MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 12/1979

Stetige und meßbare Selektionen

14. 3. bis 17. 3. 1979

Die Tagung fand, anläßlich des Deutschlandbesuches von Herrn Professor E.A. Michael, U.S.A., unter der Leitung von Herrn Professor Kölzow (Erlangen) statt. In mehreren Vorträgen wurden neuere Resultate über stetige und meßbare Selektionen sowie deren Anwendungen in verschiedenen Gebieten der Analysis, vorgestellt. Hauptzweck dieser Arbeitstagung war es jedoch, den Anwesenden Gelegenheit zu ausführlichen Diskussionen zu geben, was, durch den kleinen Teilnehmerkreis (9) begünstigt, auch mit Erfolg verwirklicht werden konnte.

Vortragsauszüge

G. NÜRNBERGER

Continuous selections in the theory of best approximation

Let G be an n-dimensional subspace of $C[a,b]$. The metric projection (associated with G) is the set-valued map P_G defined

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by $P_G(f) := \{g_o \in G: \|f-g_o\| = \inf\{\|f-g\|: g \in G\}\}$. For n = 1, BLATTER, MORRIS and WULBERT gave necessary and sufficient conditions on G for P_G to have a continuous selection. The talk treated the case n>1. (Michael's theorem cannot be applied since by a result of LAZAR, MORRIS and WULBERT P_G is lower semicontinuous only if it is single-valued). Two classes of spaces were considered: $\mathcal{G}_1 := \{G \subset C[a,b] : g^{-1}(o)$ contains no interval for $g \in G$, $g \neq o\}$ and $\mathcal{G}_2 := \{G \subset C[a,b] : G \notin \mathcal{G}_1\}$.

Theorem 1: For G in $\mathcal{G}_1$ the following are equivalent:

(1) P_G has a unique continuous selection

(2) (a) G is weak Chebyshev, i. e. each g in G changes sign at most (n-1) times

(b) Each $o \neq g \in G$ has at most n-1 zeros.

Theorem 2: If there exists a continuous selection for P_G, then G is weak Chebyshev.

Important weak Chebyshev spaces in $\mathcal{G}_2$ are the spaces $S_{N,K}$ of splines with K fixed knots and degree N.

Theorem 3: If G = $S_{N,K}$, there exists a continuous selection for P_G iff $K \leq N+1$.

(All results due to NÜRNBERGER and/or SOMMER)

P. KENDEROV

Dense strong continuity of pointwise continuous mappings

Let Y be a topological space, (Z,d) be a metric space and C(Y,Z) the space of bounded continuous functions from Y to Z. For a given topological space X and a pointwise continuous map T from X to C(Y,Z) a theorem was proved, asserting that (under certain conditions) the points where T is continuous with respect to the uniform topology on C(Y,Z) form a dense

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G_δ-subset of X. A "set-valued" version of this theorem was also proved. These results were used to get information about points of continuity of (multi-valued) monotone operators and metric projections. As corollaries some known results about Gâteaux and Fréchet differentiability of convex functions on dense subsets were obtained.

C. PORTENIER

The theorem of Choquet-Deny for lower semicontinuous functions

Let S be a convex cone of lower semicontinuous functions on a compact metric space X such that every lower bounded sequence in S has an infimum in S. Further suppose that S is closed with respect to the topology of graph convergence introduced by MOKOBODZKI. Using Michael's selection theorem the following was shown:

There exists a family (T_i) of positive operators on C(X) and a family (m_j) of positive Radon measures on X, such that S is exactly the set of lower semicontinuous functions s on X satisfying $T_i(s) \leq s$ and $m_j(s) \leq o$ for all i and j.

A.G. BABIKER

Measurability of sections for maps representing certain measures

Two types of Radon probability measures on a compact Hausdorff space X were considered: uniformly regular measures - i.e. those admitting uniform outer approximation on compact sets (Journ. reine angew. Math. 1977) - and Lebesgue measures (Mathematika 1977). Such measures are characterized by the existence of a compact metric space T and a continuous surjection

p from X onto T whose sections have certain measurability properties. It was proved that a non-atomic measure m is

(1) uniformly regular iff there exists T compact metric and p: X→T continuous, onto such that every section of p is Baire - p(m) - measurable.

(2) uniformly regular and completion regular iff for p and T as in (1) all sections are Borel - p(m) - measurable.

(3) Lebesgue iff there exists p and T as in (1) admitting a Lusin - p(m) - measurable section.

(4) topologically Lebesgue iff all sections for p and T as above are Lusin - p(m) - measurable.

G. MÄGERL

Some recent results concerning the parametrization of measurable selections

The following problem was considered: Let $(X,\mathfrak{A})$ and $(Y,\mathfrak{B})$ be measurable spaces and $F: X \to Y$ be a set-valued map. Is it possible to find a "nice" measurable space P (or a "nice" topological space P) and a map $f: X \times P \to Y$ such that

(i) $f(\cdot, p)$ is a measurable selection for F ($p \in P$)

(ii) $f(x,\cdot)$ is a Borel isomorphism (a continuous map) from P onto F(x) ($x \in X$)

(iii) f is measurable on the product space ?

The talk gave a survey of recent results due to CENZER-MAULDIN, MAULDIN, IOFFE and SARBADHIKARI-SRIVASTAVA solving this problem in some particular cases, namely - roughly speaking - when X and Y are Polish spaces with their Borel-σ-algebras and F satisfies various measurability conditions.

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