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Mathematische Stochastik

12.3. bis 18.3.1978

Die diesjährige Tagung 'Mathematische Stochastik' stand unter der Leitung von H. Heyer (Tübingen) und L. Le Cam (Berkeley).

Die Arbeitsgebiete der 46 Teilnehmer der Tagung, die aus 9 verschiedenen Ländern kamen, bildeten ein breites Spektrum der mathematischen Stochastik. Dies hatte ein dichtgedrängtes Vortragsprogramm mit Schwerpunkten in den maßtheoretischen Grundlagen sowie in der Theorie der Markoff-Prozesse mit ihren physikalischen Anwendungen und in der mathematischen Statistik zur Folge. Das Programm wurde täglich durch einen Übersichtsvortrag eingeleitet, dem in der Thematik ähnliche, speziellere Vorträge folgten. Derartige Übersichtsvorträge wurden über Zufallsmaße, Markoff-Prozesse und Potentialtheorie, Statistik und Mechanik, invariante Entscheidungstheorie und empirische Prozesse gehalten. Es ergaben sich vielfältige Anregungen zu Gedankenaustausch und Diskussionen, was zum Gelingen der Tagung entscheidend beitrug.

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Vortragsauszüge

O . KALLENBERG: The role of random measures as a universal tool in probability .

Point process theory originated with the analysis of some practical situations (queuing,reliability,particle systems, biological populations),where the state of the system is naturally described by a random collection of points in time or space . Mathematically,point process theory soon turned out to be indistinguishable from general random measure theory , the latter having developed ever since into a naturally delimited theory with its own ideas,methods,and results ,

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During the last decade, the usefulness of random measures as a general mathematical tool has been realized by a number of workers in entirely different branches of probability. Indeed, a description and analysis in terms of random measures seems appropriate and fruitful in most cases concerned with non-negative random quantities (number of objects, amount of time, length of a curve, size of a body, etc). In the present talk, some specific examples are given from a variety of fields.

B. FEREBEE: Coverings by repeated random walks

Let $R^1, R^2, \ldots$ be independent copies of the range of a random walk on the integers with positive drift.

Let $\mu_n = \min\{m : \bigcup_{i=1}^{n} R^i \supset \{1,\ldots,n\}\}$. If $P(\{x \in R\}) > 0$ for each $x \geq 0$, then $P(\{\mu_n < (1+\varepsilon)\log n / |\log(1-\alpha)|\}) \longrightarrow 1$ when $n \longrightarrow \infty$ for each $\varepsilon > 0$. If $P(\{-x \in R\}) = o(1/\log x)$ for $x \longrightarrow \infty$, then $P(\{\mu_n > (1-\varepsilon) \log n / |\log(1-\alpha)|\}) \longrightarrow 1$ when $n \longrightarrow \infty$, for each $\varepsilon > 0$.

L. BARINGHAUS: A simultaneous characterization of the Poisson and the Bernouilli distribution

Let $N, X_1, X_2, \ldots$ be non constant independent random variables with $X_1, X_2, \ldots$ being identically distributed and N being non-negative and integer valued. It is shown that the independance of $\Sigma_{i=1}^{N} X_i$ and $N - \Sigma_{i=1}^{N} X_i$ implies that N has a Poisson distribution and X_1 has a Bernouilli distribution.

B. ANGER: Darstellung von Kapazitäten

Stark subadditive Kapazitäten werden gekennzeichnet als obere Einhüllende kompakter filtrierender Mengen von Maßen. Dies verallgemeinert eine entsprechende Eigenschaft der klassischen

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Newtonschen Kapazität sowie Ergebnisse von Dellacherie und Strassen. Es klärt überdies die im Zusammenhang mit dem Neyman-Pearson-Lemma von Huber und Strassen auftretende Frage nach denjenigen Mengen von Maßen, deren obere Einhüllende stark subadditiv sind.

Beliebige normierte Kapazitäten α haben eine Darstellung der Gestalt $\alpha(K) = \inf_{M \in S(x)} \sup_{\mu \in M} \mu(K)$ (K kompakt). Dabei ist $S(x)$ ein System "minimaler" kompakter filtrierender Mengen von Maßen, welches im Falle einer stark subadditiven Kapazität aus der kleinsten kompakten filtrierenden Menge M mit $x = \sup M$ besteht. In dieser Darstellung kann man M durch die Menge der W-Maße in M ersetzen.

H. VON WEIZSÄCKER: Are liftings useful for stochastics

Liftings are basically nonconstructive (there are models of set theory without the axiom of choice, in which no liftings exist) and they seem to become useful only in "non separable" situations.

Among others, the following results have been pointed out:

Theorem 1: If X is a Hausdorff space, ν a Radon measure on X, $(\Omega, \mathfrak{A}, \mu)$ a measure space with lifting and R a continuous linear operator from $L^1(\nu)$ into $L^1(\mu)$, then there is a family $(\nu_\omega)_{\omega \in \Omega}$ of Radon measures such that $(R\varphi)(\omega) = \int \varphi \, d\nu_\omega$ holds μ-a.e. for each $\varphi \in L^1(\nu)$.

Theorem 2: Let X and Y be Hausdorff spaces. Let furthermore Q be a tight probability measure on X, μ a probabilty measure on $(\Omega, \mathfrak{A})$, $j: X \longrightarrow Y$ continuous and $g: \Omega \longrightarrow X$ $\mathfrak{A}$-measurable such that $jQ = g\mu$ is satisfied. Then there is a $\mathfrak{A}$-measurable $f: \Omega \longrightarrow X$ such that $\mu(h \circ g = h \circ j \circ f) = 1$ holds for all Borel measurable real functions h on Y.

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<u>Theorem 3</u>: Let X be a Hausdorff space, μ, ν Radon measures and $S \in L^1(\mu+\nu)$ a cone containing $s \vee s'$ for each pair s, s' of elements of S. If $\mu(s) \leq \nu(s)$ holds for every $s \in S$ then there is a family $(\nu_x)_{x \in X}$ and a Radon measure φ with $\varphi|_{S \wedge -S} = 0$ such that $\nu = \int \nu_x \, d\mu + \varphi$ and $s(x) \leq \int s \, d\nu_x$ μ-a.e. for every $s \in S$.

This generalizes results of Strassen and Rost.

G. WINKLER: <u>On the integral representation in convex sets of tight measures</u>

Let (X,t) be a topological space, $\mathcal{B}(X,t)$ the Borel σ-field on X and $\mathcal{M}^+(X,t)$ the family of positive tight measures on $\mathcal{B}(X,t)$. Then the following theorem holds:

<u>Theorem</u>: Let H be a convex weakly closed subset of $\mathcal{M}^+(X,t)$ satisfying $\sup\{\nu(X) : \nu \in H\} < \infty$. Denote by $\mathcal{M}$ the σ-field of subsets of the set $\operatorname{ex} H$ of extreme points of H generated by the family of functionals $\{\nu \longrightarrow \nu(f) : f \text{ measurable, bounded}\}$. Then for every $\mu \in H$ there is a probability measure p on $\mathcal{M}$ such that $\mu(f) = \int_{\operatorname{ex} H} \nu(f) \, dp$ holds for every Borel measurable bounded real function f on X.

<u>Corollary</u>: Let $f_1, f_2, \ldots$ be Borel measurable functions and $c_1, c_2, \ldots$ elements of $\mathbb{R}$. Then the conclusion of the theorem holds for the set $H := \{\nu \in \mathcal{M}^+(X,t) : -\infty < \int f_i \, d\nu \leq c_i\}$.

Applications to a theorem of de Finetti and to Skohorod representation have been given.

D. BIERLEIN: <u>Maßfortsetzung zu gegebener Funktion ohne meßbaren Nachbarn</u>

Es handelt sich um die Fortsetzung (Fs.) eines W-Maßes $\mu|\mathfrak{a}$ über M auf die Ziel-σ-Algebra $\mathfrak{a}_1 = \sigma(\mathfrak{a} \vee \mathcal{B}_f)$, wobei die reelle Funktion $f|M$ vorgegeben ist. $g|M$ heißt "Nachbar"

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von f ,falls $\mu^*(f = g) = 1$ gilt.

Besitzt f einen μ-meßbaren Nachbarn (m.N.),so existiert eine Fs. $\mu_1 | \mathfrak{a}_1$ von $\mu | \mathfrak{a}$, und zwar -neben anderem- zu jedem m.N. g eine Fs. μ_1 mit $\mu_1(f = g) = 1$.

Besitzt f keinen m.N. , so handelt es sich um den Fall der Existenz einer Menge $A_0 \in \mathfrak{a}$ mit $\mu(A_0) > 0$ derart,daß $\mu^*(\{f = g\} A_0) = 0$ für jedes μ-meßbare g gilt.Im wichtigsten Spezialfall hiervon ist A_0 ein μ-Atom "ohne" Kern.Auf einem μ-Atom ohne Kern lassen sich stets Funktionen mit und solche ohne zugehörige Maßfortsetzung angeben.

Für die Lösung des Maßfortstzungsproblems im gemischten Fall wurden notwendige Bedingungen angegeben .

J . LEMBCKE: Maximale Fortsetzungen von Maßen

Sei μ eine endliche Mengenfunktion auf einem Mengensystem $\mathfrak{S}$, die endlich additiv auf den von $\mathfrak{S}$ erzeugten Ring fortgesetzt werden kann,und $\mathfrak{R} \supset \mathfrak{S}$ ein Mengenring . Ferner sei $\mathfrak{K} \subset \mathfrak{R}$ $(\cup,\cap)$-stabil,so daß jede Menge aus $\mathfrak{K}$ in einer Menge aus $\mathfrak{S}$ enthalten ist . Dann ist jeder von innen $\mathfrak{K}$-reguläre Inhalt auf $\mathfrak{R}$, der μ fortsetzt,maximal in der Menge aller μ-fortsetzenden Inhalte bzgl. der durch $\mathfrak{K}$ definierten Präordnung.

Allgemein kann man zeigen,daß μ stets eine $\mathfrak{K}$-maximale Fortsetzung besitzt.Unter geeigneten Regularitätsvoraussetzungen über μ erhält man überdies,daß jede $\mathfrak{K}$-maximale Fortsetzung von μ $\mathfrak{K}$-regulär ist,und damit die Existenz $\mathfrak{K}$-regulärer Fortsetzungen von μ .

Als Anwendung ergibt sich insbesondere eine Verallgemeinerung eines Satzes von Prohorov und Kisynski über projektive Familien von Maßen.

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H. FÖLLMER: Markov processes and potential theory

The aim was to give a survey on some aspects of the interplay between Markov processes and potential theory. Using the probabilistic interpretation of the classical Dirichlet problem and of the Poisson formula as a guideline, an introduction to Dynkin's approach to probabilistic integral representation was given.

The second part concerned the following question: Have potential theoretic methods been of any use in the investigation of Markov random fields resp. of Markov processes on product spaces describing the evolution of a large system of interacting particles? In addition to some general results on the structure of the set of all fields with a given local specification, potential theoretic ideas have been very useful in the study of one-dimensional Markov fields: It reduces to a "coupling" of entrance and exit boundary of a given Markov process. (Spitzer, Yor-Dong, ...). Finally it was shown that some questions concerning Markov chains of the type $P(x,.) = \prod_{a \in A} P_a(x,.)$ on a product space $E = S^A$ (S, A countable, $P_a(x,.)$ probability on S for each $a \in A$ and each $x \in E$) have a natural interpretation in terms of space-time harmonic functions.

K. JANSSEN: Markov processes and harmonic spaces

It is well known that all decent axiomatic theories of harmonic spaces are associated with a Hunt process. This talk is concerned with the converse: Let X be a standard Markov process with a proper potential kernel such that there exists a base $\mathcal{W}$ of the topology satisfying

a) for each $x \in V \in \mathcal{W}$, $P_{V^c}(x,.) \neq 0$,

b) for $V \in \mathcal{W}$ $P_{V^c}f$ is continuous on V for bounded f,

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c) holding points are isolated .

($P_{V^c}(x,.)$ is the distribution of the process when firstly entering V^c after start in x).

As a result of joint work with V. Dembinski it can be concluded: Then there exists a harmonic structure on the state space, such that for sufficiently small open U, $P_{U^c}(x,.)$ is t the harmonic measure for x w.r. to U.

If X is a process with continuous paths the harmonic structure is a $\mathfrak{P}$-harmonic space in the sense of Constantinescu-Cornea .

C. GRILLENBERGER: Das Grenzverhalten von Warteschlangen mit nichtstationärer Poissonscher Ankunftsverteilung

Eine Warteschlange Q mit einem Schalter habe unabhängige, identisch verteilte Bedienungszeiten. (Erwartung $E(S) = 1$). Die Ankünfte seien Poisson-verteilt mit einer fastperiodischen Dichte $\lambda(t)$, so daß $\bar{\lambda} = \text{glm} \lim 1/t \int_x^{x+t} \lambda(s)\, ds < 1$. Sei $Z(t)$ die anstehende Arbeit z.Zt. t, W_n die Wartezeit des n-ten Kunden. Es gelten folgende qualitative Grenzaussagen:

1) Es gibt eine fastperiodische Schar (H_t) von Verteilungen auf $\mathbb{R}_+$, so daß $P(Z(t) \leq x) - H_t(x) \xrightarrow[t]{} 0$ für jede Anfangsbelastung des Systems Q.

2) $1/t \int_0^t 1_{\{Z(s) \leq x\}} ds \longrightarrow H(x) := \lim 1/t \int_0^t H_s(x)\, ds$

3) $P(W_n \leq x) \longrightarrow G(x) := 1/\bar{\lambda} \lim 1/t \int_0^t \lambda(s) H_s(x)\, ds$

4) $1/N \sum_{n=1}^N 1_{\{W_n \leq x\}} \xrightarrow[N]{} G(x)$

Der Beweis beruht auf dem pfadweisen Vergleich mit einer Warteschlange Q^* mit stationären Poissonankünften der Intensität $\lambda^* = \bar{\lambda} + \varepsilon < 1$. Hat nämlich Q^* eine Ruheperiode einer gewissen Länge $3T$, so hat Q wenigstens eine Ruhezeit T. Diese Beobachtung liefert einfache Beweise der obigen Aussagen.

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J . HOFFMANN-JÖRGENSEN: Cadlag modification of Markov processes

Let $(X_t, \underline{\underline{G}}_t)$ be a Markov process with values in a Polish space E and transition semigroup (P_t) satisfying $P_t f \xrightarrow[t \to 0]{} f$ for all $f \in \mathcal{C}(E)$ and $\{f : P_t f \in \mathcal{C}(E),\ t \geq 0\}$ separates measures in E. Then there exists a $(\underline{\underline{G}}_{t+})$ - progressively measurable modification (Y_t) taking values in the Alexandrov compactification $\hat{E} = E \cup \{\infty\}$ of E which is separable and closed. This modification is unique up to indistinguishability.

Theorem 1: The separable modification satisfies the strong Markov property in the following sense: If τ is a stopping time w.r.t. $(\underline{\underline{G}}_{t+})$ and ρ is a $\underline{\underline{G}}_{\tau+}$-measurable r.v. then

$P(\tau + \rho < \infty,\ \rho > 0,\ Y_{\tau+\rho} = \infty) = 0$ and

$E(1_C\, f(Y_{\tau+\rho}) \mid \underline{\underline{G}}_{\tau+}) = 1_C\, P(Y_\tau, f)$,

where $C = \{\tau + \rho < \infty,\ Y_\tau \in E\}$.

Let τ_A be the first exit time of A and define τ_∞ by $\tau_\infty = \sup_{K \text{ cpt.}} \tau_K$. A state $x \in E$ is called steady if $P^x(\tau_\infty = \infty) = 1$, and explosive if $P^x(\tau_\infty = \infty) < 1$.

Theorem 2: (X_t) has a cadlag modification in E if and only if $P(X_0 \in M) = 1$ where $M = \{x : x \text{ is steady}\}$.

Theorem 3: (X_t) has a cadlag modification in E if and only there exists a uniformly tight set T_0 such that $P(X_t \in T_0) = 1$ for all $t > 0$.

(A state $x \in E$ is called tight if: $\exists K$ cpt. : $P^x(\tau_K > 0) = 1$).

T.P . SPEED: Gaussian Markov fields on finite graphs

After some introductory remarks about Gaussian Markov fields over Z we state the basic facts characterizing Gaussian Markov fields over a finite graph in terms of the structure of the inverse covariance matrix of the field . A theorem describing a way of constructing such Markov fields was given, and we then specialized to the class of graphs having the

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the following property: every n-cycle for $n > 3$ possesses a chord. For these graphs a number of particularly special results hold, including one which asserts that an algorithm for constructing the appropriate matrices "converges" after one cycle. Further, a number of formulae can be proved for these graphs which generalize the familiar one-dimensional Markov chain results. Some comments about the estimation and testing of covariance matrices under hypotheses of these Markov types closed the talk.

U.G. OPPEL: Eine Startverteilungsinvarianzeigenschaft

Sei $(X,\mathfrak{L})$ ein Meßraum, $P := (P_n : n \in \mathbb{N})$ eine Folge von Übergangswahrscheinlichkeiten $P_n | X^n x \mathfrak{L}$ und $(\mathbb{P}_n^x | \mathfrak{L}^{\mathbb{N}}$ das zu P gehörige Maß mit der Startverteilung $x | \mathfrak{L}^n$ zur Zeit n. Sei $\mu | \mathfrak{L}$ ein Maß und $\mu_n := \mathbb{P}_1^{\mu} o pr_n^{-1}$. P heißt schwach μ-mischend, wenn für jedes $B \in \mathfrak{L}$ mit $\mu(B) = 0$ und für jedes $x \in B$ gilt: $\mathbb{P}_1^x(\bigcap_{n\in\mathbb{N}} pr_n^{-1}(B)) = 0$. Ist P Markoffsch und stationär und schwach μ-mischend, so folgt für shiftinvariantes $K \in \mathfrak{L}^{\mathbb{N}}$ aus $\mathbb{P}_1^{\mu}(K) = 0$ bereits $P_1^{\nu}(K) = 0$ für jedes $\nu | \mathfrak{L}$.

P heißt μ-mischend, wenn für jede Folge $(B_n : n \in \mathbb{N})$ mit $B_n \in \mathfrak{L}$ und $\mu_n(B_n) = 0$, für jedes $k \in \mathbb{N}$ und für jedes $(x_1,\ldots,x_k)$ aus $X^{k-1} x B_k$ gilt: $\mathbb{P}_k^{(x_1,\ldots,x_k)}(\bigcap_{m > k} pr_m^{-1}(B_m)) = 0$. Ist P Markoffsch und μ-mischend, so folgt für terminales $K \in \mathfrak{L}^{\mathbb{N}}$ aus $\mathbb{P}_1^{\mu}(K) = 0$ bereits $P_1^{\nu}(K) = 0$ für jedes $\nu | \mathfrak{L}$.

Für stationäres und Markoffsches P und P-invariantes μ ist P genau dann schwach μ-mischend, wenn P μ-mischend ist; folgt in diesem Fall für terminales K mit $\mathbb{P}_1^{\mu}(K) = 0$ auch $P_1^{\nu}(K) = 0$ für alle $\nu | \mathfrak{L}$, so ist P schwach μ-mischend.

Für einen stärkeren Begriff der μ-Mischung gilt auch für nicht Markoffsches P eine analoge Aussage für terminale K.

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H. ROST: Some remarks concerning the notion of microcanonical ensemble

Benfatto et al. recently obtained an existence theorem for Gibbs states in the case of a harmonic cristal (with super-stable formal Hamiltonian of the form $H(x) = \Sigma_{i,j\in \mathbb{Z}^d} h(j-i)x_i x_j$). (See Z.f.Wahrscheinlichkeits-theorie 41, (1978)). This theorem is modified and interpreted in such a form that any (shift invariant, with finite second moments) microcanonical state is a mixture of Gibbs states with different β's.

Theorem: The microcanonical finite volume states with parameters Λ, y (boundary condition), H energy converge to a Gaussian stationary state μ if $\Lambda \nearrow \mathbb{Z}^d$, $y^2 = o(|\Lambda|)$, $H = |\Lambda| h$.

μ turns out to be a Gibbs state for $\beta = 1/2h$; its spectral measure is equal to $h/(2\pi)^d \hat{H}(u)^{-1} du$, where $\hat{H}(u) = \Sigma_j h(j)e^{iju}$. This measure has the additional property that it has maximal prediction error $E_\mu(X_i - E(X_i | \text{past of } i))^2$ among all Gaussian measures with $\Sigma_j E(X_j X_o) h(j) = h$.

K. JACOBS: Stochastics and mechanics

Three typical models from three mathematical fields are discussed:

1) Bernoulli space from probability
2) The restricted n-body model from celestial mechanics
3) The Sinai billiard from statistical mechanics

They are first subsumed under the general notion of a dynamical system. More especially, it is shown that both the Sinai billiard and models in mechanics contain a Bernoulli shift. For the Sinai billiard this result is of a measure-theoretical nature, and due to Ornstein - Gallavotti. For mechanics, results of Sinole (the horseshoe), Kolmogorov, Arnold and Moser are

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reported .

D . SZASZ: Correlation inequalities in statistical mechanics

The conjecture that in the two-dimensional classical Ising model no non-translation-invariant states exist would follow from a certain correlation inequality . A simple transformation brings this inequality to a more symmetrical form at the cost of introducing non-ferromagnetic interactions . We generalize many classical correlation inequalities to this case.In this way we also get a simpler proof of a result of Messager and Miracle-Sole stating that the aforementioned correlation in-eqality does hold for a class of boundary conditions .

M . ROSENBLATT: Random solutions of Burgers equation

The solution of the Burgers equation $u_t + uu_x = \mu u_{xx}$ with initial condition $u(x,0) = f(x)$ a process stationary in x is considered . The asymptotic behavior of the second and third order spectra of the solution is determined as $t \to \infty$ to the first order for a class of stationary initial conditions . These estimates are related to a derived equation determinig the "dissipation of energy" and nonlinear transfer of spectral mass . Results of Hopf and Cole on the initial value problem for the Burgers equation are used . The investigation is motivated by questions in the theory of turbulence on energy decay and nonlinear transport of spectral mass . The Burgers equation is used as a model equation to test out some heuristics in the theory of homogenous turbulence .

V . DUPAC: Continuous time stochastic approximation

A survey of methods . Driml and Nedoma approach: Robbins Monro procedure described by the differential equation

$dX_t = -a(t)(M(X_t) + U_t)\,dt$, $t \geqslant 0$, $a(t) = 1$ for $0 \leqslant t \leqslant 1$ and

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$a(t) = 1/t$ for $t > 1$, $M : \mathbb{R} \to \mathbb{R}$ continuous, $M(x) \lesseqgtr 0$ for $x \gtreqless 0$, U_t continuous random function with $\lim_{t\to\infty} 1/t \int_0^t U_s\,ds = 0$. Then $X_t \longrightarrow \theta$.

Sorour's extension: noise component U_t satisfies $V_{1,t} \leq U_t(x) \leq V_{2,t}$, $1/t \int_0^t V_{i,s}\,ds \longrightarrow 0$, $i=1,2$; the same assertion. Sakrison's approach: noise component $\sum \sigma_r(x)\, dV_{r,t}$, $V_{r,t}$ bounded ergodic processes, conditions on covariances of functionals of $V_{r,t}$, mean-square convergence theorem.

Has'minskii-Nevel'son approach: noise component $\sum \sigma_r(t,x)\, dW_{r,t}$, $W_{r,t}$ independant Wiener processes, multidimensional case, the apparatus of Itô stochastic differential equations, a general convergence theorem

Dupač's application to dynamic stochastic approximation.

O. BARNDORFF-NIELSEN: <u>Hyperbolic distribution and some related distributions, theory and application</u>

A survey was given of the properties of the one- and multidimensional hyperbolic distributions, the generalized hyperbolic distribution, the inverse Gaussian distributions and a new type of distributions whose tail behaviour is like $\alpha \exp(-\beta |x|^{\gamma})$ for $|x| \to \infty$. Among the properties mentioned were infinite divisibility, self decomposability, mixture representations and regression characteristics. One of the generalized hyperbolic distributions yields an analogue of the Mises-Fisher distribution, for observations on a hyperboloid. Applications to sand particle sizes, sizes of personal income, and measurements of velocity characteristics of high Reynold's number turbulence were discussed.

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R.A. WIJSMAN: Transformation groups and probability distributions on orbit spaces

If a statistical problem with sample space $(X, \mathfrak{A}, \mathcal{P})$, $\mathcal{P} = \{P_\Theta : \Theta \in \Theta\}$, is invariant under a group G, then invariant procedures are functions on the orbit space $(X/G, \mathfrak{A}^I, \mathcal{P}^I)$, where $\mathfrak{A}^I$ are the invariant members of $\mathfrak{A}$ and $\mathcal{P}^I$ are the restrictions of the P_Θ to $\mathfrak{A}^I$. A convenient representation of the orbit space can sometimes be provided by a (global) cross-section. In the most favourable situation a cross-section Z can be taken as the orbit under a second group H and the distribution P^I_Θ induced by P_Θ on Z can be written down at once using Haar measures on G and H. If only a probability ratio $dP^I_{\Theta_2} / dP^I_{\Theta_1}$ is wanted, then one can get by with a local cross-section in each point, which exists under more general conditions. The resulting ratio of integrals over the group G has wide applications, especially in sequential analysis.

W. HUMMITZSCH: Existence of least favourable pairs and approximation of minimax tests

Two main theorems were given in the talk. The first one was a generalization of the Huber-Strassen theorem on the existence of least favourable pairs to an arbitrary topolocical space X and two sets of probability measures on the Baire σ-field, which are bounded from above by two strongly subadditive $\mathfrak{z}$-capacities, where $\mathfrak{z} = \{f^{-1}(0) : f$ continuous and bounded$\}$. The second theorem is as follows: Let X be a metric space with countable basis $\mathcal{G} = \{G_i : i \in \mathbb{N}\}$, $_o$ the Borel σ-field of X, $\mathfrak{B}_n := \sigma\{G_i : 1 \leq i \leq n\}$, ν_o, ν_1 strongly subadditive capacities and $\mathcal{P}_n := \{P|\mathfrak{B}_n : P \leq \nu_o|\mathfrak{B}_n\}$, $\mathfrak{Q}_n := \{Q|\mathfrak{B}_n : Q \quad \nu_1|\mathfrak{B}_n\}$ for every $n \in \mathbb{Z}_+$. Then $i_{(\mathcal{P}_n, \mathfrak{Q}_n)}(\alpha) \longrightarrow i_{(\mathcal{P}_o, \mathfrak{Q}_o)}(\alpha)$ for each

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$\alpha \in (0,1]$, where $i_{(\mathcal{P},\alpha)}(\alpha) := \sup_{\varphi \in \Phi_\alpha} \inf_{Q \in \mathcal{Q}} \int \varphi \, dQ$, and $\Phi_\alpha := \{\varphi : \varphi \text{ test function}, \sup_{P \in \mathcal{P}} \int \varphi \, dP \leq \alpha\}$.

I . VINCZE: On the Cramér-Fréchet-Rao inequality in the general case

Many papers pay attention to the basic and important inequality given independently by M.Fréchet (1943) , H.Cramér (1946) and C.R.Rao (1945) . In the textbooks mainly the case is considered,where the support of the underlying density functions in the sample space coincide . But there are also important investigations for the non-regular case . (Barankin (1949) , D.G. Chapman and H.Robbins (1951) , J.M. Hammersley (1950) , J.Kiefer (1952) , D.A.Fraser and I.Guttman (1951)) .

Although many of these papers refer to the possibility of further generalizations,they investigate the case of a real or perhaps vector parameter,using further restrictions . The aim of the talk was,to give a brief account on the main results for the non-regular case,pointing out that almost no assumption is needed concerning the structure of the parameter space .

W . SENDLER: On the asymptotic distribution of random permanents

Let $X_{i,j}$, $1 \leq i,j \leq n$ be i.i.d. random variables and consider the distribution of the permanent $X^{(n)} = (X_{i,j})_{i=1,\ldots,n}^{j=1,\ldots,n}$, i.e. $Y_n = \text{Per}\, X^{(n)} = \sum_{p \in \mathcal{P}(n)} X_{1,p(1)} \cdots X_{n,p(n)}$, where $\mathcal{P}(n)$ is the set of all permutations on $\{1,\ldots,n\}$. By a simple combinatorical device,the following theorem on the asymptotic behaviour of the moments of $\bar{Y}_n := Y_n (n!\, \mu_1^n)^{-1}$ is obtained .

Theorem: Suppose $P(X_{1,1} \geq 0) = 1$ and $\mu_j := E(X^j)$, j=1,2 exist . Then $E(\bar{Y}_n^k) \longrightarrow \exp\{\binom{k}{2}[\mu_2/\mu_1^2 - 1]\} = \lambda_k$, $n \to \infty$.

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The λ_k turn out to be moments of a log-normal law with parameters $-\beta$ and $\beta/2$. Since the log-normal law does not belong to a uniquely determined moment problem, we cannot prove but conjecture: $\mathcal{L}(\bar{Y}_n) \longrightarrow \log N(-\beta,\beta/2)$, $n \to \infty$ (where $\mathcal{L}(X)$ denotes the law of X) under the assumption of the theorem.

M. CSÖRGÖ: Distribution free tests of independence and normality based on the sample distribution function

Let $\mathfrak{F}$ be the class of continuous distribution functions on d-dimensional Euclidean space $\mathbb{R}^d$, and let $\mathfrak{F}_0$ be the subclass consisting of every member of $\mathfrak{F}$ which is a product of its associated one-dimensional marginal distribution functions. Let $X_1,\dots,X_n$ be independent random d-vectors with common unknown distribution function $F \in \mathfrak{F}$, and suppose that it is desired to test the null hypothesis $H_0 : F \in \mathfrak{F}_0$ against the alternitive $H_1 : F \in \mathfrak{F} - \mathfrak{F}_0$. Let F_n be the empirical distribution function of $X_1,\dots,X_n$, i.e.

$F_n(x) = F_n(x_1,\dots,x_d) = 1/n \; \Sigma_{j=1}^{n} \; \Pi_{i=1}^{d} \; \phi_{x_i}(X_{ji})$, where $X_j = (X_{j1},\dots,X_{jd})$ and $\phi_x(y) = 1$ if $y \leq x$ and $\phi_x(y) = 0$ if $y > x$. Let F_{ni} be the marginal empirical distribution function of the ith component of X_j, and let $T_n(x) = F_n(x) - \Sigma_{i=1}^{n} F_{ni}(x_i)$. Blum, Kiefer and Rosenblatt (Ann.Math.Statist. 32,(1961),485-498) studied the problem of H_0 versus H_1 via appropriate functionals of $T_n(x)$. We esthablish a strong invariance principle under $H_0 : F \in \mathfrak{F}_0$ for the empirical process $T_n(x)$. Tests of independance and a characterization based test of normality are then studied on the basis of the esthablished invariance.

P. GÄNSSLER: Selected topics on empirical processes

This was an expository lecture concerned with functional limit

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theorems (invariance principles) for empirical processes. First the classical Breiman-Brillinger-Pyke-Root construction of the uniform empirical process was reviewed. Then, based on the strong approximation theorem of Komlós-Major-Tusnády, further approximation results were derived.

Also results on empirical processes with random sample size have been presented. Finally we mentioned some results in the multidimensional case and gave an outline of Dudley's central limit theory for empirical measures (to appear in Annals of Probability).

Most of the material will be contained in a forthcoming paper (jointly with W. Stute) entitled "Empirical Processes: A survey on some results in the i.i.d. case" which will appear in the Annals of Probability.

D. POLLARD: Minimum distance tests

The problem concerns generalizations of goodness of fit tests based on the empirical distribution function. If F_n denotes the empirical distribution function obtained from sampling on some unknown member of a parametric family of distributions $\{F(.,\theta)\}_{\theta\in\Theta}$ then two possible methods of testing the fit arise: either use some estimate (e.g. Maximum Likelihood Estimate) $\hat{\theta}_n$ for θ to form the statistic $\sqrt{n}\,\sup_x |F_n(x) - F(x,\hat{\theta}_n)|$ or consider the distance of F_n from the family $\{F(.,\theta)\}$: $\sqrt{n}\,\inf_\theta \sup_x |F_n(x) - F(x,\theta)|$. A limit result for the second form of statistic was sketched. The asymptotic distribution of this statistic is obtained under a strong separation assumption plus a differentiability condition for the map $\theta \to F(.,\theta)$.

Only slight modifications are needed to generalize the result to cover the corresponding cases for the Cramér-von Mises statistics.

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G. KERSTING: The speed of Glivenko-Cantelli convergence in the Prohorov-distance

A problem of Huber is discussed which arises in the context of robust estimation. He asked to determine the class of distributions F on the real line for which $d(F,F_n) = O_p(n^{-1/2})$, where F_n is the empirical distribution based on n observations, and d is a metric on the set of all probability measures generating the weak* topology. In this context, the following theorem is presented: Let d be the Prohorov-distance and the density f of F is symmetric and convex for $|x|$ large. Further assume that the function $f(F^{-1}(t))$ is regularly variing for $t \to 0$. For each n α_n is determined by the equation $F(-\alpha_n^2) = n^{-1/2} \alpha_n$. Then, if F has an infinite support, $n^{1/2} \alpha_n^{-1} d(F,F_n) \to d > 0$ stochastically, where d is a constant. If F has a finite support, $\sqrt{n}\, d(F,F_n) \longrightarrow X$ in distribution to some non-degenerate r.v. X.

Consequently $d(F,F_n) = O_p(n^{-1/2})$ only holds in the case that F has a finite support.

P. REVESZ: Nonparametric estimation of the regression function

Let (X,Y), (X_1,Y_1), (X_2,Y_2), ... be a sequence of i.i.d. r.v. with $0 \leq X \leq 1$, $Y \in R^2$, $P(X < t) = F(t)$, $E(Y|X=x) = r(x)$. Assume that (i) $F(t)$ is absolutely continuous with $F'(t) = f(t) \geq \varepsilon > 0$, (ii) $P(Y - r(X) < t \,|X=x) = G(t)$ for all $0 \leq x \leq 1$, (iii) $E((Y - r(X))^2|X=x) = 1$ for all $0 \leq x \leq 1$, and (iv) $|r'(x)| \leq K < \infty$ holds.

Let $X_{1:n} < X_{2:n} < \dots < X_{n:n}$ be the ordered sample based on the sample $X_1, X_2, \dots, X_n$ and let $Y_{i:n}$ be the Y corresponding to $X_{i:n}$. Define the empirical regression function by $r_n(x) = k_n^{-1} \sum_{j=i-k_n/2}^{i+k_n/2} Y_{j:n}$ if $X_{i:n} \leq x < X_{i+1:n}$, where

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k_n is an increasing sequence of even numbers with $k_n^{3/2} / n \to 0$ and $k_n^{-1} \log n \to 0$.

We are interested in $M_n = k_n^{1/2} \sup_x |r_n(x) - r(x)|$. The main result states

$P\{M_n < (2 \log n\, k_n^{-1} + \log\log n\, k_n^{-1} - \log \; + y)^{1/2}\} \to \exp\, (-2\, e^{-y/2})$

for $n \to \infty$.

E. EBERLEIN: <u>An invariance principle for Banach space valued random variables and an application</u>

Let $(E, \|\cdot\|)$ be a separable Banach space. We consider triangular arrays of E-valued random variables and give a sufficient condition, in particular concerning the dependence structure, such that the invariance principle holds. Roughly spoken, we can say that for a φ-mixing array the central limit theorem implies the invariance principle.

As an application we prove a two-dimensional invariance principle for (φ_1, φ_2)-mixing lattices of real-valued random variables.

L. LE CAM: <u>Distances between experiments - applications</u> -

The approximation of experiments by simpler ones leads to the following extension of the usual method of scoring. Consider a structure $\mathcal{S}$ which consists of the following pieces: 1) a space $\mathbb{R}^k$ and an integer $r \leq k$; 2) a set Θ and a map from Θ to $\mathbb{R}^k$; 3) for each $a \in \xi(\Theta)$ a linear subspace $V(a) \subset \mathbb{R}^k$ of dimension r; 4) this $V(a)$ has a basis $u_j(a)$, $j=1,2,\dots,r$ and $u_o(a) = 0$; 5) an experiment $\mathcal{E} = \{P_\theta : \theta \in \Theta\}$; 6) a preliminary estimate $\hat{\xi}$ of ξ available on $\mathcal{E}$; 7) a number $\varepsilon > 0$.

For each $t \in \mathbb{R}^k$ let F_t be one of the measures P_θ with θ selected so that $|t - \xi(\theta)| \leq \inf\{|t - \xi(s)| : s \in \Theta\} + \varepsilon$. Let $\Lambda(t,s)$ be the log likelihood ratio $\log dF_t/dF_s$. Compute on

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$\hat{\xi} + V(\hat{\xi})$ the expressions $\Lambda[\hat{\xi} + u_i(\hat{\xi}) + u_j(\hat{\xi}), \hat{\xi}]$ for $i=0,1,2,\ldots,r$, $j=0,1,2,\ldots,r$. Fit to them, or $\hat{\xi} + V(\hat{\xi})$, a quadratic and let T be the point which maximizes the quadratic. This can be carried out without special assumptions.

To describe properties, consider not one $\mathcal{S}$ but a sequence $\mathcal{S}_n$ of such structures, with k and r fixed and with $\epsilon_n \to 0$. Otherwise all items aquire a subscript n. Then one describes assumptions which insure that the resulting T_n is asymptotically normal, asymptotically sufficient, distinguished. They also insure that the $\mathcal{E}_n$ can be approximated by Gaussian experiments with slowly varying covariance.

H. KELLERER: Markov property of point processes

Main result: To any point process $(N_t)_{t \geq 0}$ on $\mathbb{R}_+$ without multiple points there corresponds a unique point process $(N_t')_{t \geq 0}$ such that the waiting times T_n and T_n' belonging to $(N_t)_{t \geq 0}$ and $(N_t')_{t \geq 0}$ respectively are identically distributed for all $n \in \mathbb{N}$ and the version $(N_t')_{t \geq 0}$ has the Markov property, the finite-dimensional distributions of the new process can be explicitly computed from the one-dimensional distributions of the old process. The proof is based on the following result from measure theory: for p-measures μ_1, μ_2 on $\mathbb{R}_+$ with $\mu_1 \prec \mu_2$ (i.e. $\mu_1(A) > \mu_2(A)$ for all sets $A = [0,t[$ or $[0,t]$ with $\mu_1(A) > 0$ and $\mu_2(A) < 1$) there exists exactly one p-measure μ on $\mathbb{R}_+^2$ with the following properties: (1) μ has the marginals μ_1 and μ_2, (2) μ is the restriction of a product measure to the set $\{x_1 < x_2\}$.

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