MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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FINITE GEOMETRIES

21.5 bis 27.5.78

This year's Finite Geometries conference was conducted under the direction of F. Buekenhout (Bruxelles), D. R. Hughes (London), and H. Luneburg (Kaiserslautern). The topics covered in the formal lectures embraced many different aspects of finite (and indeed infinite) geometries, and its relationship to other branches of mathematics. In particular, there were relationships with group theory, combinatorics, number theory, coding theory, valuation theory, graph theory, and lattices in both senses of the word.

It is appropriate to mention here the special session of "Diagrams", for which a brief report follows later. Diagrams, as espoused by F. Buekenhout, provide a framework within which to study various classes of geometries. They are inspired by J. Tits' theory of Buildings, and it turns out that most (perhaps all) of the sporadic simple groups can be viewed as automorphism groups of geometries belonging to a diagram which is very similar to a Dynkin diagram.

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Lynn Batten: On embedding closure spaces with exchange property and certain 'partition' conditions, in (generalized) projective space of the same dimension.

Let D be a closure space with exchange property and such that $\emptyset$,D and all elements of D are closed. By an m-flat, we shall mean an m-dimensional closed set.

Let $\mathfrak{F}$ be a family of flats of D and consider the following conditions on $\mathfrak{F}$:

1. Any two elements of $\mathfrak{F}$ are in a common (m+1)-flat, and are themselves m-flats.
2. Every point is on an element of $\mathfrak{F}$.

With respect to these two conditions and weakened versions of them, we discuss known results and possible results in connection with embedding D in a (generalized) projective space of the same dimension as D.

Henry Beker: A Construction Method For Point Divisible Designs

A point division of a 1-design is a partition of the points into classes $P_1,\ldots,P_d$ such that the number of blocks through two points depends only on their point classes, and is denoted by λ_{ij}, $(1 \leq i, j \leq d)$ and further that $\lambda_{ii} = \lambda$ for all i.

The purpose of this talk is to introduce a recursive construction method for point divisible 1-designs. Under certain conditions this method also yields many infinite families of 2-designs, some of which contain strong tactical decompositions. For instance, the following theorem is proved:

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If there exists a $2-\left(m, \frac{m-1}{2}, \frac{m-3}{4}\right)$ Hadamard design with m the order of an affine plane, then there exists a $2-(4m^2(t-\gamma), m(2mt-2m\gamma+t-2\gamma), (m+1)(mt-m\gamma+t-2\gamma))$ design whenever there exists a $2-(4(t-\gamma), t, \gamma)$ symmetric design of which there are infinitely many.

Thomas Beth: On t-resolutions.

Def: Let G be an abelian group, $k \in N$, Ω_k a set, $\Omega_k \cap G = \phi$, $|\Omega_k| = k$. Let $X = G \cup \Omega_k$. A partition $(B_g)_{g\in G}$ of $\binom{X}{k+1}$ into Steiner systems $S(k,k+1,|X|)$ is called a Schreiber-Wilson-k-resolution over G if

(i) $K \in B_o \cap \binom{G}{k+1}$ implies $\sum\limits_{x\in K} x = 0$.

(ii) $B_g = B_o + g$ for all $g \in G$.

Known examples are the standard 1-factorization of K_{2n} and the 2-resolutions given by Schreiber and Wilson.

Thm.: For any $k \in N$ there exists a SW_k-resolution over $Z[1/(k+1)!]$.

Thm.: For $k = 3$ there exists a unique SW_3-resolution over $Z[1/4!]$

Thm.: Any SW_3-resolution over $GF(p)$ is obtained by taking the unique SW_3-resolution over $Z[1/4!]$ mod p.

Cor.: There is no SW_3-resolution over any finite group.

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Albrecht Beutelspacher: Blocking sets in finite projective spaces.

A t-blocking set in PG(d,q) with $d \geq t+1$ is a set B of points such that no t-dimensional subspace is contained in B and every (d-t)-dimensional subspace contains a point of B. Examples of t-blocking sets are obtained in the following way: Let U be a (t-2)-dimensional subspace, E a plane with $U \cap E = \emptyset$, and E* a Baer subplane of E. Then the set C(U,E*) consists of all points incident with a line RS, where R is a point of U and S a point of E*. The sets C(U,E*) are in fact t-blocking sets. Moreover, it holds

Theorem. Let B be a t-blocking set in PG(d,q) with $d \geq t+1$. Then

$$|B| \geq q^t+\ldots+1 + q^{t-1}\sqrt{q}.$$

Equality holds if and only if B is of the form C(U,E*).

This Theorem can be used to obtain bounds for the cardinality of maximal partial t-spreads in finite projective spaces.

F. Buekenhout: Sporadic geometries.

We discuss the action of the Janko group J_3 on the cosets of $PSL_2(16)\cdot 2$. The maximal cliques for the orbital of valency 85 have size 6 and behave as lines: any pair of points is on at most one line. Some elements of order 3 fix a set of 36 points each of which is on two pointwise fixed lines and there are 12 such lines, giving rise to a (non-thick) generalized quadrangle. The points, lines and quadrangles determine a geometry belonging to the diagram 

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P.J. Cameron: Antiflag Transitive Collineation Groups.

An antiflag in $PG(n-1,q)$ is a non-incident point-hyperplane pair. In joint work with W.M. Kantor, all collineation groups transitive on antiflags are determined. This includes the solution of the Hall-Wagner problem (determination of 2-transitive collineation groups) and the completion of Perin's result on rank 3 subgroups of $\Gamma S_p(n,q)$. Similar methods can be used to determine subgroups of classical groups transitive on pairs of non-perpendicular (isotropic or singular) points. In the lecture, the exceptional examples (related to triality), were discussed, the main ideas of the proof outlined, and some corollaries mentioned.

Frank De Clerck: Partial geometries with the diagonal axiom.

Suppose $S = (P,B,I)$ is a finite partial geometry satisfying the diagonal axiom (D).

(D): If $x_1 I L I x_2$, $x_1 \neq x_2$, $y_1 \not{I} L \not{I} y_2$, $x_i \sim y_i$, for $i,j \in \{1,2\}$ then $y_1 \sim y_2$.

These partial geometries are closely related to rank three groups and are investigated by several authors. The dual of (D) is the well-known axiom of Pasch. Let P be the set of all points of $PG(n,q)$ which are not contained in a fixed $PG(n-2,q)(n\geq 3)$; let B be the set of all lines of $PG(n,q)$ which do not have a point in common with $PG(n-2,q)$; finally let I be the natural incidence relation. Then $S = (P,B,I)$ is a partial geometry which satisfies the axiom of Pasch. This well-known dual net is denoted by H_q^n.

Theorem. The partial geometry $S = (P,B,I)$ with parameters s,t,α with $\alpha \neq 1, t + 1, s + 1$ is isomorphic to an H_q^n if and only if

(i) S satisfies (D)

(ii) S is regular

(iii) $2s > \gamma^4 - \gamma^3 + \gamma^2 + \gamma - 2$, where $\gamma = \frac{s}{\alpha}$.

Corollary. Let S be a dual net of order $s + 1$ and deficiency $t - s + 1$ (> 0). If S satisfies (D) then S is isomorphic to a partial geometry H_q^n ($q = s$ and $t + 1 = s^{n-1}$ $(n \geq 3)$).

Ingrid Debroey: Semi Partial Geometries with the Diagonalaxiom

A semi partial geometry $S = (P,B,I)$ is said to fulfil the diagonalaxiom if and only if for any element $(x,y,z,u) \in P^4$ the following implication holds:

$x \neq y$, $x \sim y$, $z \not{I} L_{x,y}$, $u \not{I} L_{x,y}$, $z \sim x$, $z \sim y$, $u \sim x$ and $u \sim y$ $\Rightarrow$ $z \sim u$, with $L_{x,y}$ the line of S which is incident with x and y.

First of all we will give some introductory theorems on semi partial geometries with the diagonalaxiom. Then we will come to some interesting characterization theorems on semi partial geometries with the diagonalaxiom using some recent results of J.A. Thas and F. De Clerck on partial geometries with the axiom of Pasch which is the dual of the diagonalaxiom.

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M. Dehon: <u>Ranks of some incidence matrices</u>.

Let S be a 2-covering in which every block has exactly 3 points and let p be a prime number. The rank modulo p of S (denoted by $RK_p(S)$) is defined as the rank in GF(p) of an incidence matrix of S. If $|S| = v \geq 3$ we have the following result:

<u>Theorem</u>. If $p \neq 3$, then

$$v - \log_p[(p-1)(p^{\frac{v-1}{2}} + 1) + 1] \leq Rk_p(s) \leq v$$

If $p = 3$, then

$$v - \log_3 v - 1 \leq Rk_3(S) \leq v - 1$$

R.H.F. Denniston: <u>Some biplanes</u>.

A Survey is given of the problem of constructing biplanes, (symmetric designs for which $\lambda = 2$). In particular, all but two of the points of a fixed block may be identified with the marks of a Galois field, all linear mappings being supposed to act as automorphisms. In the case $k = 11$, three biplanes of which two are known, arise in this way. A process of "derivation", which leads from one of the known biplanes to the other, can be repeated, to give a fourth biplane for which $k = 11$.

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Leroy J. Dickey: Existence of Symmetric Designs.

brc (λ,N) is defined to be the number of pairs (λ,N) that satisfy the Bruck-Ryser Chowla condition for the existence of a symmetric $2-(v,k,\lambda)$ design with $k = n + \lambda$. Values of brc (λ,N) are known for $1 \leq \lambda \leq 299$ and $N = 10^4$.

In the case $\lambda = 1$, there are 1280 values of $n \leq 10^4$ that are prime powers (projective planes known) and 5708 values that are not.

Other tests have been applied, including some multiplier tests and an automorphism test discovered by Hughes, against the possibility that the design has a cyclice Singer group.

λ	brc $(\lambda,10^4)$
1	6988
2	2117
3	2620
4	2009
5	1581
6	1472
7	1125
8	576
9	1658
10	835

M. Dugas: A remark on generalized André-planes over local fields.

We will show, that the generalized André-planes over local fields constructed by R. Rink in "Eine Klasse unendlicher verallgemeinerter André-Ebenen", Geo. Ded. 6(1977), 55-80, Satz 6.3 (= Typ I - planes) and Satz 6.6 (= Typ II - planes) can be represented as projective limits of finite H(jelmslev)-planes of level n, as defined by B. Artmann: "Existenz und projektive Limiten von Hjelmslev-Ebenen n-ter Stufe", Atti del Convegno di Geometria Combinatoria e sue Applicazioni, Perugia (1971), 27-41.

Let Π be a plane of Typ I or Typ II and $\Pi = \lim\limits_{\leftarrow n} H_n$, with H_n an H-plane of level n. (H-planes of level 1 are usual projective planes). If Π is of Typ I, the full collineation group of H_1 is inherited from collineations of Π. If Π is of Typ II, this is not true. All planes of Typ II can be represented as limits of different projective systems of H-planes. (The H_1's are non-isomorphic).

Michael J. Ganley: Difference sets and Projective planes.

In a paper of Dembowski and Piper, in 1968, it was shown that if a finite projective plane Π of order n admits a quasiregular collineation group Γ, of order $> \frac{1}{2}(n^2 + n+1)$, then one of 8 possibilities must occur. In the first of these cases, the group Γ acts transitively and regularly on Π, and it is well-known that this situation is equivalent to Γ containing a perfect difference set.

We extend this idea in order to obtain some information about the remaining 7 cases. With just one exception, we show that each of these other cases corresponds to the existence of a particular type of difference set in the group Γ. Furthermore, if the group Γ is abelian, then, again with one exception, the plane Π must admit a polarity. Using this fact, together with the existence of the difference set, we can obtain many interesting results concerning abelian collineation groups of finite projective planes.

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J.-M. Goethals: Spherical designs (the group case).

Spherical t-designs were introduced in [1] as a setting for various combinatorial structures. In the present lecture we consider finite groups G acting by orthogonal transformations on the unit sphere Ω in real Euclidean space, having the property that every G-orbit in Ω is a t-design. We show that a group G has this property if and only if there are no G-invariant harmonic polynomials of degree $1,2,\dots,t$. Furthermore, the representations of such a group on the harmonic spaces Harm(i) necessarily are real irreducible for $i = 1,2,\dots,[\frac{1}{2}t]$. This latter condition is also sufficient for even t.

[1]. R. Delsarte, J.-M. Goethals, J.J. Seidel, "Spherical codes and designs," *Geometric Dedicata* *6* (1977), 363-388.

William Haemers: A generalization of the "Higman-Sims technique."

Let A be a complex hermitian matrix of size n, partitioned into block matrices:

$$A = \begin{vmatrix} A_{11} & \cdots & A_{1m} \\ \vdots & & \\ A_{ml} & \cdots & A_{mn} \end{vmatrix} .$$

such that all diagonal blocks are square. Let b_{ij} be the average row sum of A_{ij}. Let $\lambda_1 \geq \dots \geq \lambda_n$ and $\mu_1 \geq \dots \geq \mu_m$ be the eigenvalues of A and $B = (b_{ij})$ respectively, then

(i) $\lambda_i \geq \mu_i \geq \lambda_{n-m+i}$, $1 \leq i \leq m$.

(ii) If $\mu_i \in \{\lambda_i, \lambda_{n-m+i}\}$ for all i then all A_{ij}'s have constant row and column sum.

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This generalizes the result: $\lambda_1 \geq \beta_i \geq \lambda_n$, which is usually used under the name Higman-Sims technique. Result (i) gives rise to several inequalities concerning substructures of combinatorial structures, such as cocliques in graphs, subdesigns in designs, etc. Because of (ii) it is then clear what happens in the case of equality.

J.W.P. Hirschfeld: Quadrics.

Let Q_n be a non-singular quadric in PG(n,q). Then, in canonical form, Q_n is one of the following:

$$\text{for } n \text{ even}, \quad P_n = V(x_0^2 + x_1x_2 + \ldots + x_{n-1}x_n)$$

$$\text{for } n \text{ odd}, \quad H_n = V(x_0x_1 + \ldots + x_{n-1}x_n)$$

$$E_n = V(f(x_0,x_1) + x_2x_3 + \ldots + x_{n-1}x_n),$$

f irreducible. Thus any quadric can be written $\Pi_{n-r-1}Q_r$, where Π_d is a subspace of dimension d, and is the set of points on the joins of Π_{n-r-1} to the points of a Q_r in a Π_r skew to Π_{n-r-1}.

The character of $\Pi_{n-r-1}Q_r$ = 2(dimension of a generator) - (dimension of singular space) - (dimension of ambient space) + 2 = 0, 1, 2, as Q = E, P, H.

Properties of the generators of Q_n were described and the number N(m,t,v;n,w) of sections $\Pi_{m-t-1}Q_t$ (of character v) of Q_n (of character w) was also given.

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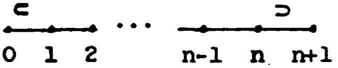
Daniel Hughes: Dually Extended Geometries.

A structure associated with the diagram



will be called a dually extended geometry (DEG). Jointly with Francis Buekenhout, we have shown: a finite DEG with $n > 2$ is one of (1) a trivial $(n+2)$ - $(n+3,n+2,1)$, (2) the complement of a $PG(n+1,2)$, (3) a "dually affined geometry" of dimension $n+2$ over $GF(2)$. In addition for $n = 2$ it is shown that a finite DEG is one of the three examples above, or is the unique geometry with 100 points and blocks, block size 22, associated with the Higman-Sims group (and the proof yields a very elementary demonstration of the existence of the group), a possible 2-(78,22,6) of a very special kind, or some possible examples associated with the existence of a projective plane of order 10. For $n = 1$ a DEG is of course a semi-biplane.

D. Jungnickel: On an assertion of Dembowski.

In his "Finite Geometries", Dembowski asserted that the class of uniform projective Hjelmslev planes whose image plane has order q and the class of symmetric divisible partial designs on two associate classes with parameters $v = b = q^2(q^2+q+1)$, $k = r = q(q+1)$, $s = q^2$, $t = q^2 + q + 1$, $\lambda_1 = q$ and $\lambda_2 = 1$ coincide. One part of this result is immediate from Kleinfeld's

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well-known counting lemma on finite H-planes. Regarding the converse, we exhibit a counter-example for $q = 2$ and prove the validity of Dembowski's assertion for all $q \geq 3$.

Reference: D. Jungnickel, On an assertion of Dembowski, J. Geom., to appear.

William M. Kantor: Some characters in finite geometry.

Most of the proof was given for the determination of all 2-transitive permutation representations of $PSL(n,q)$, $n \geq 4$. The corresponding result for primitive rank 3 representations of $PSL(n,q)$ was also stated. The rank 4 case remains open.

However, if $PSL(n,q)$ has a primitive rank r permutation representation not equivalent to its representation on subspaces of a given dimension, then $n \leq 4r$ and $q < r^2((4r)!)^2$.

J. H. van Lint: Ovals in projective designs.

Let $\mathcal{D}$ be a symmetric (v,k,λ) design with $\lambda > 1$. An arc S is a set of points, no three on a block of $\mathcal{D}$. If S has a tangent, then $|S| \leq (k+\lambda-1)/\lambda$, (I), and if not, then $|S| = (k+\lambda)/\lambda$, (II). If equality holds in one of these then we call S an *oval*. In the second case $k-\lambda$ must be even.

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Theorem 1: In Case II the exterior blocks and points not on S are the points and blocks of a 2-design.

Theorem 2: If $\mathcal{L}$ and its complement $\mathcal{L}'$ have arcs of size 3, then these are the ovals, and $\mathcal{L}$ or $\mathcal{L}'$ is a Hadamard design.

Theorem 3: In Case I, $k-\lambda$ even, the $|S|$ tangents have a set of λ points in common. Furthermore, $(k-1)|\lambda(\lambda-1)$.

Theorem 4: In Case II the rows of the incidence matrix of the design generate a code C for which $C^{\perp}$ has min. wt. $\geq (k+\lambda)/\lambda$. Equality holds only for ovals.

Theorem 5: In case II if $k \equiv 0(4)$, $\lambda \equiv 2(4)$ and the ovals form a 2-design, then $\mathcal{L}$ is 2-(7,4,2).

D. Livingstone: On Some Properties of the Monster.

A report was given on the progress of work undertaken with B. Fischer and with M. Thorne of Birmingham, and with Herr Gabrich of Bielefeld, towards determining the characters of Fischer's "Monster group" and of some of its subgroups. Ninety-one of the characters have now been determined, and most of the others have been distinguished. Some description of the subgroups used, and of computational methods was given.

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Heinz Lüneburg: Galoisfelder.

Es sei s eine primitive n-te Einheitswurzel und p sei eine Primzahl, die nicht in n aufgeht. Ferner sei Z der Ring der ganzen Zahlen. Ist r die Ordnung von p modulo n und ist P ein maximales Ideal von XXXXXX $Z[s]$, so ist $K = Z[s]/P$ ein endlicher Körper mit p^r Elementen und $s + P$ ist eine primitive n-te Einheitswurzel von K.

Diesen Satz kann man nach dem Vorgang von F. Levi (Comp. Math. 1, 303-304 (1933)) benutzen, um die Irreduzibilität der Kreisteilungspolynome zu beweisen. Indem man mit p und r beginnt und $n = p^r - 1$ setzt, erhält man für alle p und r einen endlichen Körper mit p^r Elementen, von dessen multiplikativer Gruppe man von vorneherein weiß, daß sie zyklisch ist. Schließlich folgt auch noch das Korrollar: Ist K ein Teilkörper von C, der alle Einheitswurzeln enthält, ist A der Ring alle in K enthaltenen ganzen algebraischen Zahlen und ist P ein maximales Ideal von A, welches die Primzahl p enthält, so ist A/P der algebraische Abschluß von $GF(p)$.

E. Mendelsohn: Perpendicular Arrays.

Perpendicular arrays are a natural generalization of orthogonal arrays. Their existence, as with orthogonal arrays, is linked to the existence of finite geometries. However, some additional conditions are necessary, as these conditions probably give a possible interpretation to "positive slope" applicable to a finite geometry.

Arnold Neumaier: Affine Planes and Tuple Systems.

The following Theorems are proved:

(1) If a projective plane of order n has an elation of even order d then n is divisible by $2d$, or $n = 2$.

(2) If G is a collineation group of a finite affine plane such that the stabilizer of every infinite point i contains only perspectivies with centre i then one of the following holds:

  (i) G has odd order and acts semiregular and faithfully on the infinite points,

  (ii) G consists of elations with centre i_o and finite axis,

  (iii) G contains a normal abelian subgroup of index 2 of type (i),

Case (iii) occurs only for planes of even order.

Result (1) implies well-known results of Hughes and Hall/ Paige.

The proofs use two new coordinatization methods by means of homogeneous tuple systems (2-HTS) and strong balanced tuple systems (2-SBTS).

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Udo Ott: Flag transitive planes of even order.

We outline the main steps of the proof of the following theorem:

Theorem: A flag transitive plane of even order is desarguesian or its automorphism group is sharply flag transitive.

R. Rink: Homomorphisms of special nearfield planes.

A special nearfield is a Dickson nearfield obtained from an unramified extension of local fields.

Theorem: Let N be a special nearfield which is not a field, and let $\mathfrak{A}$ be the plane coordinatized by N. If $\phi : \mathfrak{A} \to \mathfrak{A}^*$ is a homomorphism with the property that the two distinguished points on l_∞ have distinct images under ϕ, then $\mathfrak{A}^*$ is finite and uniquely determined by the structure of N.

M.A. Ronan: Generalized Hexagons.

The outline of a proof of a geometric characterization of Moufang generalized hexagons was given. Specifically, if p is a point, one defines $p^\perp = \{\text{points not opposite } p\}$, and if S is a set of points, $S^\perp = \cap \{s^\perp | s \in S\}$. Now for two points x and y which are neither opposite nor collinear one defines

$\langle x,y\rangle = \{x,y\}^{\perp\perp}$. It is easily shown that $\langle x,y\rangle$ contains at most one point on each line through z, where z is the unique point collinear with both x and y. If $\langle x,y\rangle$ contains exactly one point on each line through z, then $\langle x,y\rangle$ is said to be an <u>ideal line</u>.

<u>Theorem</u>: All such sets $\langle x,y\rangle$ are ideal lines if and only if the generalized hexagon is Moufang. In particular, in the finite cases all $\langle x,y\rangle$ are ideal lines if and only if the generalized hexagon is associated with one of the groups $G_2(q)$ or ${}^3D_4(q)$.

N.J.A. Sloane: <u>Codes over GF(4) and Complex Lattices</u>.

Self-dual codes over GF(4) (self-dual in the hermitian sense) are interesting for many reasons, for example, because they may be used to construct complex lattices, or $\mathbb{Z}[\omega]$-modules, in $\mathbb{C}^n$. <u>Extremal</u> self-dual codes have the greatest possible minimum distance, namely $2[n/6] + 2$. There are only finitely many extremal self-dual codes. The known ones were briefly described. One of the most important of these is the code e_6, which plays the same role over GF(4) as the Golay codes do over GF(2) and GF(3). This code leads to a complex lattice in $\mathbb{C}^6$ and hence to the real lattice K_{12} in $\mathbb{R}^{12}$ which is the densest known lattice packing in that dimension. The last part of the talk described the recent enumeration made by J. H. Conway, V. Pless, and the author of the self-dual codes over GF(4) of

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length 16. There are 55 inequivalent codes, of which 24 are decomposable and 31 indecomposable. Four are extremal.

Alan P. Sprague: d-nets.

We discussed incidence structures, all of whose planes are nets. A d-net is a connected semilinear incidence structure π such that (1) every plane of π is a net, (2) the intersection of two subspaces is connected, (3) two planes in a 3-space never have exactly one point in common, (4) d is the dimension of π. Theorem: Let $3 \leq d < \infty$. An incidence structure π is a d-net if and only if for some skew field F, some vector space V over F, and some subspace W of codimension d, π is isomorphic to $(S_d, S_{d-1}, \supseteq)$ where S_i is the set of i-dimensional subspaces of V whose intersection with W is the zero vector $(i = d, d-1)$. d-nets may also be characterized by the design $\circ\overset{B}{\text{———}}\circ\text{———}\circ\text{———}\;\cdots\;\text{———}\circ\text{———}\circ$ (here $\circ\overset{B}{\text{———}}\circ$ means net).

D.E. Taylor: Distance regular graphs.

Let Γ be a connected graph of diameter d and let $\Gamma_i(\alpha)$ denote the set of vertices at distance i from α. Suppose that for $\beta \in \Gamma_i(\alpha)$ the number $b_i = |\Gamma_{i+1}(\alpha) \cap \Gamma_1(\beta)|$ and $c_i = |\Gamma_{i-1}(\alpha) \cap \Gamma_1(\beta)|$ depend only on i; Γ is then called a distance regular graph. Set $k_i = |\Gamma_i(\alpha)|$. There are inequalities

$b_{i+1} \leq b_i$, $c_i \leq c_{i+1}$, $c_i \leq b_j$ for $i + j \leq d$ and $\kappa_i \leq \kappa_j$ for $i \leq j \leq d-i$. If $\kappa_1 = \kappa_2$, then Γ is either a circuit, a double cover of a complete graph (viz. a regular 2-graph) or else $d = 2$. We also have $\kappa_1 \leq b_1 + b_i + c_{i+1} - 1$ and if $\kappa_1 = 2b_1 + c_2 - 1$, then Γ is the icosahedron or a line graph.

Luc Teirlinck: Projective and affine factors in triple systems.

We construct a projective space, in the broad sense, P(w) on the set of all subspaces of order w of an $S(\lambda;2,3,v)S$. We define a linear space A(w) on the set of all non-isolated elements of P(w). Any connected component of A(w) is the set of all hyperplanes of a projective factor of order 2 or an affine factor of order 3 of S. We give conditions on w and v, which imply the connectivity of P(w) or A(w) respectively.

Henk van Tilborg: Quasi Cyclic Codes.

The connection between convolutional codes and quasi cyclic codes was shown. Examples were given of quasi cyclic codes with small constraint length ((22,11) Golay, (30,15) short. Q.R.). Further on we shall show that any Reed Solomon code is quasi cyclic.

Finally, Quasi Cyclic codes of a certain type will be shown to yield new codes with high minimum distance. e.g., (35,7,16)
(42,7,19)
(80,8,37)
(96,8,46)
(112,8,54)

Scott Vanstone: Asymptotic Properties of Locally Euclidean Designs.

An (r,1)-design D is a collection of points and lines together with an incidence relation such that every pair of distinct points determine a unique line and every point is incident with precisely r lines. D is called non-trivial if all the points do not lie on one line. D is said to be locally Euclidean at a line ℓ if D can be embedded in an (r,1) design D′ such that ℓ is embedded in a line ℓ' of D′ containing exactly r points. If D is locally Euclidean at every block, then D is locally Euclidean. It is shown that, under certain conditions, this local property of a design is global and implies that D is embeddable in a finite projective plane of order r - 1.

Richard Weiss: Symmetric Graphs.

For each finite Chevalley group G with an automorphism σ inducing an involution on its Dynkin diagram and for each pair of conjugate classes α and β of maximal parabolic subgroups exchanged by σ, let $\Gamma_{G,\alpha,\beta}$ be the undirected graph with vertex set $\alpha \cup \beta$ and edge set $\{(x,y) \mid x \in \alpha, y \in \beta, x \cap y$ parabolic$\}$. Let $\mathcal{G}$ be the family of all such graphs $\Gamma_{G,\alpha,\beta}$. For each $d \in N$, let $f(d)$ be the largest value of $|N_A(x)|$ over all $\Gamma_{G,\alpha,\beta}$ in $\mathcal{G}$ of valency d where $A = \mathrm{aut}(G)$ and $x \in \alpha$ is arbitrary (setting $f(d) = 1$ if there aren't any $\Gamma_{G,\alpha,\beta}, \in \mathcal{G}$ of valency d). We discuss the following:

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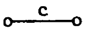
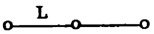
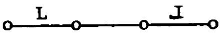
Conjecture: Let $\Gamma = (V,E)$ be an arbitrary finite undirected connected graph, $x \in V$ arbitrary. Let $A = \mathrm{aut}(\Gamma)$ and suppose that A^V is transitive and that A_x acts primitively on the set of neighbors of x. Then $|A_x| \leq \max\{d!(d-1)!, f(d)\}$ where d denotes the valency of Γ.

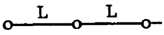
Special Session on Diagrams

An informal session lasting 1¼ hours, on Buekenhout diagrams, was held on Friday morning. The session was directed by D.R. Hughes, and the purpose was to exchange information and ideas. Amongst those taking part, W.M. Kantor expressed his belief that one should consider diagrams which involve 

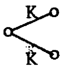
rather than 

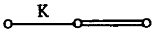
, and asked specifically what could be said about geometries belonging to the following diagrams: 

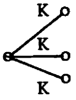
, 

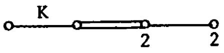
and 

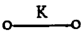
-polar space-o.

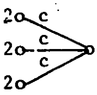
A. Neumaier pointed out that the diagram 

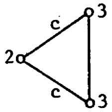
gives rise to 

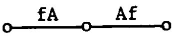
, and 

to 

, where 

is arbitrary, and also gave a specific example of 

and 

, both related to the group $PSU(3,3) \cong G_2(2)'$. D.G. Higman gave an infinite family of examples of 

, by taking points, hyperbolic lines, and planes of the generalized quadrangle associated with the group $C_2(k)$ ($Psp(4,k)$ in classical notation).

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