MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 11 / 1977

"Commutative Algebra and Algebraic Geometry"

13. - 19. März 1977

Die Tagung stand unter der Leitung von E. Kunz (Regensburg), H.-J. Nastold (Münster) und L. Szpiro (Paris).

Ziel der Tagung war es, neuere Ergebnisse aus der kommutativen Algebra und der algebraischen Geometrie darzustellen und insbesondere die gemeinsamen Gesichtspunkte beider Gebiete zu diskutieren.
Folgende Themen standen im Vordergrund des Interesses: Raumkurven, Singularitäten, Riemann-Roch-Theoreme, Vektorbündel und projektive Moduln, Determinantenideale, Exzellente Ringe, de Rham - Cohomologie usw.
Die Tagung fand großes Interesse und war mit 33 ausländischen (davon 4 aus USA, 2 aus Japan) und 20 deutschen Teilnehmern gut besucht.

Teilnehmer

Almkvist, G., Lund
André, M., Lausanne
Angermüller, G., Bonn
Berger, R., Saarbrücken
Bingener, J., Osnabrück
Boutot, J.-F., Orsay
Brodmann, M., Lausanne
Brüske, R., Münster
Bruns, W., Osnabrück
Buchweitz, Hannover
Colliot-Thélène, J.L., Orsay
Eisenbud, D., Waltham
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Flexor, M., Paris
Fossum, R.M., Kopenhagen
Foxby, H.-B., Kopenhagen
Fulton, W., Aarhus
Sprinberg, G., Paris
Greco, S., Torino
Greuel, G.M., Bonn
Gruson, Paris
Hart, R., Leeds
Herzog, J., Essen
Horrocks, G., Newcastle upon Tyne
Ischebeck, F., Münster
Iversen, B., Aarhus
Jähner, U., Bochum
Józefiak, T., Torun
Kunz, E., Regensburg
Langmann, K., Münster
Lascoux, A., Paris
Lindel, H., Münster
Lindner, M., Saarbrücken
Matlis, E., Evanston
Matsumura, H., Nagoya
MacPhearson, Paris
Moret-Bailly, L., Orsay
Nastold, H.-J., Münster
Ojanguren, M., Münster
Peskine, Ch., Oslo
Pinkham, H.C. Bures-sur-Yvette
Rotthaus, C., Münster
Saito, K., Bonn
Sernesi, E., Ferrara
Storch, U., Osnabrück
Strooker, J.R., Utrecht
Szpiro, L., Paris
Tennison, B., Cambridge
Valabrega, P., Torino
Valla, G., Genua
Vetter, U., Clausthal-Zellerfeld
Waldi, R., Regensburg

Vortragsauszüge

ALMKWIST, G.: K-theory of endomorphisms

Let $\underline{\underline{M}}(A)$ ($\underline{\underline{P}}(A)$) be the category of f.g.(projective) A-modules (A = commutative ring). Let K_o End $\underline{\underline{M}}(A)$ be the free abelian group generated by all endomorphisms $[f:M \to M]$ with M in $\underline{\underline{M}}(A)$, modulo relations $[f] = [f'] + [f'']$ whenever

$$\begin{array}{ccccccccc} 0 & \to & M' & \to & M & \to & M'' & \to & 0 \\ & & \downarrow f' & & \downarrow f & & \downarrow f'' & & \\ 0 & \to & M' & \to & M & \to & M'' & \to & 0 \end{array}$$

is commutative exact.

Theorem: $K_o(\text{End } \underline{\underline{P}}(A)) \cong K_o(A) \oplus \left\{ \frac{1+a_1t+\ldots+a_mt^m}{1+b_1t+\ldots+b_nt^n} \; ; \; a_i, b_i \in A \right\}$ where the isomorphism is given by $[P \xrightarrow{f} P] \longmapsto ([P], \lambda_t(f))$ where $\lambda_t(f) = \sum_{i \geq 0} \text{Tr}(\Lambda^i f)t^i$.

Theorem: Let A be a domain of dim 1 s.t. A' = integral closure of A is finite /A. Then

$$K_o(\text{End } \underline{\underline{M}}(A)) \cong K_o(\underline{\underline{M}}(A))/_{\mathbb{Z}} \oplus D_1 \oplus G$$

where D_1 = the free abelian group with basis <p> where $p \in \text{Spec } A[t]$, ht p = 1 and p contains a monic polynomial.

G is a group generated by all <p> where $p \in \text{Spec } A[t]$, ht p = 2, $A[t]_p$ is not regular and p contains a monic polynomial. In G there are relations $n_p\langle p\rangle = 0$ where $n_p = \underset{p' \supset p}{\text{gcd}}[k(p') : k(p)]$ (k(p) = residue field). (There may be more relations.)

Corollar: (Horstmann) If A = coordinate ring of an irred. curve over an algebraic closed field then

$$K_o(\text{End } \underline{\underline{M}}(A)) \cong K_o(\underline{\underline{M}}(A))/_{\mathbb{Z}} \oplus D_1$$

The endomorphisms $\underline{\underline{M}}(A)$ (or $\underline{\underline{P}}(A)$) form an exact category so the higher Quillen-K-theory is defined. Let S be the monic polynomials in A[t].

Theorem: $K_i(\text{End } \underline{\underline{M}}(A)) \cong K_{i+1}(S^{-1}A[t])/_{K_{i+1}A[t]}$

Corollar 1: Let k be a field. Then $K_i(\text{End } \underline{\underline{M}}(k)) \cong \bigoplus_{p \text{ max in } k[t]} K_i(k[t]/_p)$

Corollar 2: Let k be a finite field. Then

$$K_o(\text{End } \underline{\underline{M}}(k)) \cong \begin{cases} \oplus \, (k[t]/_p)^{*\otimes \frac{i+1}{2}} & \text{if i is odd} \\ 0 & \text{if i is even} > 0 \end{cases}$$

ANGERMÜLLER, G.: The value-semigroups of plane irreducible algebroid curves

Let R be the ring of a plane irreducible algebroid curve over an algebraically closed field k. In the case of char (k) = 0 the structure of the value-semigroup of R is well known (due to the existence of Puiseux-series). We show that in char (k) > 0 the value-semigroups have the same structure. Moreover, we characterize those semigroups which are the value-semigroup of a plane irreducible algebroid curve over k and show how the characteristic pairs (in the sense of Moh) are connected with the value-semigroup.

BINGENER, J.: Formale komplexe Räume, de Rham-Kohomologie und Divisorenklassengruppen

Es wird über einige Resultate berichtet, bei denen Techniken der formalen Geometrie verwendet werden. - Im Jahre 1951 stellte Zariski das folgende

Problem: Seien X ein algebraisches Schema über einem Körper k, $T \subseteq X$ eine abgeschlossene Menge und $\hat{X}$ die formale Komplettierung von X längs T. Ist dann $\Gamma(\hat{X}, O_{\hat{X}})$ ein noetherscher Ring?

Wir zeigen, daß dies nicht immer der Fall ist. Genauer gilt: Zu jedem $n \geq 3$ gibt es ein glattes n-dimensionales projektives $\mathbb{C}$-Schema X und einen irreduziblen glatten Cartier-Divisor $T \subseteq X$ derart, daß $\Gamma(\hat{X}, \mathcal{O}_{\hat{X}})$ nicht noethersch ist.

Sodann führen wir die sog. formalen komplexen Räume als analytisches Analogon zu den formalen Schemata ein. Mit ihrer Hilfe kann man den folgenden Satz beweisen, der den Vergleichssatz von Grothendieck-Deligne-Hartshorne über algebraische de Rham-Kohomologie verallgemeinert.

Satz: Seien Y ein komplexer Raum, $S \subseteq Y$ eine analytische Menge und $y \in Y$. Sei weiter $Y \hookrightarrow X$ eine abgeschlossene Einbettung in den komplexen Raum X derart, daß $X \setminus S$ nicht singulär ist. Dann gilt:

$$R_i * (\mathbb{C}_{Y \setminus S})_y = \mathbb{H}^p(\hat{X}_y \setminus S_y, \Omega^{\cdot}_{\hat{X}_y}) \text{ für alle } p \in \mathbb{N}.$$

Hierbei ist $i: Y \setminus S \longrightarrow Y$ die kanonische Injektion und $\hat{X}_y$ die Komplettierung von $X_y := \operatorname{Spec} \mathcal{O}_{X,y}$ längs der durch Y definierten abgeschlossenen Teilmenge Y_y von X_y. - Schließlich kann man die formalen komplexen Räume dazu verwenden, um Aussagen über die Divisorenklassengruppen von (kompletten) lokalen Ringen zu gewinnen.

BRUNS, W.: Embedding modules into cyclic modules

The main results:

Theorem 1: Let R be a commutative noetherian ring and M be a finitely generated R-module. Assume grade $(\operatorname{Ann}(M)) \geq 2$. Then M can be embedded into a cyclic module (for short M is cyc-emb).

Theorem 2: R as above. The following properties of R are equivalent:

(1) Every R-module of finite length is cyc-emb.

(2) For all maximal ideals $\mathfrak{m}$ of R the $\mathfrak{m}$-adic completion $\hat{R}_{\mathfrak{m}}$ has no associated prime ideal $\mathfrak{q}$ such that $\dim (\hat{R}_{\mathfrak{m}}/\mathfrak{q}) \leq 1$.

There are connections to results of M. Hochster in his paper "Purity versus cyclic purity in excellent noetherian ring".

BUCHWEITZ, R.: On deformations of monomial curves

A survey-lecture on the results known about deformations of monomial curves:

1.) Let C be a proper, reduced curve over k (k algebraically closed field of Char. 0), $s \in C(k)$ an unibranched point, B the formal ring of C at s. Then B is monomial, if there is a $\mathbb{G}_m$-action on B.

2.) One knows:

a) Monomial curve singularities aren't rigid, furthermore

b) there is at least one one-parameter family of deformations.

c) The formal, versal def. of B/k can be chosen $\mathbb{G}_m$ equivariant (Pinkham).

d) If $M^{+(-)}$ denotes the linear subspace of the base of the formal, versal def. induced by the eigenspaces of the $\mathbb{G}_m$-action to positive (neg.) eigenvalues, then there is an isomorphism of M_s^- (= set of points of M^- with smooth fibres) divided by the $\mathbb{G}_m$-action with the subset of $\mathfrak{M}_{g,1}$ (coarse moduli-space of smooth, proper curves of genus g with one section) consisting of those curves with puncture a Weierstraß point of semigroup the value-semigroup of B (Pinkham).

e) M^+ is a good candidate for equisingularity (Teissier).

f) The Theorem in d) can be extended to curves with automorphisms.

g) If there is a smooth deformation, then a component E with generic smooth fibre has dimension $2g-1+\dim \mathrm{Ext}_B^1(k,B)$.

h) The extistence of smooth deformations is only proven for: 1) complete intersections (Schlessinger), 2) edim $B \leq 3$ (Schaps), 3) edim $B \leq 4$ and B Gorenstein (Buchsbaum-Eisenbud), 4) B is an Arf-Ring.

EISENBUD, D.: The topological classification of isolated complex hypersurface singularities

An expository account of the following result of Levine, Durfee, Kato:

<u>Theorem:</u> Let $f, g : \mathbb{C}^{n+1} \to \mathbb{C}$, $n \geq 3$, be polynomials with isolated singularities at the origin. Let F and G be the Seifert forms associated to the fibres of the Milnor fibrations corresponding to f and g (so that F and G are non-singular integral bilinear forms). Then the singularities of f and g are topologically equivalent iff F and G are equivalent as bilinear forms.

EVANS, E.: <u>The Four Color Theorem</u>

The four color theorem says that given any triangulation of the sphere one can assign the numbers 1,2,3,4 to the vertices in such a way that if two vertices are joined by an edge they have different numbers. The degree of a vertex is the number of edges from it. If V_i = the number of vertices of degree i and n is the maximum degree that occurs then Kempe proved that

$$4V_2 + 3V_3 + 2V_4 + V_5 - V_7 \dots + (6-n)V_n = 12.$$

Using this formula one can produce finite lists of graphs such that every triangulation of the sphere contains at least one from each list as a subgraph. Then essay the reduction technique of Kempe they found one list which had each numbers reducible and proved the theorem. The simplest lists are

 due to Kempe

 due to Birkoff

the actual list they essed had 1879 graphs.

FLENNER, H.: <u>Relative de Rham Kohomologie</u>

Sei $A \to B$ ein Homomorphismus analytischer $\mathbb{C}$-Algebren und $\Omega^{\bullet}_{B/A}$ der relative de Rham Komplex. Es wurde der folgende Satz bewiesen: Ist $A \to B$ flach und hat die Faser $B/m_A B$ eine isolierte Singularität, so sind die Kohomologiemoduln von $\Omega^{\bullet}_{B/A}$ endliche A-Moduln. Beim Beweis dieses Satzes wurden lokale Bertini-Sätze sowie Endlichkeitssätze für singuläre Differentialoperatoren benutzt.

Eine ähnliche Beweismethode läßt sich anwenden, um Endlichkeitssätze für die relative de Rham-Kohomologie von (flachen) Deformationen vollständiger Durchschnitte herzuleiten, auch wenn die Faser keine isolierte Singularität hat. Genauer: Ist R eine analytische $\mathbb{C}$-algebra, $A = R\{f_1,\ldots,f_r\} \to B = R\{x_1,\ldots,x_n\}$ ein flacher R-Algebra-Homomorphismus, so ist die relative de Rham Kohomologie $H^p(\Omega^{\cdot}_{B/A})$ endlich über A für $p \leq \text{kodim}(\text{Sing}(B/m_A B))$. Ferner wurde der folgende Verschwindungssatz bewiesen, welcher einen entsprechenden Satz von Sebastiani/Greuel für den absoluten Fall verallgemeinert. Ist $A \to B$ eine Deformation eines vollständigen Durchschnitts, so ist $H^0(\Omega^{\cdot}_{B/A}) \cong A$ und $H^p(\Omega^{\cdot}_{B/A}) = 0$ für $0 < p < \text{kodim Sing}(B/m_A B)$, falls nur $\text{kodim}(\text{Sing } B/m_A B) > 0$!

FOSSUM, R.: Decomposition of symmetric and exterior powers in characteristic $p > 0$

Let $\nu_{p^\alpha} = \mathbb{Z}/p^\alpha\mathbb{Z}$ and p = char k, k a field. Suppose V is an indecomposable $k\nu_{p^\alpha}$-module. Consider the symmetric algebra $S_k(V) = \bigoplus_{r\geq 0} S^r_k(V)$. Then

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For $\alpha = 1$, the description is complete. Since $k\nu_{p^\alpha} \cong k[T]/(T-1)^{p^\alpha}$, there are p^α indecomposables given by $V_n := k[T]/(T-1)^n$, $1 \le n \le p^\alpha$. The representation algebra $R\nu_{p^\alpha}$ is the free $\mathbb{Z}$-module with basis $\{V_n\}$. In this algebra it can be shown that

$$\Lambda^r(V_n) = \frac{V_n \otimes_k V_{n-1} \otimes_k \cdots \otimes_k V_{n-r+1}}{V_1 \otimes_k \cdots \otimes_{k_1} V_r} =: \binom{V_n}{V_r} \text{ and that}$$

$$S^r(V_{n+1}) = \binom{V_{n+r}}{V_r}.$$

(*) Put $b_n = \dim_k \{S^r(V_{n+1})^{\nu_p}\}$ when p is "large". Then

$$\sum_{n=0}^{\infty} b_n t^n = \frac{1}{\pi} \sum_{j=1}^{\infty} \int_{\pi}^{\pi} \frac{(\sin j\varphi)\sin\varphi d\varphi}{\prod_{\nu=1}^{[r/2]} (1+t^2-2t\cos(r-2_\nu)\varphi)}.$$

Relations between these "decompositions", partitions and other combinatorial results can be given. (See above * for an example.) This work has been done jointly with G. Almkwist.

FOXBY, H.-B.: Minimal injective resolutions

Let $I^\bullet$ be a minimal injective resolution of the finitely generated module M over a commutative noetherian ring A. The i'-module I^i in this injecture resolution decomposes into the direct sum of indecomposables: $I^i = \coprod_{p \in \operatorname{Spec} A} E(A/p)^{\mu^i(p,M)}$

By the use of a version of the socalled New Intersection Theorem the following main results are obtained:

1) If A contains a field, and if depth $M_p < i < \dim A$ then $\mu^i(p,M) \ge 2$ (i.e. I^i contains at least two copies of $E(A/p)$).

2) If A contains a field, then the following conjecture (due to Vasconcelos et al.) holds: If $\mu^{\dim A}(p,A)$ where m = the maximal ideal on the local ring A, then A is a Gorenstein ring (Note: No Cohen-Macaulay assumption).

3) In general, $\mu^i(p,M) > 0$ for depth $M_p \le i \le$ injective dimension of A_p (which might be infinite).

FULTON, W.: A Finite Riemann-Roch Theorem

If X is an algebraic $\mathbb{F}_q$ -scheme, $\mathcal{M}$ a coherent sheaf on X, and φ a q-linear endomorphism of $\mathcal{M}$ (i.e. $\varphi(am) = a^q\varphi(m)$ when a is a section of $\mathcal{O}_X$, m a section of $\mathcal{M}$), there are induced linear maps $H^i(\varphi)$ on the cohomology groups $H^i(X,\mathcal{M})$ and $\varphi(p)$ on the fibre of $\mathcal{M}$ at an $\mathbb{F}_q$ -rational point P in X. We prove a simple theorem of Riemann-Roch type, inspired by the work of G. Quart, whose Hirzebruch-Riemann-Roch analogue gives

$$\sum_{i\geq 0}(-1)^i \text{ trace } (H^i(\varphi)) = \sum_{p \in X(\mathbb{F}_q)} \text{trace } (\varphi(p))$$

whenever X is proper over $\mathbb{F}_q$. When $\mathcal{M} = \mathcal{O}_X$ we recover a result of Deligne and Katz, which in turn generalizes the Chevalley-Warning theorem: If a homogeneous polynomial over $\mathbb{F}_q$ has degree less than the number of variables, then the number of solutions over $\mathbb{F}_q$ is divisible by p $(q = p^r)$.

GRECO, S.: The Gorenstein locus of an excellent ring is open

Theorem: Let A be a noetherian ring. Assume that for any $\mathfrak{p} \in \operatorname{Spec}(A)$ the Gorenstein locus of $\operatorname{Spec}(A/\mathfrak{p})$ contains a non empty open set. Then the Gorenstein locus of A is open.

Corollar: If A is an excellent ring, the Gorensten locus of A is open.

HORROCKS, G.: Vector bundles on the punctured spectrum of a local ring - the extension problem

Let x be a regular element of a regular local ring B and A = B/(x). Put V = Spec A - {m}, W = Spec B - {n} where m,n are the maximal ideals. A V-bundle is an A-module locally free on V and it extends if there is a W-bundle on W restricting as a bundle to V. The semigroup of V-bundles under $\oplus$ modulo the extendible ones is trivial and to obtain obstructions to extension by such means restrictions need to be imposed. So consider self-dual extensions and self-dual bundles. Then for dim A = 2k+1 the k-th cohomology group $H^k(V, \ )$ gives an obstruction namely its length mod 2. When A/m is algebraically closed the semi-group of self-dual V-bundles modulo self-dually extendible ones is then $\mathbb{Z}/2\mathbb{Z}$ the isomorphism being given by this obstruction. By taking the r-th exterior power of a bundle of rank 2r we obtain a self dual bundle and so an obstruction to arbitrary extension. In particular this gives for k ≡ 1,2 mod 4 an invariant of such bundles which distinguishes between topological and algebraic equivalence (inthe sense of algebraic families).

IVERSEN, B.: An isomorphism principle in the theory of motifs

Consider the category $\bar{V}$ of smooth projection varieties defined over the field k and let C denote Chow intersection theory on $\bar{V}$. From this build the category M_C whose objects are pairs (X,m), where $X \in \mathrm{Obj}\bar{V}$ and $m \in \mathbb{Z}$, $\mathrm{Mor}((X,m),(Y,n)) = C^{\dim X+n-m}(X \times Y)$, composition is that of correspondances. - For $X \in \mathrm{Obj}\bar{V}$ put $\tilde{X} = (X,0)$ and let $\mathbb{T} = (P^t,1)$ denote the Tate motif.

Put F = $\underline{\mathrm{Hom}}(V^{op}, \underline{Gr\ \mathbb{Z}})$ where $\underline{Gr\ \mathbb{Z}}$ denotes the category of quided abelian groups. Finally consider the functor

$$\Phi : M \to F$$

which transforms $(X,n) \in \mathrm{Obj}\,M_c$ into "$T \longmapsto C^{\cdot}(X \times T)[n]$". The basic properties of Φ are

"Identity principle". For morphisms α and β in $\underline{M}$,

$$\Phi(\alpha) = \Phi(\beta) \Longrightarrow \alpha = \beta$$

"Isomorphism principle". For a morphism α in M,

$$\Phi(\alpha) \text{ isomorphism} \Longrightarrow \alpha \text{ isomorphism}.$$

As an application let $Y \subseteq X$ be of codimension r and let X' denote X blow up along Y. Then we have the following decomposition in M

$$\tilde{X} = \tilde{X} \oplus \tilde{Y} \otimes \mathbb{T}^{-1} \ldots \oplus \tilde{Y} \otimes \mathbb{T}^{r+1}$$

refining a result of Manin (W.Fulton showed me how to eliminate torsion).

JÓZEFIAK, T.: Ideals generated by minors of a symmetric matrix

Let X be a symmetric $n \times n$ matrix over a commutative Noetherian ring R and $I_p(X)$ an ideal generated by all the $p \times p$ minors of X. Simple proofs were given of the following theorems:

1) If P is a minimal prime of $I_p(X)$, then

$$\mathrm{ht}\, P \leqslant \nu(p,n) := \frac{(n-p+1)(n-p+2)}{2}.$$

2) depth $I_p(X) = \nu(p,n)$ if either

a) R is a polynomial ring in the variables $\{x_{ij}\}$,

or b) $\{x_{ij}\}$, $i \leq j$, form a regular sequence in R and R is a local algebra over a field.

Moreover, an explicite construction of a free complex $\mathbb{L}(X)$ of length 3 was given and it was proved that $\mathbb{L}(X)$ is a free resolution of $R/I_{n-1}(X)$ if depth $I_{n-1}(X) = 3$.

LANGMANN, K.: Japanische und ausgezeichnete Ringe

Sei A ein noetherscher lokaler Unterring des konvergenten Potenzreihenringes $\mathbb{C}\langle z_1,\ldots,z_n\rangle$ mit $\mathfrak{m}(A) = (z_1,\ldots,z_n)A$. Dann gilt

1) Ist $p = (f_1,\ldots,f_m)A$ ein Primideal mit $\frac{\partial f_i}{\partial z_j} \in A$, so ist $p\hat{A}$ reduziert in der Komplettierung $\hat{A}$.

2) Ist $q \supset p$ ein weiteres Primideal, $q = (g_1,\ldots,g_k)$ mit $\frac{\partial g_i}{\partial z_j} \in A$, so ist genau dann $(A/p)_q$ nicht regulärer Ring, wenn $q \supset \sum (\det M_h)A$ ist, wobei M_h alle $h \times h$ = Höhe $p \times$ Höhe p Untermatrizen von $(\frac{\partial f_i}{\partial z_j})$ durchläuft.

3) Ist $\hat{q} \subset \hat{A}$ ein Primideal mit $\hat{q} \cap A = q$, so ist $(A/p)_q$ regulär genau dann, wenn $(\hat{A}/p\hat{A})_{\hat{q}}$ regulär ist.

Es folgt hieraus ein Theorem von Matsumura und Nomura: Ist für $f \in A$ stets $\frac{\partial f}{\partial z_j} \in A$, so ist A ausgezeichnet. Weiter ist A genau dann ausgezeichnet, wenn A japanisch ist und für jedes Primideal $p = (f_1,\ldots,f_m)A$ der analytisch-singuläre Ort des Nullstellenmengenkeims $v(f_1,\ldots,f_n)$ gleich dem Nullstellenmengenkeim eines Ideals $J \subset A$ ist.

LASCOUX, A.: Modules tensoriels et Syzygiés des idéaux déterminantiels

For many algebraic or geometric constructions, the family of Λ^i and S^j (exterior or symmetric powers, for a module) is not big enough. So we define, for each partition I (i.e. increasing sequence of integers), the Schur functor of index I:
S_I: Modules $\longrightarrow$ Modules (for free modules), as the image of a certain product of Λ^i into a product of S^j.

If a module E is a direct sum of modules of rk1, A,B,..., then $S_I(E)$ is the Schur symmetric polynomial of index I in the variables A,B,... . Many formulas on symmetric polynomials extend to relations between the S_I. For example, one can decompose $\Lambda^n(E \otimes F)$ in terms of the $S_I E$ and $S_J F$. (Cauchy formula). It is very useful to the study the variety Y define by the minors of a certain order of a matrix which is the image of a certain variety Z in a grassmannian and in the generic case, the sheaf $\mathcal{O}_Z$ admits for resolution a koszul complex $\Lambda^n(E \otimes Q^*)$. One can go back to Y and obtain a resolution of $\mathcal{O}_Y$; on this resolution, one can read all the already known informations on Y: Cohen Macaulay ...

LINDEL, H.: Some remarks on projective modules over Polynomial rings

The following proposition was proved: Let A be a noetherian ring and a subring of R and $T \in R$ algebraically independed over A. Let P be a f.g. projective R-module such that there exists a monic $h \in A[T]$ with the following properties: $R = A[T] + hR$, $Rh \cap A[T] = hA[T]$, there exists a submodule Q, Q projective and extended from A and $hP \subset Q$. Then $P \cong Q$. As corollaries one gets

1) Let B be a complete regular local ring, $\dim B \leq 2$ and $A = B[[X_1,...,X_n]]$ a formal power series ring over B. Then $A[T_1,...,T_m]$ projective modules are free.

2) Let B as in 1) (but not necessarely complete) and $A = (B[X])_{(m,X)}$. All the projective $A[T_1,...,T_n]$-modules are free.

Furthermore there were mentioned examples of 2-dimensional k-algebras (normal and homogenious) from type R = k[x,y,z], over which all the projective R-modules are free. Take for example $z^n = xg$, g homogenous polynomial of degree n-1 in x and y.

LINDNER, M.: Stability in positive characteristics

Let X be a smooth projective variety of dimension n defined over an algebraically closed field of arbitrary characteristic. In order to stdy moduli questions of X it is often helpful to project X generically into a projective space P^{n+1} in order to get a birational equivalent hypersurface, or onto P^n and then look at the ramification divisor.
In certain small characteristics these generic projections fail to be infinitesimal stable, and their singularities provide examples where also a certain flat-T^1-stability is violated. This notion is introduced to study global fmilies of such hypersurfaces in a fixed projective space P which all have formal isomorphic generic singularities.
To remedy this we introduce the notion of flat-T^1-stability in the fppf- or etal-topology, but even this generalization does not workkfor some examples of curves and surfaces which are given in some detail to illustrate the difference in the behavior of generic singularities when specialized to positive characteristic. We touch the question of global families of varieties with generic singularities over k not necessarily in a fixed projective space.

MATLIS, E.: Higher properties of R-sequences

Let R be a commutative ring and $\{x_1,...,x_n\}$ a fixed R-sequence. Let $I_t = (x_1^t,...,x_n^t)$ and $K = \underrightarrow{\mathrm{Lim}}\, R/I_t$ via multiplication by $x = x_1 x_2 ... x_n$. Define Γ to be the $I = I_1$-adic torsion functor and Λ to be the I-adic completion functor. Then $\mathrm{Tor}^R_{n-i}(K,-)$ is the right derived functor of Γ and $\mathrm{Ext}_R^{n-i}(K,-)$ is the i-th left derived functor of Λ. Define an R-module A to be K-torsion-free if $\mathrm{Tor}_i^R(K,A) = 0\ \forall\ i \neq 0$, and K-divisible if $\mathrm{Ext}_R^i(K,A) = 0\ \forall\ i \neq 0$. Then if A is K-torsion-free there is a natural isomorphism of functors: $\Lambda(A) \cong \mathrm{Hom}_R(K, K \otimes_R A)$; and if A is K-divisible, there is a natural isomorphism of functors: $\Gamma(A) \cong K \otimes_R \mathrm{Hom}_R(K,A)$. Thus there is an isomorphism between the category of Λ-complete, K torsion-free R-modules $\mathfrak{A}$, and the category of Γ-torsion, K-divisible R-modules $\mathfrak{B}$ given by $\mathfrak{A} \xrightarrow{K \otimes_R \cdot} \mathfrak{B}$ and $\mathfrak{B} \xrightarrow{\mathrm{Hom}_\Lambda(K,\cdot)} \mathfrak{A}$. If A is Λ-complete, then $x_1,...,x_n$ is an A-sequence $\iff$ A is K-torsion-free (i.e. $A \in \mathfrak{A}$); and if A is Γ-torsion, then $x_1,...,x_n$ is an A-cosequence $\iff$ A is K-divisible (i.e. $A \in \mathfrak{B}$). If $A \in \mathfrak{A}$, then every permutation of $x_1,...,x_n$ is an A-sequence; and if $A \in \mathfrak{B}$, then every permutation of $x_1,...,x_n$ is an A-cosequence.

PESKINE, C.: Un théoreme de Halphen

Soit C une courbe de degre d dans l'espace $\mathbb{P}^3$, supposons que C est contenue dans une surface reduitê irreducible de degre s. Soient n et k tels que $sk = d+n \quad 0 \leq n < s$. Alors

$$\text{genre de } C \leq 1 + \frac{d}{2}\left(s + \frac{d}{s} - 4\right) - \frac{n(s-n)(s-1)}{2s} .$$

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PINKHAM, H.: Equations for certain normal surface singularities

Let E be a smooth proper curve, D a divisor on E, P_i $1 \le i \le n$, (e_i, d_i) pairs of integers $e_i < d_i$, $(e_i, d_i) = 1$. Then the ring $\bigoplus_{k \ge 0} H^0(E, \mathcal{O}_E(D^{(k)}))$ where $D^{(k)}$ is the divisor $kD - \sum_{i=1}^{n} \lceil e_i/d_i \rceil P_i$, where $\{a\}$ is the least integer $\ge a$, is normal with a isolated singularity at the ideal $\bigoplus_{k>0} H^0(E, \mathcal{O}_E(D^{(k)}))$. The resolutions of this singularity can be easily reconstructed from the data D, e_i, d_i, P_i, and when E is rational or elliptic, one can explicitly compute the equation of the singularity.

SAITO, K.: Period mapping

Let $F: X, 0 \to S, 0$ be a universal unfolding of a function $f: \mathbb{C}^{n+1}, 0 \to \mathbb{C}, 0$, with isolated singularity in the fiber over 0. Let $\zeta_0 = \sum_{i=0}^{n} (-1)^i c_i dx_0 \wedge \ldots \widehat{dx_i} \ldots dx_n$ $(\Sigma c_i = 1)$ be a n-form on X. Then for $k \in \mathbb{N}$ less than any integral exponents of the classical monodromy, the integration of the n-form $(\nabla_{\frac{\partial}{\partial t}})^k \zeta_0$ over the vanishing cycles of the fiber of F, which are also satisfiing a system of differential equations on S, gives a maximal rank mapping from the base space S minus the discriminant D of the map F to a cohomology group $H^n(F^{-1}(*), \mathbb{C})$.

SPRINBERG, G.: Transformation de Nash et éventails en dimension 2

La transformation de Nash d'une variété algébrique consiste à remplacer chaque point de cette variété par l'ensemble des positions limites des espaces tangentes aux points lisses voisins. On conjecture qu'on peut désingulariser toute surface, sur un corps de caracteristique 0, avec un nombre fini de transformations de Nash suivies de normalisations. Nous demontrons ce résultat pour les éventails en dimension 2 (toroidal embeddings), c'est à dire, pour les singularités rationelles qu'on obtient comme quotient de $\mathbb{A}^2$ par l'action d'un groupe cyclique fini.

SZPIRO, L.: Sur la régularité de l'adjointe

Soit X une surface lisse et projective sur un corps k, C une courbe intègre contenue dans X, $\tilde{X}$ l'éclaté de X tel que la transformée propre $\tilde{C}$ de C soit normale. On dit que C satisfait la régularité de l'adjointe si $H^1(\tilde{X}, \mathcal{O}_{\tilde{X}}(-\tilde{C})) = 0$. Cet énoncé implique $H^1(X, \mathcal{O}_X(-C)) = 0$ et lui est équivalent en car 0 (nous avons c. ex. en car $p > 0$). Nous donnons alors un ex. dû à M. Raynaud qui montre que $H^1(X, \mathcal{L}^{-1}) \neq 0$ pour un certain faisceau ample en car $p > 0$. Ceci n'arrive jamais en car 0 (Th. de Kodaira). Nous donnons aussi notre propre démonstration de $\mathcal{L}$ ample k car 0 dim $X \geq 2$, X normale $\Longrightarrow H^1(X, \mathcal{L}^{-1}) = 0$ en réduisant mod p pour un nombre premier p bien choisi !!

VALABREGA, P.: Formal fibers and openness of loci in noetherian rings

Nagata's criterium for the openness of regular loci can be extended to the loci (R_K), (S_K), Cohen-Macaulay (shortly P-loci) in the following form (proof of C. Massaka and me);
Theorem: Let X be a locally noetherian scheme; then the P-locus of X is Zariski open if, for every $x \in P(x)$ (= P-locus of X), considered the closed reduced subscheme $\{\overline{x}\} = Y$, $P(Y)$ is an open neighbourhood of x in Y.
If P = Cohen-Macaulay, the condition is also necessary.

Good properties of openness for a P-locus on X = Spec(A) imply geometric P-property on formal fibery. Precisely, if P = regularity, Cohen-Macaulay, Gorenstein, Complete intersection, we have the following
Theorem: Assume A is m-adically complete, A/m has geometrically P formal fibers and the P-locus of A' is open for every A' = A-algebra of finite type, when the formal fibers of A are geometrically P.
The result, applied for P = regularity, allows to show excellent property for some class of rings, like completions of algebras of finite type over a field or a Dedekind domain of char. O or like some rings of restricted power series.

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