MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 24/1977

Darstellungstheorie endlich dimensionaler Algebren

12.6. bis 18.6.1977

Diese Tagung wurde von P. Gabriel (Zürich) und G. Michler (Essen) geleitet.

Die Darstellungstheorie endlich dimensionaler Algebren hat in den letzten Jahres erhebliche Fortschritte erzielt; insbesondere gelang es Roiter und Nazarowa, die lange offen gebliebenen Vermutungen von Brauer und Thrall über die Klassifikation der unzerlegbaren Darstellungen endlich dimensionaler Algebren zu lösen. Es war ein Ziel dieser Tagung, die hierzu verwandten neuen Methoden der Ring- und Modultheorie einem breiten Kreis von Algebraikern, die sich mit Darstellungen von Ringen und Gruppen beschäftigen, bekannt zu machen. Darüber hinaus sollte auch der Entwicklungsstand der Darstellungstheorie der Artin-Ringe und der endlichen Gruppen den Teilnehmern dargelegt werden. Hierzu wurden Überblicksvorträge von A.V. Roiter (Kiew), E.C. Dade (Urbana), W. Feit (New Haven) und C.M. Ringel (Bonn) über diese Arbeitsgebiete der Tagung gehalten. Herr Roiter hielt außerden an zwei Abenden zweistündige Vorträge, in denen er über neuere Ergebnisse der russischen Algebraiker berichtete und auf zahlreiche Fragen seiner Zuhörer einging.

Wesentlich ergänzt wurden diese Überblicksvorträge durch 25 Spezialvorträge, in denen über methodische und inhaltliche Fortschritte auf dem Gebiet der Darstellungstheorie endlich dimensionaler Algebren und ihrer Anwendungen berichtet wurde. Dabei zeigte sich, daß die Darstellungstheorie der Gruppen methodisch erhebliche Anregungen aus der Ring- und Modultheorie der Artin-Ringe erhalten kann, und daß sie andererseits den Ringtheoretikern als wesentliche Beispielquelle dient. Von diesen Wechselbeziehungen profitierten die Tagungsteilnehmer sehr.

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Auslander, M., Waltham
Bäni, W., Zürich
Baer, R., Zürich
Bautista Ramos, R., Waltham
Blau, H., Coventry
Bongartz, K., Zürich
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Willems, W., Mainz

VORTRAGSAUSZÜGE

Thema: Diagrams for modules

Vortragender: J. L. Alperin

The structure of modules has been described by diagrams in work over the last thirty years. We give axioms for these diagrams. We conjecture that if G is a finite group, F is an algebraically closed field of prime characteristic p, B is a p-block of G with abelian defect group, then every indecomposable projective FG-module in B has a diagram.

Thema: X-determined morphisms

Vortragender: M. Auslander

My talk was devoted to a brief introduction to the notion of morphisms determined by modules in the category of finitely generated modules over an Artin algebra. Basic definitions and existence theorems were discussed as well as some applications given.

Thema: Quadratic spaces with two subspaces

Vortragender: Werner Bäni

Betrachtet werden Quadrupel $\varepsilon = (E, \phi, U, V)$, wobei: E endlich-dimensionaler Vektorraum über k (char $k \neq 2$) mit symmetrischer Bilinearform ϕ und zwei Unterräumen U, V. Es geht um das Problem der Klassifikation solcher "Räume" bezüglich Isometrien $T : \varepsilon \xrightarrow{\sim} \varepsilon'$, d.h. Isometrien $T : (E,\phi) \to (E',\phi')$ mit $TU = U'$, $TV = V'$. Es werden zwei Funktoren D, S definiert, so daß ε in eine direkte orthogonale Summe $\varepsilon = \varepsilon_D \oplus \varepsilon_S \oplus \varepsilon_o$ zerfällt, wobei ε_D von einer Potenz von D und ε_S von einer Potenz von S annulliert werden. Ihre Struktur kann explizit angegeben werden. Die Klassifikation der Räume ε_o entspricht der Klassifikation von Paaren nicht-entarteter quadratischer Formen, ein Problem, das bekanntlich äquivalent ist zur Klassifizierung quadratischer Formen über allen monogenen algebraischen Körpererweiterungen von k.

Thema: Algebras close to hereditary Artin algebras

Vortragender: Raymundo Bautista Ramos

We consider an Artin algebra with the following condition:
*) if P and Q are indecomposable projective Λ-modules and $\varphi \in \mathrm{Hom}_\Lambda(P, Q)$, then φ is mono or zero.

We prove that Λ is of finite representation type if and only if for every indecomposable Λ-module M of finite length, there exists an integer $n > 0$ such that $(Dtr)^n M$ = some indecomposable projective, were tr is the transpose and D the usual duality for Artin algebras.

Consider for instance, I a finite partially ordered set, and k a commutative field. We take Λ the subring of matrices (a_{ij}), $a_{ij} \in k$, $i,j \in I$, $a_{ij} = 0$ if $i \nleq j$ in I. This ring satisfies the *) condition. In this case mod(Λ), the category of Λ-modules of finite length, is equivalent to the category $_k \underline{\underline{I}}_I$ of functors from I in the category of finite dimensional k-vector spaces. This category has been studied by M. Loupias.

In the general case if Λ satisfies the *) condition and is of finite representation type, the indecomposables of Λ can be described as $(trD)^n P$ with P indecomposable projective, $n > 0$.

The idea of the proof is first to reduce the general case to the case in which the following is true: If not all the proper submodules of the projective P are projectives (P indecomposable) then $\operatorname{rad} \Lambda P$ = indecomposable.

Then using the almost split sequences of Auslander-Reiten we get our result.

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<u>Thema</u>: Uniserial algebras and field extensions

<u>Vortragender</u>: H. Blau

Let A be a finite dimensional algebra over a field F, and K an extension field of F. If $A \otimes_F K$ is uniserial, then so is A. This generalizes a result of Eisenbud and Griffith.

<u>Thema</u>: 2-Darstellungen von $PSL_2(q)$

<u>Vortragender</u>: K. Bongartz

Sei $q = p^r$, p ungerade, k ein algebraisch abgeschl. Körper der Charakteristik 2. Sei $q - 1 = 2^n t$, $2 \nmid t$, und $n > 2$. Dann gilt :

<u>Satz</u>: $kSL_2(q)$ hat $1 + \frac{t-1}{2} + \frac{q-1}{4}$ Blöcke B_0G, B_1G, ..., $B_{\frac{1}{2}(t-1)}G$, P_1G, ..., $P_{\frac{1}{4}(q-1)}G$ und man hat folgende Äquivalenzen von Kategorien :

mod $B_iG \simeq$ mod $k[X]/(X^{2^n})$ $\quad 1 \leq i \leq \frac{t-1}{2}$

mod $P_iG \simeq$ mod $k[X]/(X^2)$ $\quad 1 \leq i \leq \frac{q-1}{4}$

mod $B_0G \simeq$ Darstellungen des Köchers $\overset{1}{\bullet} \underset{\beta}{\overset{\alpha}{\rightleftarrows}} \overset{0}{\bullet} \underset{\delta}{\overset{\gamma}{\rightleftarrows}} \overset{2}{\bullet}$

Mit den Relationen i) Zusammensetzen von $2^{n+1}+1$ Abbildungen ist 0

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$$\text{ii)}\quad \alpha\beta\alpha = \delta\gamma\alpha\,(\beta\delta\gamma\alpha)^{2^{n-1}-1}$$

$$\delta\gamma\delta = \alpha\beta\delta\,(\gamma\alpha\beta\delta)^{2^{n-1}-1}$$

$$\beta\alpha\beta = \beta\delta\gamma\,(\alpha\beta\delta\gamma)^{2^{n-1}-1}$$

$$\gamma\delta\gamma = \gamma\alpha\beta\,(\delta\gamma\alpha\beta)^{2^{n-1}-1}$$

(Alle Darstellungen sind endlich-dimensional)

Weiter kann man die volle Unterkategorie mod $PSL_2(q)$ klassifizieren.

Für $q \equiv 3 \ (4)$ gilt ein analoges Resultat.

Thema: Representation type and quadratic forms

Vortragender: Sheila Brenner

There is further evidence to support the conjecture, made at the Ottawa conference, that the representation type of commutative quivers is given by the appropriate quadratic form. It seems, too, that a Coxeter matrix can be constructed as a product of reflections in the form and that a corresponding factorization of the functor DTr can be found.

Thema: Remarks on blocks and subgroups

Vortragender: Michel Broué

Given a p-adic ring A, a finite group G and a subgroup H of G, we investigate some very general relations between block idempotents respectively of ZAG (centre of the group algebra of G over A) and ZAH, by studying elementary properties of the two maps

Br_H^G : $ZAG \rightarrow ZAH$, defined by

$$Br_H^G \left(\sum_{s \in G} a(s) s \right) = \sum_{s \in H} a(s) s ,$$

Tr_H^G : $ZAH \rightarrow ZAG$, defined by

$$Tr_H^G (a) = \sum_{s \in G \bmod H} s a s^{-1} .$$

Thema: On calculating almost split sequences

Vortragender: M. C. R. Butler

A method was described for calculating the almost split sequences of finitely generated modules over an Artin algebra, for which an existence theorem and partial construction had been given by M. Auslander and I. Reiten in Comm. in Algebra, 1975. The method begins with the Auslander-Reiten homological formalism and develops it further by exploiting a suitable theory of traces of endomorphisms of semisimple modules.

Thema: Group modules whose endomorphism algebras are permutation modules

Vortragender: Everett Dade

These modules arise in the study of representations of p-nilpotent groups. Over p-groups and rings of p-adic integers, they have many startling properties. Indeed, for abelian p-groups they can be completely classified in some sense. Probably they can be in other cases, too.

Thema: The present state of representations of finite groups

Vortragender: Everett Dade

At present only situations in which some part of the theory is classifiable can be handled. For example, one knows much about blocks with cyclic defect groups D because one can classify all modules over FD, where F is a field of characteristic p. The only other p-groups D for which the modules over FD are classifiable are dihedral, semi-dihedral, and (generalized) quaternion groups with $p = 2$ (see the talk of Ringel). One expects similar results in this case, too.

There are also situations in which the groups can be classified, e.g., groups of Lie type, groups with abelian 2-Sylow subgroups, etc. Here the structure of the blocks can also often be obtained.

In the general case there are three approaches, none of which has yet proved satisfactory: 1) Find a "useful" subclass of indecomposable modules,
2) Find a "useful" new equivalence relation among modules,
3) Use properties such as Auslander's stable equivalence of algebras.

Thema: Indecomposable representations of certain symmetric algebras

Vortragender: P. Donovan (Kensington 2033 Australia)

The arguments of Gelfand and Ringel classifying the indecomposable representations of two local algebras are extendable to the class of graph algebras. These algebras can be defined by means of Brauer graphs (generalized Brauer trees). This is applicable to certain blocks with dihedral defect. A more sophisticated argument does the same for twisted graph algebras (a further generalization). It is possible that an extension of Ringel's work on the representation type of local algebras [SLN 488] will lead to a classification of the isomorphism types of block algebras of blocks with dihedral, semidihedral and generalized quaternion defect group.

Thema: Self-dual modules in blocks with cyclic defect groups

Vortragender: K. Erdmann

Let G be a finite group and F a splitting field, char $F = p > 0$. For an FG-right module M, the dual (right) module is denoted by M^*. Let B be a self-dual block of FG with cyclic defect group D of order p^d $(d \geq 1)$, with e simple modules. Let f be the Green-correspondence between G and $N_G(D_o)$ where $D_o < D$ has order p. For $r = 1, 2, \ldots, p^d-1$ let

$$B_r := \{M \in B \,/\, M \text{ is indecomposable, not projective, } l(fM) = r\}.$$

Then the following hold :

Lemma. Let $r \in \{1, 2, \ldots, p^d-1\}$.

(a) If e is odd, then B_r contains a unique self-dual module.

(b) If e is even, then B_r contains two self-dual modules, if r is odd, and no self-dual modules, if r is even.

Theorem: Assume the Brauer-tree Γ of B is known. Let $M \in B_r$ be self-dual. If $r < p^d - 2$ $(2 < r)$, then one can derive from M the complete submodule structure of a unique self-dual module in B_{r+2} (B_{r-2}).

The self-dual modules in B_1 and B_{p^d-1} are the unique self-dual modules affording characters which correspond to the end-nodes of the real stem of Γ.

Thema: Some computations in the theory of modular representations

Vortragender: W. Feit, Yale University

Let $\Phi_\emptyset$ be the principal indecomposable character corresponding to the trivial character and let $C_{\emptyset\,\emptyset}$ be the corresponding Cartan invariant. The following formulas are proved:

If $G = Sp_4(2^m)$, $q = 2^m$ then $\Phi_\emptyset(1) = q^4(q^4 - q^2\, T_{2m} + 1)$

If $G = Suz(2^{2s+1})$, $q = 2^{2s+1}$ then $\Phi_\emptyset(1) = q^2(q^2 - q\, T_{2s+1} - 1)$

where $T_k = \left(\frac{1+\sqrt{5}}{2}\right)^k + \left(\frac{1-\sqrt{5}}{2}\right)^k$.

Let $q = 2^n$. Define

$$f(n) = q^3 + q^2 + q + (-1)^n 2q + q\,K_n - 2q(q+1)T_n$$

where $K_n = \alpha^n + \beta^n + \gamma^n$, $(x-\alpha)(x-\beta)(x-\gamma) = x^3 - 3x^2 - x + 5$

Then $f(2s+1) = C_{\emptyset\,\emptyset}$ for $Suz(s^{2s+1})$

$f(2m) = C_{\emptyset\,\emptyset}$ for $Sp_4(2^m)$.

The formulas for $Suz(2^{2s+1})$ are due to Chastkofsky. They imply that

$$\lim_{q\to\infty} \frac{C_{\emptyset\,\emptyset}(q)}{q^6} = 1 \quad \text{for } Sp_4(q),\ q = 2^m$$

$$\lim_{q\to\infty} \frac{C_{\emptyset\,\emptyset}(q)}{q^3} = 1 \quad \text{for } Suz(q),\ q = 2^{2s+1} .$$

Thema: Algebras of finite representation type

Vortragender: E. Green

A study of representations of quivers satisfying relations was given. The full subcategory of representations of a quiver (which is a tree) which satisfy certain relations given by paths in the quiver is equivalent to the category of finitely generated modules of a homomorphic image of a tensor algebra. A major goal in the subject is to classify when such categories are of finite representation type.

Thema: Modules with cores

Vortragender: Robert Gordon

Some of the basic properties and theorems about modules with cores were described. Also, this conjecture was made: If M is a nonlocal R-module, R a left Artin ring, such that the socle of the core of M is not simple, then R has infinite representation type. Let $f : \amalg P_i \to M$ be a projective cover of M, where the P_i are indecomposable projective R-modules. If there are indices $i \neq i'$ such that $(P_i, fP_{i'}) = 0 = (P_{i'}, fP_i)$ and $\mathrm{End}\, P_i$, $\mathrm{End}\, P_{i'}$ are both division rings, an affirmative answer to the conjecture was given.

<u>Thema:</u> Irreducible Specht Modules

<u>Vortragender:</u> G. D. James

Consider the problem : which ordinary irreducible representations of the symmetric groups remain irreducible modulo a prime p ? Since the ordinary representations are indexed by diagrams, and the p-regular diagrams give rise to all the p-modular irreducible representations, there is little loss in restricting the problem to ask which p-regular diagrams give ordinary representations which remain irreducible modulo p. R.W. Carter has conjectured that the answer is : those diagrams whose p-power diagrams have no columns containing two different numbers. Here the p-power diagram is obtained from the hook graph by replacing each integer by the exponent of p dividing that integer. The state of this conjecture is that the given condition is necessary, and is sufficient in a large class of cases; in particular for $p = 2$, or when the diagram has 2 rows. The condition in the Carter conjecture involves the p-quotient (or "star diagram"), and it appears that there are results stronger than the Carter conjecture which in particular cases associate composition factors of a diagram with composition factors of its p-quotient.

Thema: On principal 2-blocks of finite groups with an abelian Sylow 2-subgroup

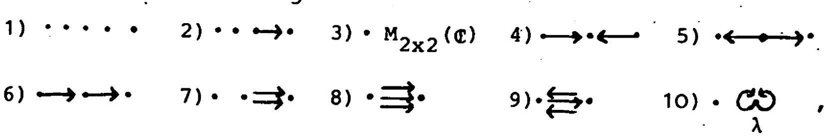
Vortragender: Peter Landrock

Only the principal 2-block of J_1 , the smallest Janko group, is not desribed by general results. The decomposition numbers have been determined by Paul Fong.
In a joint work with Gerhard Michler, we determine the vertices of the simple modules, which turn out to be the whole Sylow 2-subgroup in all cases, the Green correspondents, and the Loewy (equal to the socle-) series of all the projective modules. The Loewy length turns out to be seven in all cases and thus is bounded by 1 plus the order of the Sylow 2-subgroups.

Thema: Deformation von Algebren

Vortragender: Guerino Mazzola

Wir untersuchen die Geometrie des folgendes Schemas $Alg_n / \mathbb{C}$: Seine $\mathbb{C}$-rationalen Punkte bilden die Menge der Strukturen A auf $\mathbb{C}^{\oplus n}$, die assoziative $\mathbb{C}$-Algebren mit 1 bilden.
Die irreduziblen Komponenten für $n \leq 4$ wurden von Gabriel beschrieben. Die Alg_5-Komponenten sehen so aus :



,

wo  $\cong k \langle X,Y\rangle/(X^2,Y^2,XY - \lambda YX)$. Die Komponenten selbst sind die Zariskiabschlüsse der GL_n-Bahnen obiger Strukturen.

Das abgeschlossene Unterschema Alcom_n der kommutativen Strukturen ist irreduzibel für $n \leqslant 6$. Der Beweis beruht auf einer guten Parametrisierung dieser Strukturen.

Thema: Blocks of defect 0

Vortragender: Jörn B. Olsson

A survey on results on conditions, necessary and/or sufficient for the existence of p-blocks of defect 0 in finite groups was given. Among others, these results were discussed : Let $D_p(G)$ be the set of p-blocks of defect 0 in the group G .

1. If $O_{p'}(G) \neq 1$ and $O_{p'}(C_G(z)) = 1$ for all $z \in G$ with $|z| = p$, then $D_p(G) \neq \emptyset$.

2. If p, q are primes dividing $|G|$ and $P \in \mathrm{Syl}_p(G)$ is cyclic and TI and if $C_G(P) = P \times K$, then we have:

   If $D_q(K) = \emptyset$, then $D_q(G) \leqslant D_p(G)$.

3. If G is p-solvable with cyclic p-Sylow subgroup, then $D_p(G) \neq \emptyset$ iff $O_p(G) = 1$.

Thema: Coxeter functors without diagrams

Vortragender: Idun Reiten

Partial Coxeter functors and Coxeter functors have played an important role in the study of the representation theory of hereditary Artin tensor algebras by Gelfand, Bernstein, Ponomarev and by Dlab and Ringel. We study for basic Artin algebras Λ with a simple projective non-injective module S, the functor $\mathrm{Hom}_\Lambda(X, \ ): \mathrm{mod}\Lambda \to \mathrm{mod}\,\Gamma$. Here $X = \mathrm{TrDS} \amalg P$, where P is given by $\Lambda = S \amalg P$, and $\Gamma = \mathrm{End}_\Lambda(X)^{op}$. When $\mathrm{Hom}_\Lambda(\mathrm{TrDS}, \Lambda) = 0$, the functor $\mathrm{Hom}_\Lambda(X, \ )$ has properties similar to the partial Coxeter functors studied for hereditary tensor algebras, and there is a close connection between the functors in this case.

For hereditary algebras the functor DTr is in a certain sense a composite of functors of the above type.

The talk was based upon joint work with M. Auslander and M.I. Platzeck.

Thema: Selbstinjektive Algebren, die stabil äquivalent sind zu selbstinjektiven Nakayama-Algebren

Vortragender: Christine Riedtmann

k algebraisch abgeschlossener Körper

Λ selbstinjektive Artin'sche Algebra

N selbstinjektive Artin'sche Nakayama-Algebra

Vor. : $\underline{\text{mod}\,\Lambda} = \text{mod}\,\Lambda/\{\text{proj}\} \underset{L}{\rightleftarrows} \underline{\text{mod}\,N}$

Beh. : $\exists$ exakte Funktoren F, G

$$\text{mod}\,N \underset{F}{\overset{G}{\rightleftarrows}} \text{mod}\,\Lambda\,, \text{ so daß } \underline{F} = L\,,\ \underline{G} = L^{-1}\,.$$

Beweisidee :

Klassifikation aller Algebren, die stabil äquivalent sind zur Kategorie der Darstellungen einer Vektorraumkrone. Hilfsmittel: Gitter von irreduziblen Morphismen für die stabile Kategorie der unzerlegbaren Darstellungen der Vektorraumkrone.

Dissertation, Zürich 1977

Thema: Tame group algebras

Vortragender: C. M. Ringel

Theorem: The semi-dihedral and the generalized quaternion groups are tame in characteristic 2: a complete classification of all indecomposable representations can be given, and there exists an algorithm decomposing any given module into a direct sum of indecomposable modules.

This solves the question which p-groups are tame, since Bondarenko and the author have (independently 1975) shown that the dihedral 2-groups are tame, and these three classes are the only possible candidates. More general, one gets in this way a complete classification of tame local algebras over an algebraically closed field.

Thema: Integral representations via diagrams

Vortragender: K. W. Roggenkamp

Let T be a complete valuation ring with residue field $\overline{k}$, Λ an R-order in a separable Algebra. Let $N(\Lambda)$ denote a full set of non-isomorphic indecomposable Λ-lattices. (i) If $\overline{k}$ is infinite and $|N(\Lambda)| = \infty$, then there exist infinitely many indecomposable lattices of fixed rank n, for infinitely many $n \in \mathbb{N}$. (ii) If $|N(\Lambda)| = \infty$, then there exist infinite chains of indecomposable lattices $M_{o\lambda} \subset M_{1\lambda} \subset \ldots \subset M_{i\lambda} \subset M_{i+1\ \lambda}$, "indexed" by polynomials with $M_{o\lambda} \simeq M_{i\lambda}/M_{i-1\ \lambda}$. (iii) $|N(\Lambda)| = \infty$ iff there exists a large indecomposable R-free Λ-module.

In case Λ is a Bäckström order; i.e., $\mathrm{rad}\,\Lambda = \mathrm{rad}\,\Gamma$, when Γ in a hereditary order containing Λ, then one can associate a valued graph G with Λ and then there is a bijection between the indecomposable Λ-lattices and the non-simple indecomposable representations of Γ.
This is joint work with C. M. Ringel.

Thema: Quadratic forms and diagrams

Vortragender: A. V. Roiter

If f is a quadratic form $f(X_1 \ldots X_n) =$

$$= \sum_{i=1}^{n} a_i X_i^2 + \sum_{i<j} a_{ij} X_i X_j \quad \text{and} \quad a_i | a_{ij} \ (\forall_{i,j})$$

then under some restrictions, for example if f is positive, a vector X is a root if and only if

$$(\forall i) \quad \frac{a_i X_i}{f(X)} \in \mathbb{Z} .$$

If $f(X,Y)$ is a bilinear form corresponding to a quiver of finite type, then any positive vector X may be uniquely decomposed into the form $X_1 + \ldots + X_t$, where X_i are positive roots, and $f(X_i,X_j) \geq 0$ for all (i,j).

Thema: Pure semi-simple Grothendieck categories and rings of finite representation type

Vortragender: Daniel Simson

Let $\underline{\underline{A}}$ be a Grothendieck category. $\underline{\underline{A}}$ is pure semi-simple if every object in $\underline{\underline{A}}$ is a direct sum of finitely presented objects. Quivers Γ such that their categories of linear representations are pure semi-simple can be described.
If $\underline{\underline{A}}$ is pure semi-simple and M is a noetherian object in $\underline{\underline{A}}$ then there exists a full subcategory of $\underline{\underline{A}}$ containing M and equivalent to the category of all finitely presented modules over a ring of finite representation type (under the assumption that the endomorphism ring of any injective noetherian object in $\underline{\underline{A}}$ is an Artin algebra).
The structure of a locally finite pure semi-simple category can be described.

<u>Thema</u>: Induced representations of finite, algebraic groups

<u>Vortragender</u>: Detlef Voigt

Let $G' \subset G$ be two finite algebraic groups over an algebraically closed ground field of characteristic $p > 0$. Let M be a finite dimensional $H(G')$-module. Then we ask for the induced module $H(G) \otimes_{H(G')} M$. There seem to be many theorems concerning induced representations in the theory of finite, abstract groups which carry over to the general case, though the methods used to prove them totally break down in the infinitesimal situation. Examples of results of this type are generalizations of theorems due to Blichfeldt, Morita-Hamernik, Green, Shoda, Taketa and Dade. Besides these results there are typically infinitesimal phenomena without any counterpart in the abstract case. So for example it is possible to give representation theoretic characterizations of solvable, super-solvable and nilpotent infinitesimal algebraic groups, which become incorrect in the situation of finite, abstract groups.

K. Bongartz (Zürich) und R. Knörr (Essen)

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