MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 15 / 1977

Distributionen

12. 4. - 16. 4. 1977

Unter der Leitung der Herren Professoren J.Wloka (Kiel) und Z.Zielezny (New York) fand vom 12. 4. - 16. 4. 1977 eine Tagung über Distributionen statt. Die Dauer der Tagung wurde in Anbetracht der Fülle des Programms im allgemeinen als zu kurz empfunden.

Teilnehmer

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J. Wloka, Kiel

V. Wrobel, Kiel

Z. Zielezny, Amherst

Vortagsauszüge

N. BAZLEY: Approximation of operators with reproducing nonlinearities

Let H be a real Hilbert space and N a nonlinear operator defined on a dense domain of H. Then N is said to be "reproducing" relative to a sequence $\{u_i\}$ whenever $N(\sum_{i=1}^{n} \alpha_i u_i) = \sum_{i=1}^{m(n)} \beta_i(\vec{\alpha})u_i$, where the coefficients β_i are explicitly known. Consider the nonlinear eigenvalue problem $Au + N(u) = \lambda u$ and approximate by the Galerkin problem $P^nAP^nu + P^nN(P^nu) = \lambda P^nu$. If P^n is the orthogonal projection on the first n of the u_i, this leads to the nonlinear algebraic problem $\sum_{i=1}^{n}(Au_i,u_j)\alpha_i + \beta_j(\vec{\alpha}) = \lambda\alpha_j$ for $j = 1,2,\ldots,n$. The Ljusternick-Snirelman critical values give upper bounds to the original problem, extending the method of Rayleigh and Ritz. Further, the method can be used to separate variables for nonlinear wave equations of the form $\frac{d^2u}{dt^2} + Au + N(u) = 0$. Here one writes $u = \sum_{i=1}^{\infty} \alpha_i(t)u_i$ and approximates by $\frac{d^2P^ku}{dt^2} + P^kAP^ku + P^kN(P^ku) = 0$, where $P^ku = \sum_{i=1}^{\infty} \alpha_i^k u_i$ and the α_i^k are given as the solution of the known coupled system of ordinary differential equations $\alpha_i^k + \sum_{j=1}^{k}(Au_i,u_j)\alpha_i(t) + \beta_i(\vec{\alpha}^k) = 0$ $(i = 1,\ldots,k)$.

B. BOJARSKI: Interiour boundary value problems

Let M be a closed manifold, $X = \bigcup_\nu X_\nu$ a collection of smooth disjoint submanifolds of M of codimension $\nu \geq 2$, E, F, G-vector bundles over M and and X respectively, $\Gamma^{s}(E)$ etc. the corresponding Sobolev spaces of sections (in general distributional). The interiour boundary value problem - also called the Sobolev problem - is the problem of solving the equation $Au = f \pmod{\Delta}$, $Bu = g$, $u \in \Gamma^{s}(E)$, $f \in \Gamma^{s-m}(F)$, $g \in \Gamma^{t}(G)$, for suitable s and t. Here Δ is the subspace of distributional sections of F in $\Gamma^{s-m}(F)$ supported by X.

A theory of multiple layer potentials with "momentum" densities $\mathcal{T}$-concentrated over X is studied. These potentials are used to parametrize Δ by the densities $\mathcal{T}$, which are in general distributional sections of certain vector bundles over X. Composed with the paramatrix of A over M this gives the generalized classical multiple layer potentials for submanifolds of codimension $\nu \geq 2$. As an application we get the reduction of the Sobolev problem to a system $\hat{A}$ of pseudodifferential (elliptic) operators over X. Also the index of an elliptic Sobolev problem is expressed in terms of the index of A over M and the index of $\hat{A}$.

P. DIEROLF: Ueber zwei Räume regulärer temperierter Distributionen

Am Beispiel von $\mathcal{S}(\mathbb{R}^n)'$ = Raum der temperierten Distributionen wird untersucht, in welchem Sinn lokalintegrierbare Funktionen Distributionen erzeugen, wenn der Grundraum nicht aus stetigen Funktionen mit kompaktem Träger besteht.

1. $\Lambda_{ar}(\mathbb{R}^n) := \{ f \in L^1_{loc}(\mathbb{R}^n);\ f \cdot \varphi \in L^1(\mathbb{R}^n)\ \forall \varphi \in \mathcal{S}(\mathbb{R}^n) \}$

= Raum der absolut-regulären temperierten Distributionen.

$\langle f, \varphi \rangle := \int f \cdot \varphi \, dx \quad (\varphi \in \mathcal{S}(\mathbb{R}^n),\ f \in \mathcal{L}_{ar}(\mathbb{R}^n))$.

2. $\mathcal{L}(\mathbb{R}^n) = L^1_{loc}(\mathbb{R}^n) \cap \mathcal{S}(\mathbb{R}^n)'$ = Raum der regulären temperierten Distributionen.

$\langle f, \varphi \rangle := \lim_{k \to \infty} \int f \cdot \eta_k \cdot \varphi \, dx \quad (\varphi \in \mathcal{S}(\mathbb{R}^n),\ f \in \mathcal{L}(\mathbb{R}^n))$ für eine Approximation der Eins (η_k; $k \in \mathbb{N}$) aus $\mathcal{D}(R^n)$.

Die Räume $\mathcal{L}_{ar}(\mathbb{R}^n)$ und $\mathcal{L}(R^n)$ werden in natürlicher Weise mit LB- bzw. LF-Raum-Topologien $\mathcal{T}$ bzw. $\mathcal{F}$ versehen. Gegenstand des Vortrags sind die Struktur der Räume $(\mathcal{L}_{ar}(\mathbb{R}^n), \mathcal{T})$, $(\mathcal{L}(\mathbb{R}^n), \mathcal{F})$, ihre topologischen Eigenschaften und die Bestimmung von Multiplikatoren bzw. Convolutoren.

H.G. GARNIR: Elementary solution of boundary value problems for hyperbolic matrix operators

This talk has a triple purpose: a) to give a precise definition of the elementary solution of the general boundary value problems in hyperbolic equations with constant coefficients. b) to give an explicit expression of this elementary solution for the space $\mathbb{E}_n$ and the half-space $\mathbb{E}_{n+1}$.
c) to show an analogy of the two precedent elementary solutions and so suggest the extension from $\mathbb{E}_n$ to $\mathbb{E}_n^+$ of the different methods of finding the support (Paley-Wiener) or analytic and essential support (method of localization of Gårding-Hörmander). Solutions are proposed for a), b), c) which generalize and make more precise the last result of Sakamoto, Wakabayashi, and Tsuji.

K. GAWĘDZKI: Random distributions and their applications to quantum physics suppose that $d\mu$ is a Gaussian measure with mean zero and covariance $(-\Delta + m^2)^{-1}$ on $S'(\mathbb{R}^n)$, $m \in \mathbb{R}^1$. Moments of $d\mu$ can be considered the main object of interest in the description of a physical system namely of the system of noninteracting particles of the simplest kind. Moments of certain non-Gaussian measure would correspond to the system of interacting particles. Construction of such measures and examination of their properties is the aim of constructive quantum field theory initiated around 1966 by J. Glimm and A. Jaffe.

O. von GRUDZINSKI: <u>Remarks on the propagation of singularities by distributions with compact support</u>

A simple relation between the wave front set of a distribution f with compact support and its associated set $\mathcal{H}(f)$ of supporting functions (as introduced by Hörmander) is stated and used to answer the question as to what extent the characteristic function of the unit ball propagates singularities. The reported results were obtained in collaboration with Sönke Hansen.

S. HANSEN: <u>A uniqueness-theorem for convolution equations</u>

An extension of Holmgren's uniqueness-theorem from the case of partial differential equations to the case of convolution equations defined by a distribution $S \in \mathcal{E}'(\mathbb{R}^n)$ is given: Let $S \in \mathcal{E}'$ be a D'-invertible convolutor and $N \in \mathbb{R}^n$ a noncharacteristic vector for S (this is a condition on the zeros of the Fouriertransform $\hat{S}$ of S, which coincides for $S = P(D)\delta$ with the condition $P_m(N) \neq 0$). Then any $u \in D'(\mathbb{R}^n)$

with $S * u = 0$ in $\mathbb{R}^n$ and supp $u \subset H_N := \{x \in \mathbb{R}^n : \langle x, N \rangle \geq 0\}$ vanishes already in all $\mathbb{R}^n$.

K. KELLER: Multiplikation von Distributionen

Es wird diskutiert, in welchem Rahmen eine Produktbildung für Distributionen durchführbar ist. Dabei stellt sich heraus, daß nach Abschwächung der üblichen Stetigkeit und der Assoziativität für große Klassen von Distributionen eine Multiplikationsoperation erklärt werden kann. In verhältnismäßig einfacher Weise lassen sich z.B. alle Produkte homogener und zugeordneter Distributionen auf $\mathbb{R}$ bestimmen. Mit Hilfe von Tensorprodukten und Reduktionsformeln kann die Multiplikation in der für Quantenfeldtheorien interessanten Klasse

$$\mathcal{H}(\mathbb{R}^M, \mathbb{R}^N) := L\{|x^2|^\alpha \ln^k |x^2| \psi, \delta^{(n)}(x^2)\psi, D^\rho \delta \mid \alpha \in \mathbb{C}, k, n \in \mathbb{N}_0, \rho \in \mathbb{N}_0^{M+N}, \psi \in C^\infty\}$$
$$x^2 = x_1^2 + \dots + x_M^2 - x_{M+1}^2 - \dots - x_{M+N}^2$$

auf den zuvor behandelten Fall einer Variablen zurückgeführt werden. Der Zusammenhang der Multiplikation mit anderen, regulären Operationen legt es nahe, alle diese Operationen einheitlich mit Hilfe von "Wertfunktionalen" $E: \mathcal{E}' \longrightarrow \mathbb{C}$ aufzubauen. Durch die Festsetzung $(f \cdot g, \varphi) := E(\check{f} * g\varphi)$ lassen sich Produkte von Distributionen mit sehr allgemeinen punkt- oder flächenartigen Singularitäten konstruieren.

J. KISYNSKI: Representations of Lie groups in Hilbert spaces and cosine operator functions

Let R be a strongly continuous representation of a Lie group G in a Hilbert space H. Let $X_1, \dots, X_n$ be any set of vectors in the Lie algebra of G and let $X_0 \in \mathrm{lin}_{\mathbb{C}} \{X_1, \dots, X_n\}$. Then the operator $dR(\sum_{\nu=1}^{n} X_\nu^2 + X_o)$, defined on the set of all C^∞-vectors of R, is a pregenerator of a strongly continuous cosine operator function. As a consequence, $dR(\sum_{\nu=1}^{n} X_\nu^2 + X_o)$ is a pregenerator of a semigroup of operators which is holomorphic in the whole open right half-plane. The proof is based on a study of the wave operator $\frac{\partial^2}{\partial t^2} - \sum_{\nu=1}^{n} X_\nu^2 - X_o$ in spaces of H-valued functions square integrable on G with suitable weight functions. It slightly engages H-valued distributions on G. The develloped methods permit to prove by a purely analytical argumentation an important estimation of the decay at ∞ of probability measures on G belonging to any convolution semigroup which satisfies the Lindeberg condition.

H. KOENIG: Approximation numbers and eigenvalues of compact operators

A generalization of Weyl's inequality in Hilbert spaces to Banach spaces is shown: Let $\alpha_n(T)$ and $\lambda_n(T)$ denote the approximation numbers and eigenvalues of a compact operator T in a Banach space X. Then for any $0 < p < \infty$ there is a constant c_p such that for any $T \in K(X)$ the inequality

$$\sum_{n\in N} |\lambda_n(T)|^p \leq c_p \sum_{n\in N} \alpha_n(T)^p$$

holds. A similar inequality holds with the l_p-(quasi-)norms replaced by the Lorentz-sequence space-norms. This answers a problem of A. Pietsch and A.S. Markus - V.I. Macaev. An application is given to the eigenvalue distribution of an operator in $L_p(\Omega)$, $1 < p < \infty$, the image of which is in a Sobolev space $W_p^\lambda(\Omega)$ or a Besov space $B_{p,q}^\lambda(\Omega)$; the eigenvalues decrease of order $n^{-\lambda/\dim\Omega}$, since this is the order of the approximation numbers of the imbedding $B^{\lambda}_{p,q}(\Omega) \hookrightarrow L_p(\Omega)$. Between the single eigenvalues and the approximation numbers one has the equality $|\lambda_n(T)| = \lim_{j\to\infty} \alpha_n(T^j)^{1/j}$, which generalizes the spectral radius formula (for compact operators).

H. KOMATSU: <u>Topics from the theory of ultradistributions</u>

A survey was given of the theory of ultradistributions. Let M_p be a sequence of positive numbers satisfying suitable conditions, e.g. $M_p = p!^s$, $s > 1$. A function $\varphi(x)$ on an open set Ω in $\mathbb{R}^n$ is said to be an ultradifferentiable function of class (M_p) (resp. $\{M_p\}$) if for each compact set K in Ω and each $h > 0$ there is a C (there are constants h and C) such that $\sup_{x\in K}|D^\alpha\varphi(x)| \le Ch^{|\alpha|}M_{|\alpha|}$. The elements in the dual $\mathcal{D}^{*\prime}(\Omega)$ of the space $\mathcal{D}^*(\Omega)$ of ultradifferentiable functions of class $*$ with compact support endowed with the natural topology are called ultradistributions of class $*$. In each class $\mathcal{D}^{*\prime}(\Omega)$ of ultradistributions one can prove the analogues of major theorems in the theory of distributions, including the following:

1. Localization theorem 2. Structure theorem 3. Structure theorem of ultradistributions with support in a submanifold. 4. Kernel theorem. Lastly as an application of the third theorem the following theorem was given with a sketch of proof:

<u>Theorem</u>: For each sequence c_α of numbers of $\mathbb{R}^n$ or class $*$ there is an ultradifferentiable function φ or of class $*$ such that $D^\alpha\varphi(0) = c_\alpha$.

R. MEISE: An application of a cohomology vanashing theorem to P-convexity

The aim of this lecture was to indicate that the result on the vanishing of cohomology groups for certain product sheaves on suitable sets, given as Satz 3.3 (in Bierstedt, Gramsch, Meise: Approximationseigenschaft, Lifting und Kohomologie bei lokalkonvexen Produktgarben, manuscripta mathematica 19 (1976), 319 - 364) also has the following application:

Proposition: Let P be a differential operator with constant coefficients on $\mathbb{R}^M$ and let Δ be an open subset in $\mathbb{R}^N \times \mathbb{R}^M$ with the property that for every $t \in \mathbb{R}^N$ $\Delta_t := \{x \in \mathbb{R}^M : (t,x) \in \Delta\}$ is (either empty or) convex. Then Δ is a P-convex subset of $\mathbb{R}^{N+M}$.

J. MICHALICEK: Ueber invariante Unterräume und Räume von Halbnormen

Es sei $\mathfrak{A}$ eine abgeschlossene Algebra von Operatoren eines reflexiven Banachraumes B in sich. $\mathfrak{A}$ enthalte die Einheit. Es sei $\|b,b^*\|_{\mathfrak{A}^*} := \sup_{\|A\|\leq 1} |\langle bA,b^*\rangle|$ für alle $b \in B$ und $b^* \in B^*$. Dann gilt folgendes Kriterium:

Die Algebra $\mathfrak{A}$ besitzt in B genau dann keinen invarianten Unterraum, falls es Funktionen $p: B \longrightarrow \mathbb{R}^+$ und $q: B^* \longrightarrow \mathbb{R}^+$ gibt, so daß $\|b,b^*\|_{\mathfrak{A}^*} \geq p(b)q(b^*)$. Der Beweis wird mit Hilfe der Dualität der Räume von Halbnormen durchgeführt.

D. MITROVIC: Sur une equation singuliere de convolution

On dit qu' une fonction $f: \mathbb{R} \longrightarrow \mathbb{C}$ est Hölderienne a' l' infinie pour le constante $k > 0$ si $f(t) - f(\infty) = O(1/(|t|^k))$

ou $\lim_{t \to +\infty} f(t) = \lim_{t \to -\infty} f(t) = f(\infty) \in \mathbb{C}$.

Problème. Soient a et b deux fonctions données de $\mathbb{R}$ dans $\mathbb{C}$, indéfiniment dérivables et Hölderiennes a l' infinie avec toutes leurs derivées. On suppose que $a(t) \pm b(t) \neq 0$ soit sur $\overline{\mathbb{R}}$. Soit S une distribution données à support compact. Chercher la solution T de l' equation

$$a(t)T + \frac{b(t)}{\pi \cdot i}(T * \exp \frac{1}{t}) = S.$$

Le theorem de base. Si $T \in \mathcal{O}'_\alpha$ avec $-1 \leq \alpha < 0$ et si $\hat{T}(z) = (2\pi i)^{-1} \langle T_\tau , (\tau - z)^{-1} \rangle$, $\mathrm{Im}|z| \neq 0$, alors $\hat{T}^\pm = \lim_{\varepsilon \to +0} \hat{T}(t \pm i\varepsilon)$ exist dans $\mathcal{O}'_\alpha$ et $\hat{T}^+ - \hat{T}^- = T$, $\hat{T}^+ + \hat{T}^- = -(\pi i)^{-1}(T * \exp\frac{1}{t})$.

En combinont le théorème avec le théorème sur le prolongement analytique des distribution on trouve la solution de PROBLÈME dans une forme fermée et on démontre qu' elle appartient à l' espace $\mathcal{O}_\alpha$ avec $\alpha < 0$.

M. ORTON: Boundary values of solutions to hypoelliptic equations

Let P(x,D) denote a (properly supported) pseudo-diffential operator of the form

$$P(x,D)[u] = (2\pi)^{-n} \int_{\mathbb{R}^{n-1}_{x'} \times \mathbb{R}_{x_n}} \Big\{ \sum_{k=0}^{m-1} a_k(x', x_n, \xi')(i\xi_n)^k + a_0(x', x_n) \Big\} \cdot \hat{u}(\xi', \xi_n) e^{i\langle x, \xi \rangle} d\xi' d\xi_n$$

where $x = (x', x_n) \in \mathbb{R}^{n-1}_{x'} \times \mathbb{R}_{x_n}$, $\xi = (\xi', \xi_n) \in \mathbb{R}^{n-1} \times \mathbb{R}$ and $a_0(x', e_n) \neq 0$ in a neighorhood of $x_n = 0$. Then we prove

Theorem: If P(x,D) is as above and is hypoelliptic then every distribution $u \in D'(\mathbb{R}^n)$ satisfying $P(x,D)[u] = 0$ on $\mathbb{R}^n - \{x_n = 0\}$ has boundary values in $D'(\mathbb{R}^{n-1})$ as $x_n \downarrow 0$ $(x_n \uparrow 0)$ for all its derivatives; thus the one-side limits

$\lim_{x_n \downarrow 0} D_x^\alpha u(x',x_n)$ exist in the sense of convergence in
$(x_n \uparrow 0)$

$D'_{x'}(\mathbb{R}^{n-1})$ for all n-tuples α.

We prove this theorem using thefollowing

Lemma: There exist pseudo-differential operators Q and B_o (which we compute explicitly) such that for every $u \in D'(\mathbb{R}^n)$ with $u = 0$ on $\{x_n < 0\}$ and $u \in C^\infty(\{x_n > 0\})$

$$u = QP[u] + B_o[u]$$

where

(i) $Q[w] = 0$ on $x_n < 0$ for all $w \in D'(\mathbb{R}^n)$ with $w = 0$ on $x_n < 0$

(ii) B_o satisfies (i) with Q replaced by B_o

(iii) $B_o[u](x',x_n) \in C^{m-1}(\mathbb{R}, D'(\mathbb{R}^{n-1}))$ and

$\lim_{x_n \downarrow 0} \frac{\partial^j}{\partial x_n^j} B_o[u](x',x_n) = 0$ in $D'(\mathbb{R}^{n-1})$, $j = o,\ldots,m-1$

If $P[u] = \sum_{i=0}^{m-1} f_i(x') \otimes \delta^{(i)}(x_n)$ then $QP[u](x',x_n)$ and all its derivative w.r.t. x_n of order $\leq m-1$ have limits in $D'(\mathbb{R}^{n-1})$ as $x_n \downarrow 0$.

This work generalizes results obtained by A. Martineau for $P = \frac{\partial}{\partial \bar{z}}$ $z \in \mathbb{C}$, $z = x + iy$ (using analytic representations of distributions in $D'(\mathbb{R})$) and by P. Kree for P elliptic (using a result by L. Hörmander on parametrices for elliptic operators and the boundary operators defined by them). The work of L. Boutet de Monvel on boundary value problems for ps.d. o.'s should also be mentioned in this connection.

H.-J. PETZSCHE: <u>Darstellung von Distributionen durch Randwerte holomorpher Funktionen</u>

Jede Ultradistribution vom Beurlingschen oder Roumieuschen Typ auf dem $\mathbb{R}^N$ läßt sich darstellen als Randwert einer in $(\mathbb{C}\setminus\mathbb{R})^N$ holomorphen Funktion. Sei dazu $\{M_p\}_{p\in\mathbb{N}_0}$ eine Folge positiver Zahlen, die den Bedindungen (M1), (M2) und (M3) von Komatsu genügt. Definiert man lokalkonvexe Räume $H^N((M_p))$ im Beurlingschen bzw. $H^N([M_p])$ im Roumieuschen Fall und eine stetige Randwertabbildung T^N: $H^N((M_p)) \to D'(\mathbb{R}^N,(M_p))$ bzw. $H^N([M_p]) \to D'(\mathbb{R}^N,[M_p])$, so kann man zunächst mit dem Satz von Mittag-Leffler die Surjektivität von T^1 beweisen und mit Hilfe einer konkreten Funktionenraumdarstellung von $H^{1\prime}((M_p))$ bzw. $H^{1\prime}([M_p])$ Kern$(T^1) = H(\mathbb{C})$ berechnen. Durch Tensorproduktsätze, insbesodere den Schwartzschen Satz vom Kern für Ultradistributionen führt man dann das N-dimensionale Problem zurück auf ein eindimensional vektorwertiges und zeigt mit Hilfe von Isomorphien der Räume der periodischen ultradifferenzierbaren Funktionen zu gewissen Folgenräumen, daß die E-wertige Randwertabbildung T_E^1 surjektiv ist für geeignete lokalkonvexe Räume E. Die Surjektivität von T_E^N für beliebige (F)-Räume E, die man so erhalten hat, kann man schließlich benutzen, um Kern(T_E^N) zu bestimmen.

J. SCHMETS: Spaces of vector-valued continuous functions

Let X be a completely regular and Hausdorff space, and E be a locally convex topological vector space which system of semi-norms we denote by P. C(X;E) denotes then the space of the continuous functions on X with values in E. There are many ways to endow C(X,E) with systems of semi-norms by use of P and subsets of X, or better of the repletion υX of X. In fact for every $p \in P$ and $\varphi \in C(X,E)$, $p(\varphi)$ is continuous on X and therefore if B is a bounding subset of υX, $\|\varphi\|_{p,B} = \sup\{p[\varphi(x)] : x \in B\}$ is a semi-norm on C(X;E). The problem is then to know when such a space is ultrabornological, barrelled, bornological or evaluable, and also to characterize the respective associated spaces. To do this in some cases (for instance when endowing C(X;E) with the so-called simple or pointwise topology), we had had to introduce some rather "unnatural" topologies on C(X,E) making use of a system of semi-norms P' on E, finer than P.

C. SCHUETT: <u>The projection constant of finite-dimensional spaces whose unconditional basis constant is 1</u>

We deal with the projection constant $\lambda(E)$ of a n-dimensional normed space E and the isomorphic distance $d(E,l_n^\infty)$. We give a summary of previous results including the estimation due to Lindenstrauss and Pelczynski

$$d(E,l_n^\infty) \leq K_G^2\, \chi(E)^2\, \lambda(E)^2$$

where $\chi(E)$ denotes the unconditional basis constant of the space E. Moreover we could give the following result:

<u>Theorem</u>. Let E be $\mathbb{R}^n$ with norm $\|\cdot\|_E$, $\|\cdot\|_\infty \leq \|\cdot\|_E \leq \|\cdot\|_2$, and let the unit vectors be a basis whose unconditional basis constant is 1. Then

$$c_1 \|(1,\dots,1)\|_E \cdot (\min_{\|y\|_E=1} \|y\|_2) \leq \lambda(E) \leq \min\{(n)^{1/2}, \|(1,\dots,1)\|_E\}$$

where c_1 is the Khintchin constant for p = 1 ($c_1 = (2)^{1/2}$).

A little bit more distinct is one of the three corollaries we presented:

<u>Corollary</u>. Let E be $\mathbb{R}^n$ with the norm $\|\cdot\|_E$, $\|\cdot\|_\infty \leq \|\cdot\|_E \leq$

$\|.\|_2$. The unit vectors are a basis whose unconditional basis constant is 1. Then

$$c_1 \|(1,\dots,1)\|_E \lesssim \lambda(E) \leq d(E, l_n^{\infty}) \leq \|(1,\dots,1)\|_E$$

S. SWAMINATHAN: Normale Struktur in Banachräumen

1) Es sei X ein Banachraum, K eine beschränkte, konvexe, abgeschlossene Teilmenge von X mit int $K \neq \emptyset$. Ist X^* strikt konvex, so besitzt K einen nicht-diametralen Punkt. Der Beweis macht Gebrauch vom Begriff des Subdifferentials konvexer Funktionen.

2) Der Modul der Konvexität auf $z \neq 0$ eines Banachraumes X ist die Zahl $\delta_z(\varepsilon) = \inf\{1 - \|(x+y)/2\| : \|x\| \leq 1, \|y\| \leq 1, x-y=\lambda z, |\lambda| \geq \varepsilon\}$ für $\varepsilon \in [0,2]$ und der Koeffizient der Konvexität auf $z \neq 0$ ist die Zahl $\mathcal{E}_z(X) = \sup\{\varepsilon \in [0,2] : \delta_z(\varepsilon) = 0\}$. Es seien X_n, $n = 1,\dots$ Banachräume und $l^p(X_n)$ für $1 < p < \infty$ der Banachraum aller Folgen $x = (x_n)$, $x_n \in X_n$, mit $\sum_1^{\infty} \|x_n\|^p$ und $\|x\| = (\sum_1^{\infty} \|x\|^p)^{1/p}$. Sind für jedes n und jedes $0 \neq z \in X_n$, $\mathcal{E}_z(X_n) < 1$, so hat jede beschränkte, konvexe, abgeschlossene Teilmenge von $l^p(X_n)$ normale Struktur.

S. SZNAJDER: On some properties of convolution operators in $\mathcal{K}_1'$ and $\mathcal{S}'$

Let $\mathcal{K}_1'$ be the space of distributions of exponential growth and $\mathcal{S}'$ the space of tempered distributions in $\mathbb{R}^n$. Denote by $\mathcal{O}_c'(\mathcal{K}_1'; \mathcal{K}_1')$ and $\mathcal{O}_c'(\mathcal{S}', \mathcal{S}')$ the spaces of convolution operators in $\mathcal{K}_1'$ and $\mathcal{S}'$ respectively.

Theorem 1. Let $S \in \mathcal{O}_c'(\mathcal{K}_1', \mathcal{K}_1')$. Then the following conditions are equivalent

(a) $\exists$ N, r, C > 0 such that

$$\sup_{s\in\mathbb{C}^n, |s|\leq r} |\hat{S}(\xi + s)| \geq \frac{C}{(1 + |\xi|)^N}, \quad \xi \in \mathbb{R}^n.$$

(b) $S * \mathcal{H}'_1 = \mathcal{H}'_1$

(c) If $U \in \mathcal{O}'_c(\mathcal{H}'_1, \mathcal{H}'_1)$ and $S * U \in \mathcal{H}_1$, then $U \in \mathcal{H}_1$.

Theorem 2. Let $S \in \mathcal{O}'_c(\mathcal{S}', \mathcal{S}')$. Then the following conditions are equivalent:

(a') For every integer k there exists an integer $m \geq 0$ and constants μ, $M \geq 0$ such that

$$\sup_{\substack{|\alpha|\leq m, s\in R^n \\ |s|\leq(1+|\xi|)^{-k}}} |D^\alpha \hat{S}(\xi + s)| \geq |\xi|^{-\mu} \text{ when } \xi \in R^n, \ |\xi| \geq M.$$

(c') If $U \in \mathcal{O}'_c(\mathcal{S}', \mathcal{S}')$ and $S * U \in \mathcal{S}$, then $U \in \mathcal{S}$.

Conjecture. If $S \in \mathcal{O}'_c(\mathcal{S}', \mathcal{S}')$ and the order of zeros of its Fourier transform $\hat{S}$ is bounded, then (a') and (c') are equivalent to

(b') $S * \mathcal{S}' = \mathcal{S}'$.

W.M. TULCZYJEW: Physical field theories and partial differential equations

Let X be a vector bundle over a differential manifold M of dimension m. We introduce bundles $P = \mathrm{Hom}(X, \Lambda^{m-1}T^*(M))$, $F = \mathrm{Hom}(X, \Lambda^m T^*(M))$, $Y = P \oplus F$ and $S = X \oplus Y$. Let $\alpha: \Gamma(X) \to \Gamma(F)$ be a first order and $\beta: \Gamma(X) \to \Gamma(P)$ be a second order differential operator. The symbol $\Gamma(E)$ denotes the space of smooth sections of a bundle E. The operator $\gamma = (\alpha, \beta): \Gamma(X) \to \Gamma(Y) = \Gamma(P) \oplus \Gamma(F)$ is called formally selfadjoint if the abstract Green's formula

$$d(x\cdot\alpha(x') - x'\cdot\alpha(x)) - x\cdot\beta(x') + x'\cdot\beta(x) = 0$$

is satisfied for all sections $x, x' \in \Gamma(X)$. Here $\bullet$ denotes the natural composition of sections of X with the sections of P and sections of F and d is the exterior differential. The dynamics of a physical field is usually described by the graph $\Delta = \{x \oplus y \in \Gamma(S) = \Gamma(X) \oplus \Gamma(Y) : y = \gamma(x)\}$ of a formallyselfadjoint operator γ. Assuming that this is the case we construct symplectic vector spaces associated with compact domains in M and show that field dynamics is described by Lagrangian subspaces of the symplectic spaces. Expressing the dynamics of the field in terms of Lagrangian subspaces is an important step in the direction of understanding the quantum theory of the field.

Z. ZIELEZNY: Growth and regularity of solutions of convolution equations

Let $\mathcal{K}'_p$ be the space of distributions in $\mathbb{R}^n$ "growing" no faster than $e^{a|x|^p}$, for some a (depending on the distribution) and $p > 1$. Let Γ^d_p be the space of C^∞-functions such that

$$|D^\alpha f(x)| \leq C^{|\alpha|+1} \alpha^{d\alpha} e^{c|x|^p}$$

where $d \geq 1$ and $c, C > 0$. A convolution operator S in $\mathcal{K}'_p$ is d-hypoelliptic in $\mathcal{K}'_p$, if every solution $u \in \mathcal{K}'_p$ of the equation

$$S * u = f$$

is in Γ^d_p, when $f \in \Gamma^d_p$. We prove

Theorem. S is d-hypoelliptic in $\mathcal{K}'_p$ if and only if its Fourier transform $\hat{S}$ satisfies the following conditions:

(dh_1) For every $m = 1, 2, \ldots$, there exists $C_m > 0$ such that
$|\hat{S}(\xi)| \geq e^{-1/m\, |\xi|^{1/d}}$, when $\xi \in \mathbb{R}^n$ and $|\xi| \geq C_m$

(dh_2) There exist constants μ, $M > 0$ such that

$$\frac{|\mathrm{Im}\,\zeta|^{dq}}{|\zeta|} \geq \mu \quad , \text{ when } \zeta \in \mathbb{C}^n,\ \hat{S}(\zeta) = 0,\ |\zeta| \geq M.$$

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