MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 25/1977

Riesz Spaces and Order Bounded Operators

19.6. bis 25.6.1977

Die Tagung stand wieder unter der Leitung von Herrn Prof. Luxemburg (Pasadena) und Herrn Prof. Schaefer (Tübingen). An der Tagung nahmen 41 Mathematiker aus allen Kontinenten teil, es wurden 27 Vorträge gehalten. Die vortragsfreie Zeit wurde zu intensivem wissenschaftlichem Gedankenaustausch genutzt. Dabei wurde die einzigartige Atmosphäre des Mathematischen Forschungsinstitutes Oberwolfach von allen Teilnehmern anerkannt und geschätzt.

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VORTRAGSAUSZÜGE

C.D. ALIPRANTIS und O. BURKINSHAW: Laterally complete Riesz spaces

C.D. ALIPRANTIS: Teil I

This is the first of two lectures to be delivered separately by the authors. The lecture will be focused on the lattice properties of laterally complete Riesz spaces (.i.e., Riesz spaces having the property that the supremum of every disjoint subset of positive elements exists). The recent result of Veksler-Geiler-Bernau (every Archimedean σ-laterally complete Riesz space has the principal projection property) and the concept of dominable sets due to Fremlin (characterizing the subsets of a Riesz space which are order bounded in its universal completion) will be the main tools for understanding the lattice structure of the above spaces.

By a different approach the known results will be generalized, new ones will be obtained and at the same time some interesting examples will be presented.

O. BURKINSHAW: Teil II

This lecture will be focused on the analytical aspects of the laterally complete Riesz spaces. Specifically, the study will deal with the locally solid topologies that Archimedean σ-laterally complete Riesz spaces can carry and with the order bounded operators between locally solid laterally complete Riesz spaces. The cornerstone of the discussion will be the following theorem.

Theorem: For a Hausdorff Lebesgue topology τ on a Riesz space L the following statement are equivalent:

(i) τ has a Lebesgue extension to its universal completion L^u.

(ii) τ has a locally solid extension to its universal completion L^u.

(iii) τ is coarser than any Hausdorff σ-Fatou topology on L.

(iv) Every disjoint sequence of L is τ-convergent to zero.

(v) The topological completion $\hat{L}$ of (L,τ) is the universal completion of L, i.e., $\hat{L} = L^u$.

From this theorem the known results will be generalized and new ones will be obtained, for example, a σ-laterally complete Riesz space can admit at most one Hausdorff Fatou topology. Finally, some unsolved problems will be discussed.

G.D. ALLEN: Lorentz spaces with regular weights

It is shown that the duals of Lorentz sequence spaces have a simple explicit representation in the case the weights satisfy a regularity condition. Namely, if $\sum_{j=1}^{n} \pi_j = O(n\pi_n)$, then if $\{\alpha_j\} \in d^*(\pi,p)$, the dual of the Lorentz space $d(\pi,p)$, $\alpha_j = \eta_j \xi_j$ where $\{\eta_j\} \in \ell_q$ $(q^{-1} + p^{-1} = 1)$ and $\{|\xi_j|\}$ has a decreasing rearrangement $\xi_j^+ = O(\pi_j^{1/p})$. The result depends upon an order structure in $d^*(\pi,1)$.

T. ANDO: An operator inequality in Banach lattices

Let T be a positive linear operator on an order-complete (complex) Banach lattice E. If $|\zeta| > r(T)$ = the spectral radius of T, the operators $\zeta(\zeta - T)^{-1}$ and $T(\zeta - T)^{-1}$ are order-bounded. Our concern is the inequality between the moduli of these operators:

$$|T(\zeta - T)^{-1}| \leq |\zeta(\zeta - T)^{-1}|$$

for ζ in a suitable domain.

The inequality in question is valid for $|\zeta| > 3\cdot r(T)$ without any additional assumption on E or T . If E is atomic or if T is compact, then the inequality is valid for $|\zeta| > r(T)$.

W. ARENDT: Ordnungsspektrum und harmonische Analyse

Ist E ein ordnungsvollständiger komplexer Banachverband, so ist $\mathcal{L}^r(E)$, der Raum der regulären (d.h. ordnungsbeschränkten) Operatoren auf E , versehen mit der Norm $\|T\|_r := \| |T| \|$, eine Banachalgebra. Das Spektrum $\sigma_o(T)$ eines Operators $T \in \mathcal{L}^r(E)$ in dieser Algebra heißt O-Spektrum (H.H.Schaefer 1976). Man kann die Spektren der Elemente von M(G) , der Banachalgebra der beschränkten Maße auf einer lokalkompakten abelschen Gruppe G mit der Faltung als Multiplikation,als O-Spektren auffassen. (Genauer gilt: $\sigma(\mu) = \sigma_o(T_\mu)$ für $T_\mu \in \mathcal{L}^r(L^2(G))$ definiert durch $T_\mu f := f*\mu$ für alle $f \in L^2(G)$.)
Während für Hilbert-Schmidt-Operatoren beide Spektren übereinstimmen (H.H.Schaefer), gibt es auf $L^2(G)$ (G die Kreisgruppe) einen kompakten, hermiteschen, positiven Operator T , so daß $i \in \sigma_o(T)$. Insbesondere ist $\sigma_o(T)$ überabzählbar.

A.BELLOW: Amarts and lattice structure

The notion of amart (= asymptotic martingale) was introduced in an attempt to provide a concept general enough to include the martingale, the submartingale, the supermartingale, the quasi martingalge - with room to spare. The concept is flexible enough to permit simple unified proofs. Let $(\Omega, \mathcal{F}, P)$ be a probability space, $(\mathcal{F}_n)_{n\in\mathbb{N}}$ an increasing sequence of sub-σ-fields of $\mathcal{F}$ and let T be the set of all bounded stopping times. Definition: An adapted sequence $(X_n)_{n\in\mathbb{N}}$ of integrable r.v's is an amart if $\lim_{\tau\in T} \int X_\tau$ exists in $\mathbb{R}$. The class of L^1-bounded amarts exhibits some (probabilistically) useful properties, such as the "maximal inequality", the "optional sampling property", the "Riesz decomposition"; it also is a vector space and a lattice. We give a very simple proof of the lattice property; as a corollary we obtain a particularly simple proof of the "Doob convergende theorem" for L^1-bounded amarts.

S.J. BERNAU: Ultraproducts and local problems for Banach lattices

In a recent paper, A local characterization of Banach lattices with order continous norm, Studia Math. 58(1976) 101-128, Bernau and Lacey gave necessary and sufficient conditions, in terms of local structure, for a Banach space to be linearly isomorphic to a Banach lattice with order continuous norm.

The description was extremely complicated. The lattice structure was imposed ab initio based on the isometric situation and the isomorphic case was treated by renorming and modifying the local structure to get to the isometric situation.

In this lecture we show that everything becomes much simpler and far more transparent by the use of a natural embedding into an ultraproduct of finite dimensional Banach lattices.

A. CLAUSING: Extremal operators into simplex spaces.

The study of extremal operators from Banach spaces into simplex spaces is usually based on three tools: an operator representation theorem, a selection theorem, and the geometry of certain subsets of the dual of the domain space. We show, that stability and regularity of convex sets can be used for these purposes. (A compact convex set K is called stable, if the map $K \times K \longrightarrow K$, $(x,y) \longmapsto \frac{x+y}{2}$, is open; it is called regular, if the barycentric map $r : Max(K) \longrightarrow K$ has a w^*-continuous right inverse).
The report is based on joint work with S.Papadopoulou.

P. DODDS: Compact operators on Banach lattices

We present the following result.

Theorem: Let E, F be Banach lattices. Assume that E^*, F have order continuous norms. If $0 \leq T : E \longrightarrow F$ is a compact linear mapping and if $0 \leqslant S : E \longrightarrow F$ is a linear mapping which satisfies $0 \leqslant S \leqslant T$, then also S is compact.

This result has been obtained jointly with David Fremlin.

K. DONNER: Korovkin theorems for positive linear operators

Starting from the complete characterization of Korovkin closures for nets of positive linear operators from one topological vector lattice into another by (H,S)-affine elements, a description of Korovkin closures for equicontinuous or contractive nets of positive linear operators from a normed vector lattice E into another normed vector lattice F is given in terms of H-envelope filters. The main idea is to use bidual embeddings in order to work with sup-norms. Normboundedness conditions can be easily reformulated in terms of suitable extensions of the positive linear operator to the space generated by the bidual embedding of E (resp. F) and the constant functions on E'' (resp. F''). Using the known results on Korovkin-closures for positive linear operators we are naturally lead to the notion of H-envelope filters. Convergence of H-envelope filters characterizes Korovkin closures for nets of positive linear contractions and equicontinuous nets of positive linear operators.

M. DUHOUX: Locally solid Mackey topologies on Riesz spaces

Let E be a Riesz space. We denote by E^+ the Dedekind complete Riesz space formed with all linear functionals on E which are bounded on the intervals in E. E_c^+ (resp. E_n^+) is the band in E^+ consisting of all $f \in E^+$ such that $\lim f(x_n) = 0$ (resp. $\lim f(x_\alpha) = 0$) for every sequence $x_n \downarrow 0$ (resp. every net $x_\alpha \downarrow 0$) in E. We show that the following known theorem is an immediate consequence of Orlicz-Pettis theorem: If E is Dedekind σ-complete, then the Mackey topology $\tau(E,E_c^+)$ is locally solid. We also give a sufficient condition for that the Mackey topology $\tau(E,F)$ (F band in E^+) be locally solid. As corollary of these two results, we obtain: If E is Dedekind σ-complete, then $\tau(E,F)$ is locally solid for every band F in E_c^+. The particular case when $F = E_n^+$ is well-known. We also give some informations concerning weak compactness in bands in E^+.

H.O. FLÖSSER: Choquet boundary and harmonic functions

Let E be a Dini-lattice and $V(E)$ the set of all real valued continuous lattice homomorphisms on E. The Choquet boundary of a subset M of E relative to the closed vector sublattice $F(M)$ generated by M in E is defined as $\partial_{F(M)} M$ = the set of all $\delta \in V(F(M))$ such that the restriction of δ to the linear hull $L(M)$ of M in E is extremal in $L(M)'_+$. If E'_+ is nearly well capped, it is possible to characterize the closed vector sublattices of E. This makes it possible to prove that $\partial_{F(M)} M = V(F(M))$ implies $F(M) = \{ e \in E \mid \forall \mu \in E'_+ \; \forall \delta \in V(E) \; \mu =_M \delta \Rightarrow \mu(e) = \delta(e) \} =: E(M)$, the uniqueness closure of M. Furthermore, the same hypothesis yields $\overline{S(M)} = H(M)$, where $S(M)$ is the vector space of all strict-M-harmonic, $H(M)$ of all M-harmonic elements. If, in addition, M is finite, $S(M)$ is a vector sublattice. Conversely, if $S(M)$ is a vector sublattice, $\partial_{F(M)} M = V(F(M))$. Thus, if M is finite, we obtain as necessary and sufficient condition for the Korovkin closure of M to be E, $\partial_E M = E$ and M separates $V(E)$. In this case $\overline{S(M)} = H(M) = E(M) = E$.

B. FUCHSSTEINER: Generalized realcompactification

The notions "character", "realcompactification", "pseudocompact" etc. are generalized to arbitrary cones of bounded functions on some set. The theory which comes out of this comprises large parts of choquet's theory and of the theory of continuous functions (on noncompact sets). Very many results already known for realcompact spaces can be transferred to this general situation (under slight modifications, of course). And the transfer of these results leads to new theorems in Choquet's theory.

G. GIERZ: Two remarks on injective Banach lattices

Theorem (Gierz, Haydon): A Banach lattice E is injective if and only if there is a bundle $p : \mathcal{E} \to X$ satisfying

(i) Every stalk $p^{-1}(x)$ is an AL-space
(ii) X is Stonian
(iii) The mapping norm : $\mathcal{E} \to \mathbb{R}_+$ is continuous
(iv) If $U \subseteq X$ is open and $\sigma : U \longrightarrow \mathcal{E}$ is a bounded continuous section, then σ can be extended to a global continuous section $\bar{\sigma} : X \longrightarrow \mathcal{E}$ such that E is isometrically isomorphic to the Banach lattice $\Gamma(p)$ of all continuous sections of p .

Theorem: Let $p : \mathcal{E} \longrightarrow X$ be a bundle of Banach spaces over a compact base space X such that every stalk $p^{-1}(x)$ has the approximation property. Then the Banach space $\Gamma(p)$ of all continuous sections of p has the approximation property.

As an application of these two theorems and the fact that every AL-space has the approximation property we obtain the following corollary, which was first proven by H.P.Lotz (unpublished):

Corollary: Every injective Banach lattice has the approximation property.

A. IWANIK: Extreme operators on AL-spaces

Let E and F be AL-spaces. We denote by V and U the unit balls in $\mathcal{L}(E,F)$ and $\mathcal{L}(F',E')$, respectively. It is an easy consequence of the Banach - Alaoglu and Krein - Milman theorems that the convex hull of ex U is dense in U for the weak* operator topology $\sigma(\mathcal{L}(F',E'), F'\otimes E)$. Our aim is to present a necessary and sufficient condition for the convex hull of ex V to be dense in V for the strong operator topology in $\mathcal{L}(E,F)$. In particular we have the following result:

If F is separable then V coincides with the strong operator closed convex hull of ex V if and only if E is non-atomic or F is Banach lattice isomorphic to $\ell^1(B)$ for some (necessarily countable) index set B .

P.Y. LEE: Some sequence spaces of a nonabsolute type

Some sequence spaces of a nonabsolute type are defined by means of an infinite matrix and a modular. The norms in the spaces are nonabsolute in the sense that $\|x\|$ is not necessarily equal to $\||x|\|$ where $x = \{x_k\}$ and $|x| = \{|x_k|\}$. Furthermore the spaces may not contain all finite sequences and the associate of a norm is not necessarily a bona fide norm. We determine their associate spaces and matrix transformations mapping these spaces into ℓ_∞ , c and c_o .

H.P. LOTZ: Bounded groups of positive operators

We proved the following results:

Theorem. Let G be a group of positive operators on a Banach lattice E . If $\|T\| \leqslant M$ for all $T \in G$, then $\|T - S\| \geqslant M^{-1}$ for all $T,S \in G$ with $T \neq S$.

In particular, G is closed in the Banach space $\mathcal{L}(E)$ and discrete for the induced topology.

Corollary 1. Let G be a (norm) relatively compact group of positive operators on a Banach lattice E . Then G is finite.

Corollary 2. (Brown, Proc. AMS, 1964). Let G be a bounded group of positive n x n - matrices. Then G is finite.

W. LUXEMBURG: On a theorem of Orlicz concerning weak compactness

Let (Φ, Ψ) be a pair of complementary Young functions such that $\lim_{a \to \infty} \frac{\Psi(2a)}{\Psi(a)} = \infty$. From an earlier result of Orlicz the weakly compact subsets of L^{Φ} can be characterized in terms of equi-absolute continuity. In certain cases it can be shown that such sets are spectrally bounded.

The results are applied to compact operators.

M. MEYER: Some properties of Riesz homomorphisms

Let E and F be archimedean Riesz spaces. The following is first shown:
If T is an order bounded linear operator from E to F , these assertions are equivalent

a) $|T(f)| = |T(|f|)|$ for every f in E .

b) There exist two Riesz homomorphisms from E into F , T_+ and T_- , such that $T = T_+ - T_-$ and $(T_+(f)) \wedge (T_-(f)) = 0$ for every f in E_+ .

Some applications of this result are given:

I) The stabilisator of E is the set of all order bounded linear operators from E into E letting invariant every band of E . The stabilisator is shown to be an archimedean Riesz space. Some of its properties are studied.

II) The Riesz space F being supposed uniformly complete, let T be a positive operator from E into F . Some properties of the center Z(E) of E are given such that the following assertion may be equivalent:

a) T is a Riesz homomorphism.

b) There exists a Riesz homomorphism Π_T from Z(E) into $Z(F_T)$ such $\Pi_T(U) \circ T = T \circ U$ for every $U \in Z(E)$ (where F_T is the order ideal generated by T(E) in F and $Z(F_T)$ its center).

L.C. MOORE, Jr.: Local reflexivity in Banach lattices

Let L be a Banach lattice, let U be a finite dimensional Riesz subspace of the second dual L'', let $\{\theta_1, \theta_2, \dots, \theta_p\}$ be a finite set of elements of L', and let $\varepsilon > 0$. Then there exists a Riesz isomorphism T of U into L such that

(i) $\|T\| < 1 + \varepsilon$ and $\|T^{-1}\| < 1 + \varepsilon$, and

(ii) $|\langle Tu, \theta_k\rangle - \langle \theta_k, u\rangle| < \varepsilon \|\theta_k\| \, \|u\|$ for all u in U and $k = 1, 2, \dots, p$.

If L is Dedekind σ-complete and $v_n \downarrow 0$ implies $\|v_n\| \to 0$ ((A,i)-property), then T may be selected to satisfy (i), (ii) and

(iii) $\|Tx - x\| < \varepsilon \|x\|$ for all $x \in U \cap L$.

R. NAGEL: Dilations of positive operators

We present a new and purely functional analytic proof of the following dilation theorem of M.A.Akcoglu (1975): Let (X, μ) be a finite measure space and T a positive contraction on $L^1(X,\mu)$. There exists a new measure space $(\hat{X}, \hat{\mu})$, a lattice injection $\hat{J} : L^1(X,\mu) \longrightarrow L^1(\hat{X},\hat{\mu})$, a positive contraction $\hat{Q} : L^1(\hat{X},\hat{\mu}) \longrightarrow L^1(X,\mu)$ and a Banach lattice isomorphism $\hat{T}$ on $L^1(\hat{X},\hat{\mu})$ such that $T^k = \hat{Q}\,\hat{T}^k\,\hat{J}$ for all $k \in \mathbb{N}$. In addition, we prove that our dilation is uniquely determined by the "Markov property" and preserves all ergodic properties of the original operator.

(Joint work with M.Kern and G.Palm, to appear in Math.Z.)

SUSANNE PAPADOPOULOU: An extension problem for a positive operator

Let K be a compact convex set such that the set $\partial_e K$ of its extremal points is closed. A Dirichlet space for K is a subspace H of $C(K)$ containing $A(K)$ such that: (i) for every $f \in C(\partial_e K)$ there is a unique $\tilde{f} \in H$ with $\tilde{f}|\partial_e K = f$, (ii) $\tilde{f} \geq 0$ if $f \geq 0$.

$\tilde{f}$ is called the Dirichlet solution for f with respect to H. Given $g \in C(K)$ it is studied, whether g is the Dirichlet solution for $f = g|\partial_e K$ for a suitable Dirichlet space H. This question is equivalent to the existence of a positive extension for a certain positive operator. Sufficient conditions are given that the set of all Dirichlet solutions for f is uniformly dense in or equal to the interval $[\check{f}, \hat{f}] \cap C(K)$, as well as some counterexamples.

C. PORTENIER: Représentation des espaces de Riesz archimédiens et de leurs centres

Si F est un espace de Riesz archimédien uniformément complet, alors il existe une fibration principale π (non-nécessairement séparée) de groupe structural $\mathbb{R}_+^*$, dont la base s'identifie à l'ensemble $\mathcal{Q}$ des idéaux quasi-maximaux de F, et un isomorphisme de F sur le sous-espace coréticulé des sections continues $\xi \in \mathcal{C}(\pi_{\overline{\mathbb{R}}})$ du fibré en droites achevées $\pi_{\overline{\mathbb{R}}}$ associé à π, qui sont finies sur un ouvert dense de $\mathcal{Q}$ et telles que $\{0 < |\xi| < \infty\}$ soit compact.

De cette représentation on peut évidemment déduire toutes les représentations classiques. Comme autre application on peut citer le résultat suivant généralisant certains résultats connus:

Si T est un operateur central (i.e. si $f,g \in F$ et $|f| \wedge |g| = 0$ implique $|Tf| \wedge |g| = 0$) relativement borné, alors il existe une unique fonction continue φ_T sur $\mathcal{Q}$, finie sur un ouvert dense, telle que l'on ait, pour tout $f \in F$, $Tf = \varphi_T f$ sur l'ouvert dense $\{|\varphi| < \infty\} \cap \{|f| < \infty\}$.

En explicitant le lien entre les différentes représentations d'un L-espace F, ainsi que de son dual F', et sachant qu'il y a correspondance biunivoque entre les bandes de F et celles de F', on voit que le centre $Z(F)$ (ensembles des opérateurs centraux relativement bornés) de F est canoniquement isomorphe à F'. Le calcul explicite de cet isomorphisme fournit le théorème de Radon-Nikodym sous sa forme abstraite.

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E. SCHEFFOLD: Fixräume regulärer Operatoren

S.J. Bernau hat den Begriff eines Austausch-Unterraumes eines L^p-Raumes eingeführt und gezeigt, daß der Wertebereich einer kontrahierenden linearen Projektion auf einem L^p-Raum $(1 \leq p < \infty,\ p \neq 2)$ ein Austausch-Unterraum ist. Wir verallgemeinern diesen Begriff für Banachverbände mit ordnungsstetiger Norm und zeigen, daß die Fixräume spezieller regulärer Operatoren auf diesen Räumen Austausch-Unterräume sind. Als Anwendung bringen wir einen Korovkin-Satz für Folgen linearer Kontraktionen auf Banachverbänden mit gleichmäßig monotoner Norm.

A.R. SCHEP: Kernel operators

Let L and M be (order) ideals in $M(Y,\nu)$ and $M(X,\mu)$ respectively. Then an operator $T : L \longrightarrow M$ is called a kernel operator if $\exists\, T(x,y)$ $\mu \times \nu$-measurable such that

(i) $\int |T(x,y)\, f(y)|\, d\nu(y) \in M \qquad \forall f \in L$

(ii) $Tf(x) = \int T(x,y)\, f(y)\, d\nu(y)$ a.e. $\qquad \forall f \in L$.

Now one can prove that if T is a positive kernel operator from L into M and $0 \leq S \leq T$, then S is also a positive kernel operator. With the aid of this theorem one can prove that the kernel operators form a band in the space of order bounded (regular) operators from L into M. If the set of order continuous funtionals $L^\wedge$ is separating points of L, this band is equal to the band $(L^\wedge \otimes M)^{dd}$ generated by the finite rank operators. If again $L^\wedge$ is separating points one can derive from this description of kernel operators that for an order bounded operator T the following are equivalent:

(i) If $0 \leq u_n \leq u$ and u_n converges in measure to zero on every set of finite measure, then $Tu_n(x) \longrightarrow 0$ a.e.

(ii) T is a kernel operator.

This theorem has also been proved by A.V.Bukralov (1974-1975) and the idea of the proof is taken from a paper of H.Nakano (1953). By means of this theorem one can easily prove a lot of representation theorems of operators as kernel operators. In particular the theorem of Dunford follows easily. (Dunford's theorem states that every continuous operator from $L_1(Y,\nu)$ into $L_p(X,\mu)$, $1<p\leq\infty$, is a kernel operator).

C.T. TUCKER: The sequential mapping continuity property and Baire functions

I have shown that there exists a large class of Riesz spaces L with the property that if q is a positive linear map of L to a partially ordered directed Archimedean vector space then q is sequentially continuous. In particular, this class includes $B_\alpha(\Omega)$, the $\alpha^{\underline{th}}$ Baire class generated by Ω , where Ω is a linear lattice of functions containing the constant functions. I apply these ideas to show that if $B_1(\Omega)$ is Riesz isomorphic to a subspace of C(X) for some topological space X , then $B(\Omega)$, the set of all Baire functions is also isomorphic to a subspace of C(X) . Also if q is a positive linear map on $B_2(\Omega)$ then q can be extended to $B(\Omega)$, q is the sum of a finite number of Riesz homomorphisms, and if q is to a function space, it preserves pointwise convergence.

A.C. ZAANEN: Orlicz lattices

Let M be a modular on the Riesz space L (in the sense of Nakano) such that $M(f+g) = M(f)+M(g)$ for $f\wedge g = 0$ and $M(2f) \leq C\cdot M(f)$ for some constant $C > 0$ and all $f\in L$. Also assume that L is norm complete with respect to the norm corresponding to the modular.

Given $f \neq 0$ in L , define the function φ_f by $\varphi_f(\alpha) = M(\alpha f)/M(f)$ for all $\alpha \geq 0$. If L has a weak unit e , then L is called component invariant with respect to e (or, briefly, L is called an Orlicz lattice) if $\varphi_p = \varphi_e$ for every component p of e .

Theorem. Every Orlicz lattice is isomorphic to an Orlicz space $L_{\Phi}(X,\Lambda,\mu)$.

This theorem (with some extensions) is proved in the thesis of W.J.Claas (Leiden Univ., 1977).

Christian Rall (Tübingen)