d) over a characteristic zero field k, we give an explicit formula that permits us to compute the cyclic homology $HC_*(A,d)$ from the construction of a free model of (A,d). Explicit calculations are done for the ring of coordinates of some hypersurface. For example, if A is the ring of a projective hypersurface $(P=0)$ in $\mathbb{C}P^{r-1}$ with only zero an isolated singularity, then we have, for $n \geq r$, $HC_n(A) = HC_n(\mathbb{C})$ if $n \equiv r\ (2)$, and $HC_n(A) = HC_n(\mathbb{C}) \oplus \mathbb{C}^{\mu}$ if $n \not\equiv r\ (2)$ with $\mu = \dim_{\mathbb{C}} \mathbb{C}[x_1, \dots, x_r] / \left(\frac{\partial P}{\partial x_1}, \dots, \frac{\partial P}{\partial x_r}\right)$. If A is the ring of an irreducible affine plane curve defined in $k[x_1, x_2]$ by an equation $P = x_1^p - \lambda x_2^q = 0$, then we have $HC_{2n}(A) = k$, $HC_{2n+1}(A) = k[x_1, x_2] / \left(\partial P/\partial x_1, \frac{\partial P}{\partial x_2}\right)$.

Moreover, the formula described above, gives a natural decomposition theorem for $HC_*(A,d)$. We have $HC_*(A,d) = HC_*(k) \oplus \bigoplus_{p \geq 1} HC_*(A,d)^p$ with $HC_*(A,d)^1 = H_*(A,d)/k$. The map S sends $HC_*(A,d)^p$ into $HC_{*-2}(A,d)^{(p-1)}$. Finally, for a commutative algebra A, we prove that $HC_*(A)^2 = T_{*-1}(A|k)$ for $* > 2$, where $T_*(A|k)$ is the André-Quillen homology of the inclusion $k \hookrightarrow A$.

Positivity in cyclic cohomology (A. CONNES and J. CUNTZ)

In functional analysis one of the most important tools is positivity, for instance given an involutive algebra $\mathcal{A}$ over $\mathbb{C}$ a positive linear form φ over $\mathcal{A}$ is such that $\varphi(a^*a) \geq 0 \quad \forall a \in \mathcal{A}$ and it readily defines a Hilbert space with inner product $\langle x, y \rangle = \varphi(y^* x)$. When one develops cyclic cohomology over $\mathbb{C}$ instead of an arbitrary field the cochains are $n+1$ linear forms and one has the following notion of positivity:

$\tau \geq 0 \iff$ the following inner product on $\overset{m}{\underset{0}{\otimes}} \mathcal{A}$ is positive:

$\langle a^0 \otimes a^1 \otimes \dots \otimes a^m, b^0 \otimes b^1 \otimes \dots \otimes b^m \rangle = \tau(b^{0*} a^0, a^1, \dots, a^m, b^{m*}, \dots, b^{1*})$

Here $n = 2m$ is even.

Recall that the algebra $q\mathcal{A}$ of J. CUNTZ is constructed (as the universal differential graded algebra) as $\sum a^0 q a^1 \dots q a^n$ with the rule:

$$q(ab) = (qa)\, b + a (qb) + (qa)(qb)$$

Moreover if $\mathcal{A}$ is a $*$ algebra, then so is $q\mathcal{A}$ with $(qa^*) = (qa)^* \ \forall a$.

Given a functional T on $q\mathcal{A}$ one defines the components:

$$T^{(m)}(a^0, \dots, a^m) = T(a^0 q a^1 \dots q a^m) \qquad \forall a^i \in \mathcal{A}.$$

Thm 1) A functional T on $q\mathcal{A}$ is a trace iff a) for m even one has $b T^{(m)} = 0$, $B_0 T^{(m)} = (B_0 T^{(m)})^\lambda$ (cyclic invariance) b) for m odd one has $b T^{(m)} = T^{(m+1)}$, $B_0 T^{(m)} = T^{(m-1)\prime}$ where $\varphi' = \varphi - \frac{1}{2} b B_0 \varphi$.

2) For any positive trace T on $q\mathcal{A}$ and any even $n = 2m$ the component $T^{(n)} = T^{(2m)}$ is a positive cocycle.

3) For any positive trace T on $q\mathcal{A}$ there exists a Fredholm module $\mathcal{E}$ relative to a semifinite von Neumann algebra N such that $T(qx^0 \dots qx^n) = \mathrm{Chern}_N(\mathcal{E})$

We then show that the Dirichlet integral is a basic positive 2-cocycle yielding the conformal structure of a Riemann surface, which allows to define what is a non commutative elliptic curve. We finally explain the work of J. Bellissard on the Quantum Hall effect as an application of the integrality of Chern classes of $C_{q,\theta}$ the basic non commutative elliptic curve.

KK-theory and cyclic cohomology. (J. Cuntz)

We associate with every algebra A an algebra qA as the kernel of the natural map $id * id$ from the free product $A * A$ to A. This algebra has the following properties:

1. qA is a classifying space for KK-theory.
2. The operation of associating qA to A is "dual" to the one of associating $M_2(A)$ to A.
3. qA consists of "K-theory differential forms" over A.

The first two points throw a completely new light on Kasparov's KK-theory. The third one gives the natural link between K-homology and cyclic cohomology.

Cohomology of current Lie algebras and applications to deformations

(Claude ROGER. Universität von Metz Lothringen)

We consider the Lie algebra of sections of the associated Lie algebra bundle to a principal bundle; following Faddeev, we shall call it the current algebra of type $\mathfrak{G}$, if $\mathfrak{G}$ is the classical Lie algebra corresponding to the bundle, and denote it by $\mathfrak{G}_P$. We compute the cohomology of $\mathfrak{G}_P$ with coefficients in the adjoint representation in low degrees. For $\mathfrak{G}$ simple, the results are as follows

$H^2(\mathfrak{G}_P, \mathfrak{G}_P) \equiv 0$ if $\mathfrak{G}$ is not $\mathfrak{Sl}(n)$ for $n \geq 3$

$H^2(\mathfrak{G}_P, \mathfrak{G}_P) = \Lambda_2(V)$ the space of ~~covariant~~ contra-variant antisymmetric tensors on V, the base manifold

an explicit formula can be given for cocycles, using the symmetric bracket

$\mathfrak{G} \times \mathfrak{G} \xrightarrow{I} \mathfrak{G}$ defined by $I(A,B) = AB + BA - \frac{2}{n}\, \mathrm{tr} AB\, I_n$. as an algebraic invariant. We deduce from that computation that $\mathfrak{G}_P$ admits a lot of infinitesimal deformations, but computation of the Richardson-Nijenhuis bracket classical in deformation theory, implies that none of those deformations ~~can~~ admit prolongations, so that $\mathfrak{G}_P$ is always rigid for $\mathfrak{G}$ simple.

The computations can be made also for $\mathfrak{G}$ reductive - and then the space of deformations is much bigger; for example there exists deformations linked with local Lie algebra structures on the base manifold V, or with cyclic cohomology of functions on the manifold. One can get also formal deformations related with star products on V (there the Morita invariance for ~~cyclic~~ Hochschild cohomology is used)

Besides, one can extend those computations to the case of deformations of modules over Gp and deformations of the associated gauge group; one of the motivations are physical applications. one could try to carry over the Flato-Lichnerowicz program of deformations, which has turned out to be successful for quantum mechanics, to the case of quantum gauge theories.

All those results have been obtained by the author with the collaboration of P. Lecomte, University of Liège, Belgium.

C. ROGER.

Relative K-theory and Cyclic Homology (C. Ogle and C. Weibel)

If R is a $\mathbb{Q}$-algebra, there are two relationships between $K(R)$ and $HC(R)$. One is the chern character $K_n(R) \xrightarrow{ch} HP_n(R)$ of Connes-Karoubi, where HP_n fits into $0 \to \varprojlim_s{}^1 HC_{n+2k+1}(R) \to HP_n(R) \to \varprojlim_s HC_{n+2k}(R) \to 0$; ch factors through the Karoubi-Villamayor theory $KV_n(R)$. If $K_n^{rel}(R)$ denotes the third term of the usual long exact sequence of $K_nR \to KV_nR$, the second relationship is a secondary chern character $\nu: K_n^{rel}(R) \to HC_{n-1}(R)$. Thus we have:

$$\begin{array}{ccccccc} KV_{n+1}(R) & \to & K_n^{rel}(R) & \to & K_n(R) & \to & KV_n(R) \\ \downarrow ch & & \downarrow \nu & & \downarrow & & \downarrow ch \\ HP_{n+1}(R) & \xrightarrow{S} & HC_{n-1}(R) & \xrightarrow{B} & HC_n^-(R) & \xrightarrow{I} & HP_n(R) \end{array}$$

The map ν is not an isomorphism on rings, but in relative situations we have isos!

<u>Theorem 1</u>: (Goodwillie) If I is a nilpotent ideal in R, then ν is an isomorphism

$$K_n(R,I) \xleftarrow{\cong} K_n^{rel}(R,I) \xrightarrow[\cong]{\nu} HC_{n-1}(R,I)$$

<u>Theorem 2</u> (Ogle-Weibel) If I, J are ideals in R so that $I \cap J = 0$, then

$$K_n(R,I,J) \xleftarrow{\cong} K_n^{rel}(R\,I\,J) \xrightarrow[\cong]{\nu} HC_{n-1}(R,I,J)$$

<u>Conjecture</u> If $A \subset B$ has conductor ideal I, then the map

$K_n(A,B,I) \xleftarrow{\cong} K_n^{rel}(A,B,I) \xrightarrow{\nu} HC_{n-1}(A,B,I)$ should be an isomorphism.

These recent theorems have given new calculations of algebraic K-groups.

For example, let $R = k[x,y]/(xy=0)$ for k a field of char. 0. Then (for $\tilde{R} = R/k$)

n	0	1	2	3	4	5	etc.
$\widetilde{HC}^k_{n-1}(R)$	0	$\tilde{R}$	k	0	k	0	
$\widetilde{HC}^{\mathbb{Q}}_{n-1}(R)$	0	$\tilde{R}$	$k\oplus(\tilde{R}\otimes\Omega_k)$	$\Omega_k\oplus(\tilde{R}\otimes\Omega^2_k)$	$k\oplus\Omega^2_k\oplus(\tilde{R}\otimes\Omega^3_k)$	$\Omega_k\oplus\Omega^3_k\oplus(\tilde{R}\otimes\Omega^4_k)$	
$K_n(R)$	$\mathbb{Z}$	0	k	Ω_k	$k\oplus\Omega^2_k$	$\Omega_k\oplus\Omega^3_k$	

previously known (n = 0, 1, 2) — only known via above theorems (n = 3, 4, 5).

C. Weibel 6/15/87

A Topologist's View of Cyclic Homology (T. Goodwillie)

Waldhausen's algebraic K-theory of spaces can be thought of as the algebraic K-theory of a kind of "generalized ring". For a basepointed connected space with loop group G one thinks of

$A(X)$ as $K(R)$ where $R = \Omega^\infty \Sigma^\infty (G_+)$ is

the "group ring" of G over the universal "ring" $\Omega^\infty\Sigma^\infty S \doteq QS^0$. This heuristic point of view has been made precise in several ways by several people. In particular Bökstedt has defined a notion of "generalized ring" ~~which includes both~~ ~~Waldhausen~~ such that one can define algebraic K-theory and Hochschild homology in ~~E~~ his setting. The "rings" $Q(G_+)$ give one class of examples; their K-theory is Waldhausen's $A(BG)$. Discrete rings give another class, and in this case K-theory is Quillen algebraic K-theory. ~~The~~ Bökstedt's "topological Hochschild homology" $\underline{\underline{THH}}(R)$ of a generalized ring is a spectrum. In the case $R = Q(G_+)$ this is the unreduced suspension spectrum of the free loop space

$$\Lambda(BG) = \mathrm{Map}(S^1, BG).$$

In the case of discrete rings it is a new object. There is a map of spectra

$$\underline{K}(R) \longrightarrow \underline{\underline{THH}}(R)$$

(for a generalized ring R) which generalizes and refines the Dennis trace map and also generalizes a map

$$A(X) \longrightarrow Q\Lambda(X)_+$$

defined by Waldhausen. In fact, because $\underline{\underline{THH}}(R)$ is constructed as a cyclic object it is equipped with an S^1-action and there is (up to weak homology) a factorization of the trace map through the homotopy fixed-point spectrum:

$$\underline{\underline{K}}(R) \longrightarrow \underline{\underline{THH}}(R)^{hS^1} \longrightarrow \underline{\underline{THH}}(R).$$

This generalizes and refines a ~~lifting~~ factorization of Dennis' trace which has been defined by various people in recent years

$$K_*(R) \longrightarrow HC^-_*(R) \longrightarrow HH_*(R)$$

for R discrete (or perhaps simplicial).

The basic idea in all of this is to think of QS^0 as the ground ring, a ring more universal than $\mathbb{Z}$. ~~Thus~~ in ~~an~~ A module over QS^0 is a spectrum. The tensor product over QS^0 is the smash product of spectra. A (generalized) ring is more or less a spectrum $\underline{\underline{R}}$ together with an associative multiplication $\underline{\underline{R}} \wedge \underline{\underline{R}} \to \underline{\underline{R}}$. Bökstedt goes to much trouble to choose a notion of spectra, and of smash product, such that for any one of his rings he can make a cyclic object in the category of spectra

$$\underline{\underline{R}} \leftleftarrows \underline{\underline{R}} \wedge \underline{\underline{R}} \begin{smallmatrix}\leftarrow\\ \leftarrow\\ \leftarrow\end{smallmatrix} \cdots$$

This is the $\underline{\underline{THH}}(R)$.

Note that there is no reason to expect the Hochschild homology groups of, say,

$\mathbb{Z}$, to be the same as the ordinary $HH_*(\mathbb{Z})$ in which $\mathbb{Z}$ is the ground ring. In fact Bökstedt proved:

$$\pi_* \underline{\underline{THH}}(\mathbb{Z}) = \begin{cases} \mathbb{Z}, & * = 0 \\ \mathbb{Z}/n\mathbb{Z}, & * = 2n-1 \\ 0, & \text{else.} \end{cases}$$

He also proved

$$\pi_* \underline{\underline{THH}}(\mathbb{F}_p) = \begin{cases} \mathbb{F}_p, & * = 2n \geq 0 \\ 0, & \text{else} \end{cases}$$

In fact there is a ring structure because $\mathbb{F}_p$ is a commutative generalized ring, and one has $\pi_* T\underline{\underline{HH}}(\mathbb{F}_p) = \mathbb{F}_p[X]$, $X \in \pi_2$. Bökstedt and I have recently found the "topological HC^-" of $\mathbb{F}_p$:

$$\pi_* \left(T\underline{\underline{HH}}(\mathbb{F}_p)^{hS^1}\right) = \cancel{\mathbb{F}_p[X,T]/XT = p}$$

$$\hat{\mathbb{Z}}_p[X,T]/XT = p$$

$$X \in \pi_2 \quad T \in \pi_{-2}.$$

<u>Computing the Cyclic Homology of Curves</u> Susan C. Geller

(This is joint work with L. Reid + C. Weibel)

We are only going to compute the cyclic homology up to the cyclic homology of the normalization. By localization we need only consider rings with one singularity. By analytic isomorphisms we need only consider the analytic type of the singularity, i.e. graded rings with one singularity.

We thus assume that $R = k \oplus R_2 \oplus R_3 \oplus \ldots = k \oplus \bar{R}$ where k is a field of characteristic 0. The assumption that $R_1 = 0$ is for technical reasons and, since one can regrade the ring, causes no problems. Since $S(HC_*(\bar{R}))=0$, the SBI sequence splits into short exact sequences and we need only compute $H_*(R)$.

Our procedure, then, computes $H_*(R)$ using the Leray spectral sequence whose E_2^{pq} term is $R_p \otimes H^q(R;k)$. Since the spectral sequence is multiplicative with R along the $q=0$ axis, we need only find $H^q(R;k)$. This is obtained from the Serre spectral sequence whose E_2^{pq} terms are the same as the Leray but which converges to 0 (except for E_∞^{00}). We then explicitly compute the d_2 of the Leray spectral sequence from the generators of $H^q(R,k)$ and the Hochschild boundary map. $H_i(R)$ is then read from the rows of E_∞^{pq} whose generators have length i. The weight of the generators is used in computing $HC_i(\bar{R})$ from $0 \to HC_{i-1}(\bar{R}) \to H_i(R) \to HC_{i-1}(\bar{R}) \to 0$.

The example $A=\mathbb{Q}[t^2,t^3]$ is worked out and

$$HC_i(A) = HC_i(\mathbb{Q}) \oplus \begin{cases} \bar{R} & i=0 \\ 2\mathbb{Q} & i \text{ odd} \\ 0 & i \text{ even}, i\neq 0 \end{cases}.$$

The connection to K-theory is explained. In particular, $K_2(A\otimes k)$ maps onto $K_2(k)\oplus k\oplus k\oplus \Omega^2_k$ and $K_3(A\otimes k)$ maps onto $K_3(k)\oplus\Omega_k\oplus\Omega_k$. Other rings are then discussed.

Cyclic homology of differential forms and the Chern character John D.S. Jones
(joint work with Ezra Getzler)

Cyclic homology provides a natural model for differential forms on the (smooth) free loop space of a compact manifold - the Hochschild complex of the differential graded algebra of differential forms on the manifold X. In this model Connes B operator corresponds to the interior product i_T where T is the vector field generated by the action of the circle on the loop space LX given by rotating loops. Ideas of Atiyah and Witten have led Bismut to construct an equivariant differential form $Ch(E,\nabla)$ on the loop space LX given a vector bundle

E equipped with a connection ∇. This equivariant differential form is equivariantly closed and when restricted to the fixed point set of the circle action on LX, that is X regarded as the constant loops, it gives the usual Chern character form $\mathrm{Tr}\, e^F$ where F is the curvature of the connection ∇.

The main point of this talk is to describe this model for the differential forms on LX and to explain how to construct this equivariant differential form in terms of the model. We hope also to be able to describe the Witten current on the loop space in terms of this model. This current μ is an equivariant current on the loop space of a spin manifold and it has the property that $\langle \mu, Ch(E,\nabla)\rangle$ is the index of the twisted Dirac operator D_E.

John Jones 18 June 1987.

Cyclic Homology and the Macdonald conjectures Phil Hanlon

Let A_k denote the truncated polynomial ring $\mathbb{C}[t]/t^{k+1}$. We consider the Lie algebra cohomology of $L \otimes A_k$ for L a finite dimensional complex Lie algebra. We grade $L \otimes A_k$ by letting $L \otimes t^i$ be its i^{th}-graded piece. It is easy to check that this is a Lie algebra grading hence it extends to a grading on the cohomology of $L \otimes A_k$ which we call <u>weight</u>. Thus $H(L \otimes A_k)$ is bigraded by degree and weight and we are interested in computing $H(L \otimes A_k)$ as a bigraded module.

A simple deformation-theoretic arguement shows that $\dim(H(L \otimes A_k)) \geq \dim(H(L)^{\otimes k+1})$. We say L has <u>property</u> <u>M</u> if the dimensions are equal for all k.

<u>CONJECTURE 1</u>: If L is semisimple then L has property M.

The importance of CONJECTURE 1 is that it implies (using other known results) the Macdonald Root-System Conjectures.

In trying to prove Conjecture 1 for $L = gl_n(\mathbb{C})$ by induction on n one is naturally led to consider the case $L = \mathcal{H}_n$ where $\mathcal{H}_n$ is the $(2n+1)$-dimensional Heisenberg Lie algebra.

CONJECTURE 2: The Heisenberg Lie algebras have property M.

One might ask whether every finite-dimensional Lie algebra L has property M. The answer is no. Let $L_\alpha = \langle e, f, x \rangle$ be the three dimensional Lie algebra whose nonzero bracketts

$$[x,e] = -[e,x] = e \quad , \quad [x,f] = -[f,x] = \alpha f.$$

L_α has property M so long as α is not a negative rational. However there are examples of negative rationals α where L_α does not have property M.

Algebraic K-theory of spaces.

The main purpose of the talk was to motivate the construction of the algebraic K-theory of spaces from the point of view of the topology of manifolds, namely (1) the h-cobordism theorem and (2) the study of parametrized families of h-cobordisms, to which the so-called pseudo-isotopy theory may be reduced. Now, if one does not just want to study an individual h-cobordism (as in the h-cobordism theorem) but a parametrized family of such, it will not be enough anymore to keep track of the attaching maps of handles by (say) their homotopy classes only; rather it is necessary to keep track of such data in a more direct way. This leads to a modification of Quillen's algebraic K-theory where algebraic data (i.e. modules and isomorphisms) are replaced by more geometric data (i.e. spaces and weak homotopy equivalences), the point being that not just homotopy classes of the maps in question are used, but whole spaces of such. [For accounts for the construction of $A(X)$ from related points of view cf. (1) Proc. Conf. Alg. Top. London (Ontario) 1981, Contemp. Math. Series, AMS, and (2) Proc. Conf. Alg. Top. Durham 1985, Lond. Math. Soc. Publ.]

An offshoot of the theory is a re-interpretation of $A(X)$ as the algebraic K-theory of the "ring" $\Omega^\infty S^\infty(\Omega X_+)$. One has to cope with "rings" here which are 'multiplicative spectra' in the sense of algebraic topology. Recently Bökstedt has found a satisfactory solution to the problems that this point of view entails. For such "rings" R (which include the usual ones) he has also constructed a Hochschild homology over the "ground ring" $\Omega^\infty S^\infty$; this is called $THH(R)$, the 'topological Hochschild homology'. $THH(R)$ is a cyclic object in the usual way, and I am happy to report at this conference that the cyclic structure has proved useful in Bökstedt's amazing computation of $THH(R)$ in certain cases. Such computations, together with Bökstedt's trace map $K(R) \to THH(R)$ and its factorization (up to homotopy) through the homotopy fixed point set $THH(R)^{hS^1}$, are undoubtedly among the most promising tools in algebraic K-theory at this time.

F Waldhausen

Splitting theorems for $A(X)$, and Related Functors

Crichton Ogle

We extend the result due to Carlsson, Cohen, Goodwillie, Hsiang and (independently) the author, which proves That There is a weak equivalence $\Omega\bar{A}(\Sigma X) \simeq Q(\bigvee_{q\geq 1}\tilde{D}_q(X))$ for connected X, where $Q(-) = \Omega^\infty S^\infty(-)$ and $\tilde{D}_q(X) = E\mathbb{Z}/q \ltimes_{\mathbb{Z}/q} X^{[q]}$. These results used Goodwillie's Calculus of Functors and Goodwillie's result identifying the n^{th} derivative as $D_n\Omega\bar{A}(Y) \simeq \Sigma^n Q(\tilde{D}_n(Y))$ (Y 1-conn.).

We prove:

<u>Th 1</u> For a connected space Y and any integer $m \geq 1$, the ~~exists~~ a Goodwillie Taylor series of The functors

$Y \mapsto \Omega\bar{A}(\Omega^{m-1}\Sigma^m Y)$ and $Y \mapsto Q(ES^1{}_+ \wedge_{S^1}(\Omega^{m-1}\Sigma^m Y)/BS^1) = B(Y)$

split.

A consequence of This via Goodwillie's result above on $D_n\Omega\bar{A}(-)$ is:

<u>Cor 2</u> $\exists$ a weak equivalence $\Omega\bar{A}(\Omega^{m-1}\Sigma^m Y) \simeq B(\Omega^{m-1}\Sigma^m Y)$, Y conn. $m \geq 1$, which is natural in Y.

The techniques of proof involve constructing a weight filtration on the appropriate functors which splits The Goodwillie Taylor series. This has an interpretation in terms of Bökstedt's topological cyclic homology:

<u>Cor 2'</u> $\exists$ a weak equivalence $\Omega\bar{A}(X) \simeq THH(Q(\Omega X_+))|_{hS^1}$ for $X = \Omega^{m-1}\Sigma^m Y$, $(\ )_{hS^1}$ = homotopy orbit space.

A similar theorem exists for n-relative Waldhausen K-Theory:

<u>Th 3</u> If $I_1, \ldots, I_n$ are a family of ideals in a $\mathbb{Q}$-algebra R, R simplicial and $\pi_0(I_1 \cap \cdots \cap I_n)$ is nilpotent in $\pi_0(R)$, then there exists a weak equivalence of n-relative spaces $K^W(R, \{I_j\}) \simeq THH(R, \{I_j\})_{hS^1}$.

For $n=1$ this is due to Goodwillie, and for R discrete is joint w/ C. Weibel (c.f. Weibel's talk).

homology

Homology of Lie algebras, cyclic homology, crossed simplicial groups ...

Jean-Louis LODAY (CNRS - Strasbourg)

For a commutative unitary algebra A over a characteristic zero field k the following is known for $H_*(gl_n(A), k)$ (joint work with D. Quillen)

thm 1 $\quad H_i(gl_{n-1}(A)) \xrightarrow{\sim} H_i(gl_n(A))$ if $i<n$

$\quad H_n(gl_{n-1}(A)) \longrightarrow H_n(gl_n(A)) \to \Omega^{n-1}_{A/k} / d\Omega^{n-2}_{A/k} \to 0$ is exact (A commutative)

thm 2 $\quad \operatorname{Prim} H_n(gl(A)) = HC_{n-1}(A)$ (cyclic homology } also denoted $H^\lambda_n(A)$)

In fact H^λ_* and HC_* agree only in char. 0. The first one, that I call Connes' homology is the homology of the complex $\cdots \longrightarrow A^{\otimes n+1}/(1-t) \xrightarrow{b} A^{\otimes n}/(1-t) \to \cdots$

b: Hochschild bdry, t = cyclic operator. Cyclic homology is the homology of the (B,b)-complex

$$\begin{tikzcd} \vdots \arrow[d, "b"'] & \vdots \arrow[l, "B"'] \arrow[d] & \cdot \arrow[l] \\ A^{\otimes 2} \arrow[d, "b"'] & A \arrow[l, "B"'] \\ A \end{tikzcd}$$

where B is essentially given by

$$B(a_0, \ldots, a_n) = \sum_{i=0}^{n} (-1)^{in} (1, a_i, \ldots a_n, a_0, \ldots, a_{i-1})$$

It seems to me that it is important to think of H^λ_* and HC_* as two different theories.

thm 1 is analogous to a theorem of Suslin concerning $H_*(GL_n(F))$, F = infinite field which has recently been generalized to (non-commutative) local rings by D. Guin

thm 2 has only been proved in the GL-case in low dimension, $n=1,2$. The analogy leads to a dictionary:

multiplicative	additive (or infinitesimal)
$\mathbb{G}_m$	$\mathbb{G}_a$
GL_n	gl_n
det	trace
K_n	H^λ_n
K^M_n	$\Omega^{n-1}/d\Omega^{n-2}$
motivic cohomology	?

Motivic cohomology is a conjectural cohomology theory defined for schemes over $\operatorname{Spec} \mathbb{Z}$ (hopefully) and universal for Chern classes. Beilinson devised some axioms for this theory involving $\mathbb{G}_m$, K-theory, Milnor K-theory. Similar axioms can be envisioned in the additive case (use the dictionary). My belief is that in the additive case the answer is cyclic homology. There is a lot of evidences for this in the affine case. →

The following topological point has been dealt with in the lectures.
The simplest simplicial model for S^1 has one non degenerate 0-cell and one non deg. 1-cell, all the others are degenerate. Then there are $(n+1)$ simplices in dimension n and there is a natural bijection with $\mathbb{Z}/(n+1)\mathbb{Z}$. Though this is not a simplicial group it is almost the case. A face map (for instance) is not a group homomorphism but is a crossed group homomorphism: $d_i(xy) = d_i(x)\,(x\cdot d_i)(y)$ where x is acting on d_i by $x\cdot d_i = d_{x(i)}$ (think of $x \in \mathbb{Z}/(n+1)\mathbb{Z}$ acting cyclically on $\{0,1,\ldots,n\}$). This leads to a definition of a crossed simplicial group (joint work with Z. Fiedorowicz). Other examples ~~[illegible]~~ are the family of dihedral groups, quaternionic groups, symmetric groups, braid groups, hyperoctahedral groups. A classification of crossed simplicial groups was given. This lead to other theories mimicking cyclic homology: dihedral homology (related to $H_*(o(A))$ and $H_*(sp(A))$, joint work with Procesi) and symmetric homology.

Cyclic Homology of Enveloping Algebras

Christian Kassel (CNRS-Strasbourg)

For any Poisson manifold equipped with a Poisson bracket $\{,\}$, Brylinski constructed a degree -1 differential δ on the differential forms anticommuting with the exterior derivative

$$\delta(f_0\,df_1\ldots df_p) = \sum_{i=1}^{p} (-1)^{i+1}\{f_0,f_i\}\,df_1\ldots\widehat{df_i}\ldots df_p$$

$$+ \sum_{1\le i<j\le p} (-1)^{i+j} f_0\, d\{f_i,f_j\}\,df_1\ldots\widehat{df_i}\ldots\widehat{df_j}\ldots df_p$$

Let us consider the canonical Lie-Poisson structure on the dual of a Lie algebra $\mathfrak{g}$ over a field k of characteristic zero. We prove

<u>Theorem</u>. a) $H_*(\Omega^*_{S(\mathfrak{g})}, \delta)$ is isomorphic to the Hochschild homology of the enveloping algebra $U(\mathfrak{g})$).

b) The homology of the double complex

$$\begin{tikzcd}
\vdots \arrow[d] & \vdots \arrow[d] & \vdots \arrow[d] & \vdots \arrow[d] & \\
\Omega^2_{S(\mathfrak{g})} \arrow[d, "\delta"'] & \Omega^1_{S(\mathfrak{g})} \arrow[l, "d"'] \arrow[d] & \Omega^0_{S(\mathfrak{g})} \arrow[l, "d"'] \arrow[d] & 0 \arrow[l] \arrow[d] & \cdots \arrow[l] \\
\Omega^1_{S(\mathfrak{g})} \arrow[d, "\delta"'] & \Omega^0_{S(\mathfrak{g})} \arrow[l, "d"'] \arrow[d] & 0 \arrow[l] \arrow[d] & 0 \arrow[l] \arrow[d] & \cdots \arrow[l] \\
\Omega^0_{S(\mathfrak{g})} & 0 \arrow[l] & 0 \arrow[l] & 0 \arrow[l] & \cdots \arrow[l]
\end{tikzcd}$$

is isomorphic to Connes cyclic homology of $U(\mathfrak{g})$

c) If $\dim \mathfrak{g} < \infty$, then the periodic cyclic homology of $U(\mathfrak{g})$ is the same as the one for the ground field k.

This result allows ~~many~~ the computation of the cyclic homology of many $U(\mathfrak{g})$. It shows also that for all semi-simple Lie algebras, Connes spectral sequence for the cyclic homology of $U(\mathfrak{g})$ ~~cannot~~ never collapses at E^2 (it collapses at E^3 or beyond). In the talk, we illustrated this new and interesting phenomenon with the case of $\underline{\underline{sl}}(2)$.

DIFFERENTIAL ALGEBRAS IN FIELD THEORY.

R. STORA LAPP Annecy le Vieux (F) & CERN (CH)

The Feynman algorithm, which describes perturbative expansions in quantized local field theory, accomodates a description of exact or broken symmetries. This description goes through the construction of some differential algebras whose "local" cohomologies provide "anomalies" which are due to the breaking of the symmetry at the quantum level. This happens specifically in the case of gauge symmetries e.g. in Yang Mills theories, 1st quantized string theory.

The lectures were divided into 4 sections:

1) The Feynman algorithm
2) Current algebras
3) Quantized gauge theories
4) 1st quantized strings

Unfortunately, by lack of time, 4) was not covered.

ON BOTT-CHERN SECONDARY CHARACTERISTIC CLASSES

C. Soulé (Paris VII, IHES).

If X is a complex manifold, E a holomorphic vector bundle on X and h a hermitian metric on E, denote by $ch(E,h) \in A(X) = \bigoplus_{p \geq 0} A^{pp}(X,\mathbb{C})$ the Chern form defined using the connection ∇ attached to h: $ch(E,h) = \mathrm{tr}\exp\left(\frac{i}{2\pi}\nabla^2\right)$.

When ~~$\mathcal{E}(E,h)$~~ $0 \to S \to E \to Q \to 0$ is an exact sequence of bundles on X, give arbitrary metrics h', h, h'' to S, E, Q respectively. Bott and Chern defined classes $\widetilde{ch} \in A(X)/(\mathrm{Im}\,\partial + \mathrm{Im}\,\bar\partial)$ which can be characterized by the following properties:

i) $dd^c\,\widetilde{ch} = ch(E,h) - ch(S,h') - ch(Q,h'')$ ($d^c = \frac{1}{4\pi i}(\partial - \bar\partial)$).

ii) $\widetilde{ch}$ depends functorially of the exact sequence and its metric

iii) $\widetilde{ch} = 0$ when $(E,h) = (S \oplus Q, h' \oplus h'')$.

We give new constructions of these $\widetilde{ch}$ (joint work with Bismut and Gillet). In the talk we explain how $\widetilde{ch}$ enters in defining the regulator on $K_1(X)$ (joint with Gillet) and in expressing the variation of determinant of Laplace operators with the metrics (joint with Bismut and Gillet).

Partielle Differentialgleichungen

vom 21. – 27. Juni 1987

J. Neu, Berkeley

There is a perennial notion that matter may have a geometric-topological origin. There are nonlinear field theories originating in physics and differential geometry with topological vortex or monopole solutions. The topological invariants associated with these vortices or monopoles are refered to as "charge", as though these topological ~~invariants~~ vortices and monopoles are analogous to material charged particles.

Are these analogies meaningful *dynamically*? Do the "charged" vortices and monopoles really interact ~~the~~ like electromagnetic charged particles? We investigate the dynamics of topological vortices governed by the nonlinear wave equation $\psi_{tt} - \Delta\psi - (1-|\psi|^2)\psi = 0$. Here, ψ is a complex scalar field defined on 2+1-D Minkowsky space. The topological objects ("vortices") of the ψ field are zeros with winding numbers not equal to zero. From the full field dynamics we asymptotically derive a "reduced" dynamic for the "phase gradient tensor" $F^{ij} = \epsilon^{ijk}\partial_k(\arg\psi)$, and the world lines l of zeros of ψ. We discover that the reduced dynamic governing F^{ij} and the l's consist of Maxwell equations for F^{ij} and a Lorentz equation giving the proper acceleration of the world lines l in response to the field F^{ij}. In this "electrodynamics of vortices", the "charge" of a zero is $2\pi \times$ winding number.

3+1-D models based on O(3) Yang Mills Higgs equations or Kaluza-Klein theory are presently being examined on a similiar basis.

On Gårding's inequality

(N. Jacob, Erlangen)

An analysis of the proof of Gårding's inequality for strongly elliptic differential operators shows that a generalization of this inequality can be proved for certain non-elliptic operators, provided a generalised principal part is defined and the usual Sobolev-space is changed in an appropriate way. Moreover it turns out that we have to distinguish two classes of operators. In the most general class we cannot use a partition of unity in the proof. This implies that the coefficients of the generalised principal part are assumed to vary only slowly. The second class consists of operators which can be handled completely analogously to elliptic operators. These operators have to be formally hypoelliptic. Examples are given for both classes.

"The Cauchy problem in 3-D-thermoelasticity"

(R. Racke, Bonn)

We consider the following Cauchy problem: (homogeneous, initially isotropic 3-D-thermoelasticity)

$$\begin{cases} \dfrac{\partial^2 U_i}{\partial t^2} = C_{imjk}(\nabla U,\theta)\dfrac{\partial^2 U_j}{\partial x_m \partial x_k} + \tilde{C}_{im}(\nabla U,\theta)\dfrac{\partial \theta}{\partial x_m} \\ a(\nabla U,\theta)\dfrac{\partial \theta}{\partial t} = \dfrac{1}{f(\theta)}\operatorname{div} q(\nabla U,\theta,\nabla\theta) + \operatorname{tr}\left\{ (\tilde{C}_{im}(\nabla U,\theta))_{im} \left(\dfrac{\partial}{\partial t}\dfrac{\partial U_i}{\partial x_m}\right)_{im} \right\} \\ U(t=0)=U^0, \quad \dfrac{\partial U}{\partial t}(t=0)=U^1, \quad \theta(t=0)=\theta^0. \end{cases}$$

Global existence is shown – for small data – in the class of functions $\nabla U, \frac{\partial U}{\partial t} \in C_1(\mathbb{R}_0^+, W^{s-1,2}(\mathbb{R}^3)) \cap C_0(\mathbb{R}_0^+, W^{s,2}(\mathbb{R}^3))$, $\theta \in C_1(\mathbb{R}_0^+, W^{s-2,2}(\mathbb{R}^3)) \cap C_0(\mathbb{R}_0^+, W^{s_0,2}(\mathbb{R}^3))$ for some $s \in \mathbb{N}$, if the nonlinearity degenerates up to order 2, that is e.g. $|C_{imjk}(\nabla U,\theta) - C_{imjk}(0,0)| = O(|\nabla U|^2 + |\theta|^2)$ near the origin. Moreover the time-decay is described as well as the scattering behaviour. The proof uses the following ingredients: 1) Transformation

to a suitable first-order system: $V_t + AV = F(V, \nabla V, \nabla^2 V)$, $V(0) = V^0$, where $-A$ generates a contraction semigroup; 2) L_p–L_q-time-decay of solutions of the linearized problem; 3) local existence result (foll. Kawashima); 4) high energy estimate: $\|V(t)\|_{W^{s,2}} \le C \cdot \|V^0\|_{W^{s,2}} \cdot \exp \int_0^t \{ |V_t|^2_{L^\infty} + |V|^2_{L^\infty} + |\nabla V|^2_{L^\infty} + |\nabla^2 V|^2_{L^\infty} \}(\tau)\, d\tau$,
and weighted a priori estimate: $\sup_{0 \le t \le T} (1+t)^{2/3} \|V(t)\|_{W^{s_1,6}} \le M_0 < \infty$
for some $s_1 \in \mathbb{N}$, M_0 independent of T.
(foll. Klainerman & Ponce)

Concentration Effects in the Euler Equation

(Ronald J. DiPerna, Berkeley)

We discuss results in a joint program with A. Majda dealing with concentrations in sequences of solutions to ~~dealing~~ the incompressible Euler equations in two space dimensions. We consider sequences with uniformly bounded energy and vorticity and provide estimates on the Hausdorff dimension of the set in physical space where energy can concentrate, i.e. where L^2 strong compactness can be lost in a general L^2 weakly convergent sequence. We show that the reduced defect measure associated with a weakly convergent sequence is concentrated on a set with Hausdorff dimension ≤ 1. We study the effect of concentrations on the inertial terms and establish a concentrate-cancellation effect which asserts that the inertial terms are weakly continuous despite the possible defects in the energy field. The techniques include several generalizations of classical defect theorems from L^p to the Sobolev spaces.

Estimates for the pressure in the Navier-Stokes equations on exterior domains and their consequences

Wolf von Wahl (Bayreuth)

Let u be any weak solution of the Navier-Stokes equations $u' - \Delta u + u \cdot \nabla u + \nabla \pi = f$, $\nabla \cdot u = 0$, $u|\partial\Omega = 0$, $u(0) = \varphi$, on $(0,T) \times \Omega$, Ω an exterior domain of $\mathbb{R}^n$. Then we show that

$$\nabla \pi \in L^s((0,T), L^p(\Omega)),$$
$$\pi \in L^s((0,T), L^{p^*}(\Omega))$$

provided $n+1 \leq \frac{2}{s} + \frac{n}{p}$, $\frac{1}{p^*} = \frac{1}{p} - \frac{1}{n}$. In particular we obtain $\pi \in L^{(n+2)/n}((0,T) \times \Omega)$. The consequences are as follows: We can construct a weak solution which is bounded in (t,x) if $|x|$ is sufficiently large and fulfills at the same time the energy inequality for almost every $s > 0$, for $s = 0$ and for all $t \geq s$. As we show, this implies $\|u(t)\|_{L^2(\Omega)} \to 0$ if $t \to +\infty$ $(T = +\infty)$.
The work reported here is a joint one with H. Sohr, H. Sohr and M. Wiegner.

Explosions in chemotaxis systems
(W. Jäger, S. Luckhaus)

The following equations are a simple model for the aggregation of micro-organisms (amoeba; concentration u) caused by a chemical substance (acrasin; concentration v) produced by them:

$$\partial_t u = \Delta u - \chi \nabla(u \nabla v) \qquad \text{in } \Omega \subset \mathbb{R}^n$$
$$\partial_t v = \delta \Delta v - \mu v + \beta u \qquad n = 2, 3$$

no flux condition on the boundary. (Keller-Segel model). In case of large δ and β the system can be approximated by

$$\partial_t u = \Delta u - \chi \nabla(u \nabla v)$$
$$0 = \Delta v - \alpha(u - \bar{u}) \,, \quad \bar{u} = \int_\Omega u_0 \, dx$$

Result: There exists a $c(\Omega) > 0$ s.th. $\alpha \bar{u} \chi < c(\Omega)$ implies existence of smooth global solutions. If this condition is violated there exists blow up in finite time. Quantitative criteria are given in the radial symmetric case. The results are in agreement with the experimental observations (formation of fruiting bodies).

Existence and Singularities of nonlinear σ-model

Jalal Shatah
(Courant Institute)

We discuss existence of weak solutions and development of singularities of harmonic maps from Minkowski space time into a Riemannian manifold. The equations are given by $\partial_\alpha \partial^\alpha u^a + \Gamma^a_{bc}\, \partial_\alpha u^b \partial^\alpha u^c = 0$. The existence part is shown for the SU(2) model. The global solutions constructed there are ~~by then~~ weak. The method used is by penalization i.e. a potential which goes to ∞ outside the manifold. The singularities that are shown for this model are by exhibiting a selfsimilar solution to the equation. This is possible because of the scale invariance of the equation. The existence of harmonic maps for $S^2 \to SU(2)$ is essential and gives an idea for extending these results to various manifolds. The singularities that develop prevent the solution from having bounded first derivatives which is the requirement to show higher regularity.

Oscillations of nonlinear p.d.e and their oscillations.

Luc Tartar (C.E.A. France)

A sequence u_ε is said to have oscillations if it converges weakly and not strongly. In order to describe all weak limits of functions of u_ε one used L.C. Young's measures. If the sequence satisfies some linear differential system with constant coefficients, one can describe the weak limit of quadratic functions of u_ε by the compensated compactness lemma.

Using all "entropy" relations one can then get some constraints on the Young's measures ν_x. If ν is a Dirac mass then there is no oscillations; if not the question is to study propagation of oscillations and their interaction, this program is shown for some semilinear hyperbolic systems in one space variable.

The Stationary Vlasov-Fokker-Planck Equation

Klaus Dreßler (Universität Kaiserslautern)

We prove an existence (in C^∞) and uniqueness theorem (in the space of probability measures) for the stationary Vlasov-Fokker-Planck equation describing a steady state of a plasma. This result is also of interest for the investigation of the stationary Vlasov-Poisson system since our uniqueness theorem distinguishes one of the many solutions of that equation in some way.

A Boundary Value Problem for the Broadwell Model in Two Space Dimensions

R. Illner (Victoria)

We study a model problem for the evaporation and condensation from a surface into a rarefied gas. On the rectangle $R = [0,a] \times [0,b] \subset \mathbb{R}^2$, we look for functions $f_1,\dots,f_4 \in C_{0,+}(R)$ which satisfy

$$
(P)\quad
\begin{aligned}
\partial_x f_1 + f_1 f_2 &= f_3 f_4 , & f_1(0,y) &= \varphi_1(y) \geq 0 \\
-\partial_x f_2 + f_1 f_2 &= f_3 f_4 , & f_2(a,y) &= \varphi_2(y) \geq 0 \\
\partial_y f_3 + f_3 f_4 &= f_1 f_2 , & f_3(x,0) &= \varphi_3(x) \geq 0 \\
-\partial_y f_4 + f_3 f_4 &= f_1 f_2 , & f_4(x,b) &= \varphi_4(x) \geq 0
\end{aligned}
$$

It is shown that (P) always has a solution. The solution is unique for small data.

"Integrable Geometry, Coherence and Chaos for the Sine-Gordon PDE"

Dave McLaughlin – U of Arizona

Numerical solutions of the damped, driven sine-Gordon equation with periodic boundary conditions are summarized. These experiments display chaotic attractors with regular, coherent spatial patterns. We develop and use integrable methods to describe the sine-Gordon level sets in function space. Singular level sets, and their associated homoclinic orbits, are described. These homoclinic orbits are then correlated to the chaotic attractors in the perturbed problem. It is suggested that the damped, driven sine-Gordon equation provides a nice model of chaos in pde's, just as the driven damped pendulum has provided for ODE's.

Minimal surfaces of high genus with a nice boundary curve.

R. Böhme Ruhr-Uni-Bochum

We gave the example of a family of nice curves $\Gamma = \Gamma(g)$ in $\mathbb{R}^3$, which are Jordan curves, and a curve $\Gamma(g)$ bounds at least 2^{2g} essentially different minimal surfaces of a genus γ with $0 \leq \gamma \leq g$. The curve Γ is a perturbation of a planar curve (C), which is only immersed but the double cover of a Jordan curve C in the complex plane. (C_2 is a boundary curve for $(g+1)$– families of ~~Ri~~ pieces of Riemann surfaces (i.e. their complex embeddings into $\mathbb{P}^1$) such that any member of the family (of genus g) has $(2g+1)$ branch points varying in int$(\mathbb{C})$. ~~These~~ perturbation will be given by $2(2g+1)$ inequalities of the values of the perturbation of $(\mathbb{C})_2$ which produces Γ at specified points. The rest of the perturbation is given by monotonic but otherwise arbitrary smooth ~~perturbation~~ interpolation. The same argument will work also by other constant values of curvature.

Remarks on blow up, quenching and dead cores

B. Kawohl, Heidelberg

Nonlinear parabolic differential equations can exhibit seemingly different phenomena such as the ones listed in the title. Nonetheless these "different" phenomena can be tackled by similar techniques.
The purpose of my remarks is to point out that this convenience has a simple explanation. As an example I reduce the blow up problem

$$u_t - \Delta u = u^p - |\nabla u|^2 \qquad t>0,\ x\in\Omega,$$
$$u = 0 \qquad t>0,\ x\in\partial\Omega,$$
$$u = u_0 \geq 0 \qquad x\in\Omega,$$

to a dead core problem

$$v_t - \Delta v = -h(v) \leq 0 \qquad t>0,\ x\in\Omega,$$
$$v = 1 \qquad t>0,\ x\in\partial\Omega,$$
$$v = v_0 \leq 1 \qquad x\in\Omega.$$

There is no blow up for $p\leq 2$, but there is blow up for $p>2$, Ω large and u_0 large. This and other results were obtained jointly with L. Peletier and A. Acker.

"Collapse of periodic solutions to nonlinear Schrödinger equations."
H. Lange, Köln

We consider sufficient conditions such that any classical solution of the periodic initial-boundary value problem

$$\begin{cases} i u_t = -u_{xx} + f(|u|^2)\cdot u \\ u(x+2,t) = u(x,t) \\ u(x,0) = u_0(x) \end{cases}$$

has a finite life-time, i.e. the solution exists only on a finite time interval $[0,T)$. The conditions to ensure that are growing properties on the nonlinearity f and a relation of the type

$$E_1(0) < E^* \leq 0$$

when $E_1(0)$ is the initial energy of u_0, and E^* a certain constant depending on the data. The main ingredient for the proof of the collapse result is to follow the evolution of appropriate moments of the solution $u(x,t)$, e.g.

$$G(t,\eta) = \int_0^\eta F(y,t)\,dy, \quad F(y,t) = \int_{y-1}^{y+1} \psi(x,y)|u(x,t)|^2\,dx \quad (o<y<2)$$

with some quadratic function $\psi(x,y)$.

"Blowup of solutions of semilinear heat equations" – R. Kohn, Courant Institute.

I discuss joint work with Y. Giga on the solutions of $u_t - \Delta u = |u|^{p-1}u$ and related equations. Our goal is to analyse the local behavior of u as it blows up.

Our method is based on the scale-invariance of the equation: if $u(x,t)$ is a solution then so is $u_\lambda(x,t) = \lambda^{\frac{2}{p-1}} u(\lambda x, \lambda^2 t)$. One expects that as it blows up u should become asymptotically scale-invariant ("self-similar"). This is proved under various hypotheses on p and $\Omega \subset \mathbb{R}^n$, by using a change of variables which turns the analysis of u near blowup into the study of the large-time asymptotics of a slightly different parabolic equation.

Regularity of generalized solutions of Monge-Ampère equations — John Urbas, Canberra.

Let u be a convex generalized solution of the equation
$$\det D^2 u = f(x, u, Du) \quad \text{in } \Omega,$$
where Ω is a bounded domain in $\mathbb{R}^n$, and f is a C^∞ positive function. An example of Pogorelov shows that generalized solutions need not be smooth, even if f is analytic.
We show that generalized solutions are C^∞ smooth in each of the following cases:
i) $u \in C^{1,\alpha}(\bar\Omega)$ for some $\alpha > 1 - 2/n$;
ii) $u \in W^{2,p}(\Omega)$ for some $p \geq n(n-1)/2$;
iii) Ω is a bounded convex domain, $\partial\Omega \in C^{1,\alpha}$, and $u \in C^0(\bar\Omega)$, with $u|_{\partial\Omega} \in C^{1,\alpha}$ for some $\alpha > 1 - 2/n$. Pogorelov's example shows that these bounds on the exponents α and p are sharp.

Bifurcation of periodic solutions of Klein-Gordon equations

H. Kielhöfer, Augsburg

We consider a semilinear wave equation

$$u_{tt} - u_{xx} - c(\lambda)u - h(\lambda,x,u) = 0 \ , \ h = o(|u|) \ , \ \lambda \in \mathbb{R},$$

together with periodic, Dirichlet or Neumann boundary conditions for x and/or t. We prove bifurcation of nontrivial solutions at $\lambda = \lambda_0$ for a dense set of periods (intervals) provided $c(\lambda)$ is strictly monotone near λ_0. The main tool is a new bifurcation result for potential operators which is not proved by variational methods but uses Conley's bifurcation theory for bounded invariant sets.

"Nodal sets for solutions of elliptic equations" – Leon Simon, Stanford University. (Joint work with R. Hardt, U. Minnesota)

For solutions of the second order elliptic equation

$$a_{ij} D_i D_j u + b_j D_j u + c u = 0,$$

a_{ij} continuous, b_j, c bounded, the talk described a method for bounding the $(n-1)$-dimensional measure of $u^{-1}\{0\}$ in the neighbourhood of any point x_0 at which the solution u has finite order of vanishing d. Specifically,

$$* \quad \mathcal{H}^{n-1}\left(u^{-1}\{0\} \cap B_\rho(x_0)\right) \le c\, d\, \rho^{n-1} \ , \ \rho \le \rho_0 = \rho_0(x,u,\gamma,\mu),$$

$c = c(n)$, where γ, μ are such that

$$\gamma^{-1}|\xi|^2 \le a_{ij}\xi_i\xi_j \le \gamma|\xi|^2$$

$$|b_j| + |c| \le \mu,$$

In addition it was shown that the singular part $u^{-1}\{0\} \cap |Du|^{-1}\{0\}$ has dimension $\le n-2$, generalizing a result of Caffarelli and Friedman, and that, if the coefficients are of class C^d, this set is countably $(n-2)$-rectifiable. The bound * gives an asymptotic estimate for the measure of the nodal set $\varphi_j^{-1}\{0\}$ where φ_j is the jth eigenfunction of Laplacian on a compact manifold. The result is not as precise as the estimate of Donnelly and Fefferman in the real analytic case.

Schrödinger operators with random potential

We consider random Schrödinger operators with potential $V_\omega(x)$, $x \in \mathbb{R}^d$, $d > 1$. We are mainly interested in the case where $V_\omega(x)$ has different probability distribution for $x_1 < 0$ and $x_1 > 0$ ($x = (x_1, \dots, x_d)$). We prove that there may exist "surface states", i.e. states that "live" near the surface $x_1 = 0$. There density of state will not grow like a volume term but rather like a surface term.
(joint work with H. Englisch and B. Simon)

W. Kirsch, Bochum

Singularities of a geometric evolution equation

Gerhard Huisken, Canberra

Let M^n, $n \geq 2$ be a closed smooth hypersurface of $\mathbb{R}^{n+1}$. We deform M^n in direction of its normal vector such that the velocity at each point is given by the mean curvature of the hypersurface. It is known that convex hypersurfaces contract smoothly to a single point during this flow. Here we study singularities that occur for non-convex initial data. Using rescaling techniques it is shown that under reasonable assumptions the singularities are asymptotically selfsimilar. In some special cases the selfsimilar solutions could be classified.

Nonlinear partial differential equations of real principal type with applications to geometry

Deane Yang, Rice

In joint work with Jonathan Goodman, Courant Institute, we prove local solvability of a nonlinear partial differential equation of real principal type by proving smooth, tame estimates for the fundamental solution of a linear differential operator of real principal type and applying the Nash-Moser implicit function theorem.

The smooth, tame estimates then lead to the following geometric results:

(1) (joint work with D. DeTurck, Univ. of Pennsylvania) Given a smooth, nondegenerate tensor $R = \frac{1}{4} R_{ij,kl}(dx^i \wedge dx^j)(dx^k \wedge dx^l)$, $R_{ijkl} + R_{iklj} + R_{iljk} = 0$, on a 3-manifold, there exists in ~~the~~ a neighborhood of any point a smooth Riemannian metric g such that

$$\mathrm{Riem}(g) = R.$$

(2) Given a smooth metric $g = g_{ij}\, dx^i dx^j$ on a 2-manifold and a point p where the Gauss curvature K satisfies: $K(p) = 0$, $\nabla K(p) \neq 0$, there exists an embedding u of a neighborhood of p into $\mathbb{R}^3$ such that the induced metric is equal to g, i.e. u is an isometric embedding. This was first proved by C.S. Lin.

(3) Given a smooth, "generic" metric on a 3- or 4-manifold, there exists a smooth isometric embedding of a neighborhood of any point into $\mathbb{R}^{\frac{1}{2}n(n+1)}$, where n = dimension of manifold.

Coercive Singular Perturbations: Reduction to Regular Perturbations and Applications.

L.S. Frank

(Nijmegen, The Netherlands).

Singular perturbations appearing in the Elasticity and Diffraction theories and also in some problems in Fluid Dynamics (as, for instance, the spectral Stokes problem) belong to the class of operators with small or large parameters, which satisfy the algebraic condition of coerciveness. The coerciveness concept for one parameter families of singular perturbations has been introduced in 1976 and is a necessary and sufficient algebraic condition for the stability of singularly perturbed boundary problems, i.e. the coerciveness is a necessary and sufficient condition for the validity of two-sided a priori estimates for the solutions to singularly perturbed problems uniformly with respect to the small parameter. It turns out that the same coerciveness condition guarantees that any coercive singular perturbation can be reduced in a constructive way to a regular perturbation. As some direct applications of the reduction procedure for the coercive singular perturbations, one should mention the following ones:

1. Simple derivation of asymptotic formulae for their solutions.
2. Asymptotics for their eigenvalues and eigenfunctions.
3. Asymptotic analysis of the bifurcation phenomenon for the coercive singular perturbations
4. Construction of efficient and robust algorithms for numerical treatment of coercive singular perturbations.
5. Asymptotic analysis of some classes of singular perturbations of strictly hyperbolic operators, the singularly perturbed Boussinesq's system in the theory of waves being an example of such a perturbation.

The Korteweg de Vries Equation, Modulation, and Theta functions.

Stephanos Venakides, Duke University.

The space periodic Korteweg de Vries equation $u_t - 6uu_x + \varepsilon^2 u_{xxx} = 0$ can be solved explicitly in terms of a theta function whose period matrix is derived from a hyperelliptic function. The interest in the small dispersion limit $\varepsilon \to 0$ arises from the fact that oscillations arise out of non-oscillatory initial data. As $\varepsilon \to 0$ the genus N of the associated Riemann surface is of order $\frac{1}{\varepsilon}$ and one can show that the theta function, expressed as sum of real exponentials over the lattice Z^N, is dominated by its largest term. This allows to compute the weak limit of the solution following a procedure similar to the Lax Levermore procedure. An averaging assumption allows one to describe the local oscillations ~~up~~ up to phase shifts. Further results point toward the possibility of proving the averaging assumption rigorously.

The Yang-Mills Equation as Partial Differential Equations

Karen K Uhlenbeck, University of Chicago

The Yang-Mills equations were formulated as an extension of Maxwell's equations. The gauge ~~as an extension of~~ group for Maxwell's equations is $U(1) = S^1$ and for Yang-Mills an arbitrary compact Lie Group. The current in the inhomogeneous ~~terms~~ equations is generated by a Higgs potential in the Lagrangian

$$\int |F|^2 + |D^A\phi|^2 + \lambda(|\phi|^2 - 1)^2 \, dx .$$

A lot of intuition concerning the behavior of instantons (solution of $F = \pm * F$) comes from the t'Hooft solutions given by $A = -\mathrm{Im}\left(\frac{\partial}{\partial q}(\ln f)\, dq\right)$ when $\Delta f + \lambda f^3 = 0$ in $\mathbb{R}^4$. There are a large number of applications in fields of all sorts.

1. Geometry and topology of 3 and 4 manifolds
2. Algebraic geometry (stable bundles)
3. integrable systems
4. comparison with harmonic maps
5. physics (classical theory of vortices and monopoles).

It is a very active and exciting field. There are a lot of open problems in the technical PDE aspects.

Oscillatory solutions of partial diff. equ., and of difference approximations to them.

Peter D Lax, Courant Inst., New York Univ.

Solutions $u^\varepsilon(x,t)$ of

$$(*)_\varepsilon \qquad u_t + uu_x + \varepsilon^2 u_{xxx} = 0, \quad u(x,0) = u_0(x)$$

tend, as $\varepsilon \to 0$, to solutions of

$$(*) \qquad u_t + u u_x = 0, \quad u(x,0) = u_0(x)$$

for $t < t_{crit}$, the time for which (*) has a smooth solution. For $t > t_{crit}$, $(*)_\varepsilon$ has oscillatory solutions which converge weakly but not strongly to some limit $\bar{u}$, explicitely describable. The limit of $(*)_\varepsilon$ is

$$\bar{u}_t + \tfrac{1}{2}(\overline{u^2})_x = 0$$

where $\overline{u^2}$ is the weak limit of $(u^\varepsilon)^2$; clearly, $\overline{u^2} \neq \bar{u}^2$. Similar phenomena occur when nonlinear hyperbolic equations are approximated by dispersive difference equations: solutions converge uniformly as long as the hyperbolic equations have smooth solutions. Beyond such a critical time, the solutions of the difference equations are oscillatory, and converge weakly but not strongly. There is much evidence — analogy, analysis & numerical calculations to substantiate this. When the hyperbolic equations are in conservation form, the question is if the weak limit satisfies the conservation laws; this appears to be a surprisingly delicate problem, answerable in a few completely integrable cases.

Asymptotic Phenomena in Wave Propagation: Resonances and Instabilities

P. Werner, Stuttgart

I report on joint work with K. Morgenröther. We study the propagation of sound waves in domains with noncompact boundaries ("waveguides") and show that resonances occurring in certain unperturbed waveguides can be deleted by small perturbations of the boundary. This seems to be an interesting nonlinear phenomenon in linear wave propagation. Compare p. 140 of this book for related information.

Pseudo-Orbits of contact forms

Abbas Bahri, Ecole Polytechnique - Palaiseau —

This is an approach to the A. Weinstein conjecture, which states that a contact vector field on a $(2n+1)$-orientable, compact manifold M has a periodic orbit if $H^1(M, \mathbb{Z}) = 0$. This conjecture has, as of today, been proven under various hypotheses, mainly when M is the contact boundary of a symplectic manifold. The techniques we introduce are related to the concept of "critical points at infinity". We consider the functional $\int_0^1 \alpha_x(\dot{x})^2 dt$, α being the contact form on M, on the loop space $H^1(S^1; M)$. (x being a loop). In fact, we constrain, due to the ill-posedness of the problem, x to live in a submanifold of $H^1(S^1; M)$, namely $\mathcal{L}_\beta = \{x \in H^1(S^1; M) \text{ s.t } \beta_x(\dot{x}) \equiv 0\}$, where $\beta = d\alpha(v, \cdot)$ is a non singular differential one-form.

In this framework, the variational problem is "non-compact". We define ends to the gradient flow, which are geometric curves parametrized suitably. These ends are obtained through a convergence process, including cancellation of oscillations and persistence of others.

Existence proofs, under restricted hypotheses, are then derived using the following type contradiction argument: If there is no periodic solution, the Morse complex is described through the study of these ends. When these ends fail to represent the topology of $\mathcal{L}_\beta$, there must then be a periodic orbit —

Eigenvalue inclusions for elliptic differential operators by a numerical algorithm

Michael Plum, Köln

Object of consideration is the eigenvalue problem for linear symmetric elliptic differential operators (mainly of second order) with a discrete spectrum $\lambda_1 \le \lambda_2 \le \dots$ An algorithm and its theoretical background are presented which yield, for given n, guaranteed and close inclusion intervals for the first n eigenvalues. In particular, intervals containing no eigenvalue are computable.
The algorithm is based on Hilbert space analysis and an appropriate homotopy method. For practical numerical computations, a projection method is needed in order to calculate approximations to the eigenfunctions. Concrete examples are given to illustrate the method.

Vortex Dynamics: Numerical Analysis, Large Scale Computing, and Mathematical Theory

Andrew Majda, Princeton U.

The lecture involved the interplay between numerical analysis, large scale computing, and mathematical theory centering around ~~[illegible]~~ the phenomena in propagation of vorticity for 2-D fluid flows. First the equations of incompressible fluid flow ~~[illegible]~~ were reformulated as an integro-differential equation for the particle trajectories. Then an appropriate discretization of this formulation leads to computational vortex algorithms. A summary of the theory and numerical analysis was presented. Next results of numerical calculations for 2-D vortex sheets displaying incredible complexity were presented. Finally a recently developed theory of DiPerna and the author was described together with the predictions of this theory for the solutions ~~of~~ presented earlier through large scale simulation.

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Nilpotent groups and the spectrum of Schrödinger operators

Palle E.T. Jorgensen

Schrödinger operators with polynomial (curved) magnetic field are completly spectrally analyzed. We illustrate the results with the case $H = \frac{1}{2m}(\mathbb{P} - \frac{e}{c}A)^2$ of a spinless particle of mass m in an external field $\vec{B}(x)$, $\vec{B} = \nabla \times \vec{A}$, $\mathcal{H} = L^2(\mathbb{R}^3)$

Two cases are studied

(1) $\exists\ w \in \mathbb{R}^3 \setminus \{0\}$ st. $w \cdot \nabla \vec{B} = 0$

(2) $w \cdot \nabla \vec{B} \neq 0$ for all $w \in \mathbb{R}^3 \setminus \{0\}$

In case (1) $\exists\ \mathcal{O} \in O(3)$ st. $H = \frac{1}{2m}(\mathcal{O}\vec{P} - \vec{A}(x_1, x_2))^2$ relative to the rotated coordinates. The spectrum of H is continuous, $H = \int_{\mathbb{R}}^{\oplus} H(\xi)\, d\xi$ with $H(\xi)$ obtained from H by replacing $\frac{1}{\sqrt{-1}}\frac{\partial}{\partial x_3}$ with $\xi \in \mathbb{R}$. Each $H(\xi)$ has discrete spectrum. The spectral func. $N_\xi(\lambda) = \#\{\underset{\text{multy.}}{\text{evs}} \leq \lambda\}$ satisfies $N_\xi(\lambda) \leq \text{const.}\ \lambda^{\frac{D+1}{2}} \ln(\lambda B| + \lambda^{\frac{D+1}{2}})$ where D is the total (combined) degree.

$$\operatorname{trace}\left(e^{-tH(\xi)}\right) = \int_{\mathbb{R}^2} d^2x\ \hat{P}_t\left(0, 0, \xi - A_3(x), \vec{B}(x) + \vec{k}, D\vec{B}\right)$$

In case (2), H has purely discrete spectrum and

$$\mathrm{trace}\left(e^{-tH}\right) = \int_{\mathbb{R}^D} dy \; p_t(o,y) \int_{\mathbb{R}^D} dz \; e^{iE(o,y,z)}$$

when

$$U_g f(v) = e^{iE(g,v)} f(v+x)$$

$g=(x,y) \in$ some nilpotent Lie group, with Lie algebra generated by $\frac{\partial}{\partial x_j} - \sqrt{-1}\,A_j$,

$H = dU(X_1^2 + X_2^2 + X_3^2)$

$$\sum \tilde{X}_i^2 \, p_t(g) = \frac{\partial p_t}{\partial t}$$

$$p_t(g) = \delta(g) \,, \quad t=0$$

Spectral asymptotics is discussed in terms of the heat-kernel $p_t(g)$ on G.

Orbital Parameters for Induced Representations
Ron Lipsman, University of Maryland, 6/29/87

A general formula for the spectral decomposition of the quasi-regular representation is presented for connected Lie groups $H \subset G$. The formula -- which describes the actual spectrum, the multiplicity, and the spectral measure -- is in terms of the usual parameters in the so-called Orbit Method. The formula is proven in the nilpotent situation, and more generally when G is completely solvable. Indications are given for the general exponential solvable case. The situation of semisimple homogeneous spaces, especially symmetric spaces, is also discussed

Asymptotic estimates for some Green kernels on the Heisenberg group and the Martin boundary

D. Müller, Bielefeld

Let Δ_K denote the sub-(or Kohn)-Laplacian on the 3-dimensional Heisenberg group H_1. Δ_K has an explicit fundamental solution which had been calculated first by Folland. In contrast to this, it seems to be impossible to calculate explicit formulas for fundamental solutions of the corresponding „heat operator" $\partial/\partial s - \frac{1}{2}\Delta_K$ and the operator $\Delta_K - \mu$, $\mu > 0$. However, we can present a complete description of the asymptotic behaviour of these fundamental solutions. This enables us to determine the Martin compactification of H_1 with respect to the operator $\Delta_K - 2$. A consequence of this is the following: If $h \geq 0$ is any solution of $(\Delta_K - \mu)h = 0$, then h factorizes to a function on H_1/Z, Z the centre of H_1.
(joint work with H. Huber, Bielefeld)

Harmonic analysis and representation theory

Richard Penney, Purdue U.

"The Laplace Beltrami operator on unbounded homogeneous domains in $\mathbb{C}^n$."

Let $\Omega \subset \mathbb{C}^n$ be a domain. A Lie group G acts transitively on Ω if the action is ~~on~~ transitive and is analytic in G and holomorphic in Ω. We assume that Ω is contractible and has a G invariant volume which gives rise to a non-degenerate Koszul form. Then Ω carries a G-invariant pseudo Kahlerian structure. Under some additional assumptions on Ω, we are able to explicitly diagonalize the Laplace Beltrami operator associated with this structure. We apply this to decide when certain domains are not biholomorphic.

Norms of free operators

Tadeusz Pytlik (Wrocław)

A short proof of a theorem by Akemann and Ostrand is given with some applications to the theorem by Powers that the reduced C^*-algebra of the free group is simple. The Akemann and Ostrand theorem gives explicite formula for a convolution operator supported by a free set. i.e

$$\|\lambda(g)\|_{C^*_\lambda} = \inf_{s>0} \left(2s + \sum_{x\in \mathrm{supp}\, g} \left(\sqrt{|g(x)|^2+s^2} - s\right)\right)$$

Unitarizable highest weight representations of loop groups

Hans P. Jakobsen (Copenhagen)

Let $\mathfrak{g}$ be a simple complex Lie algebra and let $\hat{L}(\mathfrak{g}) = \mathbb{C}[z,z^{-1}]\otimes\mathfrak{g}\oplus\mathbb{C}\cdot c$ be the associated Kac-Moody algebra. We report here on joint work with Victor Kac in which we determine the full set of unitarizable highest weight modules of $\hat{L}(\mathfrak{g})$. The answer is that one should add, to the list we gave in Springer Lecture Notes in Physics, #226, the highest component of a $\otimes$-product of an elementary representation with an exceptional. Further work shows more generally how to decompose $\otimes$-products of these non-standard representations, and, using the uniqueness of highest weight modules, we show how to integrate these to projective representations of loop groups. It is finally shown how one can replace $\mathbb{C}[z,z^{-1}]$ by either the algebraic part of the irrational rotation algebra or the set of finite rank operators, and shall obtain unitarizable representations.

Irreducible Representations which cannot be Separated from the Identity

Mohammed Bekka (Tech. Univ., München)

The Cortex of a locally compact group is defined to be the set of all irreducible representations which cannot be separated from the trivial one-dimensional representation. This set has been considered by Guichardet and by Vershik and Karpushev in connection with cohomology with values in unitary representations.
We give some examples for the computation of the cortex in the case of a semi-simple and in the case of a nilpotent Lie group. For IN-groups (i.e. groups with a compact invariant neighbourhood of the group unit) we have a complete description of the cortex. If G is such a group and if G_F denotes the (open) normal subgroup consisting of all relatively compact conjugacy classes, then the cortex of G is $(G/G_F)^\wedge$. We also give an application to the cohomology theory of such a group. (Joint work with E. Kaniuth)

Differential Geometry on the dual of a Lie group.

Several notions are introduced and discussed, for a Lie group G, which generalize classical notions of differential geometry on $\mathbb{R}^n$: smooth functions, their 0-jets, n-jets, ∞-jets at a point $\pi \in \hat{G}$, the category $\mathrm{Ext}(G, \pi)$ and its Galois algebra C. The algebra of smooth functions, $\mathcal{S}(G)$, is known for nilpotent-motion- and semi-simple groups, and partially for soluble groups; it is conjectured that, if π is CCR, $\pi(\mathcal{S}(G))$ is always isomorphic to a universal algebra $\mathcal{W}$, the algebra of all infinite matrices with rapidly decreasing coefficients; this conjecture is proved by F. du Cloux for nilpotent groups, and is also true for semi-simple groups. Another conjecture claims that the algebra of ∞-jets, $\varprojlim \mathcal{S}(G)/M_{\pi}^{n+1}$, has a Taylor decomposition as $\pi(\mathcal{S}(G)) \otimes C$; it is proved in case G is nilpotent, or π is trivial.

A. Guichardet

Schrödinger operators with magnetic fields and representation theory of nilpotent Lie Groups.

by B. Helffer

We present results obtained in collaboration with Abderemane Mohamed. We consider the operator $H(A,V) = \sum_{j=1}^{n} (D_{x_j} - A_j)^2 + \sum_{j=1}^{p} V_j(x)^2$ with $V_j \in C^\infty$, $A_j \in C^\infty(\mathbb{R}^n)$ and give the following sufficient condition under which the Friedrichs self-adjoint extension starting from $C_0^\infty(\mathbb{R}^n)$ has a compact resolvent.

Let us define
$$m_\ell(x) = \sum_{\substack{j \\ |\alpha| = \ell}} |\partial_x^\alpha V_j| + \sum_{\substack{j,k \\ |\alpha| = \ell - 1}} |\partial_x^\alpha B_{jk}|$$
with $B_{jk} = \partial_{x_j} A_k - \partial_{x_k} A_j$

$$m^{(r)}(x) = 1 + \sum_{\ell=0}^{r} m_\ell(x)$$

Then if $m^{(r)}(x) \underset{|x| \to \infty}{\longrightarrow} \infty$ and $m_{r+1}(x) \le C\, m^{(r)}(x)$ then, $H(A,V)$ is with compact resolvent.

If this criterion is not satisfied, we discuss the essential spectrum in connection with the spectrum of a family of ~~expon~~ Schrödinger operators with polynomial potentials which can be seen as the image by a family of representations of a sublaplacian on a "universal" nilpotent Lie group.

Some monomial representations of exponential groups II

Hidenori Fujiwara (Fukuoka)

We consider the Frobenius reciprocity for monomial representations of exponential groups and Penney's Plancherel formula for those constructed from real polarizations.

Let $G = \exp \mathfrak{g}$ be an exponential group, $H = \exp \mathfrak{h}$ an analytic subgroup and let χ be a unitary character of H. We decompose the induced representation

$$\tau = \mathrm{ind}_H^G \chi = \int_{\hat{G}}^{\oplus} m(\pi)\, \pi\, d\nu(\pi).$$

Then, in certain cases, $m(\pi)$ is given by $\dim (\mathcal{H}_\pi^{-\infty})^{H,\, \chi\Delta_{H,G}^{1/2}}$; the dimension of some space of semi-invariant distributions.

When $\mathfrak{h}$ is a polarization, the decomposition of the Dirac measure δ_τ for τ gives

$$\langle \tau(\phi)\delta_\tau, \delta_\tau \rangle = \sum_{\Omega \in \mathfrak{g}^+/G:\,\text{finite}} \ \sum_{k:\,\text{finite}} \langle \pi(\Omega)(\phi)\, a_\Omega^k, a_\Omega^k \rangle \qquad (\phi \in C_c^\infty(G))$$

with some $a_\Omega^k \in (\mathcal{H}_{\pi(\Omega)}^{-\infty})^{H,\, \chi\Delta_{H,G}^{1/2}}$. We write these a_Ω^k in an explicit form.

Representations of the Mautner Group

Larry Baggett (Univ. of Colorado, Boulder)

Let U be an irreducible unitary representation of $G_1 \times G_2$. If either G_1 or G_2 is of type I, then U splits, i.e., U is equivalent to $U_1 \times U_2$ for U_i a representation of G_i. Examples are given of representations U that do not split, for $G_1 = G_2 = M$, the non-type I group known as the Mautner group.

Semi-group of operators on $C_0(G)$ generated by $-\sum (-1)^{n_j} X_j^{2n_j}$.

Andrzej Hulanicki (Wrocław)

Let G be a nilpotent Lie group, $X_1, \dots, X_k$ generating set of elements of the Lie algebra of G. Let

$$L = \sum (-1)^{n_j} X_j^{2n_j}.$$

The semi-group T_t generated by L has the form

$$T_t f = f * p_t ,$$

where $p_t \in C^\infty(G) \cap L^1(G)$. The map $t \to p_t \in L^1(G)$ extends holomorphically to $S_\delta = \{z : \operatorname{Arg} z < \delta\}$ and for every $z \in S_\delta$ and $\mathcal{D} \in \mathfrak{S}$ the envelloping algebra of G we have

$$|\mathcal{D} p_z(x)| \leq C_{\alpha,z} e^{-\alpha|x|}$$

for all $\alpha > 0$, where $|x|$ is the riemannian distance of x to e.

(This is a joint work with JACEK DZIUBAŃSKI).

A. Hulanicki

Completely bounded multipliers of the Fourier algeba of the free group

Ryszard Szwarc (Wrocław)

Let G be a locally compact group. ~~and~~ A function φ on G is called a multiplier of $A(G)$ if $\varphi\psi$ is in $A(G)$ for any ψ in $A(G)$. φ is called a completely bounded multiplier if the adjoint operator M_φ to $A(G) \ni \psi \longmapsto \varphi\psi \in A(G)$ is completely bounded.

Let φ be a radial function on the free group $\mathbb{F}_N$ i.e. $\varphi = \sum \dot\varphi(n) \chi_n$ where $\dot\varphi(n) \in \mathbb{C}$, and χ_n is the characteristic function of words of length n. Then φ is a completely bounded multiplier of $A(G)$ iff the hankel matrix h with entries $h_{ij} = \dot\varphi(i+j) - \dot\varphi(i+j+2)$, $i,j = 0,1,2,\dots$ is of trace class.

The formula for the completely bounded norm is given explicitly.
As a corollary one gets that every completely bounded multiplier
is represented by the integral of functions $z^{|x|}$, $|z|<1$ or $z=\pm 1$
with respect to some measure on the unit disc.
(This is a joint work with Uffe Haagerup).

Ryszard Szwarc

Bi-invariant Schwartz Multipliers on Nilpotent Lie Groups

Joe W Jenkins, the University at Albany, SUNY

Let N be a connected, simply connected nilpotent Lie group with Lie algebra $\mathfrak{n}$, and dual $\mathfrak{n}^*$. Let $\mathcal{S}(N)$ denote the Schwartz space on N and $\mathcal{S}^*(N)$ its dual.

Th$^{\underline{m}}$ A. For $D \in \mathcal{S}^*(N)$, convolution by D is a bi-invariant, continuous endomorphism of $\mathcal{S}(N)$ if, and only if, $\hat{D}$ is a smooth, Ad^*-invariant function on $\mathfrak{n}^*$ with polynomial bounds on all derivatives.

Th$^{\underline{m}}$ B. Suppose $D \in \mathcal{S}^*(N)$ that satisfies the equivalent conditions of Th$^{\underline{m}}$ A. For $\xi \in \mathfrak{n}^*$, $\pi_\xi(D*f) = \hat{D}(\xi)\pi_\xi(f)$ for each $f \in \mathcal{S}(N)$, where π_ξ is the irreducible unitary representation of N corresponding to the Ad^*-orbit of ξ.

Globally hypoelliptic systems of vector fields on nilmanifolds

Jacek Cygan, Louisiana State University, Baton Rouge, L

A system of differential operators $D_1, \dots, D_m$ on a C^∞-manifold M is globally hypoelliptic (GH) if when $D_1 f = g_1, \dots, D_m f = g_m$ with $f \in \mathcal{D}'(M)$

$g_1, \dots, g_m \in C^\infty(M)$, then $f \in C^\infty(M)$. We show that a system $\mathbb{L}$ of real vector fields on a general compact nilmanifold $M = \Gamma \backslash N$ induced by the Lie algebra $\mathfrak{n}$ of N is (GH) iff 1° The symbols of the vector fields of $\mathbb{L}$ projected onto the associated torus $\mathbb{T}^n = \Gamma[N,N] \backslash N$ as functions on the integral lattice $\mathbb{Z}^n = \widehat{\mathbb{T}^n}$ collectively decrease at infinity not faster than a reciprocal of a polynomial and 2° The Lie subalgebra of $\mathfrak{n}$ that $\mathbb{L}$ generates is not annihilated by any non-zero <u>integral</u> linear functional on any $\mathfrak{n}_j / \mathfrak{n}_{j+1}$, $j = 0, 1, \dots$ ($\mathfrak{n}_{j+1} = [\mathfrak{n}, \mathfrak{n}_j]$, $\mathfrak{n}_0 = \mathfrak{n}$). It follows that (GH) is equivalent to injectivity of the system $\mathbb{L}$ on the dual of the space of C^∞-vectors of all the non-trivial representations in the spectrum of $\Gamma \backslash N$ (a "Rockland type" condition) plus a number-theoretic condition on $\mathbb{L}$ on the associated torus (to avoid "small divisors"). (Joint work with L.F. Richardson.)

<u>Local solvability of diff. operators, and representation theory</u>

The aim of this talk is to show how technics introduced in the study of local solvability on nilpotent Lie groups can be adapted for homogeneous spaces of such groups. Let G be a graded nilpotent connected and s.c Lie group, with Lie algebra $\mathfrak{g}$, and H a closed connected graded subgroup. P is an homogeneous element of $\mathcal{U}_{\mathbb{C}}(\mathfrak{g})$, regarded as diff. operator on $_H\backslash G$. Sufficient conditions for local solvability of P are given in terms of the images of ^tP under the representations of the spectrum of the rep. of G, induced from trivial character of H, in the following cases:

1) Nilpotent symmetric space (using Benoist's Plancherel formula).
2) Grušhin-type operators.
3) $\mathfrak{g}$ is 4-dimensional with brackets: $[X,Y]=Z$, $[Y,Z]=T$ and $\mathfrak{h} = \mathbb{R}X$, or $\mathbb{R}Y$, or $\mathbb{R}Z$ or $\mathbb{R}Y \oplus \mathbb{R}Z$.

P. Lévy-Bruhl

Dual Topology of an exponential group

Jean Ludwig

Let G be an exponential group with Lie algebra $\mathfrak{g}$. If one tries to prove that the inverse of the Kirillov-map K, which assignes to every G-orbit O in $\mathfrak{g}^*$ its representation $\pi \in \widehat{G}$, one has to cope with the following situation. We are given a sequence $\pi_k \subset \widehat{G}$, such that for every k, there exists no normal connected subgroup H_k, with $\pi_k = \mathrm{ind}_{H_k}^{G} \tau_k$, for some $\tau_k \in \widehat{H}_k$. Example: $\mathfrak{g} = \langle T, X, Y, Z \rangle$, $[T,X] = -X$, $[T,Y] = Y$, $[X,Y] = Z$; $\pi_k(\exp tZ) = e^{-it\lambda_k}$, $\lambda_k \neq 0$. It is relatively easy to handle the case $\pi_\infty = \lim \pi_k$, and $d\pi_\infty(Y) \neq 0$, using variable group techniques. The case $d\pi_\infty(Y) = 0$ is more difficult. We must change the representation π_k into representations $\tilde{\pi}_k$, such that some control is conserved on $\tilde{\pi}_\infty = \lim \tilde{\pi}_k$, and such that $d\tilde{\pi}_\infty(Y) \neq 0$. This can again be done by using the method of variable groups. It is possible to extend these methods to general variable exponential groups and thus to prove that K is a homeomorphism.

Asymptotics of matrix coefficients and orbit equivalence of actions.

If G_1, G_2 are groups acting on measure spaces S_1 and S_2 respectively, leaving a measure quasi-invariant, an orbit equivalence between the two actions is a measure-class preserving Borel isomorphism between S_1 and S_2 that takes orbits to orbits.

Let G be a group of the form $GL_n(F)$, $SL_n(F)$ or $Sp_{n/2}(F)$ ($F = \mathbb{R}$ or $\mathbb{C}$) and let V_i $(i = 1, 2)$ be finite dimensional irreducible modules for G defined over F.

We examine the question: Can $G \ltimes V_1$ and $G \ltimes V_2$ have properly ergodic, orbit equivalent actions with finite invariant measure?

An integral invariant $r(V_i)$, $0 \leq r(V_i) \leq n/2$ of the module V_i is defined. The number $r(V_i)$ can be easily computed from the highest weight of V_i. Representation-theoretically, $r(V_i) = \min N\text{-rk}\, \rho|_G$, where the minimum is taken over the set of irreducible unitary representations of $G \ltimes V_i$ having no V_i-invariant vectors, and $N\text{-rk}\, \tau$

denotes the rank of the representation τ. We prove the following

<u>Theorem</u> Let G and V_i be as above, and suppose $r(V_1) < r(V_2)$ $r(V_2) \leq \frac{n-2}{3}$ $(\frac{n}{3}-2$ if $G = Sp_{n/2})$. Suppose $G \ltimes V_i$ acts essentially freely, and properly ergodically on S_i, with finite invariant measure. Assume that V_2 acts ergodically. Then the actions on S_1 and S_2 are not orbit equivalent.

Roberto Scaramuzzi (U. of Chicago)
(joint work with R. Zimmer)

The restriction theorem for K/M

Let $\underline{g}$ be a real simple Lie-algebra, $\underline{g} = \underline{p} \oplus \underline{k}$ a Cartan decomposition and K a compact Lie group with Lie-algebra $\underline{k}$, which acts on $\underline{p}$ by $Ad_G k$, G the Lie group corresponding to $\underline{g}$. Furthermore let $\underline{a}$ be a maximal abelian subspace of $\underline{p}$.

We show for the Fourier transform on $\underline{p}$ the following restriction

Theorem: If $f \in L^q(\underline{p})$, $1 \leq q < 2\,\frac{N+l}{N+3l}$, $l = \dim \underline{a}$, $N = \dim \underline{p}$, then for regular $X \in \underline{p}$

$$\int_K |\hat{f}(Ad\,k\,X)|^2\,dk \leq C_X \|f\|_q^2$$

For $q > 2\,\frac{N+l}{N+3l}$ such an inequality fails.

Gerd Mockenhaupt

<u>Eigenfunctions on Riemannian symmetric spaces</u>

Let G/K be a Riemannian symmetric space of the noncompact type with boundary $B = K/M$. Let $f \in C^\infty(G/K)$ be a joint eigenfunction for the invariant differential operators on G/K. If f has at most exponential growth, f has an asymptotic expansion, whose coefficients are distributions on B. It is shown, that knowledge of these distributions on an open subset of B determines f uniquely. This result was obtained in collaboration with Professor E. P. v. d. Ban of Utrecht.

H. Schlichtkrull

Hypergeometric functions associated with root systems

Let $R \subset \mathfrak{a}^*$ be a rank n root system, $\mathfrak{h} = \mathfrak{a} \oplus i\mathfrak{a}$ and $H = \exp \mathfrak{h}$ the complex torus with characterlattice P (the weightlattice of R). For a choice of a multiplicity function $k = (k_\alpha)_{\alpha \in R}$ we have the differential operator $L(k) = \sum_{j=1}^{n} \partial(X_j)^2 + \sum_{\alpha \in R_+} k_\alpha \operatorname{cth}(\frac{\alpha}{2}) \partial(X_\alpha)$ on H. We define the family of hypergeometric functions ass. with R as eigenfunctions of $L(k)$ which have some prescribed monodromy type (viewed as Nilsson-class functions on $W \backslash H^{reg}$) and are analytic in a neighbourhood of $e \in W\backslash H$. It turns out to be an analytic family of functions. From that fact we are able to deduce that the commutant of $L(k)$ in $(\mathcal{S} \otimes \mathcal{U}(\mathfrak{h}))^W$ ($\mathcal{S}$ the algebra of functions on H generated by h^λ $(\lambda \in P)$ and $\frac{1}{1-h^\alpha}$ $(\alpha \in R_+)$) is isomorphic to a polynomial algebra in n variables

(joint work with G. Heckman). Eric Opdam

Topological Frobenius properties of locally compact groups

A l.c. group G is said to have Fell's topological Frobenius property (FP) if for any closed subgroup H of G, $\tau \in \hat{H}$, and $\pi \in \hat{G}$, $\mathrm{ind}_H^G \tau$ weakly contains π iff $\pi|H$ weakly contains τ. The if and the only if parts of (FP) are denoted by (FP1) and (FP2), respectively. It was known that an amenable group G with open connected component satisfying (FP2) is $[FC]^-$, i.e. every conjugacy class is relatively compact. It turns out that conversely $[FC]^-$ groups have property (FP). We present several results concerning (FP1), e.g.

(1) A countable discrete group G satisfies (FP1) iff G is amenable and has a T_1 primitive ideal space; (2) Every 2-step nilpotent pro-Lie group has property (FP1); (3) Let G be a simply connected nilpotent Lie group. If all the Kirillov orbits of G are linear varieties, then (FP1) holds for G. The converse is open, but true if G is of the form $\mathbb{R} \ltimes \mathbb{R}^n$.

[(2) and (3) as well as further results are joint work with M. Bekka]

E. Kaniuth (Paderborn)

Some examples of generalized Weil representations

The classical construction of Weil representations for the groups $SL_2(k)$ (k local or finite field), via the Heisenberg group may be extended to the groups $SL_n(k)$ (at least for even n) in a very natural way as follows: Introduced the generalized Heisenberg group $H(V)$, associated to any k-vector space V, defined as the subgroup $1+\bigoplus_{i\geq 1}\Lambda^i V$ of "unipotent elements" of the multiplicative group $(\Lambda V)^\times$ of the Grassmann Algebra ΛV of V over k. For even dimensional V (say $\dim V = n$) the analogue of Stone-von Neumann Theorem holds and, since the natural action of $SL(V)$ fixes the isomorphy type of the corresponding "Schrödinger representation" of $H(V)$, we obtain a projective 2^{n-2} dimensional representation of $SL(V) \simeq SL_n(k)$, which reduces to the usual Weil representation of $SL_2(k)$ for $n=2$. Geometric variants of this method may be adapted to the case of odd n.

(joint with J. Pantoja (Valparaíso)).

J. Soto-Andrade
(Univ. Chile, Santiago)

Spectrum and multiplicity for Induced and Restricted representations of Nilpotent Lie groups.

Let $g \supseteq k$ be nilpotent Lie algebras, G, K their groups, $\sigma \in \hat{K}$, $\pi \in \hat{G}$ irreducible representations associated with Kirillov orbits $O_\sigma \subseteq k^*$, $O_\pi \subseteq g^*$. Let $P: g^* \to k^*$ be the natural map. We ~~may~~ describe the spectrum + multiplicity of $\rho = \mathrm{Ind}(K\uparrow G, \sigma)$ or $\pi|K$ in terms of geometry of coadjoint orbits. Define a "defect index" r_0 = generic value of

$$\dim G\cdot l - 2\dim K\cdot l + \dim K\cdot P(l) \quad \begin{cases} \text{for } l \in P^{-1}(O_\sigma) \text{ with induction } \rho \\ \text{for } l \in O_\pi \text{ with restriction } \pi|K \end{cases}$$

Thm 1 (Induction) $\rho \cong \int_{G^\wedge}^{\oplus} m(\pi)\pi \, d\mu'(\pi)$ where

$$m(\pi) = \# K\text{-orbits in } O_\pi \cap P^{-1}(O_\sigma)$$

Here μ' is the natural measure class μ on $P^{-1}(O_\sigma)$ pushed forward to $G^\wedge$ via $l \to \pi_l$; the spectrum is essentially $\mathrm{spec}(\rho) = \{\pi : O_\pi \cap P^{-1}(O_\sigma) \neq \emptyset\}$. If $r_0 > 0$ we have $m(\pi) \equiv \infty$ μ'-a.e. on $\mathrm{spec}(\rho)$, and if $r_0 = 0$ $\exists N$ st $m(\pi) \leq N$ μ'-a.e.

Thm 2 (Restriction) $\pi|K \cong \int_{K^\wedge}^{\oplus} m(\sigma)\sigma \, d\mu'(\sigma)$ where

$m(\sigma) = \#$ K-orbits in $\mathcal{O}_\pi \cap P^{-1}(\mathcal{O}_\sigma)$

where μ' is G-invariant measure on $\mathcal{O}_\pi$ pushed forward to $\hat{K}$ via $\ell \to P(\ell) \to \sigma_{P(\ell)} \in \hat{K}$; the spectrum is $\mathrm{spec}(\pi|K) = \{\sigma \in \hat{K} : P^{-1}(\mathcal{O}_\sigma) \cap \mathcal{O}_\pi \neq \phi\}$. If $\tau_0 > 0$ we have $m(\sigma) \equiv \infty$, μ'-a.e. $\sigma \in \mathrm{spec}(\pi|K)$, and if $\tau_0 = 0$ then $\exists N$ st $m(\sigma) \leq N$ μ'-a.e.

This result includes the decomposition of arbitrary tensor product $\pi_1 \otimes \pi_2$, $\pi_i \in G^\wedge$; just take $\pi_1 \times \pi_2$ on $G \times G$ and restrict to the diagonal subgroup Δ.

Earlier descriptions of $\mathrm{spec}(\rho)$ were given by Kirillov, in his thesis, without any attempt to give the multiplicities. Some hints of the induction orbital formula for $m(\sigma)$ occurred in work of Vergne [$\mathbb{G}$ solvable exponential, σ corresponding to a maximal subordinate subgroup K for some $\ell \in g^*$, not satisfying the Pukanszky condition. (Thus $\mathrm{Ind}(K \uparrow G, \sigma)$ is not irreducible] and of G. Grelaud (C. Rendus, 1973) [G nilpotent, K normal connected, $\sigma \in \hat{K}$] ~~However~~ ~~the~~ There was no conjecture about an orbital formula until the work of Grelaud and Corwin + Greenleaf, who independently proved Thm 1. Thm 2 is due to Corwin-Greenleaf, who have recently prepared a manuscript giving a unified and fairly canonical proof of both Thms.

By complexifying the Pukanszky parametrization of orbits in g^*, and the Kirillov theory, one can apply results from complex algebraic geometry to ~~[illegible]~~ reveal finer features of the multiplicities. One can prove, typically,

Thm 3 The parity of $m(\pi)$ (value mod 2) is constant on the spectrum for induced or restricted representations [we may have $m(\pi) \equiv \infty$]

Thm 4 Let $g \supseteq k$ be complex Lie algebras, G, K their groups regarded as real Lie groups. If $\sigma \in K^\wedge$, then $m(\pi)$ is constant on $\mathrm{spec}(\rho)$ [possibly $m(\pi) \equiv \infty$].

These results are due to Corwin + Greenleaf [Trans Amer Math Soc, to appear 1988].

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Unitary representations of a solvable Lie groups on $\bar\partial_b$-cohomology spaces

Let $(\mathfrak{g}, j, \omega)$ be a normal j-algebra and G the connected and simply connected (completely) solvable Lie group corresponding to $\mathfrak{g}$. I give a unitary representation of G in which every irreducible (up to a set of measure zero) occurs with multiplicity one, relating its construction to a certain geometric structure (= CR structure) of a nilpotent normal subgroup $N(D)$ of G. This subgroup $N(D)$ is canonically diffeomorphic to the Šilov boundary $S(D)$ of a Siegel domain D of type II on which G acts simply transitively by affine automorphisms. I will define unitary representations of G on $\bar\partial_b$ cohomology spaces on $S(D) \simeq N(D)$. Note that there is no G-invariant Riemannian metric on $S(D)$. This work will be published in Japanese Journal of Mathematics N.S. vol 13 No. 2 1987.

Takaaki NOMURA 野村隆昭.

On the symplectic structure of coadjoint orbits of (solvable) Lie groups.

Let G be a connected Lie group with Lie algebra $\mathfrak{g}$, and let $O \subset \mathfrak{g}^*$ be a coadjoint orbit of G. Suppose that $\mathfrak{h}$ is a *real* polarization at $g \in O$. Set H_0 to be the analytic subgroup corresponding to $\mathfrak{h}$, and set $H = G_g H_0$. Suppose further that $\mathfrak{h}$ is *integral*, i.e. there exists a character $\chi : H \to \mathbb{T}$ such that $\chi(\exp X) = e^{i\langle g, X\rangle}$, $X \in \mathfrak{h}$. It follows from Kostant's theory of geometric quantiation that one can define a Lie algebra homomorphism $\delta_\chi : \mathcal{Q}^1(O, \mathfrak{g}, \mathcal{G}) \to \mathcal{B}^1(G, \chi)$ from the space of quantizable functions on O defined by $\mathcal{G}$ to the first order differential operators $\mathcal{B}^1(G, \chi)$ in the homogeneous linebundle over G/H defined by χ.

<u>Theorem</u>: $\delta_\chi : \mathcal{Q}^1(O, \mathfrak{g}, \mathcal{G}) \to \mathcal{B}^1(G, \chi)$ is a Lie algebra <u>isomorphism</u>.

As an application we show that there exists on each coadjoint orbit of an exponential group coordinates $(p_1, \dots, p_{d/2}, q_1, \dots, q_{d/2})$ such that $\{p_r, p_s\} = 0$, $\{q_r, q_s\} = 0$, $\{p_r, q_s\} = \delta_{rs}$ ("canonical" coordinates).

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On the periodicity of group actions

Given a topological space X, an abstract group Γ and a homomorphism $\lambda : \Gamma \to H(X)$ into the group of all homeomorphisms of X into itself. We consider the following periodicity properties which the action of Γ on X may have or not

$P_1(\Gamma, X)$: $o_\Gamma(x) = |\Gamma x| < \infty \ \forall x \in X$ (for short $\lambda(\gamma)x = \gamma \cdot x$)

$P_2(\Gamma, X)$: o_Γ is bounded , $P_3(\Gamma, X)$: $\lambda(\Gamma)$ is finite

The following results are given

(1) $P_2(\Gamma, X) \iff \exists\ \Gamma \leq \tilde{\Gamma} \leq G$ s.t. $[\tilde{\Gamma} : \Gamma] < \infty$, $P_3(\Gamma, G/\tilde{\Gamma})$

$\iff \exists$ normal $N \trianglelefteq G$ st. $[\Gamma : \Gamma \cap N] < \infty$, $[N : \Gamma \cap N] < \infty$

(2) given a group A, $\Gamma \leq \operatorname{Aut} A$: $P_2(\Gamma, A) \iff \exists$ finite normal $E \trianglelefteq A$ s.t. $P_3(\Gamma, A/E)$

(3) $\mathfrak{A}$ Boolean algebra, $\Gamma \leq \mathrm{Aut}\,\mathfrak{A}$: $P_2(\Gamma, \mathfrak{A}) \Leftrightarrow \Gamma$ finite

(4) A Frechet space, $\Gamma \leq GL(A)$: $P_1(\Gamma, \mathfrak{A}) \Leftrightarrow \Gamma$ finite

(5) G l.c. gr., $\Gamma = \bar{\Gamma} \leq G$, $X = G/\Gamma$ compact or connected then $P_1(\Gamma, X) \Leftrightarrow P_2(\Gamma, X)$

(6) G l.c. gr., $\Gamma = \bar{\Gamma} \leq G$, $X = G/\Gamma$, G compact or connected then $P_1(\Gamma, X) \Leftrightarrow P_3(\Gamma, X)$

(7) A compact gr., $\Gamma \leq \mathrm{Aut}\,A$ then
$P_1(\Gamma, A) + P_1(\Gamma, \hat{A}) \Leftrightarrow \Gamma$ finite

(2) generalises a result of R. Baer where A is supposed to be abelian.

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Principal Series of Fuchsian Group

Let $G := SL(2, \mathbb{R})$, Γ some discrete cofinite subgroup of G. Let π be an irreducible unitary representation of G. When is $\pi|_\Gamma$ irreducible? Exactly when π is unitary or complementary series, the specific group Γ being irrelevant. This is proved, roughly, by showing that the ~~the~~ operators $(\pi(\gamma))_{\gamma\in\Gamma}$ are dense enough to effect averages with respect to $K := \left\{\begin{pmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{pmatrix};\ \theta \in \mathbb{R}\right\}$.

GELFAND PAIRS AND GENERALIZED HALF-PLANES

Let G be a separable locally compact group and K a compact group of continuous automorphisms of G. It is known that $L^1_K(G)$, the algebra of the K-invariant integrable functions on G, is commutative if and only if the trivial one-dimensional representation of K occurs at most once in every irreducible representation of the semidirect product $K \ltimes G$ (then $(K \ltimes G, K)$ is a «Gelfand pair») – I prove a characterization of the commutativity of $L^1_K(G)$ which concerns only representations of G and of certain subgroups of K, avoiding the induction process to $K \ltimes G$ – I apply this result to prove in which cases the Šilov boundary of a classical symmetric Siegel domain of type II, with a compact group of automorphisms naturally associated to it, is a Gelfand pair –

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Groups of Polynomial Growth

Let G be a locally compact group generated by a compact neighbourhood W. G is said to be of polynomial growth if $\lambda(W^n) = O(n^k)$ for some k.
If $\mathfrak{g}$ is a Lie algebra, G acts on $\mathfrak{g}$, then $\mathfrak{g}$ is said to be of type R_G if the eigenvalues of the automorphisms from G have absolute value 1.
If G is a Lie group let $\mathfrak{g}$ be the Lie algebra of G_0 (the connected component), Z the center of G_0, K the maximal compact subgroup of Z with Lie algebra k.
Theorem 1: A Lie group G has polynomial growth iff G/G_0 has polynomial growth and $\mathfrak{g}/k$ is of type R_G.

Theorem 2: If G is a compactly generated group of polynomial growth, then it has a compact normal subgroup K such that G/K is a Lie group.
Corollary: G (comp. gen.) has polynomial growth iff there exists a normal series $C \triangleleft R \triangleleft N \triangleleft G$ such that C, G/N are compact,

R/K is a (solvable) Lie group with Lie algebra of type R_G and N/R is a discrete nilpotent group.
This implies that for such G the algebra $L^1(G)$ is symmetric.
The theorems generalize results (and use techniques) of Gromov and Jenkins.

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Maximal functions on some solvable Lie groups

We consider maximal function operators of Hardy-Littlewood type. Let $\{B_\rho\}_{\rho>0}$ be a family of neighbourhoods of e in a Lie group G. Define the corresponding left-invariant (resp. right-invariant) maximal functions, for $f \geq 0$:

$$\mathcal{M}_\ell f(x) = \sup_\rho |xB_\rho|^{-1} \int_{xB_\rho} f\, d\mu_\ell \; ; \; \mathcal{M}_r f(x) = \sup_\rho |B_\rho x|^{-1} \int_{B_\rho x} f\, d\mu_\ell$$

For the 'ax+b' group, let

(1) $B_\rho^{(1)} = \{x : d(x,e) < \rho\}$ where ρ is the hyperbolic metric;

(2) $B_\rho^{(2)} = \{(a,b) : e^{-\rho} < a < e^\rho, \; |b| < \rho^Q\}$ $Q>0$

(3) $B_\rho^{(3)} = \{(a,b) : e^{-\rho} < a < e^\rho, \; -a\rho < b < a\rho\}$

(4) $B_\rho^{(4)} = \{(a,b) : e^{-\rho} < a < e^\rho, \; -ae^{Q\rho} < b < ae^{Q\rho}\}$.

THEOREM 1. $\mathcal{M}_r$, defined with respect to (1), is not of weak type (1,1)

THEOREM 2. $\mathcal{M}_\ell$ is of weak type (1,1) but $\mathcal{M}_r$ is not of type (p,p) if $+\infty > p > 1$, when $\mathcal{M}_\ell$ and $\mathcal{M}_r$ are defined w.r.t. (2).

THEOREM 3. When the system (4) is taken, $\mathcal{M}_r$ is not of weak type (1,1) and is of type (p,p) $p>1$, if $0 < Q < 1$. On the other hand, if $Q>1$, $\mathcal{M}_r$ is of weak type (1,1) and à fortiori is of strong type (p,p) if $p>1$.

This is joint work with S. Giulini, A. Hulanicki, A. Mantero, P. Sjögren.

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G-equivariant K-theory for non-compact groups.

Let G be a locally compact group which act as a group of automorphisms of a C^*-algebra A; then $C^*(G,A)$ is the full cross-product algebra and $C^*_r(G,A)$ is the reduced cross product algebra. If K is the maximal compact subgroup of G, then there is a map, called deRham induction, from $K(C^*(K,A)) \xrightarrow{D} K(C^*_r(G,A))$. We have two theorems on computing the K-groups in this situation.

Theorem 1 (Fox-Haskell-Raeburn) If $A = C(X)$ with X compact Hausdorff and G is a connected reductive group ~~[illegible]~~ that acts properly on X with finitely many orbit types. Then de Rham induction induces an isomorphism from $K(C^*(K,X)) \to K(C^*(G,X))$.

Theorem 2 (Fox-Haskell) If $G = SU(n,1)$ and A is any C^*-algebra then
$K(C^*(SU(n,1),A)) \cong K(C^*_r(SU(n,1)))$. Thus $SU(n,1)$ is K amenable.

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Random Walks on localy infinite trees.

We consider a Random walk with finitely additive transition probabilities on a free group on infinitely many generators. We prove ~~[illegible]~~ results for transient states and we classify the state space

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We define the moment map of a representation and show that for the case G semisimple compact it allows one to invert the geometric quantization procedure of Kostant & Souriau. Some functorial properties are exhibited and we show how in the special case of $SU(2)$ the notion of constellation (as used by Bacry) arises naturally as zeros of the sections of the line bundles over the 2-sphere which determine the representation.

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Eisenstein integrals for semisimple symmetric spaces.

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Let G be a real connected semisimple Liegroup with finite centre, σ an involution of G and $H = G^\sigma$. In the talk we describe the asymptotic behaviour of Eisenstein integrals for the semisimple symmetric space G/H.

There exists a Cartan involution θ s.t. $\sigma \circ \theta = \theta \circ \sigma$. Let $K = G^\theta$. The "most continuous" part of the Plancherel decomposition of $L^2(G/H)$ is expected to come from unitary principal series representations $\pi_{\xi\lambda}$ obtained by induction from a ~~minim~~ parabolic subgroup P, minimal subject to the condition $\theta P = \sigma P = \overline{P}$. To these data one can associate Eisenstein integrals: they are essentially matrix coefficients obtained by pairing K-finite vectors with H-fixed generalized vectors of $\pi_{\xi\lambda}$. Asymptotically they behave like sums of vector valued plane waves. One can arrange the associated amplitudes in vectors with common Hermitian norm. This seems to be the appropriate generalization of what Harish-Chandra calls Maass-Selberg relations in the case of a group $G = G \times G / d(G)$. We hope that the result will give insight in the "most continuous" part of the Plancherel decomposition.