MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Gruppen und Geometrien

29.5. – 4.6. 1977

Die Tagung "Gruppen und Geometrien" stand in diesem Jahr unter der Leitung von Prof. Dr. B. Fischer (Bielefeld), Prof. Dr. D.G. Higman (Ann Arbor) und Prof. Dr. H. Salzmann (Tübingen). Es haben 34 Mathematiker teilgenommen, 24 großenteils umfangreiche Vorträge wurden gehalten.

Schwerpunkte der Tagung waren Einbettungsfragen von Gruppen in Automorphismengruppen geometrischer und algebraischer Strukturen, kombinatorische Fragestellungen sowie Kennzeichnungen und Eigenschaften verschiedenartiger Gruppen und Geometrien, insbesondere von endlichen Gruppen, endlichen, projektiven, Translations- oder höher-dimensionalen topologischen Ebenen und Graphen. Neben den Vorträgen ergab sich insbesondere auch wegen der starken internationalen Beteiligung an dieser Tagung ein reger Gedankenaustausch.

Teilnehmer

J. Assion, Bielefeld
R. Baer, Zürich
B. Baumann, Bielefeld
D. Betten, Kiel
F. Buekenhout, Brüssel
B. Fischer, Bielefeld
F. Fritz, Mainz
J.-M. Goethals, Brüssel
R. Griess, Ann Arbor
Grundhöfer, Kaiserslautern
H. Hähl, Tübingen
M. Hall, Oxford
C. Hering, Tübingen
D.G. Higman, Ann Arbor
D.F. Holt, Oxford
X. Hubaut, Brüssel
C. Lefevre-Percsy, Brüssel
D. Livingstone, Birmingham
H. Lüneburg, Kaiserslautern
S. Norton, Cambridge
N. Percsy, Mons
P. Plaumann, Erlangen
O. Prohaska, Kaiserslautern
R. Rink, Kaiserslautern
H. Salzmann, Tübingen
J. Saxl, Oxford
J.J. Seidel, Eindhoven
B. Stellmacher, Bielefeld
K. Strambach, Erlangen
L. Teirlink, Brüssel
F.G. Timmesfeld, Köln
A. Wagner, Birmingham
D. Wales, Oxford
B. Weisfeiler, Penn State

Vortragsauszüge

D. BETTEN: Einige Wirkungen und Geometrien auf 3-Mannigfaltigkeiten

Es wurden die transitiven Wirkungen von zusammenhängenden einfachen Lie-Gruppen auf 3-Mannigfaltigkeiten aufgelistet und zugehörige Geometrien angegeben.

F. BUEKENHOUT: Circular extensions of spherical groups

Classical spherical Dynkin diagrams are extended by strokes of type $\circ\overset{c}{\text{---}}\circ$. The groups belonging to small rank diagrams are classified. The list of examples includes 15 of the known sporadic groups.

B. FISCHER: A generating set of involutions for the "monster"

There is a possibility that a finite simple group G exists, called the "monster". Such a group contains a set d_{ij} of involutions for $1 \leq i \leq 3$ and $1 \leq j \leq 5$ such that the following relations hold: $D_i = \langle d_{ij}, d_{r5} \mid 1 \leq j \leq 5, r \neq i \rangle \simeq W(D_6)$, and d_{i5} generates the center of D_i; furthermore, $o(d_{ij}d_{kl}) \leq 3$ and

$$\langle d_{ij} \mid j \neq 5 \rangle \simeq S_5 \times S_5 \times S_5 .$$

Various subgroups of G generated by subsets of these involutions are discussed.

F. FRITZ: A characterization of the Rudvalis group

The following theorem has been proved by O'Nan:

Theorem 1 Let G be a simple group containing a 2-local subgroup M such that $V = O_2(M)$ is elementary of order 2^6 and $M/V = G_2(2)$ acts faithfully on M. Then G is isomorphic to the Rudvalis simple group.

The author has shown the following generalization:

Theorem 2 Let G be a simple group containing a 2-local subgroup M such that $V = O_2(M)$ is elementary of order q^6, $q = 2^f$ and $M/V = G_2(q)$, $C_M(V) = V$. Then $q = 2$.

The proof uses a number-theoretic argument of Zsigmondy, Goldschmidt's theorem and the fact, that $SL(2,q)$, $q \geq 4$, $q = 2^f$ has no transitive extension as permutation group on $q+1$ letters.

J.-M. GOETHALS: Pseudo-geometric graphs with strongly regular subconstituents

(This is a continuation of the lecture given by J.J. Seidel)

Pseudo-geometric graphs (R, K, T) are investigated in connection with the Krein condition $q_{22}^2 \geqslant 0$, which reads $(R - 1)(K - 2T) \leqslant (K - 2)(K - T)^2$. In particular, we show that this condition has its consequences for Bose's theorem on partial geometries, and that, for $T = 1$, in case of equality the graph is geometric.

R.L. GRIESS, jr.: Finite Groups as Automorphisms of Lie Algebras

We give criteria for a group to be a group of automorphisms of a Lie algebra. For KG-modules, A,B, let $(A,B) = \mathrm{Hom}_{KG}(A, B)$, $T^nA = \overbrace{A \otimes \dots \otimes A}^{n}$, S^nA = n-fold symmetric tensors, Λ^nA = n-fold alternating tensors.

Th. Let V be an absolutely irreducible KG-module. Assume I or II:

I. $\dim(\Lambda^2V, V) \geq 1$, $(\Lambda^3V, V) = 0$. II. $\dim(\Lambda^2V, V) \geq 1$, $\dim(S^2V, K) = 1$, $(S^2V, V) = 0$, $(\Lambda^4V, K) = 0$. Then a nonzero map in (Λ^2V, V) makes V a Lie algebra which is the direct sum of isomorphic simple subalgebras and $G \leq \mathrm{Aut}(V)$.

Cor. $A_5 \leq PSL(2, K)$, char $K \neq 2$, if the 3-dimensional representation for A_5 can be written over K; $G_2(3) \leq E_6(\mathbb{C})$

Cor. $|H^2(G_2(3), \mathbb{Q}/\mathbb{Z})| \equiv 0 \pmod 3$.

Prop. If $\dim(\Lambda^2L, L) = 1$, $G, G_1 \leq \mathrm{Aut}\, L$, $G \sim G_1$ in $GL(L)$, then $G \sim G_1$ in Aut L.

Also, we describe a subgroup of $F_4(K)$ isomorphic to $3^3 \cdot SL(3, 3)$ if char $K \neq 3$ and a subgroup of $E_8(K)$ isomorphic to $5^3 \cdot SL(3, 5)$ (split) if char $K \neq 5$. These (and possibly other subgroups of Chevalley groups) are related to generalizations of Dempwolff decompositions of Lie algebras.

H. HÄHL: Particular properties of higher-dimensional locally compact translation planes

Let $\mathcal{P}$ be a locally compact connected topological translation plane, and denote by L_∞ the translation axis. (Considered as a set of points, L_∞ is then homeomorphic to the n-sphere S^n with $n \in \{1,2,4,8\}$.) Assume that a group Σ of continuous collineations leaving L_∞ invariant has an orbit $\mathcal{T}$ on L_∞ which is homeomorphic to S^{n-1}.

Theorem 1. If $n \in \{4,8\}$, these assumptions imply that one of the following statements holds:

(i) $\mathcal{P}$ is the classical quaternion or Cayley plane respectively

(ii) $\mathcal{P}$ has Lenz type V

(iii) There is exactly one pair of distinct points which as a set is invariant under every collineation.

By routine arguments theorem 1 can be derived from

Theorem 2. If $n \in \{4,8\}$, and if there is a collineation group Σ with the properties specified above such that moreover Σ is 2-transitive on the orbit $\mathcal{T}$, then $\mathcal{P}$ is the quaternion or Cayley plane respectively.

The proof of theorem 2 uses the Tits classification of 2-transitive actions of Lie groups. A sketch of proof was given for the case $n = 8$.

With these two theorems as important tools, all locally compact translation planes with $n = 4$ (resp. $n = 8$) and a collineation group of topological dimension at least 17 (resp. 38) have been determined explicitly. Some instances of the role of theorem 1 and 2 in this classification were discussed.

In the case $n = 2$, analogous theorems do not hold. This fact constitutes one of the fundamental differences between the theories of lower dimensional and of higher dimensional translation planes.

M. HALL: Exceptional locally affine geometries

In this talk a geometry is understood to be a system of points and lines such that every pair of distinct points lies on an unique line. If every triangle lies in an affine plane, then F. Buekenhout has shown that if a line contains 4 or more points, then the entire geometry is an affine space. But the author has shown that there are geometries in which every triangle lies in an affine 9 point plane, which are not affine spaces. These exceptional geometries can be described in at least two different ways. For each point x there is an involutory collineation a_x which fixes x and moves every other point. Also if $x \neq y$ then $(a_x a_y)^3 = 1$. The group $K = \langle a_x \rangle$ describes the geometry G. Also the points of G can be treated as the elements of a Commutative Moufang loop.

C. HERING: On collineation groups of translation planes of finite even order

The following result is proved (joint work with Chat Yin Ho): Let V be a vector space of dimension $4m$ over a field $\mathbb{K}$ of order 2^a, where m is odd and $a \geq 1$. Let G be a group of linear transformations of (V, K) which leaves invariant a congruence $\mathcal{K}$ of V. Denote the group generated by all 2-elements in G by S. If G does not contain any elation $\neq 1$, then one of the following holds:

1) $S/O(S) \simeq E_q$.

2) $S/O(S) \simeq PSL(2, q)$, where $q \equiv \pm 3 \pmod 8$.

3) $S = S_1 \times S_2$, where $S_1, S_2 \leq S$, $2 \mid |S_1| > 2$ and S_2 is elementary abelian.

4) S is elementary abelian.

5) $S \simeq SL(2, 2^b)$ for some b.

The main tool for the proof is the following

Lemma. Let β be a Baer involution in G and $H < C_G\beta$. Denote the representation of H on V_β by $-$ and $E = \{\zeta \in G \mid V_\rho = V(\zeta-1) = V_\beta\} \cup \{1\}$. Assume that $E \subseteq H$, $\bar{H}$ is generated by elations of V_β but H does not contain any elation $\neq 1$. Then $\bar{H} \simeq Sz(8)$ or $\bar{H}$ is solvable. This remains true when we drop our assumption on the dimension of V.

D.G. HIGMAN: A virtual version of a theorem of Frame

Let F be the field of fractions of an integral domain R of characteristic 0, and let $\underline{\underline{A}}$ be a finite dimensional associative algebra over F. Fix an F-basis $w_1,\dots,w_r$ of $\underline{\underline{A}}$ with $w_i w_j = \sum a_{ijk} w_k$ and assume that $a_{ijk} \in R$. Let $i \mapsto i'$ be a permutation of $\{1,2,\dots,r\}$ such that $1' = 1, 2'' = 2$.

Assume that (1) $a_{i1k} = a_{1ik} = \delta_{ij}$ (i.e. $w_1 = 1$) (2) $a_{ij1} = a_{ji1} = \delta_{ij'}v_i$, $v_i \neq 0$ (so $v_i = v_{i'}$) (3) $n := \sum_{i=1}^{r} v_i \neq 0$, and (4) $\underline{\underline{A}}$ is separable.

Then $\zeta : \underline{\underline{A}} \to F$, $w_i \mapsto \delta_{i1}n$ is a virtual trace on $\underline{\underline{A}}$ in the sense that $(x,y) = \zeta(xy)$ is a symmetric nondegenerate bilinear form on $\underline{\underline{A}}$. So $\underline{\underline{A}}$ is a symmetric algebra with dual basis $\hat{w}_i = \frac{1}{nv_i} w_{i'}$, $1 \le i \le r$.

If $\zeta_1,\ldots,\zeta_m$ are the absolutely irreducible characters of $\underline{\underline{A}}$, $\zeta_s(1) = e_s$, then we have orthogonality relations $\sum \zeta_s(\hat{w}_i)\zeta_t(w_i) = \delta_{st}\frac{e_s}{z_s}$ and we have $\zeta = \sum_{s=1}^{m} z_s\zeta_s$. We call z_s the virtual multiplicity of ζ_s. Assume there exists a subring S of the algebraic closure $\bar{F}$ of F such that $S \cap F = R$ and the absolutely irreducible representations of $\underline{\underline{A}}$ can be written in S. Let $Q = n^r \prod_{i=1}^{r} v_i \,/\, \prod_{\alpha=1}^{m} z_\alpha e_\alpha^2$. Then $Q \in R$ and, if $\zeta_s(w_i) \in R\ \forall s,i$, then $Q = d^2$, $d \in R$. In case $\sum_k a_{ijk} = v_j\ \forall i,j$, Q can be replaced by $Q_o = n^{-2}Q$. Such results can be applied to generic algebras of systems of configurations.

D.F. HOLT: On the local control of Schur-multipliers

The following result will be proved in outline:

Theorem : Let G be a finite group, and P a Sylow p-subgroup of G having nilpotency class less than p/2. Then the Sylow p-subgroups of the Schur multipliers of G and $N_G(P)$ are the same.

This can be regarded as a generalization of a result of Wielandt:

Let G, P be as above, where P is regular (this holds, in particular, if $cl(P) < p$). Then the Sylow p-subgroups of G/G' and $N_G(P)\,/\,N_G(P)'$ are the same.

X. HUBAUT: Strongly regular graphs having $PO_n(q)$ as a transitive automorphism groups

Known rank 3 graphs with an orthogonal group on given by

1) $PO_n(q)$ acting on the points of a quadric in $PG(n-1,q)$

2) $PO^+_{10}(q)$ acting on the isotropic V_4 of a quadric in $PG(9,q)$

Exceptional representations occur with

3) $PO^{\pm}_{2n}(2)$ acting on the points outside a quadric of PG(2n-1,2)

4) $PO^{\pm}_{2n}(3)$ acting on an orbit of points not on a quadric of PG(2n-1,3)

5) $PO_{2n+1}(3)$ acting on one of the two blocks of points not on a quadric.

A strongly regular graph may be obtained in the following way:

Vertices are hyperbolic (resp. elliptic) sections of a quadric in PG(2n,q), adjacency is tangency. $PO_{2n+1}(q)$ acts on this graph as a rank $\frac{q+3}{2}$ for q odd, or $\frac{q+2}{2}$, for q even, permutation group. This result has been obtained with R. Metz.

C. LEFEVRE-PERCSY: Hermitian conics and unitals in desarguesian projective planes

Characterization problems about unitary groups are related to the study of unitals, a concept generalizing the notion of hermitian conic.
The class of known unitals embedded [U is a unital (embedded) in a projective plane $P_2(q^2)$ of order q^2, if U is a set of $q^3 + 1$ points of $P_2(q^2)$ such that each secant line meets U in q + 1 points] in a projective plane is not very large; in a desarguesian plane, the only examples are the unitals of BUEKENHOUT-METZ, which extend the class of hermitian conics.
We give geometric characterizations of these unitals and of the hermitian conics.

Theorem 1. Let U be a unital in a desarguesian projective plane $P_2(q^2)$ of order q^2 and let I be some tangent to U.
If all Baer sublines of $P_2(q^2)$ having a point on I intersect U in exactly 0, 1, 2 or q + 1 points, then U is a unital of Buekenhout-Metz.

Theorem 2. Let U be a unital in a desarguesian projective plane $P_2(q^2)$ of order q^2. If all secant line to U intersect U in a Baer subline, then U is a hermitian conic.

S. NORTON: On the largest Fischer group

A proof was given of the existence and uniqueness of the triple cover of F_{24}' , by using its 783 dimensional representation. By passing to characteristic 3 it was shown that there is a subgroup properly containing half the stabiliser of an octad of transpositions. This group, $^2D_5(2)$, has 3 orbits on the transposition of F_{24}' . As it contains A_{12} as a subgroup, which has another 2 normalizing it in F_{24}' , it is possible to find convenient names for the transpositions, leading to a system of generators and relations for F_{24}. Furthermore, in terms of them, one can find generators for the Held group, thereby proving its containment in F_{24}' and giving an independent existence proof. Although the Brauer trick can be used to prove that they generate a proper subgroup, it is necessary to exhibit a set of 2058 transpositions (not in the group) on which they act, in order to identify it. However, this can be done fairly elegantly. Finally, generators for the subgroup $3^7 . O_7(3)$, and the decomposition of the 783 representation over several subgroups, were given.

N. PERCSY: Finite Minkowski planes in which every circle-symmetry is an automorphism

The known finite Minkowski planes are:

(1) the planes Mn(GF(q)) associated to a field GF(q) (and isomorphic to the geometry of a ruled quadric in PG(3,q)):
(2) the planes Mn(N(q)) associated to a regular nearfield N(q) of odd square order.
A common useful definition of both classes of planes is given and their full automorphism group is determined.
Our main result is the following characterization of these planes:
Theorem: Let M be a finite Minkowski plane. M is isomorphic to Mn(GF(q)) or Mn(N(q)) if and only if, for every circle c, there exists a non identical automorphism fixing c pointwise.

P. PLAUMANN: Minimal locally compact semigroups and rings

Following an idea of Pollák and Rédei (Publ. Math. Debrecen 6 (1959)). Karl Strambach and I determined all locally compact semigroups resp. rings, which satisfy the property that every closed proper subsemigroups (subrings) is contained in a subgroup (subfield).

R. RINK: Homomorphisms of translation planes

Let q be a prime power. Then there exists an infinite translation plane R such that every translation plane of order q is a homomorphic image of R.

H. SALZMANN: 8-dimensional Planes

Let $\mathcal{P}$ be a compact 8-dimensional topological projective plane and Δ a connected closed subgroup of the automorphism group of $\mathcal{P}$, taken with the compact open topology. Assume dim $\Delta \geq 18$. Then Δ is a Lie group. If, moreover, Δ is semi-simple, then Δ is actually simple, and Δ is either the full collineation group of the arguesian plane $\mathcal{H}$ over the quaternions, or $\Delta \simeq PU(3, \mathbb{H}, r)$, the elliptic or hyperbolic motion group of $\mathcal{H}$. In both cases, $\mathcal{P}$ is also arguesian, and the action of Δ is equivalent to the classical one.

J.J. SEIDEL: Strongly regular graphs with strongly regular subconstituents

Margaret Smith considered the problem of permutation groups whose rank and subrank is 3. The present work (jointly with P.J. Cameron and J.-M. Gothals) generalizes her results to strongly regular graphs satisfying the local condition that the subconstituents with respect to some vertex are strongly regular.

In addition, the global significance of this condition is described in terms of spherical 3-designs, and the vanishing of a Krein parameter.

B. STELLMACHER: On groups having a strongly 3-embedded subgroup

A subgroup H of a finite group G is said to be strongly 3-embedded if $H \neq G$, $3 \mid |H|$ and $3 \nmid |H \cap H^g|$ for $g \in G \setminus H$.

Some special cases of the following problem have been discussed:

Problem. Classify all finite groups G with the following properties:

(a) G is of characteristic - 2 - type,

(b) Sylow - 3 - subgroups of G are not cyclic,

(c) G contains a strongly 3-embedded subgroup H,

(d) $3 \mid |O(H)|$.

F.G. TIMMESFELD: On the extraspecial problem

Let G be a finite simple group, z an involution of G and assume $F^*(C(z))$ is extraspecial. It was discussed, why it is interesting to characterize groups with this property. Further, a theorem was discussed, which should give the final characterization of these groups. That is the problem was reduced to the problem of solving several specified centralizer problems.

A. WAGNER: Finite reflection groups

Let $G \subseteq PGL(V,F)$, $|G| < \infty$, G is irreducible on V and G is generated by reflections. (A reflection is an element which may be written as $\mathrm{diag}(-1,1,1,\ldots,1)$). Denote the characteristic of F by p.

For $F = \mathbb{C}$ this problem was solved by Mitchell (1914). Using other methods Coxeter (1934) and Witt (1941) gave treatments of the case $F = \mathbb{R}$.

Denote by R_p the subgroup of G generated by all reflections which have centre in P. Then for some P the group R_p acts irreducibly on P. Now Mitchell (1911) determined all finite irreducible groups of a plane, when $p \neq 2$. A case by case discussion of the different R_p that may occur then leads to a determination of G for any F with $p \neq 2$.

As an application one may determine the subgroups of $PGL_5(q)$. Besides the above one needs the classification theorem for simple groups containing no elementary abelian group of order 8.

D. WALES: Linear Groups over the complex numbers generated by bireflections and applications

Finite quasiprimitive linear groups over the complex numbers containing a matrix with an eigenspace of codimension 2 have been determined by the author and W.C. Huffman. The most difficult case involving a matrix of order two was discussed as well as applications to linear groups of small degree. Linear groups containing a matrix of order 2 were classified by showing any two had a product or order 1,2,3,4 or 5 and if 4 the square was in $O_2(G)$ or conjugate to the original involution. Such groups have been determined by Aschbacher, Fischer, and Timmesfeld. The classification of such groups has led to the determination of complex linear groups of degree 8 and 9. Linear groups of smaller degree had been previously determined. This determination was instrumental in all the proofs.

B. WEISFEILER: On the isomorphism problem for graphs

The problem is to find an algorithm for fast identification of finite graphs. The process of stabilization of graphs was described which in an invariant manner in $\leq n^4$ steps constructs for any graph Γ with n vertices a coherent configuration of D.G. Higman, say $A(\Gamma)$.
To do this we use technically more convenient notion of a graph as a matrix whose entries are partially ordered independent variables. With every $A(\Gamma)$ there is associated an algebra whose properties were studied in detail by D.G. Higman in Geom.Dedic., 1975. This provides us with algebraic invariants. However they are not sufficient to distinguish graphs.
So we use stabilizations of depth 1 (when we fix the graph at some vertices). For some graphs which are stable of depth 1 we are able to prove that they have a large automorphism group.
Theorem. Let X be a graph which coincides with its stabilization. Then X decomposes into block form, $X = (X_{ij})$.
Let us take the block X_{11} and let V_1 be the set of indices such that X_{11} occupies all positions (i, j) with $i, j \in V_1$. Suppose next that X is stable of depth 1 w. r. to V_1 and that a fixation of every vertex from V_1 leads to a graph with n^2 variables. Then Aut X is of order $|V_1|$ and acts transitively on each V_i, where V_i is constructed for X_{ii} as V_1 for X_{11}.
This permits one to construct an algorithm of graph canonization. Some modifications and difficulties were described. The material is from Lecture notes in Math. 558, edited by me.

B.Baumann (Bielefeld)

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