MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 12/1977

Mathematische Stochastik

20.3. bis 26.3.1977

Die unter der Leitung von Prof. Dr. D. Bierlein (Regensburg) und Prof. Dr. J. Wolfowitz (Urbana) stehende Tagung wurde von 48 Teilnehmern aus den Vereinigten Staaten, Israel und acht europäischen Ländern besucht. Mit lebhaftem Bedauern wurde registriert, daß einer größeren Anzahl von eingeladenen Kollegen aus Prag, Budapest, Bukarest und Jaşi eine Teilnahme an der Tagung nicht möglich war.

Die 36 gehaltenen Vorträge erstreckten sich thematisch über den gesamten Bereich der Mathematischen Stochastik; die Mathematische Statistik und die Theorie der stochastischen Prozesse waren dabei am dichtesten vertreten. Durch die Vorträge, in den daran anschließenden Plenardiskussionen und in zahlreichen Gesprächen in kleinem Kreis fand ein für die wissenschaftliche Arbeit aller Teilnehmer fruchtbarer Gedankenaustausch statt, der für die Atmosphäre von "Oberwolfach" typisch ist und diese Institution international auszeichnet.

Die Vortragspausen vor dem Nachmittagstee und nach dem Abendessen wurden aber auch zur Entspannung auf Spaziergängen, am Klavier und beim Tischtennis genutzt; der Ausflug am Mittwochnachmittag zum Wasserfall bei Bad Rippoldsau mit Variante Glaswaldsee fand regen Zuspruch.

Tagungsleiter und Teilnehmer sind Herrn Barner und Frau Oeckinghaus für die großzügige Unterstützung bei der Vorbereitung der Tagung und für den technisch perfekten Ablauf zu großem Dank verpflichtet.

Teilnehmer

R. Ahlswede, Bielefeld
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W.J. Bühler, Mainz
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K. Daniel, Bern
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U. Dieter, Graz
H. Dinges, Frankfurt
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W. Eberl, Hagen
V. Fabian, East Lansing
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Z. Govindarajulu, z.Z. Mannheim
E.Henze, Braunschweig
H. Heyer, Tübingen
U. Herkenrath, Bonn
J. Hoffmann-Jørgensen, Aaarhus
G. Hübner, Hamburg
U. Krengel, Göttingen
J. Lehn, Karlsruhe
J. Lembcke, Erlangen
R. Lerche, Heidelberg
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U. Oppel, München
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P. Révész, Budapest
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M.P. Schützenberger, Paris
E. Siebert, Tübingen
D. Siegmund, Stanford
W. Vogel, Bonn
G. Wahba, Madison
H.G. Weidner, Erlangen
H. v. Weizsäcker, München
H. Witting, Freiburg
J. Wolfowitz, Urbana
R. Zmyslony, Wrocław

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Vortragsauszüge

R. AHLSWEDE: Elimination of correlation in random codes for arbitrarily varying channels

The author determines for arbitrarily varying channels

a) the average error capacity $\overline{C}_1$.

b) the maximal error capacity in case of randomized encoding C_2.

c) the everage error capacity in case of randomized encoding $\overline{C}_2$.

The identities $\overline{C}_1 = C_2 = \overline{C}_2$ always hold. In case capacities are positive they are equal to the correlated random code capacity $\overline{C}_3$ found in [1]. A formula for $\overline{C}_2$ was announced several years ago in [2]. Under a mild regularity condition this formula turns out to be valid. To find the maximal error capacity is by far the hardest problem. An answer is known only for binary output alphabets [3].

[1] Blackwell, D.; Breiman, L.; Thomasian, A.J.:
The capacities of certain channel classes under random coding.
Ann. Math. Stat. 31, 558 - 567 (1960).

[2] Dobrushin, R.L.:
Unified information - transmission schemes for discrete memoryless channels and messages with independent components.
Dokl. Akad. Nauk. SSSR 148, No. 6, 1245 - 1248 (1963).

[3] Ahlswede, R.; Wolfowitz, J.:
The capacity of a channel with arbitrarily varying channel probability functions and binary output alphabet.
Zeitschrift f. Wahrsch. u. verw. Geb., 15, 3, 186 - 194 (1970).

R.E. BARLOW: Inference for Multivariate Life Distributions

A theory of multivariate life table analysis is developed. Estimators are developed for various multivariate interval failure rates. These estimators are multivariate extensions of the usual estimators in the univariate case. Under appropriate conditions, the estimators are shown to be asymptotically normal and asymptotically independent.

R.H. BERK: On an asymtotically optimal sequential test

It is observed that for sequentially testing one-side hypotheses about the mean of an normal distribution (variance known), a sequential test first proposed by Schwarz has the following desirable asymptotic property: Among all sequential tests that control the error rates at two parameter values, this test asymptotically has minimal expected sample size in every direction (i.e., for every parameter value). Some implications are given for the Kiefer-Weiss problem of minimizing the maximum expected sample size. Using a technique due to Woodroofe, an asymptotic expansion for the power function of the test is obtained.

W. BÜHLER: Order statistics stability

Let g by a strictly increasing function mapping the unit interval into itself and with at most one fixed point in the interior of [0,1]. Let q be this fixed point or 0, if there is no such fixed point. If $g(y) < y$ for $y < q$ and $g(y) > y$ for $y > q$, then it is easily seen, that for any $a > 1$ there are (too) many distribution functions F satisfying the Poincaré functional equation $F(ax) = g(F(x))$. There are even many such F, which are "as nice as g" (i.e. differentiable if g is etc.) outside of 0. A regularity condition at the origin, namely $\lim_{x \to o} F(x)/x^{\alpha}$ (for $\alpha > 0$) being positive and finite, reduces the variety of solutions to a one-parameter family. The tail of the distributions are under investigation. The special case $g(y) = 1-(1-y)^n$ yields a known characterisation of the Weibull families. This can be generalised to distributions for which the order statistics $X_{k:n}$ has the same distribution as X_1 up to a scale factor a.

M. CSÖRGÖ: Strong approximations of the quantile process

The talk is a report on a joint paper with P.Révész. Let $X_1, X_2, \ldots$ be i.i.d.r.v. with a continuous distribution function F. Let $X_{1:n} < \ldots < X_{n:n}$ be the order statistics of the first n oberservations. Define $Q_n(y) = X_{k:n}$ if $\frac{k-1}{n} < y \leq \frac{k}{n}$, $k = 1,2,\ldots,n$, and

$q_n(y) = n^{1/2}(Q_n(y) - F^{-1}(y))$, $0 < y \leq 1$, the so called sample quantile process. Given some regularity conditions on F, which is assumed to be twice differentiable on the whole real line, we prove that there exist a sequence of Brounian Bridges $B_n(y)$ and a Kiefer process $K(y,t)$ such that

$$\sup_{0 \leq y \leq 1} |f(F^{-1}(y))\, q_n(y) - B_n(y)| = O(n^{-1/2} \log n) \text{ a.s.}$$

and

$$\sup_{0 \leq y \leq 1} \left| f(F^{-1}(y)) q_n(y) - \frac{K(y,n)}{\sqrt{n}} \right| = O(n^{-1/4} (\log \log n)^{1/4} (\log n)^{1/2})$$

a.e. The two given rates of nearness are best possible.

M. DENKER: A.S. Convergence of Sums of Banach Space Valued Random Vectors

Let $(\Omega, \mathfrak{A}, P)$ be a probability space, E a Banach space with norm $\| \;\|$ and $L^P(E) := \{X: \Omega \to E$: X is strongly measurable and $\| X \| \in L^P\}$.

Theorem:

Let $(X_n : n \in \mathbb{N})$ be an independent, E-valued process with $EX_n = 0$, and let $a_n \downarrow 0$. Define Y_n on the product probability space by $Y_n(\eta, \omega) = X_n(\eta) - X_n(\omega)$. If (a) $\sum a_\kappa Y_\kappa$ converges in $L^P(E)$ for some $p \geq 1$ or (b) $\lim_{n\to\infty} a_n \sum_\kappa Y_\kappa = 0$ a.e. and $\sup \| a_\kappa X_\kappa \| \in L^P$,

then $\lim_{n\to\infty} a_n \sum_\kappa X_\kappa = 0$ a.e.

The proof uses the following proposition.

Proposition:

If $a_n \sum_\kappa X_\kappa \to 0$ a.e., X_κ independent, $a_n \downarrow 0$ and $\sup \| a_n X_n \| \in L^P$,

then $a_n \sum_\kappa X_\kappa \to 0$ in $L^P(E)$.

The method of proof of this proposition gives a new proof of a theorem of Hoffmann-Jørgensen:

Theorem:

If $(X_n : n \in \mathbb{N})$ is independent. Then (1) and (2) are equivalent:

(1) $\sum_{i=1}^{n} X_i$ is stochastically bounded and $\sup_n \| X_n \| \in L^p$.

(2) $\sup_n \| \sum_{i=1}^{n} X_i \| \in L^p$.

U. DIETER: The Lattice Structure of Linear Congruential Random Number Generators

Pseudo-random numbers are usually generated by linear congruential methods: A sequence $\{z_i\}$ is constructed by $z_i = a_1 z_{i-1} + \ldots + a_r z_{i-r} (\mathrm{mod}\ m)$, $m, a_1, \ldots, a_r$ being integers. Then $u_i = z_i/m$ is taken as a sample from the uniform distribution in $[0,1]$. To ensure maximal period length of the generator, the a_i have to fulfill some algebraic conditions, i.e. if m is prime and r=1 the a_1 has to be a primitive root, or if $m = 2^e$, a_1 has to be congruent to 5 mod 8.

For most applications of random numbers one has to study the distribution of $P_n = \{U_i\}$ of points $U_i = (u_{i+1}, \ldots, u_{i+n})$ in the n-dimensional hypercube $[0,1)^n$. It is easy to see that P_n is equal to $G \cap [0,1)^n$ where G is a lattice generated by n vectors $e_1, \ldots, e_n$. Associated to this lattice is the so-called dual lattice G^* of parallel hyperplanes which cover all points of G. Its basis is given by e_i^* where $e_i^* e_j = \delta_{ij}$ holds. The two lattices enable one to answer the following questions: (1) Determine the maximum distance of parallel hyperplanes covering P_n. (2) Determine the minimum number of parallel hyperplanes covering P_n. (3) Find upper and lower bounds for the discrepancy of P_n, i.e. the maximum deviation of P_n from the uniform distribution.

For (1) and (2) one has to calculate Min $\{ \| x^* \| \ | x^* \in G^*, x^* \neq 0\}$ where the norm is eather the l_1 norm or the Euclidean norm.

For the lower bound in (3) one has to calculate Min $\{R(x^*) \mid x^* \in G^*, x^* \neq 0\}$ where $R(x) = \prod \max(1, x_i)$. For upper bounds one has to determine sums of the form $\Sigma R(x^*)^{-1}$ where the summation has to be carried out over a special bounded part of G^*. It seems to be difficult to derive the exact values of the discrepancies.

H. DINGES: Some approximations of the binomial distributions

There exist practical rules (for instance in quality control), in which ranges of the parameters the hypergeometric and the binomial distribution can be approximated by the Poisson - or the normal distribution. These seem fascinating to the students. There is interest in the way practical people really behave. However the students feel insecure when it comes to application of such rules. Asymptotic theory is considered more appropriate for a student of mathematics. In this talk some approximations are suggested, all starting from the equality

$$\frac{n(n-1)\ \dots\ (n-k-1)}{n^k} = -n\cdot g\left(\frac{k-\frac{1}{2}}{n}\right) + A(k,n),$$

where

$$A(k,n) = n\cdot g\left(-\frac{1}{2n}\right) + \frac{1}{24}\left(\frac{1}{n-k+\frac{1}{2}} - \frac{1}{n+\frac{1}{2}}\right) - \left(\frac{1}{24}\right)^2 \left[\frac{1}{(n-k+\frac{1}{2})^3} - \frac{1}{(n+\frac{1}{2})^3}\right] c(k,n)$$

with $1 \leq c\ (k,n) \leq 2$ and $g(x) = (1-x) \log (1-x) + x$.

A. DVORETZKY: Amarts

Generalization of asymptotic martingale are developed. In particular: A sequence of extended real-valued random variables X_n $(n=1,2,\dots)$ is an S-martingale if EX_t is defined for every finite stopping time t and $\lim_{t\to\infty} EX_t$ exists (on the extended real line). A sequence of extended real-valued random variables is an $\bar{S}$-martingale if the truncated random variables $(-H) \vee X_n \wedge H$ form an amart for every $0 < H < \infty$.

Sample theorems:

1) X_n is a.s. convergent $\Leftrightarrow$ X_n is an $\bar{S}$-martingale.

2) X_n amart and $E|X_n|$ bounded $\Rightarrow$ X_n^t is an S-martingale.
(Reference: Bull.Amer.Math.Soc. 82, 347 - 349, 1976)

A simple proof of a remarkable inequality recently discovered by Krengel and Sucheston is given. Let $Y_1,\dots,Y_n$ be independent non-negativ random variables. The inequality asserts that there exists a stopping time t for which $EY_t \geq \frac{1}{2}\ E \bigvee_{i=1}^{n} Y_i$. This is proved by showing the existence of t with $EY_t \geq G(\bigvee_{i=1}^{n} Y_i)$ where $G(x)$ is defined by $E(X - G(X))^+ = G(X)$ for every random variable X.

V. FABIAN: A new set estimate of a real parameter, and a comparison with hypotheses testing

For every θ in a set $\Theta \subset (-\infty,+\infty)$ let P_θ be a probability. Let $a \in (-\infty,+\infty)$, $\alpha \in (0,1]$ and let c, C be random variables with the property, for every θ

$$P_\theta(c < \theta) = 1 - \frac{\alpha}{2},\ P_\theta(\theta < C) = 1 - \frac{\alpha}{2}.$$

The set estimate S, defined by

$$S = \begin{cases} (c,C) \text{ on } \{a \in (c,C)\}, \\ (c,a) \text{ on } \{C \leq a\}, \\ [a,C) \text{ on } \{a \leq c\} \end{cases}$$

satisfies $P_\theta\{\theta \in S\} = 1 - \alpha$. Estimate S will be compared with conventional tests of hypotheses concerning θ.

W. FIEGER: On Gaussian Markoff processes

Let $\{X(t) : t \in \mathbb{R}^1\}$ be a stochastic process with continuous sample paths and $E\,X(t) \equiv 0$, $E|X(t)| \equiv c > 0$; $\mathfrak{A}(t_o)$ denotes the σ-algebra generated by $X(t)$, $t \leq t_o$. Then the following two statements are equivalent:

(i) $\{X(t) : t \in \mathbb{R}^1\}$ is a Gaussian Markoff process,

(ii) there exist a real function $\rho|(0,+\infty)$ and to every $h > 0$ and every convex function $s(\cdot)$ a real number $\gamma(h,s(\cdot))$ such that $\rho(h)\,X(t_o) + \gamma(h,s(\cdot))$ is the best prediction for $X(t_o + h)$, when the $X(t)$, $t \leq t_o$, is known and the loss is measured by the function $s(\cdot)$, and it is $\gamma(h,s^-(\cdot)) = -\gamma(h,s(\cdot))$ where $s^-(u) := s(-u)$.

Z. GOVINDARAJULU: The Secretary Problem with Interview cost

The paper deals with the problem of optimal stopping of the random permutation $x_1,\ldots,x_n$ of the numbers $1,\ldots,n$, when the admissible information at each stage is only the relative rank of the element observed last with respect to the preceeding ones. In part I the considered payoff is: receive a,b or nothing ($a \geq b \geq 0$) if one stops at the best, second best or any other element, and one pays $C_k - C_{k-1}$ for taking the k-th observation. It is shown that the optimal stopping rules are redically different depending on wether the costs of observations are "low" or "high". In the former case, the character of the optimal rule is the same as in the case involving no costs, while in the latter, it is best to stop at the first element. Methods of obtaining asymptotic optimal solutions in the case of linear and quadratic costs are also given. In part II the case when the payoff is the absolute rank of the k-th element and one pays $C_k - C_{k-1}$ for taking the k-th observation is considered.

U. HERKENRATH: The associated Markov-Process of generalized random systems with complete connections

The associated Markov-process $(\zeta_n)_{n\geq 1}$ of generalized and ordinary random systems with complete connections (GRSCC) is studied. This is done by making assumptions on the elements of the (G)RSCC, which ensure the validity of the Doeblin-condition (D) or certain continuity properties for the transition probability Q of the process $(\zeta_n)_{n\geq 1}$. The continuity properties of Q imply again (D) for a compact metric state space W. The problem of regularity of the process $(\zeta_n)_{n\geq 1}$ is discussed too. Finally possibilities of application of the theory of (G)RSCC, which go beyond the traditional applications in learning theory are indicated.

J. HOFFMANN-JØRGENSEN: Stochastic integration for vector valued function

Let (S,Σ,μ) be a finite positive measure space and $(B,\|\ \|)$ a Banach space. If $\{W(A):A \in \Sigma\}$ is a "white noise" with covariance μ, we can define the stochastic integral $\int_S f\,dW$ with $f \in \mathcal{S}_B$ (the B-valued simple functions) in the usual way. Closing $\mathcal{S}_B$ in the norm: $|f|_p = \|f\|_p + E\,\|\int_S f\,dW\|$ gives the space $L^p_B(W)$ $(0 \leq p < \infty)$, and the integral is extended in the natural way to $L^p_B(W)$.

Theorem 1: Contraction principle.

Let $f \in L^p_B(W)$, $\varphi \in L^\infty(\mu)$ with $|\varphi| \leq 1$, then $\varphi f \in L^p_B(W)$ and if $g \uparrow$ and is convex, then

$$Eg(\|\int_S \varphi f\,dW\|) \leq Eg(\|\int_S f\,dW\|)$$

and if $g \uparrow$, then

$$Eg(\|\int_S \varphi f\,dW\|) \leq 6\,Eg(6\,\|\int_S f\,dW\|).$$

Theorem 2:

$f \in L^p_B(W)$ if and only if: (i) $f \in L^p_B(\mu)$

(ii) $\langle x',f\rangle \in L^2(\mu)$ für alle $x' \in E'$

(iii) $R_f(x',y') = \int_S \langle x',f\rangle \langle y',f\rangle\,d\mu$ is the covariance of a gaussian Radon measure.

G. HÜBNER: Weak ergodic nonhomogeneous Markov chains and their application to dynamic optimization

We consider a stationary Markov decision model (with or without discounting). Let $V^n(s)$ be the optimal expected total return for n stages when starting in state s. ($V^n(s)$ does exist under very weak assumtion). Knowing $V^o, V^1, \ldots, V^n$ there are estimates of V^n for $N > n$ and of the asymptotic behaviour of the sequence (V^n) (cp. e.g. MaxQueen 1966). These estimates may be improved if some contraction factor γ_* is less than one. Here $1 - \gamma_*$ is a generalization of the ergodic coefficient from the theory of weak ergodic Markov chains (cp. e.g. Dobrushin 1956), Hajnal 1958). Similar estimates may be used to recognize future non-optimal decisions in the early stages. In the case of finite transition measures (e.g. if there is an absorbing state) related estimates can be found by other means.

U. KRENGEL: An inequality on betting with a prophet

In the last couple of years a lot of work has been done on a generalized notion of martingale, called asymptotic martingales or amarts. Let T be the collection of bounded stopping times for an increasing sequence $\mathcal{F}_n$ of σ-algebras and $(X_n)_{n\in\mathbb{N}}$ a process adapted to $(\mathcal{F}_n)$. Assume $X_n \in L_1$ for all n. (X_n) is a martingale iff $EX_\tau (\tau \in T)$ is constant, an amart iff $\lim_\tau EX_\tau$ exists. I report on recent extensions of a number of martingale results. The investigation of relations to independent sequences (X_n) or sequences $X_n = n^{-1} \sum_{i=1}^{n} Y_i$ with independent, non-negative Y_i has led to inequalities, which has an interesting interpretation in gambling language. One can show that there is a universal finite constant $K \leq 2(1 + \sqrt{3})$ such that $E(\sup_n X_n) \leq K \cdot \sup_\tau EX_\tau$.

J. LEHN: Premeasurable Functions

Let $(X, \mathfrak{A}, p)$ be a complete probability space. A function $f: X \to \mathbb{R}$ is called premeasurable iff there is a measure extension of p to the σ-field generated by $\mathfrak{A}$ and all preimages of Borel sets. Denote by $\mathfrak{M}$ the class of measurable functions and by $\mathfrak{F}$ the class of premeasurable functions. A function f belongs to $\mathfrak{F}_S$ iff there are sets X_i and functions $f_i \in \mathfrak{M}(i\in\mathbb{N})$ such that $\sum X_i = X$ and $f = f_i$ on X_i. $\mathfrak{F}_A$ is the class of all functions, whose graph admits on approximation by some product measurable set (in a certain sense used in literature). A function f belongs to $\mathfrak{F}_*$ iff there is some $g \in \mathfrak{M}$ such that the set $\{f \neq g\}$ has inner measure zero. Using the Measurable Choice Theorem one can prove $\mathfrak{M} \subset \mathfrak{F}_S \subset \mathfrak{F}_A = \mathfrak{F}_* \subset \mathfrak{F}$. Moreover there exists a probability space with $\mathfrak{M} \neq \mathfrak{F}_S \neq \mathfrak{F}_* \neq \mathfrak{F} \neq \mathbb{R}^X$ under the continuum hypothse.

J. LEMBCKE: A set function without σ-additive extension having finitely additive extensions arbitrarily close to σ-additivity

An example is given of a compact metrizable space X, a system $\mathfrak{F}$ of closed subsets of X and a set function $\mu: \mathfrak{F} \to \mathbb{R}_+$ having the following properties:

(1) $X \in \mathcal{F}$ and $\mathcal{F}$ generates the Borel σ-algebra on X.

(2) μ has a finitely additive but no σ-additive extension to the Borel σ-algebra $\mathcal{B}(X)$.

(3) For arbitrary ε > 0 there are positive set functions φ_ε and σ_ε on $\mathcal{F}$ with $\sigma_\varepsilon + \varphi_\varepsilon = \mu$ and $\varphi_\varepsilon(X) \leq \varepsilon$

such that σ_ε has an extension to a σ-additive measure on $\mathcal{B}(X)$ whereas all measures on $\mathcal{B}(X)$ extending φ_ε are purely finitely additive. A set function μ satisfying (2) and (3) would not exist if $\mathcal{F}$ were closed under finite intersections.

R. LERCHE: Sequential Bayesian Tests with Power 1

Ausgehend von der Situation bei der Brownschen Bewegung werden sequentielle Tests mit Macht 1 in solchen Fällen konstruiert, in denen keine erschöpfenden Partialsummen unabhängiger Zufallsgrößen existieren. Es sei $t \rightsquigarrow W(t)$ die k-dimensionale Brownsche Bewegung mit unbekanntem Trend $\theta \in \mathbb{R}^k$ und Kovarianz $(t \wedge s)I$. Der Prozeß der Aposteriori-Verteilungen von θ bezüglich dem Lebesquemaß auf dem $\mathbb{R}^k$ $t \rightsquigarrow N\left(\frac{W(t)}{t}, \frac{1}{t} I\right)$ erfüllt das folgende Gesetz des Iterierten Logarithmus (dabei ist N(a,B) die k-dimensionale Normalverteilung mit Mittelwert a und Kovarianz B, $B_{\theta_o}(a)$ die Kugel um θ_o mit Radius a und θ_o der wahre Parameter):

$$\overline{\lim_{t\to\infty}} N\left(\frac{W(t)}{t}, \frac{1}{t} I\right) \left(\complement B_{\theta_o}\left(d \sqrt{\frac{\log \log t}{t}}\right)\right) = \begin{cases} 1 & \text{für } d < \sqrt{2} \\ 0 & \text{für } d > \sqrt{2} \end{cases} .$$

Unter den Bedingungen des Bernstein-von Mises-Le Cam Theorems kann man für die Aposteriori-Verteilungen $F_{x,n}$ ein Gesetz des einfachen Logarithmus beweisen:

$$\lim_{n\to\infty} F_{x,n} \left(\complement B_{\theta_o}\left(\sqrt{\frac{\log n}{n}}\right)\right) = 0 \quad P_{\theta_o}\text{-fast sicher.}$$ Dies gestattet einem Test mit Macht 1 und trennscharfe Konfidenzbereiche zu konstruieren. Ähnlich wie es in der Arbeit von Robbins und Siegmund (Ann. Math. Stat. 41, 1410 - 1429, 1970) für Tests basierend auf Partialsummen durchgeführt wurde, kann man mit einem Invarianzprinzip

für Aposteriori-Verteilungen das asymptotische Niveau solcher Tests bestimmen. In der Grenze tritt der Prozeß der Aposteriori-Verteilungen der Brownschen Bewegung auf.

K. MARTI: Lösungen stochastischer Optimierungsprobleme mittels "Stochastischer Dominanz" und Stochastischer Penalty-Methoden

There are presented two new Methods for the approximate solution of stochastic programming problems. The first method is based on the calculation of directions of decrease by means of the order relation $\lambda_1 \precsim_C \lambda_2$ if and only if $\int u d\lambda_1 \leq \int u d\lambda_2$, $u \in C$ between probability measures λ_1, λ_2, where C is a suitable set of convex functions. In the second case we obtain approximate solutions by minimizing the sum of the given objective function of the problem plus $\frac{1}{s} E\psi(g(\omega,x)$ where s is a small positive number, E denotes the expectation operator, $g(\omega,x) \leq 0$ a.s. is the original constraint and $\psi : \mathbb{R}^m \to \mathbb{R}_+$ is a penalty function. By this second method it becomes possible to find calculable approximations to the price-system related to the non-anticipativity constraints in two- and multy-stage stochastic optimization problems.

D.W. MÜLLER: Separation of measures whose log-likelihood ratio is approximately normal

Let P,Q be two probability measures on the same space. Let $\pi = \int dP \wedge dQ$ and $\gamma = \int \sqrt{dP\, dQ}$. The inequality $\gamma \leq \sqrt{\pi(2-\pi)}$ is improved for those P,Q whose "standard measure" is close to the standard measure S_θ of some $\left(N(-\frac{\theta}{2}, 1), N(\frac{\theta}{2},1)\right)$. One of the results is as follows: among all standard measures U on [0,1], subject to $U \leq (1+\varepsilon)\, S_\theta$ and $\int s \wedge (1-s)\, U(ds) = \pi$, there is a "least favorable" one of the form $U_o = (1_{[A,B]} + 1_{[1-B,1-A]})\,(1+\varepsilon)\, S_\theta$ maximizing γ. γ_{max} can be determined form the following equations, where I denotes an interval:

$$2\,(1+\varepsilon)\, N\left(-\frac{\theta}{1}, 1\right)(I) = \pi$$

$$2\,(1+\varepsilon)\, N\left(\frac{\theta}{2}, 1\right)(I) = 2-\pi$$

$$2(1+\varepsilon)\, N(0,1)\,(I) = \gamma_{max}/\gamma_{gauss}$$

(here $\gamma_{gauss} = \int \sqrt{s(1-s)}\; S_\theta(ds)$). Similar results holds for π_{min} given γ.

U. OPPEL: Random search and evolution

Zur Beschreibung irreversibler Prozesse und zur Gewinnung von Optimierungsverfahren wird ein auf dem Zufallssuchverfahren (random search) basierender Evolutionsbegriff eingeführt. Zu einem Evolutionsprozeß gehört ein Zustandsraum $(X, \mathfrak{A})$, eine Folge $\Pi := (\pi_n : n \in \mathbb{N})$ von Mutationsübergangswahrscheinlichkeiten $\pi_n : X^n \times \mathfrak{A} \to [0,1]$, ein Qualitätsniveau $G: X \to \mathfrak{A}$ und eine Folge $P := (P_n : n \in \mathbb{N})$ von Selektionsübergangswahrscheinlichkeiten $P_n : X^n \times \mathfrak{A} \to [0,1]$ mit

$$P_n(x_1,\ldots,x_n;A) := \pi_n(x_1,\ldots,x_n;A \cap G(x_n)) + \pi_n(x_1,\ldots,x_n;X \setminus G(x_n))\delta_{x_n}(A).$$

P bestimmt zu jedem $x \in X$ einen in x startenden kanonischen Evolutionsprozeß $(X^{\mathbb{N}}, \mathfrak{A}^{\mathbb{N}}, \mathbb{P}^x; Y_n : n \in \mathbb{N})$. Für solche Prozesse werden Bedingungen für die Gültigkeit der Erreichbarkeitsaussage

"$\mathbb{P}^x\left(\bigcup_{n\in\mathbb{N}} \bigcap_{n \geq m} Y_n^{-1}(B)\right) = 1 \quad \forall \quad x \in X$" angegeben.

J.W. PITMAN: Symmetric events for independent random variables

Recent work on canonical Gibbs states draws attention to the question of when the symmetric σ-Field generated by a sequence of random variables is trivial. For an independent sequence (X_n) with countable range space S, $B \subset S$, let $\alpha_n(B)$ be the minimum of $P(X_n \in B)$ and $P(X_n \notin B)$. Then a necessary and sufficient condition for the symmetric σ-field to be trivial is that $\sum_n \alpha_n(B)$ equals either 0 or ∞ for every $B \subset S$. Unlike the Hewitt-Savage 0-1 law this result fails to extend to more general range space. For real valued random variables the condition for all Borel sets B is still necessary but no longer sufficient for a trivial symmetric σ-field.

R.D. REISS: Approximate distributions of a large number of order statistics

Let $Z_{1:n} \leq \ldots \leq Z_{n:n}$ denote the order statistics of n i.i.d. exponential random variables. Let $1 \leq r_1 < \ldots < r_k \leq n$ and $\underline{r} = (r_1, \ldots, r_k, n)$. Let $P_{\underline{r},i}$ denote the standardized distribution of $Z_{r_i:n} - Z_{r_{i-1}:n}$.

Starting from an asymptotiv expansion over the Borel-algebra $\mathfrak{B}$ to (l + 1) terms of $P_{\underline{r},i}$ it can be proved (by means of the stochastical independence of $Z_{r_i:n} - Z_{r_{i-1}:n}$, i=1,...,k) that

$$(1)\ \sup_{B\in\mathfrak{B}^k} \left| \left(\mathop{X}_{i=1}^{k} P_{\underline{r},i} \right)(B) - \int_B \prod_{i=1}^{k} \left(1 + \sum_{j=1}^{l} L_{\underline{r},j}(x_i) \right) dN^k_{(0,1)}(x_1, \ldots, x_k) \right|$$

$$= O\left(\sum_{i=1}^{k+1} (r_i - r_{i-1})^{-\frac{l+1}{2}} \right)$$

(with $r_o = 0$ and $r_{k+1} = n$) where $N_{(0,1)}$ is the standard normal distribution and $L_{\underline{r},j}$ are polynomials of degree $\leq 3j$. Using suitable transformations one can derive from (1) an analogue result for the joint distribution of the order statistics $Z_{r_1:n}, \ldots Z_{r_k:n}$. For $l > 1$ this result extends the Theorem in [Reiss, R.D. (1975). The asymptotic normality and asymptotic expansions of the joint distribution cf several order statistics, in: 11.Limit Theorems Probab.Theory (Colloq. Math. Soc. Janos Bolgai), ed. P. Révész, 297 - 340].

P. RÉVÉSZ: On the multi-parameter Wiener Process

Let W(x,y) ($x \geq 0$, $y \geq 0$) be a two-parameter Wiener Process, and let $b_T \geq T^{1/2}$ be a nondecreasing function of T and define

$$\gamma_T = (2T\,[\log(\log b_T\, T^{1/2} + 1) + \log\log T])^{-1/2}$$

$$D_T = \left\{ (x,y): xy = T,\ 0 \leq x \leq b_T,\ 0 \leq y \leq b_T \right\}.$$

Suppose that: (i) γ_T is a non-increasing function of T, (ii) for any $\varepsilon > 0$ there exists a $\theta_o = \theta_o(\varepsilon) > 1$ such that

$$\limsup_{k\to\infty} \gamma_{\theta^k} / \gamma_{\theta^{k+1}} \leq 1 + \varepsilon$$

if $1 < \theta \leq \theta_o$. Then $\limsup_{T\to\infty} \sup_{(x,y)\in D_T} \gamma_T |W(x,y)| = 1$ with probability 1.

This theorem is a generalization of a law of iterated logarithm of Zimmermann (Ann.Math.Statist. 1972, 43, 1235 - 1246).

U. RÖSLER: Über die Trivialität der tail σ-Algebra bei Diffusionsprozessen

Die Stoppzeit t_y bezeichne das erstmalige Erreichen des Punktes y. Die Verteilung von t_y besitzt, bei Start in einem Punkt, eine Lebesguedichte, die erst monoton steigend, dann monoton fallend ist. Hieraus läßt sich ein Kriterium für die Gültigkeit des 0-1 Gesetzes der tail σ-Algebra bei transienten Diffusionsprozesses ableiten. Rekurrente Diffusionsprozesse erfüllen stets das 0-1 Gesetz.

H. ROST: Infinite systems of interacting diffusions

In an analogous way to results of Marchioro, Pellegrinotti and Presutti, who showed that the infinite systems of particles moving according to a Hamiltonian equation of motion

$$\frac{dp_i}{dt} = - \sum_{j\neq i} \text{grad}\, \Phi(q_i - q_j), \quad \frac{dq_i}{dt} = p_i \quad \text{for } i=1,2,...$$

admits a meaningful time evolution, one consideres the stochastic differential equation

$$(*)\ dX_i = - \left(\sum_{j\neq i} \text{grad}\, \Phi(X_i - X_j)\right)dt + dW_i \quad \text{for } i=1,2...$$

where the W_i are independent Wiener processes. The existence and uniqueness of solutions of (*) within a suitable class of configutations is shown; the invariant measures are characterized as canonical Gibbs stats for the potential Φ.

W. RUPP: Mengenwertige Maße und Fortsetzungen

Seien $(\Omega, \mathfrak{A}, \mu)$ ein Wahrscheinlichkeitsfeld, $\varphi:\Omega \Rightarrow \mathbb{R}^m$ eine Korrespondenz (d.h. $\emptyset \neq \varphi(\omega) \subset \mathbb{R}^m$) und $\mathcal{L}_\varphi(\Omega, \mathfrak{A}, \mu)$ die Menge aller μ-integrierbaren

Funktionen $f:\Omega \to \mathbb{R}^m$ mit $f(\omega) \in \varphi(\omega)$ μ-fast sicher. Weiter sei $\Phi_\varphi^\mu(A) := \int_A \varphi \, d\mu := \left\{ \int_A f \, d\mu : f \in \mathcal{L}_\varphi(\Omega, \mathfrak{A}, \mu) \right\}$ das Integral von φ über $A, A \in \mathfrak{A}$. $\Phi_\varphi^\mu | \mathfrak{A}$ stellt ein mengenwertiges Maß dar. Betrachtet man zwei W-Felder $(\Omega, \mathfrak{A}_i, \nu_i)$ mit $\mathfrak{A}_1 \subset \mathfrak{A}_2$, so stellt sich die Frage, ob ν_2 genau dann eine Fortsetzung von ν_1 ist, wenn $\Phi_\varphi^{\nu_2}$ eine Fortsetzung von $\Phi_\varphi^{\nu_1}$ ist? Eine Antwort liefert der folgende

<u>Satz:</u>

Ist φ abgeschlossenwertig und schwach meßbar bezüglich $\mathfrak{A}_1$, d.h. ist $\{\omega \in \Omega : \varphi(\omega) \cap G \neq \emptyset\} \in \mathfrak{A}_1$ für alle offenen $G \subset \mathbb{R}^m$, dann gilt:

α) Falls $(\Omega, \mathfrak{A}_i, \nu_i)$ atomfrei ist und φ ν_1-integrierbar beschränkt ist, dann gilt

$$(\nu_2 \text{ Fortsetzung von } \nu_1 \Rightarrow \Phi_\varphi^{\nu_2} \text{ Fortsetzung von } \Phi_\varphi^{\nu_1}).$$

β) Falls φ konvexwertig und ν_i-integrierbar beschränkt ist für $i=1,2$ und $\nu_1\left(\left\{\omega:\varphi(\omega) = \{0\}\right\}\right) = \nu_2\left(\left\{\omega:\varphi(\omega) = \{0\}\right\}\right) = 0$ ist, dann gilt

$$(\Phi_\varphi^{\nu_2} \text{ Fortsetzung von } \Phi_\varphi^{\nu_1} \Rightarrow \nu_2 \text{ Fortsetzung von } \nu_1).$$

N. SCHMITZ: <u>Fixed-width confidence intervals</u>

Für das Schätzen des Parameters μ einer $N(\mu,\sigma^2)$-Normalverteilung gibt es bei unbekanntem σ^2 und vorgegebenen γ und d keinen festen Stichprobenumfang n, so daß ein Konfidenzintervall C_n zum Niveau γ der Breite $2d$ existiert. Die von den Praktikern benützten sequentiellen Konfidenzintervalle

$$N_T(x) = \min\left\{n \geq 2 : s_n^2(x) \leq \frac{n \cdot d^2}{t^2_{n-1,(1+\gamma)/2}}\right\} \quad \hat{C}_n(x) = [\bar{x}_n - d, \bar{x}_n + d]$$

haben den Nachteil, daß sie das Niveau nicht einhalten. Es wird eine Methode angegeben, durch eine (nur vom Niveau γ und dem Anfangsstichprobenumfang abhängende) Anzahl von Zusatzbeobachtungen ein asymptotisch konsistentes und asymptotisch effizientes sequentielles Verfahren zu konstruieren, das auch finit das Niveau einhält; einige Resultate von Computer-Berechnungen werden angegeben.

M.P. SCHÜTZENBERGER: On Banach match box problem

Let for given n, X be the random variable equal to the number of returns to zero in the classical coin tossing game of duration 2n. Also let Y = -1 + the number of matches remaining in the equally classical Banach match box problem (with n as number of matches at epoch 0 in each box). The two variables have the same probability distribution. D. Foata has made the remarkable discovery that in an appropriate geometric set up, the distribution of Y given X + Y is uniform. Proof is by a simple combinatorial (i.e. geometric) argument.

E. SIEBERT: Limit theorems for distributions on locally compact groups

Our aim are analogous results for distributions on a locally compact group G of the following classical results:

I. Chintschin's theorem on the accompanying laws.
II. Central limit theorem.
III. Convergence to Poisson laws.

Our method of proof is a rigorous use of infinite dimensional Fourier transformation. Let $\mathfrak{J} = (\mu_{n\kappa})_{1\leq\kappa\leq n; n\geq 1}$ be a triangular array of distribution on G, which is infinitesimal, i.e. $\lim_n \max_\kappa \mu_{n\kappa}(\complement U) = 0$ for all U in the neighbourhood system $\mathfrak{U}(G)$ of the unit e in G, and commutative, i.e. $\mu_{n\kappa} * \mu_{ne} = \mu_{ne} * \mu_{n\kappa}$. I holds for $\mathfrak{J}$ under an additional boundedness condition (B) for the Fourier transforms. If (B) is satisfied and we have $\lim_n \sum_\kappa \mu_{n\kappa}(\complement U) = 0$ for all $U \in \mathfrak{U}(G)$ then the seuqence $\mu_n := \mu_{n1} * \ldots * \mu_{nn}$ has only Gaussian accumulation points. If we have $\overline{\lim_n} \sum_\kappa \mu_{n\kappa}(G \setminus \{e\}) < \infty$ the sequence (μ_n) converges to a Poisson measure.

D. SIEGMUND: Repeated significance tests for a normal mean

Let $x_1, x_2, \ldots$ be independent $N(\mu,\sigma^2)$ random variables, $s_n = \sum_{\kappa=1}^{n} x_\kappa$. To test $H_o: \mu = 0$ against $H_1: \mu \neq 0$, with σ^2 known, consider the following sequential procedure. For $c \geq 0$, $a > 0$, $n_o = 1,2\ldots$ stop sampling at T = first $n \geq 1$ such that $|s_n| \geq \sigma \sqrt{2a(n+c)}$ or at n_o, whichever is smaller. If $T \leq n_o$, reject n_o; if $T > n_o$, do not reject H_o. On Sequential Medical Trials, 2nd ed., P. Armitage has studied the test numerically. The present work gives asymptotic approximations as $a \to \infty$ for $P_{o,\sigma}\{T \leq a\ t_o\}$, $P_{\mu,\sigma}\{T > a\ t_o\}$ $(|\mu| > \sigma \sqrt{2/t_o})$, and for $E_{\mu,\sigma}\{\min(T, a\ t_o)\}$ $(\mu = 0$ or $|\mu| > \sigma \sqrt{2/t_o})$. The asymptotic results provide good approximations to the significance level, power, and expected sample size for small and moderate sample sizes. An analogous test for the case of unknown σ^2 is proposed and similar results obtained.

G. WAHBA: Optimal Smoothing

We discuss various optimal smoothing problems when the noise is "white", the "signal" is concentrated at "low frequency" and the signal to noise ratio is unknown but must be estimated from the data. It can be shown that an appropriate scaling factor, which controls the smoothness of the estimated signal, can, asymptotically, be estimated from the data by the method of generalized cross-validation (GCV). Some applications are to 1) Smooth curve approximation and numerical differentiation of noisy experimental data, 2) Optimal smoothing of density and spectral density estimates, 3) Ridge regression, 4) The approximate solution of ill posed linear operator equations when the data are noisy.

H. von WEIZSÄCKER: Choquet representation of probability measures

Theorem:

Let $(\Omega, \mathfrak{B})$ be a measurable space. Let M be a convex set of probability measures on $\mathfrak{B}$. Assume either

i) Ω is a topological space, $\mathfrak{B}$ its Borel σ-Field and M is closed in the narrow topology (weak topology of Topsøe) in the class of tight probabilities

or

ii) $(\Omega, \mathfrak{B})$ is isomorphic to a universially measurable subset of [0,1] with its Borel σ-field and M satisfies the closedness condition

$$(C)\quad \begin{cases} \text{there is a countable set } F \subset \bigcap_{\mu \in M} \mathcal{L}^1(\mu) \text{ such that for any} \\ \text{probability measure } \rho \text{ on } \mathfrak{B} \text{ the following is true} \\ \mu_n \in M \;\; \forall\, n \in \mathbb{N},\ \int f\, d\mu_n \to \int f\, d\rho \;\; \forall\, f \in F \Rightarrow \rho \in M. \end{cases}$$

Then for all $\mu \in M$ exists a probability measure p on the σ-algebra $\sigma\{\text{ex } M \ni v \;\; \to v(B) | B \in \mathfrak{B}\}$ such that

$$\mu(B) = \int_{\text{exm}} v(B)\, dp(v) \qquad \text{for all } B \in \mathfrak{B}.$$

J. WOLFOWITZ: The Rate Distortion Function for Source Coding with Side Information at the Decoder

Let (X_i, Y_i), $i=1,2,\ldots$, be independent chance variables with the same distribution as (X,Y). X and Y take values in the respective finite sets A and B. Let $C = \{1,\ldots,c\}$, C_n be the cartesian product of n C's, and similarly for A_n and B_n. A distortion function d_0: $(A \times C) \to$ positive numbers, is given. We also define d: $(A_n \times C_n) \to$ positive numbers, as follows: $d\left(\left[a_1,\ldots,a_n\right], \left[c_1,\ldots,c_n\right]\right) = \frac{1}{n} \sum_{i=1}^{n} d_0(a_i, c_i)$. Write $X^n = (X_1,\ldots,X_n)$ and similarly for Y^n. Let $\varepsilon > 0$ be arbitrary but fixed, and let n be sufficiently large. Suppose that there exists a function f: $A_n \to D = \{1,2,\ldots, ||f||\}$, and a function γ: $(D \times B_n) \to C_n$, such that $Ed(X^n, \gamma(f(X^n), Y^n)) < z + \varepsilon$. The problem is: how small can $||f||$ be? More precisely, we want to find

$$\lim_{\varepsilon \to 0} \lim_{n \to \infty} \frac{1}{n} \log (\text{minimum } ||f||) = R(Z), \text{ say.}$$

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The answer is as follows: Let Z be a chance variable with values in C such that Y, X, Z, in this order, form a Markov chain, and such that $Ed_0(X,Z) \le z$. Then

$$R(z) = \min \left[I(X,Z) - I(Y,Z) \right],$$

where I is the usual information function between two chance variables, and the minimum is over all Z which satisfy the above conditions.

R. ZMYSLONY: Completness for a family of normal distributions

Let y be an R^n valued random vektor. Let $Ey=Xb$ and $\operatorname{cov} y = \sum_{i=1}^{m} c_i V_i$.

Matrices $X, V_1, \ldots, V_m$ are known, $V_m = I$, vectors $b=(b_1,\ldots,b_p)$ and $c=(c_1,\ldots,c_m) \in \Omega$ are unknown. It is assumed that $\Omega \subset R^m$ contains a nonempty set and $V(c) = \sum_{i=1}^{m} c_i V_i > 0$ for each $c \in \Omega$.

Let $P=XX^+$ and let $M=I-P$, where X^+ stands for the Moore-Penrose general inverse matrix of X.

Theorem. The family of distributions $N(Xb, V(c))$ admits a complete and sufficient statistcs if and only if $\mathcal{W} = \operatorname{span} MV_iM$, $i=1,\ldots,m$ is a quadratic subspace and $PV_i = V_iP$ for $i=1,\ldots,m$.

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