MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 19/1978

Konvexe Körper

30.4. bis 6.5.1978

Die diesjährige Tagung über "Konvexe Körper" stand unter der Leitung von R.Schneider (Freiburg i.Br.) und G.C.Shephard (Norwich). Da den Geometrischen Ordnungen, die früher gemeinsam mit den Konvexen Körpern Tagungsgegenstand waren, diesmal eine gesonderte Tagung gewidmet war, konnten jetzt vermehrt Vorträge (insgesamt 40) aus dem weiteren Gebiet der geometrischen Konvexität stattfinden. Der Ausweitung dieses Gebietes konnte damit Rechnung getragen werden. Ein größerer Block von Vorträgen befaßte sich mit Themen aus der Theorie der konvexen Körper im engeren Sinne und mit diesbezüglichen Ungleichungen, eine fast gleich große Zahl war der kombinatorischen und metrischen Theorie der Polytope und Zellkomplexe gewidmet. Weitere Themen waren der Diskreten und Kombinatorischen Geometrie zuzurechnen, einige behandelten verallgemeinerte Konvexitätsbegriffe. Insgesamt ergab sich ein weites Spektrum geometrischer Fragestellungen.

Auch dieses Mal wurde während der Tagung wieder eine Sammlung offener Probleme zusammengestellt, die den Teilnehmern zur Verfügung gestellt wird. Über die Fragen aus den Problemlisten von 1974 und 1976, die inzwischen gelöst werden konnten, wurde zu Beginn der Tagung ein kurzer, vervielfältigter Bericht verteilt.

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Vortragsauszüge

A. ALTSHULER: 3-pseudomanifolds with preassigned links

A 3-pseudomanifold is a finite connected simplicial 3-complex K such that every triangle in K belongs to precisely two 3-simplices of K, the link of every edge in K is a circuit, and the link of every vertex in K is a closed 2-manifold. After exhibiting some particularly interesting 3-pseudomanifolds, we prove that for every finite set Σ of closed 2-manifolds, there exists a 3-pseudomanifold K such that the link of every vertex in K is homeomorphic to some $s \in \Sigma$, and every $s \in \Sigma$ is homeomorphic to the link of some vertex in K. We also discuss the possibility of embedding 3-pseudomanifolds in the boundary complexes of convex polytopes.

U. BETKE: Bewertungen auf Gitterpolytopen

Sei $\Lambda \subset E^d$ ein Gitter, $\mathfrak{P}^d$ der Raum der Polytope mit Ecken in Λ und $G(P)$ die Gitterpunktanzahl von P für $P \in \mathfrak{P}^d$. Es gilt dann bekanntlich $G(nP) = \sum_{i=0}^{d} n^i \cdot G_i(P)$, $n \in \mathbb{N}$. Sei B die Gruppe der unimodularen Transformationen: $\beta \in B \Leftrightarrow \beta: \beta(x) = (a_{ij})x + a_i$ mit $a_{ij}, a_i \in \mathbb{Z}$, $\det(a_{ij}) = \pm 1$.
Es wird dann folgender Satz gezeigt:
Genügt eine Bewertung $\varphi: \mathfrak{P}^d \to \mathbb{R}$ $\varphi(P) = \sum_{i=1}^{n} \varphi(P_i) - \sum_{i \neq j} \varphi(P_i \cap P_j) + \ldots + (-1)^{n-1} \varphi(P_1 \cap \ldots \cap P_n)$

für $P = \bigcup_{i=1}^{n} P_i \in \mathfrak{P}^d$, $\bigcap_{j=1}^{k} P_{i_j} \in \mathfrak{P}^d$, $1 \leq k \leq n$, und gilt

$\varphi(P^\beta) = \varphi(P)$, dann gilt $\varphi(P) = \sum_{i=0}^{d} \alpha_i G_i(P)$ mit $\alpha_i \in \mathbb{R}$.

R. BLIND: Convex polytopes with congruent regular facets

In E^3, Freudenthal and v.d. Waerden have found all convex polytopes with congruent regular facets: there exist five besides the regular ones. The analogous problem is studied in higher dimensions and it is proved, that if such a polytope is not regular, then in E^4 its facets are regular simplices and in $E^{\geq 5}$ it is the union of two regular simplices.

J. BOKOWSKI: Zerlegung konvexer Körper durch minimale Trennflächen

Aus einer gemeinsamen Arbeit mit Emanuel Sperner jr. werden Beweismethoden skizziert, die z.B. den folgenden Satz liefern:

Sei $K \subset \mathbb{R}^n$ offen, beschränkt, konvex, $\neq \emptyset$, $n \geq 2$, $B(x_o, r) := \{ x \mid \|x-x_o\| < r \} \subset K \subset B(x_o, R)$, $\lambda := r/R$ und $\varkappa \in (0,1)$. Dann erfüllt jede abgeschlossene Menge $A = \bar{A} \subset \bar{K}$ mit rektifizierbarem Rand, deren n-dimensionales Lebesgue-Maß durch $\mathcal{L}^n\langle A \rangle \leq \varkappa\, \mathcal{L}^n\langle K \rangle$ beschränkt ist, die isoperimetrische Ungleichung

$$\mathcal{H}^{n-1}\langle \partial A \cap K \rangle \geq c(\lambda, \varkappa) \cdot (\mathcal{L}^n\langle A \rangle)^{\frac{n-1}{n}}.$$

($\mathcal{H}^{n-1}$ = (n-1)-dimensionales Hausdorff-Maß)

$c(\lambda, \varkappa)$ wird angegeben. Das Gleichheitszeichen wird im Fall der Kugel ($\lambda = 1$) für bestimmte Mengen $A_\varkappa$ angenommen.

M. BREEN: The dimension of the kernel of a starshaped set

Let S be a subset of $\mathbb{R}^d$. For $T \subseteq S$, the set $\{ s : [t,s] \subseteq S$ for every t in T $\}$ is called the kernel of T relative to S, denoted $\ker_S T$. If $\ker_S S \equiv \ker S$ is not empty, then S is said to be starshaped. An interesting problem is that of determining necessary and sufficient conditions for S to be a starshaped set whose kernel is k-dimensional, $0 \leq k \leq d$. We will examine two types of theorems related to this problem:

Theorem 1. For S a subset of $\mathbb{R}^d$, $d \geq 2$, assume that for every (d+1)-member subset T of S, there corresponds a (d-2)-dimensional convex set $K_T \subseteq S$ such that every point of T sees K_T via S and (aff K_T) $\cap$ S = K_T . Further, assume that when T is affinely independent, then K_T is the kernel of T relative to S. Then S is starshaped and ker S is (d-2)-dimensional.

Theorem 2. Let S be a compact subset of $\mathbb{R}^2$. For $1 \leq k \leq 2$, the dimension of ker S is at least k if and only if for some $\epsilon > 0$, every f(k) points of S see via S a common k-dimensional neighborhood having radius ϵ, where f(1) = 4 and f(2) = 3. The number f(k) is best possible.

U. BREHM: Polyeder gegebenen Geschlechts mit minimaler Eckenzahl

1) Aus der Eulerschen Polyederformel ergeben sich theoretische minimale Eckenzahlen für Polyeder gegebenen Geschlechts. Die Frage ist nun, für welche dieser Zahlen ein Polyeder (linear, ohne Selbstdurchdringung) im $\mathbb{R}^3$ realisierbar ist.

Für 7 Ecken und Geschlecht 1 existiert bekanntermaßen ein solches Polyeder. Für 10 Ecken und Geschlecht 3 habe ich ein solches Polyeder (und ein symmetrisches 3-dim. Modell davon) konstruiert. Durch leichte Abänderung ergibt sich ein weiteres solches Polyeder von anderem kombinatorischen Typ.

2. Es werden notwendige Bedingungen angegeben, die ein 3-dimensionaler berandeter simplizialer Komplex erfüllen muß, um eine Zerlegung eines Polyeders in Tetraeder sein zu können. Es wird ein solcher Komplex für das Polyeder vom Geschlecht 6 mit 12 Ecken angegeben, der als erster Schritt für eine Realisierung des Polyeders dienen kann.

3. Es werden Polyeder vom Geschlecht 1 vorgestellt, deren Gaußsche Krümmung in allen Ecken gleich Null ist ("flacher Torus").

A. BRØNDSTED: Intersections of translates of convex sets

A finite family $(C_i)_{i \in I}$ of at least two convex subsets of $\mathbb{R}^n$ is said to have the intersection property provided that $\bigcap_{i \in I}(a_i + C_i)$ is non-empty for all families $(a_i)_{i \in I}$ of "translation vectors" $a_i \in \mathbb{R}^n$. Families $(C_i)_{i \in I}$ with the intersection property can be characterized in terms of the barrier cones of the sets C_i. For details, see Archiv der Mathematik 30(1978), 99-103.

A. DESSARD: Simplicial convexity

In a convexity space as defined by Bryant and Webster, the j-simplicial convex hull $S_j(A)$ of a set A, where j is an integer at least equal to 1, is the union of all the simplexes of dimension less or equal to j-1 and with apexes in A. A is j-simplicially convex if there exists M such that $A = S_j(M)$ and simplicially convex when there is an integer $j \geq 1$ such that A is j-simplicially convex. Let A be a j-simplicially convex set: $\mathcal{M} = \{ M : \exists\ k \text{ such that } S_j k(M) = A \}$ is not empty and the integer (if finite) $\sup_{\mathcal{M}} \min \{ k : A = S_j k(M) \}$ is the j-order of A. If the linear dimension of A is n (finite), and if $j > 1$, the convex hull [A] of A is equal to $S_j^k(A) = S_j k(A)$ when $k \geq \log_j(n+1)$ and $w_j(A) \leq 1 + E(\log_j n)$. If A is convex and $j,k > 1$,

$$\log_k(j^{w_j(A)-1}+1) \leq w_k(A) \leq E(\log_k(j^{w_j(A)}-1)) + 1 .$$

The j-core $J_j(A)$ is the set $\{ x : \exists \{x_1,\dots,x_{j+1}\}$ independent such that $x \in x_1 \dots x_{j+1} \subset [x_1,\dots,x_{j+1}] \subset A \}$. A point x belongs to $J_j(A)$ if there exists a linear set V with $d(V) = j$ and $V \subset \langle A \rangle$ such that $x \in {}^{i(V)}A$, the core of A relative to V; if A is convex, the converse is also true. $[A] \setminus J_j(A) \subset S_j(A)$, and if $d(A) = n$, A is a simplex if and only if there exists a set B such that $A = [B]$ and $S_j(B) = A \setminus J_j(A)$ for every $j \leq n$.

J.-P. DOIGNON: Independences of points

The results are taken from a joint paper with G. Valette (Mathematika 24):

- There exist strongly independent sets which are maximal for the inclusion; in fact, such maximal sets are characterized as not being fully independent.
- The full independence of a finite set S may be geometrically described by the following condition: for every pairwise disjoint subsets $S_1, S_2, \ldots, S_{n+1}, \{y\}$ of S:

$$y \notin \text{proj}\,[(\bigcap_{i=1}^{n} \text{proj}\, S_i) \cup S_{n+1}]$$

(except when this subspace is the space).

So there does not exist maximal fully independent sets. This fact makes full independence more appropriate than strong independence for proving the extension of Tverberg's theorem to affine spaces over any ordered skew-field.

J. ECKHOFF: On a problem of Hadwiger and Mani

Sei C^d die Klasse der endlichen Vereinigungen von abgeschlossenen konvexen Mengen im $\mathbb{R}^d$, und sei C_n^d die Teilklasse der Vereinigungen von höchstens n solcher Mengen. Hadwiger und Mani haben 1974 die Extremalwerte der Euler-Charakteristik auf der Klasse $C_n = \bigcup_{d=1}^{\infty} C_n^d$ bestimmt und allgemeiner die Frage nach den entsprechenden Extremalwerten auf den Klassen C_n^d aufgeworfen. Wir definieren zwei Funktionen q(d,n) und p(d,n) und zeigen, daß die ganze Zahl χ genau dann die Euler-Charakteristik einer Menge $X \in C_n^d$ ist, wenn $q(d,n) \leq \chi \leq p(d,n)$ gilt. Darüber hinaus charakterisieren wir die Mengen $X \in C_n^d$ mit $\chi(X) = q(d,n)$ bzw. $\chi(X) = p(d,n)$. Die Lösung des oben genannten Problems ergibt sich als Spezialfall.

G. EWALD: Bemerkungen zum Steinitzproblem

Sei C ein durch sein Eckenschema gegebener Zellkomplex; |C| eine k-Sphäre. Das Steinitzproblem besteht in der Suche nach Bedingungen, unter denen C zum Randkomplex eines geeigneten Polytops isomorph, also "polytopal" ist. Bisher weiß man allgemein nur, daß nach einem Satz von Tarski ein Algorithmus existiert, mit dem von jedem vorgegebenen C entscheidbar ist, ob es polytopal ist oder nicht. Wir geben einen expliziten Algorithmus dieser Art an. Er bettet zunächst C in das k-Skelett eines Simplex T^n ein ($n+1$ = Eckenzahl von C) und stellt dann durch Projektion auf einen $(k+1)$-dimensionalen Unterraum in einer repräsentativen, nur von n abhängigen Zahl von Richtungen fest, ob C polytopal ist. Es ist zu hoffen, daß der Algorithmus durch Verbesserungen auch auf konkrete Probleme anwendbar wird; einstweilen ist die Zahl der Schritte noch sehr hoch.

W.J. FIREY: Inner contact measures

Let K be a convex body in Euclidean n-space which rolls freely inside another such figure K'. This means that, given any rotation R and any boundary point x of K', there is a translation t such that x is in $RK + t = gK$ and gK lies in K'. Paint sets σ, σ' on the respective boundaries of K,K' which are inverse spherical images of Borel sets ω, ω' of directions. What is the probability, after a random rolling of K in inner contact with the boundary of K', that the contact is paint-to-paint? The answer is given by a certain motion-invariant measure of the set of those motions g such that gK lies in K' and $g\sigma$ meets σ'. The measure has the representation, up to normalization,

$$\Sigma_{p=0}^{n-1} \binom{n-1}{p} (-1)^p \cdot S_{n-p-1}(K';\omega') S_p(K;\omega)$$

in terms of the area functions of W. Fenchel and B. Jessen. This aids in proving that, in the plane, K rolls freely inside K' precisely when $S_1(K';R\omega) \geq S_1(K;\omega)$ for all R,ω as an extension of the classical curvature condition.

R. FOURNEAU: Nonclosed simplices

A non empty convex subset of a real vector space is a Choquet simplex if

$$S \cap (\alpha S+x) = \begin{cases} \emptyset \\ \text{or} \\ \beta S+y \ (\beta \geqq 0,\ y \in E) \end{cases}$$

for any $\alpha \geqq 0$ and $x \in E$. Various authors gave a description of the closed simplices of R^d, Simons described the bounded simplices of R^d in 70, and recently we found the complete characterization of all the simplices of R^d. The aim of the talk will be to present this characterization.

P.R. GOODEY: A characterization of planar convex sets of constant width

If S_1, S_2 are two plane convex curves which are not coincident nor externally tangent then $\alpha(S_1,S_2)$ is defined to be the number of connected components of $S_1 \cap S_2$. The aim of the talk is to prove that if S is a convex curve such that for every circle C of diameter w, $\alpha(S,C)$ is even or infinite, then S has constant width w. This confirms a conjecture of B.B. Peterson.

P. GRUBER: Isometrien des Raumes der konvexen Körper

Es sei $\underline{C}$ der Raum der eigentlichen konvexen Körper im E^d. Dann sind die Isometrien von $\underline{C}$ bezüglich der Symmetrischen-Differenz-Metrik genau die Abbildungen von $\underline{C}$ in sich, die durch volumstreue Affinitäten des E^d erzeugt werden.

R.E. JAMISON: Zwei Klassen von konvexen Körpern, die universelle offene Abbildungsbedingungen erfüllen

Sei K eine kompakte konvexe Menge in einem Hausdorffschen topologischen Vektorraum. Man möchte zwei mögliche Eigenschaften von K untersuchen:

(P) für jede kompakte konvexe Menge A ist jede stetige affine Abbildung von A auf K eine offene Abbildung.
(S) für jede kompakte konvexe Menge A ist jede stetige affine Abbildung von K auf A offen.
Es ist nicht schwer zu zeigen, daß K die Eigenschaft (P) genau dann besitzt, wenn K ein Polytop ist. Es ist auch möglich - aber dieser Satz liegt tiefer - zu beweisen, daß kein unendlich-dimensionales K die Eigenschaft (S) besitzen kann. Die Klasse der konvexen Körper, die (S) erfüllen, enthält u.a. alle Polytope, alle streng-konvexen Körper, sowie endliche Durchschnitte und Produkte von solchen. Die Körper, die (S) erfüllen, können durch eine etwas technische Forderung an die Extremalpunkte charakterisiert werden.

B. JESSEN: The algebra of polytopes

The results of a joint paper with A. Thorup: "The algebra of polytopes in affine spaces", to appear in Mathematica Scandinavica, will be described.

M. KATCHALSKI: Helly's theorem with fractions

Let A denote a finite family of n convex sets in $\mathbb{R}^d$. The family A is E(c,r), for $0 < c \leq 1$ and an integer r, if $n \geq r$ and there are at least $c\cdot\binom{n}{r}$ subfamilies B of A satisfying $|B| = r$ and $\cap B \neq \emptyset$. The family A is $S(\beta)$, for $0 \leq \beta \leq 1$, if there is a subfamily B of A for which $\cap B \neq \emptyset$ and $|B| \geq \beta n$. Helly's theorem asserts that if A is E(1,d+1) then A is S(1). It is possible to prove a "fractional" related result:
Theorem: For a fixed $0 < c \leq 1$ there is a $\beta = \beta(c,d) > 0$ such that for each A: If A is E(c,d+1) then A is $S(\beta)$. Also $\beta \to 1$ as $c \to 1$.
For the case d = 1 (segments on the line) the best possible value of $\beta = \beta(c,2)$ is $\beta = 1 - \sqrt{1-c}$. It may be of interest to find the best possible value of $\beta = \beta(c,d)$ for $d \geq 1$.

There are related problems (and results) for boxes with sides parallel to the axis.
Results have been obtained with Abbott and Liu from Edmonton.

P. KENDEROV: Hausdorff approximation of plane convex compacta by polygons

Let CONV be the set of all convex compact subsets of the usual plane $\mathbb{R}^2$. For every positive integer n denote by Δ_n the subset of CONV consisting of all polygons with not more than n vertices. The polygon $P_o \in \Delta_n$ is said to be a best Hausdorff approximation in Δ_n for a given $C \in$ CONV if $d(C,P_o) = \inf_{P \in \Delta_n} d(C,P)$, where $d(A,B)$ stands for the Hausdorff distance between A and B (both in CONV). In general, an arbitrary compact convex subset of $\mathbb{R}^2$ may have more than one best approximation in some Δ_n. But it turns out that the set of all those C in CONV which have more than one best approximation in some Δ_n is a "small" one. It is of the first Baire category in (CONV, d(.,.)):
Theorem. The set of those C in CONV having just one best Hausdorff approximation in Δ_n (for every positive integer n) is dense and G_δ in the complete metric space (CONV, d(.,.)).

P. KLEINSCHMIDT: Shellings of polytopes and spheres

Bruggesser and Mani proved that the boundary-complex of a convex polytope P can be shelled, i. e. there is a nice way of assembling $\mathcal{B}(P)$ from its component parts.
We give a geometric proof for the existence of a special shelling procedure for polytopes.
This special shelling turns out to be important for the characterization of polytopal spheres, for there are combinatorial spheres which don't allow such a shelling.
We discuss another way of shelling which has consequences in algorithmic tests for the shellability of complexes.

M. KÖMHOFF: An isoperimetric inequality for simplicial 3-polytopes

Let $V(P)$, $A(P)$ and $L(P)$ denote the volume, surface area, and the total edge length, respectively, of a 3-polytope P. We begin with a survey of the known inequalities relating the pair $A(P)$ and $L(P)$ and the pair $V(P)$ and $L(P)$ and present the new inequality

$\frac{L(P)^3}{V(P)} \geq 6\sqrt[4]{2}$ for all simplicial 3-polytopes P with equality if and only if P is a regular tetrahedron. Those isoperimetric problems generalize in a meaningful way to higher dimensions, but so far only very weak results had been obtained and most of the questions are still open.

D.G. LARMAN: A triangle free graph which cannot be $\sqrt{3}$ imbedded in any unit Euclidean sphere

Let G be a finite graph without loops. Say that G is λ realisable in S^d (the unit sphere of E^{d+1}) if its vertices can be imbedded in S^d in such a way that two vertices are joined by an edge in G if and only if they are at a distance greater than λ in the imbedding. Here we give a counter example to the following conjecture of Erdös - Rosenfeld:

Every triangle free graph can be $\sqrt{3}$ imbedded in S^d for some d.

In fact we show that for every $\lambda > 2\sqrt{2}/\sqrt{3}$ there is a triangle free graph which cannot be λ imbedded in S^d for any d. It remains open whether $2\sqrt{2}/\sqrt{3}$ is the best possible. I believe that $\sqrt{2}$ is the best possible.

E. LUTWAK: The Brunn-Minkowski inequality as a complementary Minkowski inequality

The following analytic version of the Brunn-Minkowski inequality is presented.

If $f,g: S^{n-1} \times S^{n-1} \to (0,\infty)$, then

$$\left\{ \int_{S^{n-1}} \inf_{v\in S^{n-1}} [f(u,v)+g(u,v)]^n du \right\}^{\frac{1}{n}} \geq$$
$$\left\{ \int_{S^{n-1}} \inf_{v\in S^{n-1}} f(u,v)^n du \right\}^{\frac{1}{n}} + \left\{ \int_{S^{n-1}} \inf_{v\in S^{n-1}} g(u,v)^n du \right\}^{\frac{1}{n}}$$

where f and g satisfy certain given conditions.

P. McMULLEN: Sets homothetic to intersections of their translates

Let S be a compact set in some euclidean space, such that every homothetic copy λS of S, with $0 < \lambda < 1$, can be expressed as the intersection of some family of translates of S. It is shown that such sets S are precisely the compact star-shaped sets, such that every point in the complement of S is visible from some point of the kernel of S. Alternatively, such a set can be characterized as a compact star-shaped set whose maximal convex subsets are cap-bodies of its kernel.

P. MANI: A characterization of the ellipsoid

Let $K \subset E^n$ be a convex body, and assume that there are two points p_1, p_2 in E^n such that, for all hyperplanes $H \subset E^n$ through the origin, $(H+p_1)\cap K$ is directly homothetic to $(H+p_2)\cap K$. Then K is an ellipsoid.

CHR. MEIER: Stützfunktionen ebener Eibereiche als Kegel eines Hilbertraumes

Der positive Kegel $\mathfrak{H}^+$ der Stützfunktionen $h(A,\varphi)$ ebener Eibereiche A mit Minkowski-Addition und Dilatation, versehen mit der Blaschke-Hausdorff-Metrik, erzeugt einen Unterraum $\mathfrak{H}$ der stetigen Funktionen $C(S^1,\mathbb{R})$ mit der üblichen Norm. $\mathfrak{H}$ ist linear isomorph und homöomorph zu einem Unterraum der Fourierreihen

$$H_\alpha = \Big\{ f(\varphi) = \frac{a_o}{2} + \sum_1^\infty (a_k \cos k\varphi + b_k \sin k\varphi) \;\Big|\; \sum_0^\infty (1+k^2)^{1+\alpha}(a_k^2 + b_k^2) < \infty \Big\}$$ für $0 < \alpha < \frac{1}{2}$; d.h. $\mathfrak{H}^+$ ist Kegel im Hilbertraum H_α mit Skalarprodukt $\langle f_1, f_2\rangle =$

$\sum_{0}^{\infty} (1+k^2)^{1+\alpha} (a_{1k}a_{2k} + b_{1k}b_{2k})$. Dies folgt aus einer Abschätzung der Koeffizienten a_k, b_k der fourier-entwickelten Stützfunktion $h(A,\varphi) = \frac{a_0}{2} + \sum_{1}^{\infty} (a_k \cos k\varphi + b_k \sin k\varphi)$. Es gilt $|a_k|, |b_k| \leq \frac{c_1 \ln k + c_2}{1 + k^2} R$, wo c_1, c_2 von A unabhängige Konstanten und $R = \max_{\varphi} \{ |h(A,\varphi)| \}$ Radius einer Kreisscheibe ist, die A enthält.

U. PACHNER: <u>Bistellare Äquivalenzen</u>

Eine bistellare k-Operation ist eine Abänderung der Gestalt $\mathfrak{K}' = (\mathfrak{K} \setminus A \cdot \mathfrak{B}(B)) \cup \mathfrak{B}(A) \cdot B$. Hierbei ist $\mathfrak{K}$ ein Simplizialkomplex, A eine k-Zelle aus $\mathfrak{K}$ und B ein Simplex mit $B \notin \mathfrak{K}$, $\mathfrak{B}(B) = \mathrm{link}(A;\mathfrak{K})$. Es gilt:

(1) Zwei geschlossene p.l. Mannigfaltigkeiten M_1, M_2 sind genau dann p.l. homöomorph, wenn (beliebige) Triangulierungen von M_1, M_2 durch bistellare Operationen ineinander überführt werden können.

Für Polytope läßt sich diese Aussage verschärfen zu:

(2) Randkomplexe simplizialer Polytope gleicher Dimension n und gleicher Eckenzahl lassen sich stets durch bistellare k-Operationen ineinander überführen, so daß bei jedem Schritt die Eckenzahl erhalten bleibt, d.h. $0 < k < n-1$.

J.R. REAY: <u>Several results on Tverberg's Theorem</u>

A list of problems generated by the July 1977 Oberwolfach conference on Discrete Geometry includes the following: "Given 12 points in general position in space, can one always put them into 3 sets of 4 each so that the three tetrahedra have an interior point in common?" The answer is "Yes". The proof uses a theorem of Tverberg: Any set S of at least $[(d+1)(r-1)+1]$ points in R^d has an r-partition $S = S_1 \cup \ldots \cup S_r$ (into pairwise dis-

joint subsets) so that $\bigcap_{i=1}^{r} \operatorname{conv} S_i \neq \emptyset$.
Another question which generalizes Tverberg's theorem is: How large must $S \subset R^d$ be to assure that S has an r-partition $S = S_1 \cup \ldots \cup S_r$ so that each m members of the family $\{\operatorname{conv} S_i\}_{i=1}^{r}$ have a non-empty intersection? The answer is 3r-2 in the plane. Partial results and bounds are given in higher dimensions.

G.T. SALLEE: Squaring the Euler relation

By using Rota's incidence algebra over the lattice of faces of a convex d-polytope, new relations among the numbers of faces may be obtained. Two such are
$\sum_{i=-1}^{d}(-1)^i \sum_{F^i \subseteq P} \nu(\emptyset, F^i) = (-1)^d$ and
$\sum_{i=-1}^{d}(-1)^i \sum_{F^i \subseteq P} \nu(\emptyset, F^i)\nu(F^i, P) = 0$ where the inner sums are taken over all i-faces of P and $\nu(F^i, F^j)$ is the number of faces G such that $F^i \subseteq G \subseteq F^j$. These relationships are essentially unique for the class of all polytopes, but analogues of the Dehn-Sommerville relations hold for simplicial polytopes.

R. SCHNEIDER: The curvatures of a typical convex body

It was proved that every convex body in Euclidean space E^d, except those of a certain set of first category in the space of all convex bodies, has the following property: Its boundary contains a dense subset such that for each point in this set and each tangent direction, the lower curvature is 0 and the upper curvature is ∞.

CHR. SCHULZ: Dichte Klassen konvexer Polytope

Das folgende Problem wurde 1975 von G.Ewald gestellt: Für welche Dimensionen d gibt es eine Klasse $\mathfrak{P}$ konvexer d-Polytope, die (bezüglich der Hausdorff-Metrik) dicht in der Menge $\mathfrak{K}^d$ der konvexen Körper des E^d liegt, und nur von d abhängige Konstanten c_1, c_2, so daß für alle $P \in \mathfrak{P}$ gilt:

(i) Die Eckenzahl jeder Facette von P ist durch c_1 beschränkt.

(ii) Für jede Ecke x von P ist die Anzahl der Facetten, die x enthalten, durch c_2 beschränkt.

Die gemeinsam mit J.Bokowski gefundene positive Antwort im Fall d=4 wird im Vortrag dargestellt.

G.C. SHEPHARD: Eberhard's Theorem

This talk will describe joint work by Branko Grünbaum and the speaker.

The theorems of Euler and Eberhard for convex polyhedra (3-polytopes) are well-known. Here we shall describe several analogous theorems that hold for tilings in the plane. It turns out that the main difficulty in this endeavour is to restrict the sort of tilings under consideration in such a way that meaningful results are obtained. To this end we introduce the concepts of normal, balanced and strongly balanced tilings. The proofs are by explicit construction of tilings that satisfy the required conditions.

G. SIERKSMA: The exchange number of a convexity space

R.H.K. THOMAS: A characterization of the simplex

If K is a convex body, some translate of $-\frac{1}{d}K$ is always contained in K, where d is the dimension of the space. If $\frac{1}{d}$ is the largest factor for which this is true, then K is a simplex. Several proofs of this fact are known, but they are difficult. An elementary proof will be presented.

H. TVERBERG: On the existence, for any 5 mutually disjoint convex plane sets, of a line separating ≧2 sets from ≧1 set, and related questions

It is proved that, given 5 sets as described, there is a line such that one of the two closed halfplanes which it defines contains at least 2 of the sets, while the other contains at least 1 of the sets. It is deduced from this that for any $k \in N$, there is a $K \in N$ such that the preceding sentence is true when (5,2,1) is replaced by (K,k,1). Examples are given to show that for every K, however large, it is false when (5,2,1) is replaced by (K,2,2). In dimensions >2 one doesn't even have (K,2,1) for any K (now hyperplanes and halfspaces replace lines and halfplanes, of course). The proofs are elementary and use Caratheodory's, Radon's and Ramsey's theorems.

G. VALETTE: Hyperbolic tilings

It is known that there is no topologically uniform map of type (3.5.5.5) in the plane. But does it exist an Archimedean map with all vertices of type 3.5.5.5? Equivalently, does it exist an Archimedean tiling of type (3.5.5.5) in the hyperbolic plane? The answer is yes: more precisely, the following Archimedean tilings of type (3.5.5.5) exist in the hyperbolic plane:

1) a tiling whose symmetry group is generated by a hyperbolic translation T and a reflection with respect to the line fixed by T.

2) a tiling whose symmetry group is generated by two central symmetries.

3) a tiling whose symmetry group is generated by a glided reflection.

4) a tiling whose symmetry group is generated by a rotation through $\frac{2\pi}{5}$.

5) a tiling whose symmetry group is generated by a rotation through $\frac{2\pi}{3}$.

W. WEIL: Random contacts for convex bodies

The following two situations in E^n are discussed:
I. There is a fixed convex body K and a variable q-plane E touching K, $E \in \mathfrak{E}_q^n$ (space of q-planes in E^n).
II. There is a fixed convex body K and a body L moving randomly around such that gL touches K, $g \in \mathfrak{G}_n$ (motion group of E^n).
In both cases Firey and Schneider used surface area measures resp. curvature measures to describe the natural measure of contact positions with prescribed outer normal directions resp. prescribed set (sets) of contact points in the boundary of K (resp. K and L). Thus there are numbers assigned to certain sets of touching q-planes (resp. touching-motions). It is shown that in both cases these numbers come from a natural contact measure, which is introduced as a finite Borel measure on the set of q-planes touching K, resp. the set of motions g such that gL touches K, and which is characterized by certain properties. As applications, kinematic formulas are obtained.

J.M. WILLS: Gitterzahlen und innere Volumina.

Zu einem konvexen Körper $K \subset E^d$ sei $G(K) = \text{card}\ (K \cap Z^d)$ seine Gitterpunktanzahl. Ist $P \subset E^d$ ein Gitterpolytop, so läßt sich bekanntlich G entwickeln:
$G(nP) = \sum_{i=0}^{d} n^i G_i(P)$, $n \geqq 0$ ganz.
Die Gitterzahlen G_i verhalten sich zum Teil wie die normierten Quermaßintegrale bzw. inneren Volumina $V_i = \binom{d}{i} W_{d-i}/\omega_{d-i}$: Sie sind homogen, additiv, translationsinvariant (gegen Gittertranslationen) und dimensionsinvariant.
Im Vortrag wird gezeigt, inwieweit sich die G_i von den V_i unterscheiden. Die G_i sind "extrem" nicht monoton, nicht definit, es gilt nicht $G_i \leqq V_i$, die G_i gehorchen keiner isoperimetrischen Ungleichung, jedoch Unglei-

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chungen wie z.B. $\binom{d+1}{2}G_d \geq G_{d-1}$. Abschließend wird auf Hadwigers erstaunliches Gegenbeispiel zu $G \leq \sum_{i=0}^{d} V_i$ eingegangen, und es werden einige Ansätze für obere Schranken für G diskutiert.

J. ZAKS: Bounds of neighborly families of polytopes and an example of a non-Hamiltonian simple 3-polytope with faces having at most seven edges

A family of d-polytopes in E^d is called neighborly if every two of its members meet in a (d-1)-dimensional set. Let f(d,k) be the maximum number of d-polytopes having at most k facets in a neighborly family. Extending earlier results of Bagmihl ($8 \leq f(3,4) \leq 17$) and of Baston ($8 \leq f(3,4) \leq 9$), we prove that $f(d,k) \leq 2k!$ and $f(d,d+1) \leq \frac{2}{3}(d+1)!$. It is conjectured that $f(d,d+1) = 2^d$.

Solving a problem of B.Grünbaum, H.Walther and G.Ewald, we shall present a non-Hamiltonian simple 3-polytope which has faces having at most seven edges; in Ewald's terms, we will prove that G(7,3) is non-Hamiltonian. In fact, we will show that $\rho(G(7,3))$ is less than 1, where ρ is the shortness coefficient (defined first by B.Grünbaum and H.Walther) and G(7,3) is the family of all simple polytopal graphs having faces with at most seven edges. The Hamiltonicity of G(6,3) is still open.

T. ZAMFIRESCU: A bad example of a Grünbaum spread

Spreads were introduced by B.Grünbaum in 1966. They generalize a number of families of segments associated with a planar convex body. Under a certain additional continuity condition, we have among others $\overline{M}_2 \neq D$, where $M_n = \{x \in D : x \text{ lies on} \geq n \text{ curves in the spread } \mathfrak{S}\}$ and D is the domain where $\mathfrak{S}$ is defined. The question arises whether M_2 can be dense in D or not. We give here an example of a planar convex body K, the family of diameters of which is a spread having $M_{\aleph_0}$ dense in K. The existence of the employed body is assured by Baire category arguments and follows from results of V.Klee (independently P. Gruber), R.Schneider, and myself. We prove moreover that most points of K lie in $M_{\aleph_0}$, in the sense that $\complement M_{\aleph_0}$ is of 1. Baire category in K. We also show that almost no points of K lie in $M_{\aleph_0}$, in the sense that the Lebesgue measure of $M_{\aleph_0}$ is zero.

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