MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 3|1978

"Local Algebra and Local Analytic Geometry"

15.1. - 20.1.1978

Die Tagung stand unter der Leitung von R. Berger (Saarbrücken) und G. Scheja (Bochum).

Ziel der Tagung war es, lokale Fragen der Kommutativen Algebra und damit in Zusammenhang stehende Fragen und Methoden einerseits der algebraischen Geometrie, andererseits der analytischen Geometrie darzustellen und vom Gesichtspunkt beider Abreitsrichtungen gemeinsam zu diskutieren.

Folgende Einzelberichte wurden vor allem behandelt:

Lokale Dualität, lokale de Rham-Cohomologie algebraische und transzendente Singularitäten und Deformation von Singularitäten unter verschiedenen Gesichtspunkten.

Teilnehmer

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Nastold, H.-J., Münster
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Riemenschneider, O., Hamburg
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Si, M., Bochum
Stenrich, M., Essen
Storch, U., Osnabrück
Ulrich, B., Saarbrücken
Verdier, J.-L., Paris
Vetter, U., Vechta
Waldi, R., Regensburg
Wiebe, H., Bochum

RIEMENSCHNEIDER, O.: A Generalization of the Eagon-Northcott-Complex

A result of J. Wahl (1977) says that the "Bett- numbers" of a two-dimensional rational singularity over an algebraically closed field k are given by $b_i = i\binom{e-1}{i+1}$ where e is the embedding dimension of the singularity.
This suggests a connection with determinantal singularities defined by 2xm matrices, m=e-1.
However, as was shown by Wahl, very few rational singularities are determinantal.

Therefore, one has to generalize the notion of a determinantal singularity. We consider a commutative ring R with unit 1, and elements $a_1,..,a_m, b_1,..,b_m, c_{1,2},..,c_{m-1,m} \in R$.

Let $c_{i,j} = \prod_{k=i}^{j-1} c_{k,k+1}$, $1 \leq i < j \leq m$, and let $\mathfrak{A}$ be generated by the elements $f_{i,j} = a_i b_j - b_i c_{i,j} a_j$.

Then there exists a canonical complex over R associated to $R/\mathfrak{A}$ which specializes to the Eagon-Northcott-complex if all $c_{i,j}=1$.
Moreover this complex is a minimal projective resolution, if R is noetherian and grad $\mathfrak{A}$ = m-1 (joint work with J. Herzelhoy).

It can be shown that almost all two-dimensional quotient singularities can be written in this "weak determinantal" form.

BERNDT, R.: The canonical Module for Cyclic Quotient Singularities

The Ring $C = k[x_1,..,x_e]$ with the relations

$$f_{\delta\varepsilon} = -x_\delta x_\varepsilon + x_{\delta+1} c_{\delta,\varepsilon-1} x_{\varepsilon-1} = 0 , \quad 1 \leq \delta < \varepsilon - 1 \leq e-1$$

$$c_{\delta,\varepsilon-1} = \prod_{i=\delta+1}^{\varepsilon-1} x_i^{a_i-2}$$

$$\frac{n}{n-q} = a_2 - 1/1 - \ldots - 1/a_{e-1} \quad \text{(continued fraction)}$$

$$0 < q < n , \quad (n,q) = 1$$

denotes for $k = \mathbb{C}$ the Ring of invariants of the Group $G = \langle \varphi \rangle$ operating by $\varphi : \mathbb{C}^2 \longrightarrow \mathbb{C}^2$

$$(u,v) \longmapsto (\zeta u, \zeta^q v), \quad \zeta = \sqrt[n]{1} \text{ primitive.}$$

(s. Riemenschneider: Math. Ann. 209 (1974)).
By explicit calkulations the equality of the Dedekind-complementary module $\mathcal{C}^o(C/C_o)$ and the inverse $\vartheta_o(C/C_o)^{-1}$ of the Kähler -different $\vartheta_o(C/C_o)$ is shown.

This equality can be interpreted as an example for the equality of Kunz's version of the canonical sheaf (for $X = \operatorname{Spec} C$) with special case of a rather general notion of a sheaf of entire differentials defined by the author (Abh.Math.Sem. Hamburg 47 , to appear).

HAMM, H.A.: Relative De Rham Cohomology

Let $f: (\mathbb{C}^m,o) \to (\mathbb{C}^k,o)$ be holomorphic. $(C,o) \subseteq (\mathbb{C}^m,o)$ and $(D,o) \subseteq (\mathbb{C}^k,o)$ germs of analytic subsets with $f(C) \subseteq D$ such that $f: C \to D$ is finite. We take suitable representatives of all the germs and assume that there is a Zariski open dense subset D' of D such that the singular points of $f^{-1}(D') - C$ form a closed subset of $f^{-1}(D')$, D' and $f^{-1}(D')$ being pure-dimensional of the same codimension. Then there is a Zariski open dense subset D'' of D such that:

1.) $\mathcal{H}^i(f_* \Omega^{\cdot}_f \mid C) \mid D''$ is a coherent $\mathcal{O}_{\mathbb{C}^k} \mid D''$ - module,

2.) $\mathcal{H}^i(f_* \hat{\Omega}^{\cdot}_f) \mid D'' \cong \mathcal{H}^i(f_* \Omega^{\cdot}_f \mid C)^\wedge \mid D''$ for all i .

Here $\Omega^{\cdot}_f$ denotes the relative holomorphic de Rham complex, and the completion is formal along C and D respectively.

KIYEK, K.: 1 - Dimensional CM-Rings: A New Proof of Some Results of Matlis

Let R be a local CM-Ring with maximal ideal $\mathfrak{m}$, $\dim R = 1$, Q full ring of quotients of R. A ring A between R and Q is called a strongly unramified extension, if $A = R + \mathfrak{m}A$.
We give a direct proof of the following fact, established by Matlis: There is a 1-1-correspondence between the set of strongly unramified extensions of R and the set of ideals of $Q(\hat{R})$, the full ring of quotients of the completion $\hat{R}$.

$K = Q/R$ is an artinian R-module. If $A_1, \ldots, A_n$ denote the maximal extensions, V_i the integral closure of A_i in Q, then V_i is a finite A_i-module, V_i is a pseudovaluation ring and $\bigcap V_i$ is the integral closure of R in Q.

BECKER, J.: Application of Functinal Analysis to the Solutions of Power Series Equations

It is wellknown that any algebra homomorphism between complete local rings is both open and closed in the respective Krull topologies, and that the corresponding statement for analytic rings is false. The main result of this talk is that every homomorphism between analytic rings which is closed is necessarily open (the converse is not true); this can be interpreted as a statement about the solutions of certain power series equations. This result is proven by deriving certain connection between the Krull topology and the simple and inductive topologies on the respective analytic rings, and applying some functional analytic results about the automatic continuity of linear operators and the uniqueness of topologies. For example, it is shown that the algebra of formal power series modulo any ideal carries a unique Frechet algebra topology (namely the simple topology).

BECKER, J.: Topological and Differentiable Properties of Analytic Varieties

We study the equivalence, embedding dimension and multiplicity of complex analytic varieties and the effect on these of continuous C^0, Lipschitz L, differentiable D, continuously differentiable C^1, C^k, C^∞, real analytic A, and complex analytic $\mathcal{O}$ changes of coordinates. For instance, a classical result of Zariski stats that two complex analytic planar curves are topologically equivalent iff they have the same characteristic exponents. This can be improved to differentiable equivalence but not C^1 equivalence. Two such curves are C^1 equivalent if and only if they have the same characteristic coefficients (up to conjugation). For curves in $\mathbb{C}^n$, $n \geq 3$, ther is an equally interesting theory.

BINGENER, J.; FLENNER, H.: Nichtalgebraische Singularitäten

Ein komplexer Raumkeim (X,x) heiße kompaktifizierbar, wenn es einen kompakten komplexen Raum Z und einen Punkt $z \in Z$ gibt, so daß die Keime (X,x) und (Z,z) isomorph sind.

In unserem Vortrag wurden Beispiele von komplexanalytischen Singularitäten konstruiert, die nicht kompaktifizierbar sind. Solche Singularitäten sind insbesondere nichtalgebraisch. Wir geben sodann für jeden bewerteten Körper k analytische k-Algebren mit Restekörper k an, die nicht algebraisch sind. Analoge Beispiele wurden für den Übergang vom analytischen zum kompletten Fall konstruiert. Die Beweise dieser Aussagen stützen sich auf die Deformationstheorie projektiver Schemata.

Mit ähnlichen Methoden lassen sich kompakte formale komplexe Mannigfaltigkeiten konstruieren, die nicht isomorph sind zur Komplettierung einer komplexen Mannigfaltigkeit längs eines analytischen Unterraumes.

COWSIK, R.C.; NORI, M.:

We prove the following theorem:
Let k be an infinite perfect field of positive charakteristic and let C be a curve in A_k^n , then C is a set theoretic complete intersection.

The proof depends on the corresponding theorem in the infinitesimal case, which in turn is proved by projecting onto a plane curve (birationally, finite). Then one proves that C is set theoretically a local complete intersection. The theorem (affine) follows from the theorems of L. Szpiro and N. Mohan Kumar.

GALLIGO, A.: Stability and Division Theorem

I compute the equations of the basis of the semi-universal flat deformation of the point x_o which algebra is $\mathbb{C}\{x,y,z\}/(x,y,z)^2$.

Using the division theorem one get the equations by taking the local flatner of the universal unfolding of the map germ $f: (\mathbb{C}^3,o) \longrightarrow (\mathbb{C}^6,o)$ defined by

$$(x,y,z) \longmapsto (x^2,y^2,z^2,xy,yz,zx) \quad .$$

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GRANGER, J.H.: The Irreducibility of $\mathrm{Hilb}^n\, \mathbb{C}\{x,y\}$
(after J.Briançon)

The Hilbert scheme $\mathrm{Hilb}^n A$ of the ring $A = \mathbb{C}\{x,y\}$ is the set of Ideals $I \subsetneq A$ such that $\dim_{\mathbb{C}} A/I = n$. It can be identified with an algebraic subset of a grassmannian ($n(n-1)/2$ planes of $A/\mathfrak{m}^n$) and we prove the following theorem: $\mathrm{Hilb}^n A$ is an irreducible algebraic set of pure dimension.

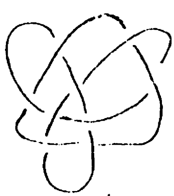
We outline the principal steps of the proof first by showing how to reduce to deform complete intersections. The proof is built on three technical lemmae which reduce the problem to the crucial point of showing how to deform $Axy + A(x^p \pm y^q)$ to an order 1 ideal.

EISENBUD, D.: Iterated Torus Links and Degeneration of Plane Curves (joint work with W. Neumann)

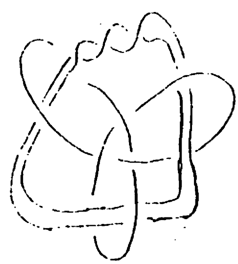
We consider curves in a neighbourhood of $0 \in \mathbb{C}^2$. Suppose $f(x,y) = 0$ has an isolated singularity. Then to study the degeneration of the smooth curves $f(x,y) = \delta$ to $f(x,y) = 0$ is the same as to study the (unique) fibration of the complement of the iterated torus link L corresponding to $f(x,y) = 0$.

We sketch a method for determining the fibration of the complement of any iterated torus link that has one. From this one gets a criterion for fiberability, and geometric information about the monodromy; for example, a power of the monodromy is a product of Dehn twists along disjoint closed curves in the fiber. These constructions generalize those of A'Campo for iterated torus knots (analytically irreducible curves).

e.g.:



fibers, but



does not.

STEURICH, M.: Golod Ideals of the Type $\mathfrak{a} \cap \mathfrak{b}$
(joint work with J. Herzog)

The following theorem was proved: Let R be a noetherian local ring, $\mathfrak{m}$ its maximal ideal, k the residue class field $R/\mathfrak{m}$, and $\mathfrak{a}, \mathfrak{b} \subseteq \mathfrak{m}$ two ideals of R such that $\mathrm{Tor}_i^R(R/\mathfrak{a}, R/\mathfrak{b}) = 0$ for all $i > o$, then $\mathfrak{a} \cap \mathfrak{b}$ is a Golod ideal and the equation

$$P^k_{R/\mathfrak{a}\cap\mathfrak{b}} = P^k_R / P^{R/\mathfrak{a}}_R + P^{R/\mathfrak{b}}_R - P^{R/\mathfrak{a}}_R \, P^{R/\mathfrak{b}}_R$$

holds, where P^M_S denotes the Poincaré series

$$\sum_{i=o}^{\infty} \dim_k \mathrm{Tor}_i^S(M,k)\, z^i$$

for a finitely generated module M over a local ring S.

PREUSS, D.: Residuen symmetrischer differentialformwertiger Bilinearformen

Seien k ein Körper, char k = $p \geq 2$, X eine über k definierte, irreduzible, vollständige, normale Kurve, L der Funktionenkörper der Kurve X, k in L algebraisch abgeschlossen, J die Weilschen Differentiale von X/k. Es wurde am Beispiel eines Körpers k mit $[k:k^p] < \infty$, $p > 2$, und eines abgeschlossenen Punktes x von X mit separabler Körpererweiterung $k \subseteq k(x)$ ($k(x)$:= Restklassenkörper von x) gezeigt, wie man die in einer Arbeit von Geyer, Harder, Knebusch und Scharlau ("Ein Residuensatz für symmetrische Bilinearformen" Inv. Math. Bd. 11) konstruierten Residuen $\mathrm{Res}_x : W(J) \longrightarrow W(k)$ mit Hilfe der von Kunz und Waldi... Nastold untersuchten Differentialformen höchster Stufe berechnen kann.

ELZEIN, F.: The Relative Fundamental Class of a Cycle
(joint work with B. Angeniol)

We consider these three questions:
Let S be a scheme and X locally of finite type, equidimensional of relative dimension p over S.

1.) Let $i: X \hookrightarrow Y$ be a closed immersion in a smooth scheme Y over S of relative dimension n. Determine an element $c_{X/S} \in H_X^{n-p}(Y, \Omega_{Y/S}^{n-p})$ which induces (if X is flat over S), at any point of S with value in a field K, the fundamental class $c_{X_K} \in H_{X_K}^{n-p}(Y_K, \Omega_{Y_K}^{n-p})$ by base change.

2.) Let $h: X \longrightarrow Y$ be an S-morphism finite of finite Tor dimension on Y smooth over S. We have a trace morphism $\mathrm{Tr}\, h : h_* \mathcal{O}_X \rightarrow \mathcal{O}_Y$ which induces for any integer e a trace $\mathrm{Tr}\, h: h_* h^* \Omega_{Y/S}^e \longrightarrow \Omega_{Y/S}^e$. Can we define $\mathrm{Tr}\, h$ on the whole sheaf $h_* \Omega_{X/S}^e$? - Even in the case S-spectrum of a field, this problem is not trivial.

3.) Let $u_1, \ldots, u_p$ be sections of $\mathcal{O}_X$ on X, which define a subscheme $V(u_1, \ldots, u_p)$ of X, proper and surj. over S, and $\omega \in \Gamma(X, \Omega_{X/S}^p)$. Can we define the Grothendieck Residue Symbol $\mathrm{Res}_S^X \left[{\omega \atop u_1, \ldots, u_p} \right] \in \Gamma(S, \mathcal{O}_S)$ when X is not necessarily smooth over S ? - To understand the links and answer the problems we use the duality theory by Grothendieck.

We give solutions when $S = \mathrm{Spec}\, k$, in any charakteristic, or X is locally complete intersection; and in characteristic 0, when X is of finite Tor dimension over S (a particular case: X flat over S).

FALTINGS, G.: Dualisierende Komplexe

Es wird folgender Satz bewiesen:
Sei A ein noetherscher Ring, $I \subsetneq A$ ein Ideal, so daß A I-adisch komplett ist. Wenn dann A/I einen dualisierenden Komplex besitzt, dann besitzt A ebenfalls einen.
Anschließend werden einige Existenzbedingung für dualisierende Komplexe gegeben, wobei der Fall der henselschen Ringe sich in vielen Fällen als einfacher erweist.

VERDIER, J.-L.: Local Terms of the Lefschetz Formula

We describe the local terms for the Lefschetz formula for smooth curves equipped with an etale sheaf with singularities. The general formula for the local terms obtained by Allbert generalizes formulae found by Nielsen-Wecken, Grothendieck, Bucur.

JUNG, G.: Strikte Homomorphismen lokaler k-Algebren

Motiviert durch die Untersuchungen über die Erzeugung formaler Relationen von konvergenten Reihen durch analytische Relationen (A.M. Gabrielov) werden im allgemeinen Rahmen der lokalen k-Algebren (k Körper) (es sind solche lokalen k-Algebren A mit maximalem Ideal $\mathfrak{m}_A$, endlich erzeugt, für die die kan. Abb. $k \longrightarrow A/\mathfrak{m}_A$ bijektiv ist) solche Homomorphismen $\varphi : A \longrightarrow B$ betrachtet, für die $\mathrm{Ker}\,\hat{\varphi} = \hat{A}\cdot\mathrm{Ker}\,\varphi$ ist. Es gilt: $\mathrm{Ker}\,\hat{\varphi} = \hat{A}\cdot\mathrm{Ker}\,\varphi \Leftrightarrow \varphi$ strikt $\Leftrightarrow \overline{\varphi(A)} = \hat{\varphi}(\hat{A})$. Nach Artin-Rees gilt: φ endlich, A noethersch $\Longrightarrow \varphi$ strikt. Beispiele zeigen, daß man auf die Voraussetzung "A noethersch" i.a. nicht verzichten kann. Es gilt jedoch: $\varphi : A \longrightarrow B$ endlich, $\mathrm{Ker}\,\varphi \subseteq \mathfrak{m}_A^{\infty}$, $\hat{A}$ Integritätsring $\Longrightarrow \varphi$ strikt. Zum Beweise wird eine über Parametersysteme definierte Dimensionstheorie für die nicht notwendig noetherschen lokalen k-Algebren herangezogen.

HERZOG, J.: A Cohen-Macaulay Criterium with Applications to the Conormal Module and Module of Differentials

Let R be a local integral domain and CM-ring, M a f.g. R-module with dim M = dim R, then the following conditions are equivalent:
a) $l(M/(\underline{x})M) = l(R/(\underline{x})R)\cdot \mathrm{rk}\, M$ ($\underline{x}$ is a system of Parameters)
b) M is a CM-module.

Using this charakterisation of CM-modules we show:
1.) Let $\mathfrak{p} \subseteq A$ prime ideal, height $\mathfrak{p} = 2$, dim A = 3, A regular, then the following conditions are equivalent:
 a) $\mathfrak{p}/\mathfrak{p}^2$ is torsion free
 b) $\mathfrak{p}$ is locally complete intersection.
2.) Let $\mathfrak{p}$ be a Gorenstein ideal in a CM-ring, then $\mathfrak{p}/\mathfrak{p}^2$ is CM-module for height $\mathfrak{p} \leq 3$.
3.) Let $R = k[\![X_1,\ldots,X_n]\!]/\mathfrak{p}$ 1-dimensional integral domain, k field, and let T = torsion of the module of differentials $\Omega_{R/k}$. Then $T \neq 0$ for edim R = 3, and for edim R = 4 and R Gorenstein.

LANGMANN, K.: Japanische und ausgezeichnete Unterringe des Tateschen Ringes

Ein Ring R heißt japanisch, wenn für alle Primideale $\mathfrak{p} \in R$ das Erweiterungsideal $\mathfrak{p}\hat{R} \subseteq \hat{R}$ (=Komplettierung von R) reduziert ist. Dann ist R genau dann japanisch, wenn in allen Faktorringen $R/\mathfrak{p}$ der normale Ort offen ist und für alle Primideale $\mathfrak{p} \in R$ mit eindimensionalem $R/\mathfrak{p}$ auch $\widehat{R/\mathfrak{p}}$ reduziert bleibt. Ist weiter A ein noetherscher regulärer ausgezeichneter Jacobsonring, und ist R ein Unterring von A, so daß A treuflach über R ist und die maximalen Ideale von R durch Elemente von A erzeugt werden, so ist R genau dann japanisch, wenn in allen Faktorringen $R/\mathfrak{p}$ der normale Ort offen ist. R ist genau dann ausgezeichnet, wenn in allen Faktorringen $R/\mathfrak{p}$ der reguläre Ort offen ist. Diese Sätze können für Unterringe des Tateschen Rings noch verschärft werden.

LINDEL, H.: Projective Modules over Polynomial Extensions over Regular Noetherian Rings

Let B be a regular noetherian ring with dim $B \leq 2$, $C = B[X_1,..,X_n]$, $\mathfrak{P} \in \operatorname{Spec} C$, $A = C_{\mathfrak{P}}$. It was proved that finitely generated projective $A[T_1,..,T_s]$-modules are free. Further if $B = D[[Z_1,..,Z_t]]$, D a complete regular local ring with dim $D \leq 2$, C, $\mathfrak{P}$, as above, then finitely generated projective $A[T_1,...,T_n]$-modules are free. The local statements implies Quillen's conjecture in the case that A is an unramified complete regular local ring.

The main tool to prove these results is the following lemma: Let R',R noetherian rings and $h \in R'$ such that $R = R' + Rh$, $Rh \cap R' = R'h$, and let Q, P be finitely generated projective modules such that $h^e P \leq Q$ for all $e \in \mathbb{N}$, Q extended from a projective R'-module. Then P is extended from a finitely generated projective R'-module P'.
If $R' = R''[X]$ and h is monic in X, one has $P \cong Q$.

Masumi Si (Bochum)