MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 25 /1978

Ergodentheorie

11.6. bis 17.6.1978

In diesem Jahr fand erstmals seit 10 Jahren wieder eine Tagung über Ergodentheorie in Oberwolfach statt. Sie stand unter der Leitung von M.Denker (Göttingen) und K.Jacobs (Erlangen).

Im Mittelpunkt der Tagung - in den Vorträgen und zahlreichen Diskussionen - standen das Isomorphieproblem für maßtreue Transformationen und Flüsse, das Klassifikationsproblem für topologische Markov-Ketten, die Ergodensätze und das Variationsproblem für die topologische Entropie. Die große Anzahl von 34 Vorträgen, die Bedeutsamkeit der präsentierten Resultate und die fruchtbaren Diskussionen trugen dabei der schnellen Entwicklung der Ergodentheorie in letzter Zeit Rechnung. Zwei zusätzliche Diskussionsabende und eine abendliche "problem session" rundeten die Tagung ab.

Die idealen Möglichkeiten, die das Forschungsinstitut nicht zuletzt auch zur Zerstreuung und Unterhaltung bietet, ließ die Tagung zu einem vollen Erfolg werden. Die Teilnehmer der Tagung danken dem Direktor des Institutes, Herrn Prof.Dr. M.Barner, und seinen Mitarbeitern für die organisatorische Unterstützung bei der Vorbereitung und Durchführung der Tagung.

Teilnehmer

J.Aaronson (Rennes)
R.Adler (Yorktown Heights)
M.A.Akcoglu (Toronto)
St.Alpern (London)
J.Auslander (Maryland)
A.Beck (Madison)
A.Bellow (Evanston)
J.R.Blum (Tucson)
S.G.Dani (Bombay)
M.Denker (Göttingen)
Y.Derriennic (Rennes)
J.Feldman (Berkeley)

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F.Schweiger (Salzburg)
C.Series (Cambridge)
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W.Szlenk (Warschau)
M.Thaler (Salzburg)
J.-P. Thouvenot (Paris)
K.M.Wilkinson (Nottingham)

Vortragsauszüge

J.AARONSON: About transformations preserving infitite measures

Let (X,B,μ,T) be a conservative ergodic transformation preserving an infinite measure. We study properties of the form

$$(*)\quad \frac{1}{a_n}\sum_{k=0}^{n-1} f\circ T^k \to \int_X f\,d\mu \quad \text{in some sense } \forall f \in L^1$$

where $a_n > 0$ are constants. Although the property (*) in the pointwise (a.e.) sense is eliminated by the condition $\mu(x) = \infty$, it does occur in a weaker sense quite frequently:

$$(**)\quad \forall\, n_k \to \infty\ \exists\, m_l \to n_{k_l} \to \infty \text{ s.t. } \frac{1}{n}\sum_{l=1}^{n}\frac{1}{a_{m_l}}\sum_{j=0}^{m_l-1} f\circ T^j \to \int f\,d\mu \text{ a.e. } \forall f\in L^1$$

even though this (**) is not a consequence of ergodicity. The constants $\{a_n\}$ are uniquely defined up to asymptotic equivalence, and can be combined with Krengel entropy to yield a fairly strong invariant for the isomorphism of null recurrent Markov shifts (which satisfy (**)). Some other examples of transformations satisfying (**) are:

$$\left.\begin{array}{l} Tx = x + \sum_{k=1}^{N} \frac{p_k}{t_k - x} \quad p_k \geq 0,\ t_k \in \mathbb{R} \\ Tx = \alpha x + (1-\alpha) \tan x \end{array}\right\} a_n \sim c\sqrt{n}$$

$$Tx = x + \tan x \qquad\qquad a_n \sim \log n.$$

R.ADLER: Topological Entropy and Equivalence of Dynamical Systems

Let (X,φ) be a compact dynamical system. Unless otherwise stated all dynamical systems will be topologically transitive nonwandering. Let N_X denote the set of nondoubly transitive points. $(X,\varphi) \approx (Y,\psi)$ denotes topological conjugacy between two systems. $\approx$ is an invariant and topological entropy $h(X,\varphi)$ is an invariant. Let $(S^{\mathbb{Z}},\sigma)$ denote the full shift on a finite symbol set S. Let $(X(T),\sigma)$ denote a topological Markov subshift of $(S^{\mathbb{Z}},\sigma)$ where T is a 0-1 transition matrix on the symbols from S. It is well known that entropy is not a complete invariant for topological Markov shifts. Therefore, we introduce another equivalence relation slightly weaker than topological conjugacy. We say (Y,ψ) is an almost conjugate extension of (X,φ) if $\exists$ a continuous boundedly finite to one map π of Y onto X which is 1-1 on the doubly transitive points. The map π makes $(Y-N_Y,\psi) \approx (X-N_X,\varphi)$. Two dynamical systems are said to be almost top. conjugate, $(X,\varphi) \sim (Y,\psi)$, if there exists a common almost conjugate extension. Now $\sim$ is an equivalence relation and topological entropy is an invariant.

Homeomorphism Theorem (Adler, Marcus)

For aperiodic topological Markov shifts topological entropy is a complete invariant.

Cor. Two hyperbolical toral automorphisms are almost topologically conjugate iff they have the same entropy.

Many questions naturally arise for this equivalence relation.

M.AKCOGLU: Pointwise Ergodic Theorems

We shall discuss the theorems related to the existence of a.e. $\lim_{n\to\infty} A_n(T)f$, where $A_n(T) = \sum_{i=0}^{n-1} \frac{T^i}{n}$ are the Césaro averages of a linear contraction $T : L_p \to L_p$ on the L_p space of a σ-finite measure space (X,F,μ) and $f \in L_p$. Also we shall give a somewhat simplified version of the Menchoff-Burkholder counter-example for p=2 and indicate why this example does not seem to be adaptable to the other values of p.

St.ALPERN: Generic Properties of Measure Preserving Homeomorphisms

Let μ be a nonatomic Borel measure on a compact connected metric space (X,d), with support $\mu = X$. Let $G = G(X,d,\mu)$ denote all invertible μ-preserving transformations on X, and let $H \subset G$ be the subset consisting of (μ-preserving) homeomorphisms. We define two metrics on G

$$\varphi_{weak}(F,g) = \inf\{\lambda : \mu(x : d(F(x),g(x))>\lambda) < \lambda\}$$

$$\varphi_{sup}(F,g) = \operatorname{ess\,sup}_{x\in X} d(F(x),g(x)).$$

Let $G^{\varepsilon} = \{g \in G : \varphi_{sup}(g, \text{identity}) < \varepsilon\}$, and $H^{\varepsilon} = H \cap G^{\varepsilon}$.

Definition: (X,d,μ) is a weak Lusin measure space if $\forall\ \varepsilon\ \exists\ \delta$ such that H^{ε} is dense in G^{δ} in the weak topology.

Theorem 1: If (X,d) is a manifold of dimension at least 2, then (X,d,μ) is a weak Lusin measure space.

Theorem 2: Let (X,d,μ) be a weak Lusin measure space.
Let $S \subset G$ be a weak topology G_{δ} subset which is self-conjugate in G. Then the following are equivalent: (i) S dense in G, weak topology; (ii) $S \cap$ aperiodics $\neq \emptyset$; (iii) $S \cap H$ dense in H, sup topology.

J.AUSLANDER: Disjointness in ergodic theory and topological dynamics

Let (X,T) and (Y,S) be minimal flows, with (X,T) point distal. Let μ and ν be ergodic invariant measures on X and Y respectively such that the processes (X,T,μ) and (Y,S,ν) are disjoint. Then (X,T) and (Y,S) are topologically disjoint. The proof uses the Veech structure theorem for point-distal minimal flows, as well as results of Glasner and the author on lifting topological disjointness. Similar techniques are used to obtain examples of topologically disjoint minimal flows with positive topological entropy.

A.BELLOW: Another look at a.s. convergence

Let $(\Omega,\mathcal{F},P)$ be a probability space and $(\mathcal{F}_n)_{n\in N}$ an increasing sequence of sub-σ-fields of $\mathcal{F}$. We denote by T the set of all bounded stopping times and by T_f the set of all stopping times (relative to the filtration $(\mathcal{F}_n)_{n\in N}$) that are finite a.s. We introduce the notion of "very rich" set of stopping times. A "very rich" set $S \subset T_f$ is in particular a lattice. Examples of such sets S are of course $S = T$ and $S = T_f$, but there are many other examples that arise naturally. In terms of this notion one may characterize a.s. convergence as follows: Theorem. Let $(X_n)_{n\in N}$ be an adapted, L^1-bounded sequence of real r.v.'s. The following are equivalent assertions:

(1) $\lim_{n\in N} X_n(\omega)$ exists a.s.

(2) There is a "very rich" set $S \subset T_f$ such that the set $\{X_\tau | \tau\in S\}$ is L^1-bounded and such that $\lim_{\tau\in S} \int X_\tau dP$ exists.

In other words, a.s.convergence is equivalent to an "amart type" property.

J.R.BLUM: Pointwise ergodic summability methods on LCA groups

Let G be σ-compact LCA group of measure-preserving transformations on a probability space (Ω,F,μ). Let φ be a probability density on G with respect to Haar measure and let φ^{*J} be J-fold convolution of φ with itself. We obtain

1) Assume $|\hat{\varphi}(\gamma)| < 1$ for $\gamma \in \hat{G}$, $\gamma \neq e$. Then there is a set D dense in $L_2(\Omega)$ such that for $f \in D$

   $$\int_G f(g\omega)\, \varphi^{*N}(g)\,dg \to Sf, \text{ for a.a. } \omega$$

   where S is the projection on the manifold of f invariant under G.

2) Assume $\hat{\varphi}(\gamma) \neq 1$ for $\gamma \in \hat{G}$, $\gamma \neq e$. Then for $f \in L_1(\Omega)$ we have

   $$\frac{1}{n}\sum_{J=1}^{n} \int_G f(g\omega)\, \varphi^{*J}(g)dy \to \bar{f} \text{ a.e., where } \bar{f} \text{ is invariant order } G,$$

   and $\bar{f} = Sf$ for $f \in L_2(\Omega)$.

S.G.DANI: Invariant measures of horospherical flows

For a compact homogeneous space G/Γ where G is a semisimple Lie group and Γ is a discrete subgroup the horospherical flow is uniquely ergodic; i.e. admits a unique invariant probability measure. We now consider non-compact homogeneous spaces (then the above is no more true) and classify the invariant measures, for a suitable large class of homogeneous spaces. The study enables us to identify minimal subsystems of some of the horospherical flows. In particular we consider the natural action of $SL(n,\mathbb{Z})$ on $\mathbb{R}^n$ and describe all invariant measures and minimal sets.
Proofs will be indicated for the special case of $SL(2,\mathbb{Z})$ (as a subgroup of $SL(2,\mathbb{R})$).

J.FELDMAN: Reparametrization of probability-preserving n-flows

Quite recently, striking progress was made in the study of Kakutani-equivalence of probability-preserving transformations, in works of Feldman, Katok, Ornstein, Rudolph, Satayev, and Weiss on "Loosely Bernoulli" transformations. This may be interpreted as the study of the classifications of probability-preserving flows up to reparametrization and isomorphism. Similar results have now been obtained (by Feldman, Nadler and Ornstein) for probability-preserving actions of $\mathbb{R}^n$. However, new phenomena arise, and new techniques are required: especially in the positive entropy case, where an "approximate entropy" plays a critical role. Some basic results on reparametrization by Rudolph are used, as well as a version for n-flows of Abramov's formula, due to Nadler.

T.HAMACHI: Fundamental homomorphism of normalizer group of ergodic non-singular transformation

Let G be a countable ergodic group of non-singular transformations of a Lebesgue space (Ω,F,P). Let N[G] be the normalizer group consisting of all non-singular transformations φ satisfying $Orb_G(\varphi\omega) = \varphi\, Orb_G(\omega)$ a.e. ω where $Orb_G(\omega) = \{g\omega \mid g\in G\}$. Let [G] be the full group of G consisting of all non-singular transformations φ satisfying $\varphi\omega \in Orb_G(\omega)$ a.e. ω. [G] is a subgroup of N[G]. Normalizers R_1 and R_2 are outer conjugate with each other if there exists a normalizer φ such that $R_1\varphi R_2^{-1}\varphi^{-1}\in[G]$. We consider invariants for outer conjugacy. For $R \in N[G]$ let p(R) be the least positive integer p such that $R^p \in [G]$ and if there does not exist such p let $p(R) = \infty$. Set $\tilde{R}(\omega,u) = (R\omega,\ u - \log \frac{dPR}{dp}(\omega))$ for $(\omega,u) \in \Omega \times R$. Let $\xi(\tilde{G})$ be the ergodic decomposition of $\Omega \times R$ with respect to $\tilde{G} = \{\tilde{g} \mid g \in [G]\}$. Let mod R be the factor transformation of $\tilde{R}$ on the quotient space $\Omega \times R/\xi(\tilde{G})$. We say mod R is the fundamental homomorphism of the normalizer group N[G]. Then p(R) and mod R are invariants for the outer conjugacy.

G.HELMBERG: On ε-independence and topological Rohlin-sets.

Let $P = \{P_1,\dots,P_n\}$ and $Q = \{Q_1,\dots,Q_m\}$ be measurable partitions of a probability space (X,F,μ). ε-independence of P and Q ($P\perp^{\varepsilon}Q$ in the sense of Ornstein) implies

$$h(P) - h(P/Q) \leq \frac{3-\varepsilon}{2}\, \varepsilon \log n + \eta(\tfrac{\varepsilon}{2}) + \eta(1-\tfrac{\varepsilon}{2})$$

There are partitions P and Q such that $P \perp^{\varepsilon} Q$ and

$$h(P) - h(P/Q) \geq \frac{3-\varepsilon}{2}\, \varepsilon \log n - (1-\varepsilon)\,\eta(\tfrac{\varepsilon}{2}) + o(1) \quad \text{as} \quad n \to \infty$$

The statement of proposition 15.4. in Lecture Notes 527 (Denker/Grillenberger/Sigmund) may be connected to serve as well for the proof of the following assertions. The original statement, however, is invali-

dated by a counterexample.
(joint work with E.Fleischmann, B.Roider, P.Wagner).

F.HOFBAUER: Das maximale Maß für die Transformation $T:x\to\beta x+\alpha \pmod 1$

We consider the transformation $T : x \to \beta x + \alpha \pmod 1$. $([0,1],T)$ is isomorphic to a shiftspace $\sum_T^+ = \{\underline{x} \in \{1,\ldots,n\}^N : \underline{a} \leq \sigma^m \underline{x} \leq \underline{b}$ for all $m \geq 0\}$ for some $\underline{a}$ and $\underline{b}$. Its natural extension $\sum_T$ can be written as disjoint union $X \cup N$. Every measure concentrated on N has entropy 0 and X is isomorphic to a shift $\sum_M$ of finite type over a countable state space. One shows that $\sum_M$ has unique measure m with maximal entropy. m is Markov on $\sum_M$. One can bring back m to $([0,1],T)$ to get the unique maximal measure for T. It is absolutely continuous with respect to Lebesgue measure and its support is a finite union of intervals. One can use the above methods to determine those T's, whose maximal measure has all of $[0,1]$ as support.

A.KATOK: Lyapunov exponents, entropy and invariant foliation in smooth ergodic theory

Connections between Lyapunov characteristic exponents and entropy with respect to an invariant measure were discussed: estimation of entropy from above for every Borel invariant measure, an exact formula for a measure which generates family of absolutely continuous conditional measures on expanding manifolds, example of a smooth dynamical system for which equality is not true for every measure. In a case of a smooth invariant measure several consequences of the formula for entropy were presented: upper-semicontinuity of the entropy, continuity on the set of ergodic diffeomorphisms, continuity in the zero-entropy case and for some partially hyperbolic systems. There is also an example of a flow without non-trivial zero exponents such that there exist an arbitrary small perturbation which is periodic on the set of full measure. Some conjectures about the structure of the set of continuity of the entropy and about genericity of some metric properties of measure-preserving diffeomorphisms have been formulated.

A.KATOK: Generalized rotation numbers for Anosov flows

Main problem: when the homeomorphism h which conjugates two Anosov diffeomorphisms or Anosov flows with a smooth invariant measure can be chosen absolutely continuous?

We describe a countable number of invariants of absolutely continuous equivalency in the cases of dimension 2 (for diffeomorphism) and 3 (for flows). Typical results:

Theorem 1. Let φ_t, ψ_t be two geodesic Anosov flows on 2-dimensional manifold M, γ - a closed trajectory of φ_t $\gamma' = h\gamma$ - the corresponding trajectory of ψ_t. If there exists an absolutely continous h, then the length $l(\gamma) = l(\gamma') + a([\gamma])$ where $[\gamma] \in H_1(M,\mathbb{Z})$ is a homology class of γ and a some linear function on $H_1(M,\mathbb{Z})$

Theorem 2. If in the same case $l(\gamma) = l(\gamma')$ and h is Hölder then h is measure-preserving.

M.KEANE: Bernoulli schemes of the same entropy are finitarily isomorphic

We show that two Bernoulli schemes with equal entropies are finitarily isomorphic. The proof is of an elementary nature, using only the weak law of large numbers and a marriage lemma with "weights" and the isomorphism can be explicitly constructed. This is joint work with M.Smorodinsky, submitted for publication in the Annals of Mathematics.

G.KELLER: Piecewise monotonic functions and exactness

We consider piecewise monotonic functions T on the unit interval, as they have been considered before by Lasota & Yorke, Bowen, and Kowalski e.g., and prove the following theorem:
The support of the T-invariant measure $\mu = h \cdot \lambda$ (see Lasota & Yorke) splits into a finite number of T-ergodic components A for each of which there exists a natural number $p = p(A)$ such that A splits into a finite number of subcomponents (T^p-invariant), on each of which $(T^p)_{|A}$ is exact. Each of these subcomponents is a finite union of open intervals.

F.LEDRAPPIER: A variational principle for topological conditional entropy

Let X be a compact space and T a continuous map. We show that the topological conditional entropy of the system can be expressed in terms of the metric entropy of the invariant measures on the Cartesian square of the system. Namely it is equal to the maximum of defect of upper semicontinuity of some relative metric entropy. This shows the invariance of the top.cond. entropy under some equivalences.

M.LIN: Weak mixing

We obtain a weak mixing theorem for transformations and Markov operators which need not have a finite invariant measure:

"If P is an ergodic Markov operator, the following are equivalent:

(i) P has no unimodular eigenvalues except 1.

(ii) For every $u \in L_1$ with $\int u\, dm = 0$ and every f in L_∞

$$\lim_N \frac{1}{N} \sum_{k=1}^{N} |\langle u, P^k f\rangle| = 0.$$

(iii) For every Q ergodic with finite invariant measure, $P \times Q$ is ergodic."

Ergodicity of $P \times P$ implies these conditions, but is not equivalent.

D.A.LIND: Specification for compact group automorphisms

A weak version of Bowen's property of specification for certain automorphisms of compact groups plays a key role in the splitting of certain skew products. Unfortunately, there is an ergodic automorphism of an eight-dimensional torus that does not have the weak specification property, and one of a four-dimensional torus that has the weak but not the strong specification property (dimensions eight and four are the lowest possible).We will discuss the extent to which general group automorphisms have one of these properties.

B.MARCUS: Topological Entropy of some skew products

Let $B : X \to X$ and $F : Y \to Y$ be aperiodic shifts of finite type. Let ψ be an integer-valued continuous function on X. Let P_1 be the pressure of $h(F)\psi$ and μ_1 its unique equilibrium state. Let P_2 be the pressure of $-h(F)\psi$ and μ_2 its equilibrium state. Define $T : X \times Y \to X \times Y$ $T(x,y) = (B(x), F^{\psi(x)}(y))$. The following is joint work with Sheldon Newhouse.

Theorem: $h(T) = \max(P_1, P_2)$. Moreover,

1) if $P_1 \neq P_2$, then T has a unique measure of maximal entropy.

2) if $P_1 = P_2$ and $\mu_1 \neq \mu_2$ then T has exactly two ergodic measures of maximal entropy.

3) if $P_1 = P_2$ and $\mu_1 = \mu_2$ then T has infinitely many ergodic measures of maximal entropy.

Example: If B and F are both the full 2-shift and

$\psi(x) = \begin{Bmatrix} 1 \text{ if } x_o = 1 \\ -1 \text{ if } x_o = 0 \end{Bmatrix}$, then $h(T) = \log(5/2)$ and its only ergodic measures of maximal entropy are $B(\frac{4}{5},\frac{1}{5}) \times B(\frac{1}{2},\frac{1}{2})$ and $B(\frac{1}{5},\frac{4}{5}) \times B(\frac{1}{2},\frac{1}{2})$.

M.MISIUREWICZ: Invariant measures for continuous transformations on [0,1] with zero topological entropy

Let $f : [0,1] \to [0,1]$ be a continuous mapping with zero topological entropy and let μ be an ergodic f-invariant probability measure on [0,1]. Then either μ is concentrated on some periodic orbit of f or the system $([0,1],\mu,f)$ is isomorphic to the system (Σ,ν,φ), defined as follows:

$$\Sigma = \prod_{0}^{\infty}\{0,1\},$$

$$\nu = \prod_{0}^{\infty} \tilde{\nu}, \text{ where } \tilde{\nu}(\{0\}) = \tilde{\nu}(\{1\}) = 1/2,$$

$$\varphi(0,0,\ldots,0,1,\varepsilon_1,\varepsilon_2,\ldots) = (1,1,\ldots,1,0,\varepsilon_1,\varepsilon_2,\ldots).$$

J.MOULIN OLLAGNIER: A new proof of E.Følner s result: Countable amenable groups have an ameaning filter

The existence of an ameaning filter for countable groups with the fixed point property is proved by a new method, completely different from Følner's very combinatorial one. It depends on the study of a particular dynamical system, the topological dynamical system of all total orders on G.

J.NEVEU: On the filling scheme and a simple proof of the Chacon-Ornstein Theorem

For any positive linear contraction of an L^1-space which is conservative ergodic and for any $h \in L^1$ with $\int h < 0$, the filling scheme ($h_o = h$, $h_{n+1} = T h_n^+ - h_n^-$) is such that $\sum_n h_n^+ < \infty$ a.s. Hence if $f,g \in L_+^1$ are such that $\int f < \int g$, then since

$$\sum_{k<n} T^k f = \sum_{k<n} h_k^+ + \xi_n, \quad \sum_{k<n} T^k g = \sum_{k<n} T^{n-1-k} h_k^- + \xi_n$$

for some positive ξ_n, one gets that

$$\limsup_{n\to\infty} \frac{\sum_{k<n} T^k f}{\sum_{k<n} T^k g} \leq 1$$

from which the Chacon-Ornstein follows immediately. The ergodicity hypothesis can be easily dispensed with.

R.NÜRNBERG: Constructions of Strictly Ergodic Systems which are not Loosely Bernoulli

Sei ω ein strikt ergodischer Punkt in $\{0,1\}^{\mathbb{Z}}$ mit topologischer Entropie 0, der mit jedem Block auch seinen gespiegelten enthält. Es werden notwendige Bedingungen angegeben, unter denen das einzige Maß auf dem Bahnabschluß nicht "loosely Bernoulli" ist. Bis jetzt ist nicht bekannt, ob es Folgen der Form $b_1 \times b_2 \times \ldots$, b_i ein Block aus Nullen und Einsen, gibt, die diesen Bedingungen genügen.

W.PARRY: Generic properties of endomorphisms
(Joint work with Rachel Palmer and Peter Walters)

We consider E(x), the class of endomorphisms of the unit interval $(X,\mathcal{B},m)$ endowed with two topologies, viz. the weak topology inherited from the weak or strong topologies on isometrics of $L^2(X)$ and the strong adjoint topology which requires $T_n f \to Tf$ for all $f \in L^2(X)$ when $T_n \to T$. In the former A(x), the set of automorphisms, is a dense G_δ and strong mixing and zero entropy is "rare" (forming sets of first category.) In the latter, A(x) is closed and nowhere dense and exactness and infinite information $(I(\mathcal{B}|T^{-1}\mathcal{B}))$ is the general case. The basic result which establishes these theorems is that Markov endomorphisms are dense with respect to the strong adjoint topology.

K.PETERSEN: Balancing ergodic averages

Let $\{T_t : -\infty < t < \infty\}$ be a measure-preserving flow on a probability space $(X,\mathcal{B},\mu)$, and $g \in L^1(X,\mathcal{B},\mu)$. Let $\mathcal{O} = \{x \in X : \int_0^t g(T_s x)ds = 0$ for some $t > 0\}$. In joint work with Brian Marcus, we prove the following:

Theorem: $\int_{\mathcal{O}} g d\mu = 0.$

Corollary: Let $g^*(x) = \sup_{t>0} \frac{1}{t} \int_0^t g(T_s x)ds$ be the ergodic maximal function. If $\{T_t\}$ is ergodic and $\alpha \geq \int g d\mu$, then

$$\int_{\{g^*>\alpha\}} g d\mu = \alpha\mu\{g^* > \alpha\}.$$

For a single measure-preserving transformation $T : X \to X$ and

$f \in L^1(X,\mathcal{B},\mu)$, let $S_n f(x) = \sum_{k=0}^{n-1} f(T^k x)$, $S_* f(x) = \inf_{n\geq 1} S_n f(x)$, and $A(f) = \{x \in X : S_n f(X) > 0$ for all $n \geq 1\}$. In this situation the corresponding result is:

Theorem: If T is ergodic and $\int_X f d\mu \geq 0$, then $\int_{A(f)} S_* f d\mu = \int_X f d\mu$.

K.SCHMIDT: Unique Ergodicity

Given a nonsingular ergodic automorphism T of a Lebesgue space, (X,S,μ) under what conditions can one assume that μ is the only measure on (X,S) satisfying certain conditions. The Krieger-Jewett-theorem shows, for example, that one may assume to be the only T-invariant probability measure on the space. In general, one can specify unique probability measures with given Radon-Nikodym derivative, but measures with given type or infinite measures with a given R.N.derivative (i.e. there always exist many such measures).

F.SCHWEIGER: The "jump transformation" and its applications

Es sei (M,S) ein "fibered system" (Mich.Math.J.22(1975),181-187). Sei $\mathcal{A}$ eine Klasse von Zylindern. Wir setzen

$$\mathcal{B}_n = \{B(i_1,\ldots,i_n) \mid B(i_1,\ldots,i_s) \notin \mathcal{A},\ 1\leq s\leq n-1,\ B(i_1,\ldots,i_n)\in\mathcal{A}\}$$

$$B_n = \bigcup_{E\in\mathcal{B}_n} E,\quad P = \bigcup_{n=1}^{\infty} B_n$$

$$\mathcal{D}_n = \{B(i_1,\ldots,i_n) \mid B(i_1,\ldots,i_s) \notin \mathcal{A},\ 1\leq s\leq n\}$$

$$D_n = \bigcup_{E\in\mathcal{D}_n} E,\quad W = \bigcap_{n=1}^{\infty} D_n,\quad V : P \to M$$
$$Vx = S^n x \Leftrightarrow x \in B_n$$

$$M^* = M \setminus \bigcup_{j=0}^{\infty} V^{-j}W,\quad S^* = V|_{M^*}$$

Das neue "fibered system" (M^*,S^*) erweist sich als nützlich (Ergodizität, invariante Maße); Anwendungen auf ß-Entwicklungen, $x \to x - \frac{1}{x}$, Gütings Algorithmusfolgen.

C.SERIES: Foliations and ergodic equivalence relations

Let $X,\mathcal{B},\mu$ be a Lebesgue space and R an equivalence relation on X with countably many points in each orbit. R is classified by the associated flow and is of type II, III_1, III_λ, III_0 according as the flow is $\mathbb{R}$ on $\mathbb{R}$, $\mathbb{R}/\mathbb{R}$, $\mathbb{R}/\mathbb{Z}\log\lambda$, properly ergodic respectively. We compute the associated flow for the relations H,A induced on transversals to the stable and weak stable foliations of an Anosov diffeomorphism, with respect to any equilibrium state.

Theorem: 1) H,A are never III_0

2) The following possibilities occur:

H	A
II	$III_{\text{top.entropy}}$
III_λ	$\{ III_{\lambda^{1/n}}$
	III_1
III_1	III_1

M.SMORODINSKY: Bernoulli factors that span a transformation

Theorem (Smorodinsky, Thouvenot): Let T be an invertible ergodic measure preserving transformation on probability space $(X,\mathcal{B},\mu)$. There are three Bernoulli factors $\mathcal{B}_1,\mathcal{B}_2,\mathcal{B}_3$ such that $\bigvee_{i=1}^{3} \mathcal{B}_i = \mathcal{B}$.

Question: Are there always two Bernoulli factors that span $\mathcal{B}$?

Remark: All the known examples of transformations T have the weak Pinsker property, in which case two Bernoulli factors do span $\mathcal{B}$.

L.SUCHESTON: Operator ergodic theorems for superadditive processes

Let T be a sub-Markovian operator on L^1. Let $f_n \in L^1_+$, $s_n = \sum_0^{n-1} f_i$. Assume (s_n) superadditive, i.e., such that

$s_{n+k} \geq s_n + T^n s_k \;\; \forall\, n,k$. $\delta \in L^1_+$ is called a dominant if

$\sum_0^{n-1} T^i\delta \geq s_n \;\; \forall\, n$, an exact dominant if δ has the minimal integral among dominants.

Theorem: The following are equivalent (a) $\underline{\lim}\, \frac{1}{n} \sum_0^{n-1} \int (s_i - Ts_{i-1}) < \infty$; (b) $\overline{\lim}\, \frac{1}{n} \sum_0^{n-1} \int (s_i - Ts_{i-1}) < \infty$; (c) there exists a dominant; (d) there exists an exact dominant.

In the Markovian case, the implication (a) $\Rightarrow$ (d) was proved with

M.A.Akcoglu, together with a ratio ergodic theorem, a common generalization of the Chacon-Ornstein theorem and Kingman's subadditive theorem. The result as stated above was obtained with A.Brunel.

W.SZLENK: An example of a dynamical system with positive topological sequence entropy and zero metric sequence entropy.

Let (X,φ) be a topological dynamical system. For any increasing sequence of integers $A = (n_1, n_2, \dots, n_k, \dots)$ and for any open cover α of X we set $h_A(\varphi,\alpha) = \limsup_k \frac{1}{k} N\left(\bigvee_{i=1}^{k} \varphi^{-n_i}(\alpha)\right)$. The number

$h_A(\varphi) = \sup_\alpha h_A(\varphi,\alpha)$ is called the sequence topological entropy of the system (X,φ) (Goodman 1975). Goodman constructed an example of a dynamical system (X,φ) such that for any φ-invariant finite measure μ we have $h_{\mu,A}(\varphi) = 0$ and $h_A(\varphi)$ is positive for a sequence A.

We construct an example of a dynamical system admitting this property and satisfying some other conditions. Namely, we construct a subshift of finite type (X,σ) such that

1. X is countable
2. the center of the system consists of one point
3. for a sequence of integers A and for the natural cover α of X the number $h_A(\sigma,\alpha)$ is positive.

M.THALER: Ergodische Eigenschaften von reellen Transformationen

Es werden spezielle Transformationen $T : \mathbb{R}^+ \to \mathbb{R}^+$ betrachtet, die zu Ziffernentwicklungen mit monoton wachsenden Ziffernfolgen gehören. Klassische Beispiele sind die Engelschen und Sylvesterschen Reihen. Die Engelschen Reihen haben lineares Ziffernwachstum und sind ergodisch, die Sylvesterschen Reihen haben quadratisches Ziffernwachstum und sind nicht ergodisch (bzgl. des Lebesgueschen Maßes) (cf. F.Schweiger, W.Vervaat). Derselbe Zusammenhang zwischen Ziffernwachstum und Ergodizität besteht zwischen den folgenden beiden Algorithmen:

$Sx = (n+1)(x-n)^{-1}, \quad x \in]n, n+1] \qquad (n \geq 0)$

$Kx = (xn+1)(x-n)^{-1}, \quad x \in]n, n+1] \qquad$ (Kotangensalgorithmus von D.H.Lehmer)

K.M.WILKINSON: <u>Stopping Times and Towers</u>

If τ is an ergodic automorphism of the probability space $(\Omega,\mathcal{B},\mu)$ and ν is a measurable map : $\Omega \to \mathbb{N}$ such that τ^{ν} is again an automorphism of $(\Omega,\mathcal{B},\mu)$ we call ν a stopping time for $(\Omega,\mathcal{B},\mu,\tau)$ and τ^{ν} a stopping time transformation. Let $S(\tau)$ be the set of stopping time transformations derived from τ . We restrict attention here to those τ defined by means of a tower construction (which includes all Bernoulli shifts). A generalization of the tower construction is introduced which allows us to investigate properties of τ^{ν}. In particular we show that if τ is a Bernoulli shift, the set of τ^{ν} which are again Bernoulli is dense with respect to the uniform metric in $S(\tau)$.

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