MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 4|1979

Mathematical Economics

21. 1. bis 27. 1. 1979

Tagungsleiter: G. Debreu (Berkeley)
W. Hildenbrand (Bonn)
D. Sondermann (Hamburg)

This was the second meeting in Oberwolfach on Mathematical Economics. As for the first one in 1977, the main purpose of the meeting was to further the active interaction between mathematicians and economists. To this end, the conference was limited again to those areas of Economic Theory and Game Theory which are challenging both from an economic and a mathematical point of view.
We scheduled eight invited papers, all of them by leading specialists, some of which have a survey character reviewing well developed fields, while others gave an exposition of current lines of research in specific fields. In addition to the eight invited papers shorter communications on special topics were presented: - regular economies, rate of convergence of the core, fixed price analysis, statics and dynamics, experimental and theoretical game theory, dynamical systems, aggregation of demand, ... -.
It would have been impossible to gather this impressive group of specialists without the financial support of the Mathematisches Forschungsinstitut. The excellent facilities created a stimulating atmosphere which was appreciated by all the participants.

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Teilnehmer

J.P. Aubin, Paris
R. Aumann, Jerusalem
Y. Balasko, Paris
M. Beckmann, München
M.C. Blad, Kopenhagen
B. Booss, Roskilde
H. Bühlmann, Zürich
H.-Ch. Cheng, Louvain-la-Neuve
E. Dierker, Bonn
H. Dierker, Bonn
J.H. Dreze, Louvain-la-Neuve
I. Ekeland, Paris
H. Föllmer, Zürich
J.H. van Geldrop, Eindhoven
J.M. Grandmont, Paris
J. Greenberg, Blacksburg
B. Grodal, Kopenhagen
O. Hart, Cambridge, U.K.
J. Harsanyi, Berkeley
M. Hellwig, Bonn
R. Henn, Karlsruhe
R. John, Bonn
H. König, Saarbrücken
D.M. Kreps, Cambridge
A. Mas-Colell, Berkeley
J.F. Mertens, Louvain-la-Neuve
R. Myerson, Evanston
W. Neuefeind, St. Louis
H. Neumann, Saarbrücken
D. Pallaschke, Bonn
D. Puppe, Heidelberg
D. Rand, Coventry
N. Reif, Hamburg
J. Rosenmüller, Bielefeld
D. Schmeidler, Tel Aviv
R. Selten, Bielefeld
Y. Taumann, Louvain-la-Neuve
W. Trockel, Bonn
K. Vind, Kopenhagen
C. Weddepohl, Tilburg
J. Weinberg, Bonn
E.-A. Weiss, Bonn
H. Wiesmeth, Hamburg
Y. Younes, Paris

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Vortragsauszüge

J.P. AUBIN: Behavior of Agents acting on the Environment and Formation of Coalitions

If $c(t) \in [o,1]^n$, we consider the differential equations

$$(1) \quad \begin{cases} x'(t) = g(t,x) + \sum_{i=1}^{n} c_i(t) f_i(t,x) \\ x(o) = x_o \end{cases}$$

whose solutions are denoted by $x_c(t)$. If a family of compact convex subsets $K(t) \subset \mathbb{R}^n$ is given, we look for "viable" solutions, i.e., solutions satisfying

$$(2) \quad \forall\, t \geq o \ , \quad x(t) \in K(t) \ .$$

We define the tangent cone to $K(t)$ at x by

$$T_{K(t)}(x) = \text{closure} \, \Big(\bigcup_{\lambda \geq o} \lambda (K(t) - x) \Big)$$

and the derivative from the right at $(t,x) \in$ graph $(K(\cdot))$:

$$K'(t,x) = \lim_{h \to o+} \frac{\pi_{K(t)} x - x}{h} \ , \quad \pi_{K(t)} = \text{projector onto } K(t) \ .$$

We essentiably prove that if

$\forall\, (t,x) \in$ graph $(K(\cdot))$ the set

$$C(t,x) = \{ c \in [o,1]^n \mid g(t,x) + \sum_{i=1}^{n} c_i f_i(t,x) \in K'(t,x) + T_{K(t)}(x) \}$$

is non empty, there exists a trajectory $x_c(\cdot)$ of system (1), (2). We also prove the existence of dynamic equilibria

$$\forall\, t \quad \exists\, x(t) \in K(t) \quad \exists\, c(t) \in [o,1]^n \text{ such that}$$

$$g(t,x(t)) + \sum_{i=1}^{n} c_i(t) f_i(t,x(t)) = K'(t,x(t))$$

R. AUMANN: Aplications of the Shapley Value

A survey covering developments of the last three or four years. Topics covered were as follows: The Asymptotic and Partition Values; solution of the diagonal conjecture by Neyman and Tauman; the space DIFF defined by Mertens; Political applications covering mainly weighted majority games and culminating in the recent proof by Neyman that non-atomic weighted majority games have asymptotic values; applications to cost allocation in airport landing (Littlechild and Owen) and internal telephone billing rates (Billera, Heath, and Raanan). Economic applications including the value equivalence theorem and recent extensions thereof by Champsaur, Mas-Colell, and Hart. Politico-Economic applications, especially to problems of redistribution and public goods (Kurz, Neyman, Osborne, Rosenthal, Gardner, and others). Finally, mention was made of a value approach to coalition-forming recently developed by Myerson and the undersigned, and to work on the axiomatic foundations of value theory by Roth.

Y. BALASKO: Number and Definiteness of Economic Equilibria

It is shown that if the number of equilibria is known for every initial endowments in pure exchange economies, then, provided some technical assumptions are verified, the equilibria associated with every economy or initial endowments are determined.

M. BLAD: Exchange of Stability

The content of my talk was the introduction of a simple dynamical disequilibrium model, based on Malinvaud's macro economic model. The evolution of the economy is described as the result of an interaction between a relatively fast adjustment process in quantities and a relatively slow adjustment process in prices. It is shown that equilibria exhibit "exchange of stability" whenever the prices cross a boundary between two equilibrium regions. A generic long-term description is obtained by introducing parameters which are usually "hidden" in this type of analysis. Depending on the values of these parameters the equilibrium state will either be of the well known type (Keynesian Classical or Repressed inflation) or of a "dual" type where equilibrium is determined by the long side of the market.

B. BOOSS: From Kepler's Planetary Motion to the " Quark Confinement": Empirical Investigations in the Problems of Model Transfer and Analogy

A key-word in methodological discussions of mathematical economics is "models". The general problem in these discussions may be formulated as a question: What is the relation between the model and (in this case social) reality?

There might be at least so much in common between physical matter and social systems and between the brain activity of physicists and e.g. economists, that natural science experience on the relevance of models for the understanding of reality

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

may have a certain transfer value for social science discussions of the same question.

A fable, dealing with episodes from the development of astronomy and physics, and concentrated on the application of the Kepler-Newton model of the solar system in a wide range of physical situations may serve to present a moral for which it would not be quite as easy to argue on empirical foundations inside the closed domain of the social sciences:
(1) Reality possesses a sufficient unity of structure, regularities or laws for as to carry over models from one domain to another, at least on the level of analogies.

(2) Transfer of more mathematical precision than permitted by the essential relations in the real world is counterproductive.

(3) One may ask whether the indubitable theorems from the theory of the private economy offer when transferred more than a few fade elements of the total question of the economic activity of a society.

H. BÜHLMANN: Optimal Risk Exchanges

Let X $(o \leq X \leq k)$ be a real random variable, n agents must "split X". For this purpose they use a risk exchange (REX) $\underline{Y} = (Y_1, Y_2, \ldots, Y_n)$ with $Y_i \geq o$ for all i

$$\sum_{i=1}^{n} Y_i = X$$

The motivation for the study of this problem comes from

insurance where $X(w)$ is interpreted as "pooled claims if w happens" and $Y_i(w)$ as payment of paticipant i in the pool.

A REX $\underline{\tilde{Y}}$ is <u>Pareto optimal</u> if there is no other REX $\underline{Y}$ with $E[v_i(\underline{Y}_i)] \leq E[v_i(\underline{\tilde{Y}}_i)]$ for all i with strict inequality for some i, where $v_i(x)$ measures the disutility to i of a payment x ($v_i'>o$; $v_i''\geq o$).

A REX $\underline{\tilde{Y}}$ is <u>θ-fair</u> ($\theta = (\theta_1,\theta_2,...,\theta_n)$, with $o \leq \theta_i \leq 1$, $\Sigma\theta_i = 1$) if $E[\underline{\tilde{Y}}_i] = \theta_i E[X]$ for all i.

<u>Any Pareto optimal REX $\underline{\tilde{Y}}$ is characterized as:</u>

There exist constants $k_1,k_2,...,k_n$ ($k_i > o$) and a random variable C such that

$$\left.\begin{array}{llll} k_i v_i'(\tilde{Y}_i(w)) & \geq C(w) & \text{for all} & i \\ & = & \text{if} & \tilde{Y}_i(w) > o \end{array}\right\} \text{ for almost all } w$$

<u>Pareto Optimality and Fairness</u> (Assume that all $\theta_i > o$)

Let $\Phi(\underline{Y}) = \sum_{i=1}^{n} E[\psi_i(Y_i)]$ where $\psi(z) = \int_o^z \ln v_i'(x)\,dx$

Then $\underline{\tilde{Y}}$ minimizes Φ among all fair REX $\underline{Y}$ if and only if $\underline{\tilde{Y}}$ is both Pareto optimal and fair.

From this it follows immediately that there is almost one REX which is both Pareto optimal and fair provided that at least one of the $v_i'' > o$. As existence of the minimizers of Φ can be proved we have exactly one REX which is both Pareto optimal and fair.

H.C. CHENG: What is the normal rate of convergence of the core?

In Debreu's smooth regular framework, it was proved that for regular economies the rate of convergence of the core is at least $1/m$, where m is the number of agents in the economies. But he has a boundary condition which is hardly normal (meaning natural). When this condition is replaced by a natural boundary condition, a (harmless) indecomposability condition is needed to insure the rate $1/m$. However in piecewise smooth regular framework, we normally don't have unique supporting prices at equilibrium, hence we can only prove the rate $1/\sqrt{m}$. Examples are given to show that in certain neighborhood of economies, $1/\sqrt{m}$ cannot be improved generically, $1/m$ is a degenerate rate, and furthermore, almost everywhere the rate of convergence is about $1/m^{2/3}$. This result is proved through the help of the measure theory of continued fractions and the rate of convergence of the core is related to the order of approximation of real numbers by rational numbers.

G. DEBREU: Existence of Competitive Equilibrium

A survey of proofs of existence of a competitive equilibrium by means of fixed point theorems. The first part of the paper was devoted to the simultaneous optimization approach in which every agent in the economy (including a fictitious market operator) maximizes his utility function in a set

depending on the actions of the other agents. The second part was concerned with the properties of the excess demand correspondence ζ of an economy and in particular with the conditions under which there is a price-vector p such that the origin of the commodity space belongs to $\zeta(p)$. The third part dealt with the existence of a competitive equilibrium for an economy with a measure space of agents.

E. DIERKER, H. DIERKER, W. TROCKEL: Smoothing Demand by Aggregation

If the agents in the consumption sector of an economy are described by non-convex preferences and wealth, individual demand sets in general are not singletons. Looking for a measure which describes dispersedness of characteristics one finds a natural candidate on the wealth space only. It is analysed which level of smoothness of demand one arrives at if one fixes a preference and integrates with respect to wealth only. Continuity of mean demand can be shown for preferences represented by a residual subset U_{res} in the set of all C^{∞} utility functions fulfilling some conditions. The topology on the space of utility functions is the Whitney C^{∞} topology. For a fixed preference there exists a closed null set of prices such that the mean demand outside this null set is a C^1 function.

For one of the phenomena distroying differentiability the vanishing Gaussian curvature of indifference surface in a point x of demand, the following can be shown: If

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

demand is unique and there are no other disturbant phenomena except the vanishing Gaussian curvature, $\kappa(x) = o$, the mean demand is a C^1 function at x .

J. DRÈZE: Non-Walrasian Economies

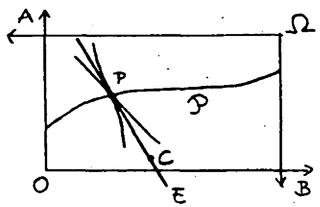
A survey of some theory of resource allocation in an economy where all exchanges must take place at predetermined prices and where quantity constraints are used to bring about compatibility of the actions of individual agents.

For a pure exchange economy with ℓ goods, the budget constraints at fixed prices can be used to eliminate one commodity (with positive price) and thereby define a new exchange economy with $\ell-1$ goods. All the "relevant" properties of the first economy are preserved, except monotonicity of preferences. It is verified that monotonicity of preferences plays an important role in defining equilibria and relating them to optima and cooperative solutions.

The theories of demand under quantity rationing and of continuity and cardinality of equilibrium sets for Walrasian and Non-Walrasian economies are largely comparable, but the Non-Walrasian framework offers additional possibilities to treat such questions as imperfect competition, public goods or increasing returns.

I. EKELAND: The Balasko box

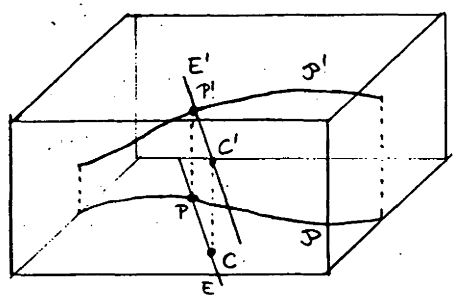
It is possible to give an elementary account of Balasko's and others' work on general equilibrium theory in the simple setting of a two-goods, two-agents exchange economy. This leads to the well-known Edgeworth box:



𝒫 Pareto optima

C initial endowment

Adding as a third dimension the equilibrium price p_1 (the normalized slope of E then being $-\frac{1-p_1}{p_1}$) one gets a three dimensional box, which I propose to call Balasko box, and which has the Edgeworth box as its floor:



𝒫 lifts to curve 𝒫' , E to a horizontal line E' , P and C to P' and C' . As P moves along 𝒫 , P' moves along 𝒫' , and the line E' builds up a two dimensional surface ℰ within the Balasko box.

The study of this surface is fundamental to equilibrium theory, leading to existence, uniqueness, results. With the simple assumption that the utilities u and v of both agents are monotonic and concave in a strong way (i.e.

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

u_1' and $u_2' > \begin{pmatrix} u''_{11} & u''_{12} \\ u''_{21} & u''_{22} \end{pmatrix}$ positive definite) the essentials can be proved in an elementary fashion, namely that $\mathcal{E}$ is a well-defined surface, the tangent plane to which is never vertical above $\mathcal{P}$.

Detailed proofs and statements are to be found in my forthcoming book, "Elements d'èconomie mathèmatique", Herman.

H. FÖLLMER: Some macroscopic aspects of large stochastic systems

Let A be a countable set of agents. If each $a \in A$ can assume states (e.g. preferences) in some measurable state space $(E_a, \mathcal{E}_a)$, then $E = \prod_{a \in A} E_a$ and the σ-field $\mathcal{E} = \prod_{a \in A} \mathcal{E}_a$ contains the detailed "microscopic" information. But if A is infinite then we may also introduce the tail field $\hat{\mathcal{E}}$ containing the "macroscopic" information. Now consider a stochastic dynamics given by a Markov chain $P(x,dy)$ on $(E, \mathcal{E})$ and the induced "macroscopic" process. We discuss the interplay between the two. In particular we characterize the condition that it is enough to observe the macroscopic process in order to make the best possible predictions on the long run behavior of the whole system.

J.v. GELDROP: Sufficient second order conditions for local optimality

This lecture was devoted to proofs of a well-known theorem, stated by Smale and Wan. The first proof used an indirect argument in a very general situation. The second proof was concerned with a situation in pure exchange economies with fixed total resources. The implicit function theorem was applied to utility functions in points satisfying the first order critical Pareto set condition.

J.M. GRANDMONT: Applications of Fixed Price Equilibrium Models to Macroeconomics

The goal of this lecture was to show that the fixed price method was very useful in unifying theories of employment which appeared fundamentably distinct before hand. Classical economists would say that unemployment is due to a too high real wage. They would accordingly advocate a policy aiming at curbing down wages in order to cure unemployment. On the other hand, they deny that increases in governmental spending have a stimulating effect on economic activity. On the contrary, Keynesians claim that the origin of unemployment is a lack of demand. Consequently they advocate an increase of the wage rate and/or of government spending in order to stimulate aggregate demand and therefore to lower the unemployment rate.

This talk presented a simple fixed price model with three commodities (good, labour, money) and three agents (firm,

household, government) due to Barro-Grossman, Benassy, Younès and well presented by Malinvaud. It was shown that the plane $\left(\frac{M_o}{p} , \frac{w}{p} \right)$ could be partitioned in three regions:

- Keynesian unemployment where there exists an excess supply of good and labour
- Classical unemployment where there is excess supply for labour and excess demand for good
- Repressed inflation where there is an excess demand on both markets.

It turns out that under some specifications of the model, the Keynesian policies (increase of the wage rate of government spending) must be implemented in the region of Keynesian unemployment in order to cure unemployment, while the reverse must be done in the Classical unemployment region. The model appears thus to be a useful pedagogical device in presenting in a synthetic way theories that appeared antagonistic at first.

J. GREENBERG: Core and Competitive Allocations in Finite Economies

By focusing attention on core allocations in economies with a finite number of agents, explicit bounds on the difference between the value (using the efficiency prices) of each individual's final and initial bundle, are derived. (Recall that when these two values coincide, the core allocation is also a competitive allocation). In particular,

in the limit these bounds yield the equivalence theorem for replica economies and the exploitation theorem for economies with a measure space of agents with both atoms and an atomless part. Not only finite economies seem to describe reality better than an economy with an infinite number of agents, but also it turns out that the proofs are extremely simple and rely only on separating hyper-plane arguments.

J.C. HARSANYI: Difficulties in Classical Game Theory and Their Resolution by Means of a New Solution Concept

In classical game theory, the theories of cooperative and of noncooperative games have little logical connection. Indeed, the various solution concepts proposed for cooperative games are also largely idnependent from a logical point of view. Moreover we are given no clear criteria for using one particular solution concept, rather than another. More importantly, some empirically interesting types of games are left without any usable solution concept at all; these include games intermediate between fully cooperative and fully noncooperative games; cooperative games having a sequential structure; and cooperative games with incomplete information.

To overcome these difficulties, Reinhard Selten and I have developed a new solution concept. For any noncooperative game, we always obtain one specific equilibrium point as solution. For any cooperative game, the solution will be one specific equilibrium point of a suitably chosen noncooperative bargaining game, which models the bargaining process among the players.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

O. HART: Perfect Competition and Optimal Product Differentiation

The question addressed in this paper is: Will a Pareto optimal allocation of goods be achieved in situations where firms choose not only the quantities of goods to produce but also the qualities of these goods? The answer to this question is of course no if firms have some monopoly power. We show that the answer is also in general no even when competitive conditions prevail. We consider an economy in which there is free entry of firms subject to the payment of a set-up cost. Markets are assumed to exist only for goods which are actually produced and for a numeraire good which is consumed by all consumers. Firms choose the quantities and qualities of goods to produce so as to maximize profit, taking the quantity decisions of other firms as given (This is the Cournot-Nash assumption). The economy is made competitive by replicating consumers. It is shown that even in the limit, as the size of the economy tends to infinity, the market allocation may not be Pareto optimal. The reason is that the gains (both private and social) from several firms simultaneously producing new commodities may be much greater than the sum of the gains to each firm producing a new commodity by itself. Thus the economy may get stuck in a situation where several new commodities should be produced but where it is not in the interest of any individual firm to change its production plan.

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

M. HELLWIG: Effective Demand and Economic Coordination Failures

The paper proposes an alternative approach to the concept of fix-price equilibrium. The standard approach relies on an extra-economic agent, the keynesian auctioneer, who makes rationing constraints in different markets mutually consistent. The need for this construct disappears in a dynamic context, in which behaviour on one market is basedon expectations about constraints in other markets. Overtime, a divergence between expectations and actual experience of constraints leads to learning. A fix-price equilibrium with rational expectations arises when expected and actual constraints coincide.

However, actual constraints will also affect the accumulation and decumulation of inventories of goods and money balances. Rationality of expectations can only be expected when inventories have settled to a stationary state. This leads to stationarity of inventories as an additional condition for fix-price equilibrium.

For the 3-good model of Malinvaud, the paper analyses existence and uniqueness of a stationary state of inventories. It is shown that the comparative statics for the money wage and money price differ from the static model of Malinvaud. The existence of equilibrium with rational expectations and stationary inventories is shown. Reasons for nonuniqueness of such an equilibrium are discussed. This raises difficult problems for disequilibrium dynamics.

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

W. HILDENBRAND: On the uniqueness of mean demand for dispersed families of preferences

Let A be an open subset of a finite dimensional Euclidean space E and X a compact metric space. Let $U: X \times A \to R$ be a continous function.

Define $\varphi(a)$ as the set of elements in X which maximizes $u(.,a)$ on X. The problem is to find conditions on the family $\{u(.,a)\}_{a \in A}$ of utility functions which imply that the set

$$\{ a \in A \,|\, \# \varphi(a) > 1\}$$

of parameters in A for which there is more than one maximizer is of Lebesgue measure zero.

Def.: A family $\{u(.,a)\}_{a \in A}$ is said to be dispersed if for every $x,y \in X$ and every $a \in A$ with $x \neq y$ and $x,y \in \varphi(a)$ there exist a vector $q \in E$ and neighborhoods U_x, U_y and V_a such that for every $a' \in V_a$, $x' \in U_x$ and $y' \in U_y$ the function $u(x',.) - u(y',.)$ restricted to the segment $(L_q + a') \cap V_a$ has at most one zero where it has neither a maximum nor a minimum.

Theorem: Let the function $u: X \times A \to R$ be continous. If the family $\{u(.,a)\}_{a \in A}$ is dispersed then

$$\lambda\{a \in A \,|\, \# \varphi(a) > 1\} = 0 .$$

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

D.M. KREPS: Arbitrage and Martingales

Given a probabilistic model of a multiperiod securities market, there a no opportunities to make pure arbitrage profits if and only if there is a measure equivalent to the original measure that makes the security price process into a martingale. One can also use such "equivalent martingale measures" to say when a contingent claim's price is determined by arbitrage.

A. MAS-COLELL: Regular Economies

The lecture was a mix of introduction and survey to the theory of regular economies as initiated by Debreu and Smale and developed by many researchers (E. and H. Dierker, Balasko, K. Hildenbrand, etc.). It was presented as a development of the classical theme of the counting of equations and unknowns and it was pointed out that the reintroduction of differential methods and the generic viewpoint are having an impact on general economic theory.

The notion of regular economy was introduced first in the exchange case with emphasis on the excess demand approach. Work by Debreu, Dierker, Balasko, Smale was touched upon. Next the production case was considered in the special case of polyhedrical conical convex production sets (Kehol). Applications of the regular economies concept to obtain sharpenings of the Core and Value convergence theorem were mentioned.

R.B. MYERSON: Optimal Auction Design

An auction can be thought of as a noncooperative game with imperfect information played by the bidders. We can ask how to design an auction so as to maximize the expected revenue for the seller, and the problem turns out to be a mathematical programming problem. This talk derived the form of this problem and exhibited optimal auctions for a wide class of problems.

D. RAND: Survey of dynamical systems

This seminar was a brief survey of (i) the attempts to introduce (deterministic) dynamical systems into general economic models and (ii) the state of the art in the qualitative theory of smooth dynamical systems with particular emphasis on those areas of likely interest to mathematical economists.

J. ROSENMÜLLER: Values of Non-Sidepayment Games, Location Conflicts, and Public Goods

Values for Non-Sidepayment Games can be extended such that they are defined for games that are not "weakly superadditive". In this context, there are no "threats", but rather the players are forced to retreat from a "bliss point" to some "fair" point on the feasible set. A typical situation where this occurs, the "location conflict", is defined by a

tripel $\Sigma = (\Omega, B, U)$, where $\Omega = \{1,\ldots,n\}$, $B \subseteq \mathbb{R}^{\ell}$ (the planning area) , $U = (U^i)_{i\in\Omega}$, $U^i: B \to \mathbb{R}$ (the utility functions) such that U^i has maximizers within the planning area B (the blisspoints). It is then possible to define "fair values" for location conflicts by applying ideas from Game Theory. Consider now an Economy with public goods. Given prices $p > o$ for private goods and a taxation c^i of public goods (for each player), the players maximize utility with respect to private goods and budget constraints given by their initial holdings and their tax. This defines a utility on public goods (given prices and taxes) which turns out to feature satiations or bliss points. Thus, there is a mapping $(p,c^1,\ldots,c^n) \to \Sigma^{p,c^1,\ldots,c^n}$, assigning a location conflict to a given price and tax structure. A quadrupel $(\bar{p},\bar{c},\bar{x},\bar{y})$ = (prices, taxes, bundles of private goods, bundle of public goods) is said to be an equilibrium (with respect to a value Ψ and a system of possible taxation $\mathfrak{C}$) if

1. Utilities and taxes are compatible (and hence generate a location conflict).
2. The public goods are a "fair location" with respect to the assigned location conflict at prevailing taxes and prices.
3. Players maximize private utility within their budget constraints.
4. The public bundle can be obtained by the economies' production structure.

Theorem 1: Given proper conditions to utilities and production, for taxes arbitrary but linear the Lindahl equilibrium is a $\Psi - \mathbb{C}$ (= linear functions)- equilibrium no matter what Ψ is. (trivial)

Theorem 2: (Equal taxation). Let $\mathbb{C} = \{(q_1,\ldots,q_\ell) \mid q \in \mathbb{R}^\ell\}$ (taxation arbitrary but equal). Given proper conditions for utilities and production, there exsists a $\Psi - \mathbb{C}$ - equilibrium provided the mapping

$$\Psi : (p,q) \to \Sigma^{p,q} \to \Psi(\Sigma^{p,q})$$

is upper hemi continuous.

Note: in general the $\Psi - \mathbb{C}$ - equilibrium will not be Pareto optimal.

D. SCHMEIDLER: Informationally Decentralized Allocation Mechanisms

It is a short survey of recent results obtained by Hurwicz, Maskin, Postlewaite and the speaker: (i) There is a strategic outcome function s.t. its Nash performance correspondence coincides with the Walrasian correspondence on pure exchange standard Arrow-Debreu economies. Under the additional assumption of differentiability each Nash equilibrium is also a strong equilibrium.

(ii) If the Nash performance correspondence is Pareto efficient and individually rational, it contains the Walrasian correspondence.

(iii) Under the assumption that initial endowments are centrally

known a feasible strategic outcome function exist s.t. its Nash equilibria coincide with the constrained Walrasian equilibria.

R. SELTEN: Some Results on Experimental Economics

Experimental economics can be described as the performance and evaluation of laboratory experiments on human economic decision making. The research in this area has the purpose to test known theories and to develop new hypotheses on economic behavior.

It is the aim of this paper to present a selection of results, which seem to be important from the point of view of future theory development. No attempt is made towards completeness.

The following topics are discussed among others: intransitivity of choice behavior - presentation effects - the casuistic character of decision behavior - the short run orientation of decision behavior - the dynamics of risk taking - significance of structure, communication and information for oligopoly experiments - the expansionist bias - the equity principle - the principle of balanced aspiration levels.

R. SELTEN: Comparison of three theories for three person characteristic function experiments

Predictions derived from the bargaining set, from the equal division core and a new theory of "equitable payoff bounds" are compared with each other. The data are taken from experiments performed by Maschler, by Rapoport and Kahan and by Kahan and Rapoport.

A measure of predictive success is used which can be described as relative frequency of correct predictions minus relative size of the area covered by the theory. The equal division core gives better results than the bargaining set for games with three person coalitions and for those games without three person coalitions, where there are relatively big differences between values of two person coalitions. The bargaining set predicts better for games without the grand coalition and with relatively small differences between two person coalition values. The new theory of equitable payoff bounds gives better predictions than both of the two other theories for all categories of zero normalized three person game experiments evaluated in this study.

D. SONDERMANN: Continuity of Mean Demand

I. Mathematical Preliminaries

Theorem: Let $f: R^n \rightarrow R$ be C^∞. Then there exists a closed subset $M_o \subset M = f^{-1}(o)$ with $\lambda(M_o) = \lambda(M)$ such that for all $x \in M_o$, for all $K = 1,2,3,\ldots$, $D^K f(x) = o$

A consequence of this theorem is the following

Theorem: Let F be an n-dimensional smooth manifold of real C^2 functions defined on an open set $X \subset R^m$, satisfying:

(i) $\Phi : F \times X \rightarrow R^m$

$(f,x) \mapsto D_x f(x)$ is transversal to zero

(ii) For any $x,y \in X$, $x \neq y$, there exists an integer $K \geq o$ such that

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

$$D_f^K[f(x)-f(y)] \neq o$$

Then the set

$$C = \{f \in F: \exists\, x,y \in X\ ,\ x \neq y\ ,\ \text{such that}$$
$$D_x f(x) = D_y(f(y) = o \ \text{ and } \ f(x) = f(y)\}$$

is null in F .

(Proofs see CRMS-WP, U. Berkeley, 78)

II. Application to Mean Demand of a Large Economy.

$P = R^{\ell}_{++}$

$S = S^{\ell-1} \cap P$

$\mathcal{P}$ all C^2-preferences on P, BC , monotone

$\mathcal{P}^n$ n-dimensional smooth $C(^{\infty})$ manifold $\subset \mathcal{P}$

<u>Definition:</u> $\mathcal{P}^n$ " rich enough" $\underset{\text{dfn.}}{\longleftrightarrow}$ the map. $g: \mathcal{P}^n \times P \to S$ defined by

$$(\precsim_\rho, x) \mapsto \frac{Du(\rho,x)}{||Du(\rho,x)||}$$

is regular

(i.e. rates of substitution vary sufficiently.)

(TC) For any $x,y \in P$, $x \neq y$, there exists an integer $K \geq o$ such that

$$D^K[u(\cdot,x) - u(\cdot,y)] \neq o$$

<u>Theorem:</u> Assume $\mathcal{P}^n$ is rich enough and satisfies (TC). The

$$p \mapsto \Phi(p) = \int_A \varphi(\precsim, pe, p)\, d\mu$$

is a continuous function for any dispersed distribution μ on $A = \mathcal{P}^n \times R^{\ell}_{+}$.

Y. TAUMAN: A Characterization of Vector Measures Games in pNA

The space pNA plays a central role in the theory of non atomic-games. Aumann and Shapley in their book "Values of Non Atomic Games" ([A-S]) have proved the existence of a unique value on pNA and presented a formula which enable us to compute the value for games in pNA of the form $f \circ \mu$ where μ is a finite vector of probability NA-measures (non atomic measures) and f is a real function defined on the range $R(\mu)$ of μ with $f(o) = o$. It is natural to ask which games of the form $f \circ \mu$ are in pNA ?

A characterization for the case where μ is a scalar probability NA measure has given by Aumann and Shapley [A-S], and a characterization for the case where μ is a sign measure in NA has given by E. Kohlberg (1973).

In this paper we introduce a full characterization for all games in pNA , having the form $f \circ \mu$. We define the concept of "f is continuous at μ" and prove the following

Theorem: A necessary and sufficient condition for $f \circ \mu$ to be in pNA is that f is continuous at μ .

K. VIND: A Generalization of Additive Functions

Let $f : Y_1 \times \ldots \times Y_n \to \mathbb{R}$ be an increasing continuous function. We look for conditions on (the level surfaces of) f such that there exist functions F, (f_i), h and (g_i) such that

$$F(f(y_1,\ldots,y_n)) = \Sigma\, f_i(y_i) + h(g_1(y_1),\ldots,g_n(y_n))$$

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

where $h(z_1+k, \ldots, z_n+k) = h(z)$. Special cases of interest are $f_i = g_i$ or g_i known. Conditions under which $h = o$ are well-known (essentially independence). Economic justifications for the interest in the problem were given.

Several problems equivalent to this problem or the special cases were given. A useful lemma turns out to be

Lemma. Let $\mathcal{C}$ be a family of curves in $\mathbb{R}^n$ $(n > 2)$ (One curve through each point, curves increasing). The projection of the curves on two dimensional coordinate subspaces is again a family of curves $\Longleftrightarrow$ there exists a transformation of the axis $y_i \mapsto f_i(y)$ such that $\mathcal{C}$ becomes a family of parallel straight lines.

C. WEDDEPOHL: Rationing Schemes and Price Theory

In a model of a market for a single commodity with a finite number of firms and a decreasing demand function, fixed-price-equilibria are considered, under the assumption that the rationing scheme (in the case of excess supply) is determined by market shares. These market shares are a result of the consumer's choice of a firm. Secondly, equilibrium prices are analysed, using the fixed-price-results, where each firm determines its own price and these prices are an equilibrium if no firm can profitably change its price under certain assumptions on the behaviour of competitors.

E.-A. WEISS: Finitely-additive exchange economies

In this talk, a finitely additive exchange economy $\mathcal{E}$ is defined to be a map of the set A of agents into the space $\mathcal{P} \times \mathbb{R}^{\ell}$ of agents' characteristics such that mean endowment $\int \mathrm{proj}_2 \circ \mathcal{E}\, d\nu$ is finite, where ν is a finitely additive probability measure on the algebra of all subsets of A .

The Equivalence Theorem $W(\mathcal{E}) = C(\mathcal{E})$ is proven for an atomless (i.e. ν is atomless) finitely additive exchange economy $\mathcal{E}: A \to \mathcal{P}_{mo} \times \mathbb{R}^{\ell}_{+}$ with $\int \mathrm{proj}_2 \circ \mathcal{E}\, d\nu \gg o$.

This is interesting because

(i) A might be a countable set.

(ii) All subsets of A are admitted as coalitions.

H. WIESMETH: Non-Tâtonnement and Stability

Non-Tâtonnement-Processes with a 'fast' quantity adjustment and a 'slow' price adjustment require the notion of a 'slow manifold', the set of fixed-points of the quantity adjustment process. Problems and difficulties in constructing convergent processes arise both from the topological structure of this slow manifold and from the chosen adjustment rules.

In the framework of an exchange economy, the set of fixed-price equilibria corresponding to a given rationing scheme can be considered as the slow manifold of an adjustment process, where quantities adjust infinitely fast, and where the initial endowments depend on prices via implicit production activities. This set of fixed-price equilibria is

stratified, i.e. composed of differentiable manifolds. A further property is given by the 'parametrization property', which says that generically in each maximal stratum the fixed-price allocations depend differentiably on prices and endowments in a neighborhood of a competitive equilibrium. Prices are adjusted according to Smale's price adjustment rule in the direction of some positively weighted sum of the 'short-run'-demands of the agents. Processes should satisfy the following condition: If a process reaches a point in a boundary stratum, then it should stay in the closure of this stratum. Of course, the price adjustment rule has to be changed appropriately. To avoid stagnation in a process, negative adjustment parameters have to be allowed.

The local existence and convergence proof is again based on an appropriate choice of these adjustment parameters, resulting in an 'effective' price adjustment. By means of a Lyapunov function, asymptotic stability of the process is shown. Then, one can proceed as follows: Choose an initial fixed-price equilibrium sufficiently close to a competitive equilibrium. There exists a solution curve satisfying these initial conditions in this stratum. If this curve reaches the boundary, change the adjustment rule such that the corresponding solution curve stays in this stratum. Either you reach the competitive equilibrium or another boundary stratum, where you have to repeat this procedure. There are only finitely many strata, hence the process approaches the equilibrium.

Berichterstatter: W. Hildenbrand (Bonn)