MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 23|1977

Allgemeine Gruppentheorie

5.6. bis 11.6.1977

Die in diesem Jahr wieder einteilige Gruppentheorietagung stand unter der Leitung der Professoren W. Gaschütz, K. W. Gruenberg und B. Huppert, die etwa 50 Teilnehmer - davon über die Hälfte aus dem Ausland - für die zweite Juniwoche eingeladen hatten.
Das Spektrum der 30 Vorträge war sehr breit und berührte fast alle aktuellen Fragen der Gruppentheorie. Trotz eines dichten Vortragsprogramms blieb weitreichend Möglichkeit zu wissenschaftlichem Gedankenaustausch in kleiner Gesprächsrunde. Als besonders erfolgreich wurde eine allgemeine Diskussion am letzten Nachmittag über Stand und Bedeutung wichtiger Probleme der Gruppentheorie angesehen.

Teilnehmer

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Vortragsauszüge

J. L. ALPERIN: Modules for SL(2,q)

We study the tensor product structure of modules for SL(2,q). Let $G = SL(2,q)$, $q = p^e$ and let F be an algebraically closed field of characteristic p. In joint work with L. Kovács, we show that there are only finitely many isomorphism types of indecomposable summands of the polynomial algebra in the standard FG-module. In other work, I've proved that there are only finitely many isomorphism types of simply generated FG-modules.

T. R. BERGER: Form Primitive Modules of Solvable Groups

Let K be a finite field, G a solvable group, and V an irreducible $K[G]$-module having a nonsingular classical form g. The object is to study the inductive structure of V as a $K[G]$-module with a form g. Such results can be applied to study the structure of primitive solvable linear groups.

R. BIERI: Poincaré Duality Pairs

This is a report on joint work with Professor B. Eckmann. A pair $(G,\underline{S})$ consisting of a group G and a family of subgroups $\underline{S}$ is said to be a PD^n-pair if it satisfies duality relations similar to Alexander-Poincaré duality of compact manifolds-with-boundary: $H^k(G;A) \cong H_{n-k}(G,\underline{S};\tilde{A})$, all k,A. It is conjectured that all PD^2-pairs can be obtained by taking for G the fundamental group of a closed surface from which m open disks are removed, and for $\underline{S}$ the m boundary cycles. The best result in this direction obtained so far is the

<u>Theorem:</u> If $(F,\underline{S})$ is a PD^2-pair then

(1) F is a free group of rank n;

(2) $\underline{S}$ consists of $m \leq n+1$ infinite cycles $\langle r_i \rangle$;

(3) there is a basis $\{x_1,\ldots,x_n\}$ of F such that x_i is conjugate to r_i for $i = 1,2,\ldots,m-1$;

(4) $G = F/\langle \underline{S} \rangle^F$ is a one-relator group whose minimum number of generators is $d(G) = n-m+1$;

(5) if $n = m-1$ then $G = 1$; if $n = m$ then $G = Z/2\,Z$; and if $n > m$ then G is an absolute Poincaré duality group.

N. BLACKBURN: <u>Schur Multipliers of p-Groups</u>

Let G be a p-group. $M = M(G) = H_2(G,Z)$.

(1) The best possible upper bound for the number of generators of $M(G)$ in terms of the number of generators d of G and the class c of G is

$$m_2(d) + \ldots + m_{c+1}(d),$$

where $m_i(j)$ is the rank of the lower central factor $\gamma_i(F_j)/\gamma_{i+1}(F_j)$ and F_j is a free group of rank j.

(2) The Schur multiplier of G is completely known if G is of class 2 and G/G' is elementary Abelian.

I. M. CHISWELL: Metrics which satisfy the 4-Point Condition

If G is a group with a length function (mapping $L:G \to \mathbb{R}$ satisfying certain axioms of Lyndon), and L is integer-valued, then one can construct a tree on which G acts. In general, one obtains a metric space on which G acts. Recently W. Imrich has given an elegant proof of this, which also generalises results about tree realisations of metric spaces to spaces with infinitely many elements.

D.E. COHEN: Groups of Cohomological Dimension One

Dunwoody has recently obtained the structure of groups of cohomological dimension one over an arbitrary commutative ring with identity. We give an introduction to his results and a survey of the earlier results on this problem.

D. J. COLLINS: One-Relator Groups with Centre

A survey of the theory of one-relator groups with centre.

M. J. COLLINS: On Blocks of Characters of Finite Groups

Some character-theoretic characterisations of p-blocks of ordinary characters of a finite group were discussed, together with their extension to the concept of a π-block, π a set of primes.

E. C. DADE: Some Classifiable Modules

The modules in question are OP-modules M such that $End_O(M) \cong M \otimes_O M^*$ is a permutation OP-module. Here P is a finite p-group and O is a p-adic ring. When P is abelian these modules can be completely classified (see my paper "Endo-Permutation Modules over p-Groups" to appear in the Annals of Mathematics). They arise naturally in the study of representations of p-nilpotent groups.

U. DEMPWOLFF: Some Subgroups of GL(n,2)

Let V be a vectorspace over F_2 of dimension n. In GL(V) define the following subsets:

I_k = set of involutions u with $\dim\langle v(1+u) | v \in V\rangle = k$

T_k = set of elements x of order 3 with $\dim\langle v(1+x) | v \in V\rangle = k$.

McLauglin determined all pairs (V,Y), Y subgr. of GL(V), Y irred. with $Y = \langle Y \cap I_1\rangle$.

Problem. Determine all groups Y with $Y = \langle Y \cap (I_1 \cup I_2)\rangle$.

In trying to do this also groups X with $X = \langle X \cap T_2\rangle$ come into play. Also results of Stellmacher and Timmesfeld are applied.

W. GASCHÜTZ: Zu einer erweiterungstheoretischen Frage von Herrn Baer

Frage (R. Baer): Gegeben sei eine endliche Gruppe E und eine Primzahl p mit den folgenden Eigenschaften: Es gibt einen und nur einen minimalen Normalteiler M von E; M ist nicht abelsch; und p ist ein Teiler der Ordnung von M. Gibt es dann eine Gruppe G derart, daß $G/\phi(G) \cong E$, $\phi(G)$ eine elementar-abelsche p-Gruppe und $\phi(G) \supseteq C_G(\phi(G))$ ist?

Antwort: ja.

F. GROSS: Permutable Subgroups

Let H be a subgroup of G such that $HK = KH$ for every subgroup K in G. Assume that $\bigcap_{x \in G} x^{-1}Hx = 1$. Many results are known about the structure of H and its embedding in G. (For example:

(1) (Maier and Schmid) If G is finite, then $H \leq Z_\infty(G)$.

(2) (Stonehewer) H is ascendant in G.

(3) (Gross) H is the extension of a locally finite nilpotent group by a torsion-free group.)

A standard technique has been to analyze very carefully the special case when $G = H\langle x \rangle$ and $|x| = p^n$, p a prime. In this special case, I have shown that there is a largest example which contains all others. As a first application of this result, a group is constructed showing that a core-free permutable subgroup need not be locally nilpotent. The questions of whether H must be locally solvable and of generalizing the Maier-Schmid result to infinite groups also will be discussed.

H. HEINEKEN: Endliche Automorphismengruppen

Es scheint nicht genau bekannt zu sein, welche endlichen Gruppen als Automorphismengruppen endlich erzeugter unendlicher Gruppen auftreten können. Durch Beispiele wird in diesem Vortrag belegt, daß von den elementarabelschen p-Gruppen $(p \neq 2)$ jedenfalls diejenigen des Ranges nk auftreten können, wobei $n \geq 5$ und k eine Zahl zwischen n und $2n-3$ ist.
Die dazugehörigen endlich erzeugten unendlichen Gruppen sind Erweiterungen einer unendlich zyklischen Gruppe durch eine von n Elementen erzeugte p-Gruppe der Nilpotenzklasse 2, die die zusätzliche Eigenschaft hat, daß nur Untergruppen U mit $UZ(G)/Z(G)$ zyklisch abelsche Untergruppen sind.

B. HUPPERT: Invariant Quadratic Forms in Characteristic 2

Let K be a field of char. 2 and V a selfdual irreducible KG-module.

(1) If V does not carry a regular G-invariant quadratic form then V is a composition-factor of

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$P_1 J(KG)/P_1 J(KG)^2$; in particular V belongs to the principal block.

(2) (by W. Willems) If G is solvable, then V carries a G-invariant regular quadratic form.

K. JOHNSEN: Automorphismengruppen mit lauter zyklischen Fixgruppen

Sind H und G endliche Gruppen und $G \leq \mathrm{Aut}(H)$, so definieren wir für $n \in \mathbb{N} \cup \{0\}$

$$(H,G) \in \mathcal{R}_n \iff \left.\begin{array}{ll}(1) & C_H(g) \text{ abelsch} \\ (2) & m(C_H(g)) \leq n\end{array}\right\} \text{ für alle } g \in G^{\#}$$

und

$$\mathcal{G}_n = \{G \mid \text{ex } H \text{ mit } (H,G) \in \mathcal{R}_n\}.$$

Die Klasse $\mathcal{G}_0$ ist von H. Zassenhaus untersucht worden. Im Vortrag wurde die Klasse $\mathcal{G}_1$ bestimmt.

G. A. JONES: Connections between Permutation Groups, Fuchsian Groups, and Maps on Surfaces

There is a natural bijection betwwen isomorphism classes of finite maps $\mathcal{M}$ of type (m,n) and conjugacy classes of subgroups M of finite index in $\Gamma := gp\langle X,Y,Z \mid X^2 = Y^m = Z^n = XYZ = 1\rangle$. Given such a subgroup M, Γ leaves invariant a tessellation of a simply-connected Riemann surface $U(= \mathbb{C} \cup \{\infty\}, \mathbb{C}$, or H as $\frac{1}{m} + \frac{1}{n} > \frac{1}{2}, = \frac{1}{2}$, or $< \frac{1}{2})$, and $\mathcal{M}$ may be obtained from the corresponding triangular tessellation of U/M.

$\mathfrak{M}$ is regular (in the sense of Coxeter and Moser) iff $M \trianglelefteq \Gamma$, and $\mathfrak{M}$ is quasi-regular (all vertices have valency m, all faces valency n, and either all edges or no edges are free) iff M is torsion-free. Aut $\mathfrak{M} \cong N_\Gamma(M)/M$ acts conformally on U/M, so $|\mathrm{Aut}\,\mathfrak{M}| \leq 84(g-1)$ where U/M has genus $g \geq 2$ (Hurwitz). For each $g \geq 0$, by triangulating a surface of genus g in infinitely many non-isomorphic ways, one obtains an easy proof that the modular group $PSL(2,Z) = (2,\infty,3)$ has infinitely many subgroups of genus g. (Joint work with D. Singerman)

A. KERBER: <u>On Multiply Transitive Groups</u>

A matrix $T := (t_{ik})$, $i,k \in \mathbb{N}$, $t_{ik} \in \mathbb{N}$, is introduced, which satisfies the following theorem:

<u>Theorem:</u> If $G \leq S_n$, $k \leq n$, and $a_i(g)$ denotes the number of cyclic factors of length i of $g \in G$, then G is k-fold transitive if and only if for each $r_1,\ldots,r_k \in Z_{\geq 0}$, where $\sum_1^k ir_i = k$, we have

$$\frac{1}{|G|} \sum_{g \in G} a_1(g)^{r_1} \ldots a_k(g)^{r_k} = \prod_{i=1}^{k} \frac{t_{r_i,i}}{i^{r_i}} .$$

The first coefficients of T read as follows:

$$T = \begin{array}{rrrrrrl} 1 & 1 & 1 & 1 & 1 & 1 & \cdots \\ 2 & 3 & 4 & 5 & 6 & 7 & \cdots \\ 5 & 11 & 19 & 29 & 41 & 55 & \cdots \\ 15 & 49 & 109 & 201 & 331 & 505 & \cdots \\ 52 & 257 & 742 & 1657 & 3176 & 5497 & \cdots \\ 203 & 1539 & 5815 & 15821 & 35451 & 69823 & \cdots \\ 877 & 10299 & 51193 & 170389 & 447981 & 1007407 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \end{array}$$

The first column contains the Bell numbers.

L. G. KOVACS: Groups of Prime Power Order with Cyclic Frattini Subgroup

This is a report on joint work with T. R. Berger and M. F. Newman.

A well-known theorem of P. Hall describes the structure of the groups of prime power order in which every abelian characteristic subgroup is cyclic. Van der Waall has recently observed that for odd primes p the same groups G arise if one merely assumes that $Z(\Phi(G))$ and $Z(\Omega_1(G))$ are cyclic. The first result to be dicussed is that for all primes p the same groups G are obtained by asking that just one abelian characteristic subgroup, namely $Z(\Omega_1(C_G(\Phi(G))))$, be cyclic. The second result gives the structure of the groups of prime power order in which the centre $Z(\Phi(G))$ of the Frattini subgroup is cyclic.

A. LICHTMAN: The Necessary and Sufficient Conditions for the Residual Nilpotency of Free Products of Groups

Theorem 1. Let $G = \prod^{*}_{j \in J} G_j$. Then G is a residually nilpotent group iff one of the following two possibilities holds.

I. All the groups G_j, $j \in J$, have no periodic elements.

II. For any finite set of non-trivial elements

$$g_1, g_2, \dots, g_l,$$

that are taken from some free factors G_{j_i}, $i = 1,2,\dots,r$ a prime p (depending on $g_1, g_2, \dots, g_l$) can be found such that none of the elements (1) is an element of infinite p-height in the corresponding free factor G_{j_i}.

We construct an example of a group H, which is an extension of an abelian group by a free abelian group and satisfies the following conditions:

1) H contains a periodic element.

2) H is not residually "nilpotent and of finite p-exponent".

3) H can be discriminated by nilpotent p_i-groups of finite exponents where $\{p_i\}$, $i = 1,2,\dots,$ is the set of all the prime numbers.

4) The group $G = H*H$ is residually nilpotent.

This example shows, in particular, that Malcev's Sufficient Conditions for the residual nilpotence of free products of groups do not coincide with the necessary ones.

TH. MEIXNER: Power Automorphisms of Finite p-Groups

Let $A(G) = \bigcap_{U \leq G} N_{AutG}(U)$ be the group of power automorphisms of the finite p-group G. A survey and some generalisation of Cooper's results (Cooper, Power automorphisms of a group, MZ 107, 1968) is given and the example of some 2-group having $r(A(G)) > 2$ is introduced. Further the following

Theorem: Let G be a 2-generated, metabelian group of exponent p^2. Then

(i) if $A(G) \neq 1$, then $c(G) \leq 2(p-1)$

(ii) if $1 \neq \alpha \in A(G)$ is universal, then $c(G) \leq p-1$

(iii) if $|A(G)| \geq p^2$, then $|A(G)| = p^2$ and $c(G) = p$.

M. F. NEWMAN: Presentations of Groups of Prime Power Order

A brief survey of results on presentations of finite p-groups will be given to set in context the following result:

For every positive ϵ, and every prime p, there is a finite p-group which has a presentation as a pro-p-group with d generators and r relations such that

$$r < (\tfrac{7}{24} + \epsilon)d^2.$$

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A. REIFART: A General Characterisation of the Group $^2E_6(2)$

We prove the following

Theorem: Let G be a finite simple group containing an involution z such that $C_G(z) = M$ has the following properties

(i) $F^*(C_G(z)) = Q$ is a extraspecial group of width 10

(ii) M/Q is isomorphic with $U_6(2)$

Then G is isomorphic with $^2E_6(2)$.

The proof is more or less standard with the exception of the final characterisation where we prove that M acts on the conjugates of z with 5 orbits. We can thus easily see that G is a $\{3,4,3^+\}$-grp in the sense of Timmesfeld and so we finally get $G \cong {}^2E_6(2)$.

L. RIBES: Subgroups of Finite Index of Profinite Groups

A profinite group is said to be topologically finitely generated (top. f. g.) if it contains a dense subgroup which is finitely generated as an abstract group. Let G be a prosupersolvable group (projective limit of finite supersolvable groups), whose order involves only finitely many primes; then we show that G is top. f. g. iff its Frattini subgroup is open in G. If a prosupersolvable group G is top. f. g., then so is each Sylow p-subgroup of G. If G is a top. f. g. prosupersolvable group, then every subgroup of G of finite index is open in G.

D. SEGAL: <u>Polycyclic Groups with Isomorphic Finite Quotients</u>

<u>Thm 1:</u> Let G be an abelian-by-cyclic polycyclic group. The polycyclic-by-finite groups H with $J(H) = J(G)$ lie in $< \infty$ isomorphism classes ($J(G) =_{def}$ set of iso. classes of finite quotients of G).

<u>Thm 2:</u> Let $x \in GL_n(Z)$. Then there are only $< \infty$ conjugacy classes of elements $y \in GL_n(Z)$ such that the images in $GL_n(Z\ mZ)$ of the cyclic gps. $\langle y \rangle$ and $\langle x \rangle$ are conjugate (in $GL_n(Z/mZ)$) for all $m \neq 0$.

I briefly outline the steps in proving Thm 2 and "Thm 2 => Thm 1".

U. STAMMBACH: <u>Cohomological Characterisations of Finite Solvable and Nilpotent Groups</u>

Let G denote a finite group. Let p be a fixed prime and let $k = Z/pZ$ denote the field of p elements. If M is a kG-module, CM denotes the centralizer of M in G.

<u>Thm 1:</u> (Stammbach) G is p-solvable if and only if $H^1(G/CM, M) = 0$ for all simple kG-modules M.

Let $\mathfrak{F}$ denote the formation which is locally defined by the non-empty formation $\mathfrak{C}$ at the prime p and the formation of all groups at the primes $q \neq p$.

<u>Thm 2:</u> (Barnes, Schmid, Stammbach) G is in $\mathfrak{F}$ if and only if $H^1(G,M) = 0$ for all simple kG-modules with $G/CM \notin \mathfrak{C}$.

These two theorems immediately yield cohomological characterisations of p-nilpotent groups, of p-supersolvable groups, of p-solvable groups of p-length ≤ 1, etc.

G. STROTH: Some Groups of Characteristic 2-Type

A group G is called of characteristic 2-type iff $F^*(M)$ is a 2-group for all 2-locals M of G. The following theorem is proved:

Let G be a finite group, $z \in G$ an involution s. t. $F^*(C_G(z)) = Q \cong F \times E$, where F is extraspecial with $z \in F'$ and E is elementary abelian with $|E| \geq 2$. Further $C_{C_G(z)}(Q/Z(Q)) = Q$. Then one of the following holds:

(i) $z^G \cap Q \subseteq Z(Q)$

(ii) $\exists R : |Z(Q):R| = 2$ and $C_G(R) \nsubseteq C_G(z)$. The groups of characteristic 2-type with this property are determined.

(iii) The structure of $C_G(z)$ is determined.

The known finite simple groups occuring in (iii) are Co_2, $F_4(2)$, $M(22)$, $M(23)$ and $Sp_{2m}(2)$. If you drop the assumption $C_{C_G(z)}(Q/Z(Q)) = Q$ then there are further groups (f. e.: $\Omega_7(3)$, A_{12}). It seems so that in groups of characteristic 2-type $C_{C_G(z)}(Q/Z(Q)) = Q$ is fulfilled.

D. WALES: Finite Linear Groups of Small Degree over the Complex Numbers

Quasiprimitive complex linear groups of degree at most nine have now been determined. The degrees 8 and 9 have been classified during the last three years by Dora, Feit, Huffman, and myself. This work utilizes group order formulaes from Blichfeldt, modular character theory and several recent classification theorems of finite group theory. Very crucial in the arguments is the classification, due to Huffman and myself of, quasiprimitive linear groups containing a matrix with eigenspace of codimension 2. This enables one to assume there is no such matrix in a supposed linear group G. Now various configurations are impossible and the groups can be listed.

B. A. F. WEHRFRITZ: Nilpotence in Groups of Semilinear Maps

Let M be a finitely generated module over the finitely generated abelian U and denote the group of semi-U-automorphisms of M by S. Then locally nilpotent subgroups of S are nilpotent. More generally if G is any subgroup of S then G has finite central height, and this central height is bounded in terms of U and M only. Further the sets of left and right Engel elements of G are what one would expect. These results depend on some recent and highly nontrivial work of J. E. Roseblade on groups acting on rings.

H. WIELANDT: Cosubnormal Pairs of Subgroups

Call two subgroups A,B of a group cosubnormal if they are subnormal in their join: A csn B :<=> A sn ⟨A,B⟩, B sn ⟨A,B⟩.

Theorem: Let A,B,C be finite subgroups of a group such that

A csn B, B csn C, C csn A. Then

(i) the nilpotent residuals $^{\mathfrak{N}}A$, $^{\mathfrak{N}}B$, $^{\mathfrak{N}}C$ are subnormal in ⟨A,B,C⟩;

(ii) if A is permutable with ⟨B,C⟩ then A sn ⟨A,B,C⟩.

Corollary: If A,B,C are finite subgroups such that A sn ⟨A,B⟩, A sn ⟨A,C⟩, B sn G and C sn G then A sn ⟨A,B,C⟩. Similar results hold for an arbitrary number of finite subgroups. The proofs depend on the properties of distributive functions on the class of finite groups, i.e. maps $\delta: \mathfrak{F} \to \mathfrak{F}$, $X \to {}^{\delta}X$ such that

(i) ${}^{\delta}X \leq X$,

(ii) ${}^{\delta}(X^{\sigma}) = ({}^{\delta}X)^{\sigma}$ for any isomorphism σ of X,

(iii) ${}^{\delta}\langle A,B\rangle = \langle {}^{\delta}A, {}^{\delta}B\rangle$ whenever A csn B (see Abh. Hamburg 21 (1957)).

C. R. B. WRIGHT: Splitting Revisited

Several results on splitting of a group over a normal subgroup which seemed originally to be formation-theoretic can be viewed as special cases of the following general theorem of Shemetkov (1970):

If $H \trianglelefteq G$ and for each prime p dividing $|G/H|$ the p-Sylow-groups of H are abelian, and the p-Sylow-groups of G split over H, then G splits over H.

For example:

If $\mathfrak{F}$ is a locally induced formation, if $K \trianglelefteq G$, if $H = K^{\mathfrak{F}}$, the $\mathfrak{F}$-residual of G, and if H has abelian p-Sylow-groups for p dividing $|G/H|$, then G splits over H.

Other methods must still be used when the Sylow-groups are non-abelian.

Th. Meixner (Erlangen)

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