MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 11/1979

Mathematische Stochastik

4.3. bis 10.3.1979

Die diesjährige Tagung 'Mathematische Stochastik' stand unter der Leitung von H.Föllmer (Zürich) und P.J.Huber (Cambridge,MA). In 33 Vorträgen wurde ein breites Spektrum von aktuellen Arbeitsrichtungen abgedeckt. Dabei ergaben sich Schwerpunkte in der Wahrscheinlichkeitstheorie und Statistik räumlicher stochastischer Modelle. Zu den folgenden Themenkreisen wurden längere Uebersichtsvorträge gehalten: Stochastische Felder (Zusammenhänge mit Markoffschen Prozessen, Self-Similarity, Anwendungen in der Geostatistik), Punktprozesse und ihre statistische Analyse, Diffusionsmodelle in der mathematischen Biologie, Grenzwertsätze für empirische Verteilungen, Robustheit. Neben den Vorträgen gab es an zwei Abenden informelle workshops über stochastische Felder, Robustheit und Martingaltheorie.

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Vortragsauszüge

R. AHLSWEDE

A correlation inequality

The following result was obtained jointly with D.F. Daykin (Z. Wahrsch. Verw. Geb. 43, 183-185, 1978).

Theorem: Let S be the family of all subsets of the set $\{1, 2, .., n\}$. If $\alpha, \beta, \gamma, \delta$ are non-negative real valued functions on S such that (1) $\alpha(a)\beta(b) \leq \gamma(a \cup b)\delta(a \cap b)$ for all $a, b \in S$, then (2) $\alpha(A)\beta(A) \leq \gamma(A \vee B)\ \delta(A \wedge B)$ holds for all $A, B \subset S$, where $\alpha(A) = \sum_{a \in A} \alpha(a)$, $A \vee B = \{a \cup b: a \in A, b \in B\}$ etc.

The result easily extends to (also infinite) distributive lattices. It contains as special cases - and is often sharper than - inequalities obtained by combinatorialists (Kleitman, Masica-Schönheim, Seymour, Daykin, West, Green and others) and it also implies certain correlation inequalities used in Statistical Physics (FKG, Holley).

Several other inequalities - also for other binary operations - can be found in subsequent work by the same authors (Math.Z.165, 267 - 289, 1979).

T. BEDNARSKI

A new proof of the Neyman-Pearson Lemma for capacities

Huber and Strassen 1973 have proved that for two "approximately" known simple hypotheses there exist minimax tests of the Neyman-Pearson structure. The word "approximately" is there formally expressed in terms of Choquet capacities of order two. This result known as the Neyman-Pearson Lemma for capacities is fundamental in robust testing. We look at the same problem from the view point of Le Cam's experiment theory, applying there methods of the theory. The following result is obtained:

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For two sets of probability measures $\mathcal{P}_0$ and $\mathcal{P}_1$ generated by two alternating capacities there exists a least informative binary experiment in $\mathcal{P}_0 \times \mathcal{P}_1$.

This result easily implies the Huber and Strassen theorem.

E. CARNAL

Propriétés markoviennes d'une classe de processus indexés par R^n.

Un processus gaussien indexé par R^n dont la covariance est un produit tensoriel de covariances markoviennes au sens classique du terme a de bonnes propriétés markoviennes. Il en va de même pour le processus $\{N_z\}$ $z\in\mathbb{R}^n_+$ associé à la mesure de Poisson.

H. DINGES

A renewal theorem

Die erwartete Anzahl der Besuche einer Markoff-Kette in einer Menge A hängt bekanntlich nicht sehr von der Startverteilung ab, wenn nur A weit genug davon entfernt ist.Es wird gezeigt, dass die Anzahl der Besuche (von der Startverteilung μ aus), die nach einer Stoppzeit $\tau(\mu,\nu)$ stattfinden, dieselbe ist wie die Anzahl der Besuche von der Startverteilung ν aus nach einer Stoppzeit $\tau(\nu,\mu)$.

Der Erneuerungssatz wird also hier zu einem Existenzsatz für Stoppzeiten.

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R.M. DUDLEY

Limit theorems for empirical measures

Let $(X,\mathcal{A},P)$ be a probability space, $X_1, X_2, \ldots$ independent with law P, $P_n := \frac{1}{n}(\delta x_1 + \ldots + \delta x_n)$, and $\mathcal{C} \subset \mathcal{A}$. We say the central limit theorem holds uniformly on $\mathcal{C}$ iff there exist T_n with $\mathcal{L}(T_n) = \mathcal{L}(\sqrt{n}(P_n - P))$ for all n and $\lim_{n\to\infty} \sup_{A \in \mathcal{C}} |(T_n - G)(A)| = 0$ a.s. where G is a Gaussian process, $EG(A) = 0$ and $EG(A)G(B) = P(A \cap B) - P(A)P(B)$ for all $A, B \in \mathcal{C}$. Earlier results are in Ann. Prob. 6 (1978) 899-929. We say the log log law holds uniformly on $\mathcal{C}$ iff $\lim \sup_{n\to\infty} \sup_{A \in \mathcal{C}} |\sqrt{n}(P_n - P)(A)/\sqrt{\log \log n}| < \infty$ a.s.. J. Kuelbs and I have shown that under some (P ϵ Suslin) measurability conditions, the central limit theorem uniformly on $\mathcal{C}$ implies the log log law. Recent results of Mark Durst and the author: 1) if $X = \mathbb{N} = \{0,1,2,\ldots\}$, $\mathcal{C} = \mathcal{A} = 2^X$, the limit theorems hold uniformly iff $\Sigma_n \sqrt{P\{n\}} < \infty$; 2) if the central limit theorem holds uniformly on $\mathcal{C}$ for all P on the σ-algebra $\mathcal{C}$ generates, then for some n, no set F with n elements has all its subsets of the form $A \cap F$, $A \in \mathcal{C}$; i.e. $\mathcal{C}$ is a Vapnik-Červonenkis class (Theor. Probability Appls., 1971). Open problems: 1) Can measurability conditions be removed ? 2) Extend results from classes of sets to classes of functions, with a possible application to consistency of maximum likelihood estimators; 3) If $\mathcal{C}$ = convex sets in $\mathbb{R}^3$, P= uniform law on the unit cube, does the log log law hold ?

V. DUPAC

Parameter Estimation in Poisson Fields of Circles

Discs of random size are placed at random on the plane, the centres of the discs being distributed in a Poisson field of intensity λ and the radii being distributed normally with unknown mean and variance ρ and σ^2, ($\sigma/\rho<1/3$). The effect of clumping is considered. It is assumed that circular shaped clumps can be distinguished from the others. Circular shaped clumps are formed

either by isolated discs or by discs containing other discs wholly inside. A method of estimating λ, ρ, σ^2 is suggested, based on observed area covered by clumps and on sample mean and variance of the radii of circular shaped clumps.

E. DYNKIN

Markov processes and random fields

With each Markov process X_t a random field ξ_x on the state space can be associated (intuitively, this is the total time a particle spends at x). Introducing a white noise in the path space, we construct a Gaussian field with the same second order moments. In this way, the free Euclidean field of quantum field theory can be obtained from Brownian motion. Generally, Markov property of the field constructed can be described in terms of hitting probabilities of original Markov process.

W. EHM

Local times and equidistribution properties of stable sheets

Es werden Eigenschaften der Lokal- und Aufenthaltszeiten von Multiparameterprozessen mit unabhängigen und homogenen Zuwächsen diskutiert, die charakteristisch sein dürften für Prozesse mit mehrdim. "Zeit". (1) Lokalzeiten von Mulitparameterprozessen sind glatt. Die Lokalzeit des N-Parameter Wienerprozesses z.B. ist (N-1)-mal stetig differenzierbar in der Ortsvariablen. (2) Es gelten Gleichverteilungsaussagen eines Typs, bei dem die Rolle der Zeitpunkte 0 und ∞ gegenüber der herkömmlichen Form vertauscht ist. (3) Dies hängt damit zusammen, dass im Multiparameterfall Prozesse rekurrent sein können, auch wenn sie ein lokalendliches "Potential" besitzen.

Th. EISELE

A Martingale Approach to Branching Diffusions

Infinitesimal operators of branching diffusion type $K_t = L_t + G_t$ on $\mathbb{R}^d$ are regarded, where L_t is the usual diffusion generator with drift and diffusion term and $G_t\, f\,(x) = \rho(t,x)\ \Sigma_n\, q_n\,(t,x)\ f\,(x)$ with $0 \leq \rho(t,x) \leq M$ (the branching rate) and $0 \leq q_n(t,x)$ $(n=0,2,3,\ldots n_0)$, $\Sigma_{n \neq 1}\, q_n\,(t,x) = 1$, $q_1 = -1$, (the branching law).

Transfering the operator K_t by a product rule to the higher dimensional space of finite configurations $\langle x_i \rangle$ on R^d, i.e. $\overline{K}_t\ (\prod_i f(x_i)) = \sum_i K_t f(x_i) \prod_{j \neq i} f(x_j)$ $f \in \mathcal{C}^2(\mathbb{R}^d)$, $||f|| \leq 1$ we show the existence and uniqueness of the martingale problem for $\overline{K}_t$. This result is of interest in the theory of stochastic controls of branching diffusions.

M.P. Ershov

Topics in the theory of stochastic equation

By general stochastic equations we mean equations of various kinds (algebraic, differential, integral etc.) containing random elements. In particular, stochastic differential equations. Any general stochastic equation can be reduced to the form $\underline{F}\ (m) = n$ where n is a given measure, F is a given function from one measurable space into another (the domain of n), $\underline{F}$ is the mapping induced by F which acts on measures and m is a measure on the first measurable space to be found. The "price" of this reduction is possibly a complicated form of F and the measurable spaces.

Considering this "standard" equation, it is possible to obtain useful results on the existence, uniqueness and structure of its solutions under rather general conditions.

J. FRITZ

A Remark on Lattice Dynamics

Consider the system $dx_i = v_i dt$, $dv_i = - \Sigma_{j:|i-j|=1}\,(x_i - x_j)dt - m_0 x_i dt - \lambda v_i dt + \sigma dw_i(t)$ of stochastic differential equations, where $x_i,\ v_i \in R$, $i \in Z^d$,

$m_o > 0$, $\lambda > 0$, $\sigma > 0$, $|.|$ denotes the Euclidean norm and $w_i(t)$ $(i \in Z^d)$ is a sequence of standard Wiener processes. It is easy to show that this system defines a Markov process P_t with state space $\Omega_o = \{(x_i, v_i) \mid \sup (x_i^2 + v_i^2) e^{-|i|} < \infty\}$, and that the Gibbsian lattice field μ with Hamiltonian

$$H = \frac{2\lambda}{\sigma^2} \sum_i \frac{1}{2} v_i^2 + m_o^2 x_i^2 + \sum_{j:\,|j-i|=1} (x_i - x_j)^2$$

is an invariant distribution of P_t. In the one-dimensional case (d=1) the above μ is the only translation invariant distribution of P_t such that Ω_o is of full measure. Moreover, μ is ergodic, i.e. $\mu = \lim \mu_t$ for any translation invariant μ_o with $\mu_o(\Omega_o) = 1$. If $d > 1$ then the invariant and translation invariant distributions of P_t satisfying $\nu(\Omega_o) = 1$ are described as follows: Let X^* denote the set of such (x_i) that satisfy

$$\left(\frac{m}{2d} + 1\right) x_i = \frac{1}{2d} \sum_{j:\,|j-i|=1} x_j \quad \text{and} \quad \sup x_i^2 e^{-|i|} < \infty .$$

Then the x_i in such a distribution ν are obtained as $x_i = a_i + z_i$ where $(a_i) \in X^*$, $(z_i) \in \Omega_o$ and (z_i) is distributed according to μ, while (a_i) is a measurable function of (z_i) and the set of Wiener trajectories we have. The velocities v_i are independent Gaussian variables with respect to ν, $\nu(v_i) = 0$, $\nu(v_i^2) = \frac{\sigma^2}{2\lambda}$ and (v_i) is independent of (x_i).

P. GAENSSLER

On the Functional Central Limit Theorem for Martingales

Sufficient conditions for the functional central limt theorem for martingale difference arrays are presented which are weaker than the previous ones in the literature. Also necessary and sufficient conditions are found extending results of Rootzen (Z.Wahrscheinlichkeitstheorie verw.Gebiete 38,199-210 (1977)) and giving the solution to a remaining problem posed in his paper. The present results were obtained jointly with E.Hänsler (Univ. of Munich).

H.O. GEORGII

Local and single-point characterizations of mixed Poisson and Gibbs point processes

Nguyen X.X. and H. Zessin (1976) have shown that the (grand canonical) Gibbsian point processes are characterized by an integral equation describing their behaviour if a single point is removed or added. We show that the canonical Gibbs processes admit a similar characterization by means of an equation which describes what happens if the position of a single point is changed. If the interaction potential is sufficiently homogeneous then the canonical Gibbs processes are exactly the mixtures of Gibbs processes with different values of a certain parameter. This result generalizes a characterization of the mixed Poisson processes (due to K. Nawrotzki 1962 and D.A. Freedman 1962) which is the point process counterpart of de Finetti's representation theorem.

X. GUYON

Propriétés-Hida markoviennes pour les processus spatiaux

On generalise ici les propriétés de Hida etudiées pour les processus gaussiens à indice dans $\mathbb{R}$ (Hida (1961), Mandrekar (1974), Pitt(1975)) aux processus du second ordre, centrés, à paramètre dans $\mathbb{R}^2$. On dira que le processus Y_t est Hida si $\mathcal{E}_t^a = Sp\{E(Y_{t'}|\mathcal{F}_t), \ t'\geq a\}, \ a\geq t$ est indépendant de a, de dimension finie, $\mathcal{F}_t$ étant la tribu naturelle de Y_t. Alors, si Y_t est Hida, on peut partitionner E en domaine connexes maximaux sur lesquels Y_t sera n-Hida, admettant donc une n-représentation de Goursat. On introduit ensuite une propriété $\mathcal{G}$-Hida relativement aux tribus $\mathcal{G} = \mathcal{F}^1 \vee \mathcal{F}^2$. Si un processus admet une representation de Goursat, l'une des martingales n'etant pas forte, il n'est jamais $\mathcal{G}$-Hida. Reciproquement, un processus $\mathcal{G}$-Hida est Hida, admettant une representation de Goursat forte.

F. HAMPEL

Some recent results in robustness

The problem of minimizing the asymptotic covariance matrix of an estimator under a given bound on its "gross-error-sensitivity" has been solved for vector parameters and vector observations. This result has many applications, including estimation of location and scale, covariance matrices, and regression with random "carriers".

In robust regression, a number of proposals have been made on how to diminish the influence of "leverage points" in unbalanced designs. The two classes deriving naturally from theory were also the best ones empirically. Yet there are still open questions about details.

A small sample asymptotic formula has been found for M-estimators of location which gives excellent results far into the extreme tails, e.g. for a Huber-estimator under the Cauchy distribution, down to about n=3. The method is a variant of the saddlepoint method which expands f'/f; since with this operator an n-fold convolution turns into smoothing and multiplication by n, and a normal distribution turns into a straight line, the method can be of interest also in other areas of probability theory, such as the central limit problem.

J. KENT

Infinite divisibility and diffusions

All hitting times for one-dimensional diffusions are infinitely divisible. This general result can be used to given simple proofs of infinite divisibility for some common distributions. Examples include the inverse of the gamma distribution (and hence also the t-distribution) the von Mises-Fischer distribution.

This technique cannot, in general, be used in higher dimensions, but it can be used in the following specific problem: Consider Brownian motion in the plane with constant drift, started at the origin and stopped at the unit circle. Then this hitting time is infinitely divisible.

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K. KRICKEBERG

Some problems and the statistical analysis of point processes

A survey on some basic ideas of statistical inference based on the observation of a point process. We are given a point process $\tilde{\mu}$ whose realizations are point measures $\mu = \tilde{\mu}(\omega)$ in a locally compact space X with a countable base, and another random element $\rho = \tilde{\rho}(\omega)$ in some set such that $\tilde{\mu}$ and $\tilde{\rho}$ are jointly distributed. Let P and V be the laws of $\tilde{\mu}$ and $\tilde{\rho}$, respectively. Take a bounded (i.e. relatively compact) domain K in X and observe one realization $\tilde{\mu}(\omega)$ within K. The problem is either that of inference about V (ordinary inference), or that of inference about $\tilde{\rho}(\omega)$ (filtering). Both the one of a fixed domain K and the asymptotic theory for K X are considered.

The following models are treated:

1. Families of Poisson processes, Cox processes and inference about their intensities.
2. Methods based on the natural order in $X = \mathbb{R}_+$: inference about the predictable intensity.
3. Stationary models in $\mathbb{R}^d$: methods connected with the ergodic theorem by Nguyen Xuan Xanh and H. Zessin.
4. Models given by "specifications" (conditional probabilities).
5. A Model by T. Schweder for estimating the intensity of an animal population in motion by observations from a line transect.

S. KUSUOKA

Markov fields and local operators

Many examples of Markov fields with continuous multidimensional parameters which have been known, have the form like $L^{-1}W$ where L is a linear local operator and W is a white noise. Here we show the proposition, in some sense, that if Y is a Markov field and L is a local linear operator, then $L^{-1}Y$ is Markov. So we can construct many new examples of Markov fields.

G.LEHA

Conditional Probabilities and Gibbs Representation

A necessary and sufficient condition is stated in order to have a Gibbsian representation for a given collection of (strictly positive) conditional probabilities in discrete stochastic fields. The potentials used here may have a "macroscopic", i.e. a tail-measurable, component.

P. MAJOR

Some new results on self-similar fields

We are interested in the limit distribution of partial sums of random variables. No independence or "quasi-independence" is assumed. We investigate a modified problem: Let X_n, $n= ..-1,0,1,..$ be a stationary sequence, and define the sequence

$$Y_n^N = A_N^{-1} \sum_{j=(n-1)N}^{nN-1} X_j \qquad n= ..-1,0,1,.., \; N=1,2,..,$$

where A_N is an appropriate norming constant. We want to prove limit theorems for Y_n^N as $N \to \infty$. We have proved such theorems if X_n is of the form $X_n = H(\xi_n)$, where $\ldots \xi_{-1}, \xi_0, \xi_1, \ldots$ is a stationary Gaussian sequence, $E\,\xi_0 \xi_n \sim n^{-\alpha}$, $H(x)$ is a real function, $E\,H(\xi_0) = 0$, $E\,H(\xi_0)^2 < \infty$. If $\alpha > 0$ is small and the expansion of $H(x)$ by the Hermite polynomials does not contain $H_1(x) = x$ then the limit belongs to a class of non-Gaussian self-similar fields constructed by Dobrushin.

P. MANDL

Adaptive Replacement Policies

Renewal of a component with preventive and service replacements is considered. The failure distribution is assumed to be unknown. Adaptive replacement policies to minimize the average cost are presented. The method of cost potentials is used to investigate their asymptotic behaviour.

R. MARONNA

Asymptotic behaviour of general M-estimates for regression and scale with random carriers

Let $(\underset{\sim}{x}_i, y_i)$ be a sequence of independent identically distributed random variables, where $\underset{\sim}{x}_i \in R^p$ and $y_i \in R$, and let $\underset{\sim}{\theta} \in R^p$ be an unknown vector such that $y_i = \underset{\sim}{x}_i{}'\underset{\sim}{\theta} + u_i$, where $\underset{\sim}{u}_i$ is independent of $\underset{\sim}{x}_i$ and has distribution function $F(u/\sigma)$, where $\sigma > 0$ is an unknown parameter. This paper deals with a general class of M-estimates of regression and scale, $(\underset{\sim}{\theta}^*, \underset{\sim}{\sigma}^*)$, defined as solutions of the system $\Sigma_{i=1}^n \; \phi(\underset{\sim}{x}_i, (y_i - \underset{\sim}{x}_i{}'\underset{\sim}{\theta})/\sigma) \; \underset{\sim}{x}_i = 0$ and $\Sigma_{i=1}^n \chi(|y_i - \underset{\sim}{x}_i{}'\underset{\sim}{\theta})|/\sigma) = 0$, where $\phi : R^p \times R \to R$, and $\chi : R \to R$. Several proposals for the robust estimation of $(\underset{\sim}{\theta}, \underset{\sim}{\sigma})$ are contained in this class, e.g. classical M-estimators, Mallows, Schweppes, and Hampel estimators. The consistency and asymptotical normality of the general M-estimators are proved assuming general regularity conditions on ϕ, χ and F.

P. A. MEYER

A predictable protection theorem for two parameter processes

First it is shown that given a bounded measurable process depending on a parameter $u \in U$, that is $X(u,t,\omega) : U \times R_+ \times \Omega \to R$ measurable of the

triple, then it has a predictable projection $Y(u,t,\omega)$, $\mathcal{U}\times\mathcal{P}$-measurable, such that for any bounded predictable random measure $\mu(\omega, dt\times A)$ $E[\mu(X)]=E[\mu(Y)]$. Here predictable μ means that for any $A\in\mathcal{U}$ $\mu(\omega, dt\times A)$ is a predictable random measure in the ordinary sense. Such Y is unique in a very strong sense.

The result is applied to processes indexed by $\mathbb{R}_+^2$ (The details are given in vol. XIII of the Strassb. Seminar).

M. NAGASAWA

Segregation and Spatial Patterns of a Population in an Environment

A population shows typical spatial patterns in its equilibrium distributions when it is in an environment that can affect individuals in the population. What we are considering is a population of animals, insects, bacterium etc. We assume that the movement of individuals can be approximated as sample paths of a diffusion process. We define Kinetic energy, Intensity energy in terms of an equilibrium distribution density ϕ (or ψ such that $|\psi|^2=\phi$); $K(x)=\frac{1}{2}(\frac{1}{\psi}\frac{\partial\psi}{\partial x^i})(\frac{1}{\psi}\frac{\partial\psi}{\partial\psi^j})\,a^{ij}(x)$, $Q(x)=\frac{1}{2}(\frac{-1}{\phi}\Delta\,\phi)$. Introducing an environment potential $V(x)$, we define the total energy by $H(x) = K(x) + Q(x) + V(x)$. Then we say that a population is in the equilibrium state of λ (excitation parameter) if $H(x) = \lambda$. By means of "Lemma: $\{K(x) + Q(x)\}\psi = -\frac{1}{2}\Delta\psi$ ", we have, "Theorem: ψ and λ satisfy $\frac{1}{2}\Delta\psi-(V(x)-\lambda)\psi=0$". Thus we obtain a variety of excited states. However, the obtained diffusion process of an excited state is singular. To discuss this singular diffusion, Theorem of segregation is crucial. Let T be the first hitting time to $\partial D=\{x|\phi(x)=0\}$, then there exists a neighbourhood U of ∂D and $P_x[T<\infty$, or $X_t \in U$ $\forall t>\exists t_0]=0$. Several examples are explained, distributions of monkey populations, splitting of Coli (and a mutant Coli), distributions of insects around favourable and unfavourable areas. Remarks are given to Nelson's paper (1966).

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J. NEVEU

On point processes

Three questions have been discussed in this talk.

a) by the use of a simple ergodic lemma for point processes many convergence theorems for point processes constructed by branching procedures can be proved very simply.

b) When is a point process formed by jumps of a Markov process, i.e. when is $N_t = \Sigma_{s \le t} 1_F(X_{s-}, X_s)$ where $F \subseteq (E \times E) \setminus (\text{diagonal})$, a Poisson process. Several sufficient conditions are given, among others 1) $\int A(x,dy)\, 1_F(x,y)$ is constant in x (then N_t^F is a $(\mathcal{F}_t)$-Poisson process for every initial point), where A denotes the generator of the process. 2) $\int_x \mu(dx)\, A(x,dy)\, 1_F(x,y) = \alpha \cdot \mu(dy)$ if μ is the invariant measure of the Markov process which is supposed recurrent irreducible (then for the initial measure μ, $(N_s^F, s \le t)$ is a Poisson process and furthermore independent of $(X_s, s \ge t)$). But these conditions can also be combined with reduction of Markov process, i.e. by considering $(\phi(X_t), t \in R_+)$ for a ϕ which preserves the Markov character.

c) The problem of choosing optimal disciplines in queueing systems with several servers was discussed; an example where the cyclic discipline is "better" than the "minimal load" discipline considered in Borokov's book was constructed.

F. PAPANGELOU

On the entropy of point processes

Given a stationary point process on $(-\infty,\infty)$ with finite intensity denote by $N[a,b)$ the number of points of the process in $[a,b)$, by $\mathcal{F}_t$ the σ-field of events generated by the process in $(-\infty,t)$, and by P_c the Palm probability of the process. For any $\varepsilon > 0$ consider the random variables $\dots, \xi_{-1}^{\varepsilon}, \xi^{\varepsilon}, \xi_1^{\varepsilon}, \dots$ where

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ξ_n^ε is 1 or 0 according as $N[n\varepsilon,(n+1)\varepsilon)$ is ≥ 1 or $=0$, and let $p_\varepsilon(i|\xi_{-1}^\varepsilon,\xi_{-2}^\varepsilon,\ldots) = P(\xi_0^\varepsilon=i|\xi_{-1}^\varepsilon,\xi_{-2}^\varepsilon,\ldots)$, $i=0,1$. If $P_0 \ll P$ on $\mathcal{F}_0$ then (1972) $\lim \frac{1}{\varepsilon} P(N[t,t+\varepsilon)\geq 1|\mathcal{F}_t) = \Lambda_t$ exists in L_1 for every t and if in addition $E[\Lambda_0 \log \Lambda_0]<\infty$ then one can prove that $\frac{1}{\varepsilon}\log p_\varepsilon(\xi_0^\varepsilon|\xi_{-1}^\varepsilon,\ldots) = \frac{1}{\varepsilon}\xi_0^\varepsilon(\log \Lambda_{\tau_1} - \log\frac{1}{\varepsilon}) - \Lambda_0 + o(1)$ in L_1 as $\varepsilon \to 0$, where τ_1 is the first non-negative point of the process. A number of limit theorems can be derived from this. For example

$$\log p_{\frac{1}{n}}(\xi_0^{\frac{1}{n}},\xi_1^{\frac{1}{n}},\ldots,\xi_{n-1}^{\frac{1}{n}}|\xi_{-1}^{\frac{1}{n}},\ldots) + (\log\frac{1}{\varepsilon})\sum_{k=0}^{n-1}\xi_k^{\frac{1}{n}} \xrightarrow[L_1]{} \int_0^1 \Lambda_t dt + \sum_{0\leq\tau_i\leq 1}\log\Lambda_{\tau_i}$$

where the τ_i's are the points of the process in $[0,1]$ and the first term on the left has the obvious meaning. The right-hand side is closely related to the logarithm of the Radon-Nikodym density of the distribution of the given point process in $[0,1]$ with respect to the distribution of the the standard Poisson process in $[0,1]$ (likelihood function; I.Rubin, D.L.Snyder 1972).

C.PRESTON

A note on specifications

Let $(X,\mathcal{F})$ be a measurable space and $(\mathcal{F}_\Lambda)_{\Lambda\in I}$ a decreasing collection of sub-σ-algebras of $\mathcal{F}$ (I being a partially ordered index set). Also let $\mathcal{V} = (\pi_\Lambda)_{\Lambda\in I}$ be a specification corresponding to $(\mathcal{F}_\Lambda)_{\Lambda\in I}$ (in the sense of Föllmer). We can construct new specifications from $\mathcal{V}$ as follows: for each $\Lambda\in I$ let $w_\Lambda : X \to R^+$ be $\mathcal{F}$-measurable; let $z_\Lambda^w(x) = \int w_\Lambda(y)\,\pi_\Lambda(x,dy)$ and define $\pi_\Lambda^w : X\times\mathcal{F} \to R^+$ by

$$\pi_\Lambda^w(x,F) = \begin{cases} [z_\Lambda^w(x)]^{-1}\int_F w_\Lambda(y)\,\pi_\Lambda(x,dy) & \text{if } 0 < z_\Lambda^w(x) < \infty \\ 0 & \text{sonst} \end{cases}$$

Wir haben die Frage: Gegeben μ ein W-mass auf $(X,\mathcal{F})$, können wir $(w_\Lambda)_{\Lambda\in I}$ finden, so dass $(\pi_\Lambda^w)_{\Lambda\in I}$ eine Spezifikation ist, und auch μ ein Gibbsmass bezüglich (π_Λ^w) ist? Notwendig ist, dass wir $\mu \ll \mu\pi_\Lambda$ für alle $\Lambda\in I$ haben. Nehmen wir an, dass dies gilt. Dann haben wir mit nur einer zusätzlichen Bedingung, dass (w_Λ) gefunden werden kann.

P. RÉVÉSZ

On the non-differentiability of the Wiener sheet

Let $(W(x,y): x \geq 0, y \geq 0)$ be a Wiener sheet. The following is proved:

$$\lim_{h\to 0} \inf_{(x,y)\in I^2} \inf_{0<\alpha\leq 2\pi} \sup_{0<s\leq 2\pi} \gamma_h |W(x+s\cos\alpha, y+s\sin\alpha)-W(x,y)| = 1$$

with probability 1, where $I^2 = [0,1]\times[0,1]$ and $\gamma_h = \left(\frac{8\log h^{-1}}{\pi^2 h}\right)^{1/2}$. This result clearly implies that a Wiener sheet is nowhere differentiable with probability 1.

D. RIPLEY

Goodness of fit tests for spatial point processes

Spatial point processes can be sampled in several ways: by quadrats, by nearest neighbour distances, by interpoint distances and by estimating moment measures. All these approaches lead to tests of "randomness", i.e. of a Binomial or Poisson process but some are better suited to test the fit of other processes and to summarize the data. Edge effects are an important ingredient of the problem which are often ignored yielding erroneous conclusions. The field will be surveyed and the results of some comparison studies presented.

K. SCHUERGER

On the asymptotic geometrical behaviour of a class of contact-interaction processes

For a class of contact-interaction processes (introduced by T.E. Harris (1974)) having contact intensities $c_i > 0$, $i \geq 1$, and in which particles cannot die, a "strong law of large configurations" (SLLC) could be proved. To formulate it, let $T(x)$, $x \in Z^d$, denote

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the first instant at which x is occupied, and put $\tau(y) = \tau(x)$ if $y \in Q_x$ $(x \in Z^d)$, where $Q_x = \{y | x^i - \frac{1}{2} < y^i \leq x^i + \frac{1}{2}\}$. Then we have

Theorem (SLLC) let $c_1 \leq c_2 \leq \ldots \leq c_{2d}$. Then there exists a norm $N(.)$ on R^d such that for all finite nonvoid configurations ξ and for all $0<\varepsilon<1$ we have a.s. (P_ξ) for all sufficiently large t

$$\{x | N(x) \leq (1-\varepsilon)t\} \subset \{x | \tau(x) \leq t\} \subset \{x | N(x) \leq (1+\varepsilon)t\}.$$

P. SWITZER

Application of Spatial Random Fields to Geostatistical Problems

The concentration of metals in ore is modelled as a linear function of control variables plus a spatially correlated random field. The concentration data occurs on a finite subset of the three-dimensional field. One problem is to interpolate the field from the data at each point. Linear unbiased estimators using second-order properties of the field are usually used, although certain kinds of non-linear estimators are now being proposed which more nearly approximate conditional expectation estimators. These non-linear estimators involve a transformation of the field to an approximate Gaussian field, pointwise estimation in the Gaussian context, then transformation back to the original scale. The next problem is to estimate the conditional distribution of the field at each point given the data and finally the estimation of the distribution of average concentration over missing blocks (a particular convolution of the field) using the available data.

D. SZASZ

On the motion of an infinite subsystem of particles

The diffusion problem is investigated for the simultaneous motion of several, possibly an infinite number of, particles. These particles are situated in an equidistant way at time 0. They are understood as being inserted into the Harris collision

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model, which has been defined on R^1. If the initial distances between the particles increase in a suitable way, then one obtains a non-trivial limit theorem. The insertion of particles at increasing distances is an analogue of the renormalization in cases when we are interested in a vertical behaviour.

W.R. VAN ZWET

On a Theorem of Hoeffding

Let $X_1, \ldots, X_n$ be independent random variables taking values in ^{m}, let X_i have distribution F_i and write $F = (F_1, \ldots, F_n)$, $F_. = n^{-1} \Sigma F_i$, $\bar{F} = (F_., \ldots, F_.)$ and E_F (or $E_{\bar{F}}$) for expectations under the model F (or $\bar{F}$). Define

$\mathcal{F}_k = \{F=(F_1,\ldots,F_n): \exists A \subset \mathbb{R}^m$ such that card(A)=k and

$F_i(A)=1$ for $i=1,\ldots,n\}$, let $S_n = \Sigma X_i$ and $g: \mathbb{R}^m \to \mathbb{R}$, measurable. Finally let $\mathcal{G}_k$ be the class of functions g for which $E_F g(S_n) \leq E_{\bar{F}} g(S_n)$ for all $F \in \mathcal{F}_k$ and all n, and let $\mathcal{G}_\infty = \cap \mathcal{G}_k$. A general characterization of $\mathcal{G}_k$ is given and for various m and k this leads to an explicit description of all functions in $\mathcal{G}_k$. The results generalize a theorem of Hoeffding on Poisson-binomial trials (Ann. Math. Statist. 27 (1956), 713-721) and were obtained in collaboration with P.J. Bickel.

A. WAKOLBINGER

Desintegration w.r. to the Tail Field

Starting from the question which families of point processes may be characterized as Gibbs processes to a common local specification (to which a partial answer is given at least for Σ-processes - i.e. processes with "non degenerate vacuum"), various concepts of singularity for a family of processes are discussed. This is motivated by the fact that the extremal Gibbs processes (or "pure phases") to a local specification

possess a tail-measurable "support function", which attaches in a measurable way to each configuration the pure phase it originates from. A family of processes having a support function forms the extreme points of a family of processes having an H-sufficient statistic (in the sense of Dynkin 1978) and vice-versa. Finally, the tail desintegration of Cox processes is considered and a charäcterization theorem for processes being conditionally Poisson under the tail field is given.

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