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Algebraische Gruppen

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Die international zusammengesetzte Tagung "Algebraische Gruppen" wurde von T.A. Springer (Utrecht) und J. Tits (Paris) geleitet.
Die Vorträge konzentrierten sich um die folgenden Schwerpunkte:

- Studium der Darstellungen halbeinfacher algebraischer Gruppen über Körpern positiver Charakteristik: Weyl-Moduln, Bestimmung der Kompositionsfaktoren, Berechnung von Multiplizitäten, kohomologische Methoden, Zusammenhänge mit der Invariantentheorie
- Operationen reduktiver Gruppen auf Varietäten: Konjugationsklassen in reduktiven algebraischen Gruppen, Singularitäten in Abschlüssen von Konjugationsklassen
- Arithmetik algebraischer Gruppen: Kohomologie arithmetisch definierter Gruppen, Beziehungen zur Theorie der automorphen Formen
- Darstellungen von Weylgruppen: Lusztig's Vermutung, Zusammenhänge mit der Theorie der Singularitäten .

Weitere Vorträge beschäftigten sich mit Anwendungen der Theorie der Gebäude, eindimensionalen Gruppenschemata, der Geometrie von Schubertvarietäten sowie kommutierenden Varietäten in halbeinfachen algebraischen Gruppen.
Die vorgestellten neuesten Ergebnisse ersehe man aus den Vortragsauszügen.

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Vortragsauszüge

C. Procesi: Normality of closures of conjugacy classes

Let $X \in M_n(\mathbb{C})$ be a nilpotent matrix and C_X its conjugacy class.

Theorem: The closure $\overline{C_X}$ is normal, Cohen-Macaulay with rational singularities.

The main point is to prove normality. Let $V_o = \mathbb{C}^n, V_1, V_2, \ldots$ be vectorspaces of dimension $\dim V_i := \dim \mathrm{Im} X^i$ and consider the subvariety Z of $\mathrm{Hom}(V_o,V_1) \times \mathrm{Hom}(V_1,V_o) \times \mathrm{Hom}(V_1,V_2) \times \mathrm{Hom}(V_2,V_1) \times \ldots$ defined by the equations

$$A_oB_o = B_1A_1 \ , \ A_1B_1 = B_2A_2 \ , \ \ldots \ , \ A_{s-1}B_{s-1} = o \ :$$

$$V_o \underset{B_o}{\overset{A_o}{\rightleftarrows}} V_1 \underset{B_1}{\overset{A_1}{\rightleftarrows}} V_2 \rightleftarrows \ \cdots \ \underset{B_{s-1}}{\overset{A_{s-1}}{\rightleftarrows}} V_s \underset{o}{\overset{o}{\rightleftarrows}} V_{s+1} = o \ .$$

It turns out that Z is a complete intersection and smooth in codimension 1, hence normal. Furthermore the map $\pi : Z \to M_n(\mathbb{C}), (A_o,B_o,A_1,B_1,\ldots) \longmapsto B_oA_o$, is a quotient with respect to the natural action of $GL(V_1) \times GL(V_2) \times \ldots$ with image $\overline{C_X}$, which implies that $\overline{C_X}$ is normal too. The proofs are based on the classification of nilpotent pairs $U \underset{B}{\overset{A}{\rightleftarrows}} V$ with respect to $GL(U) \times GL(V)$, in particular on a dimension formula relating the dimension of such a nilpotent pair orbit with the dimensions of the conjugacy classes AB and BA.

The method applies also to the other classical groups. In this case $V_o = \mathbb{C}^n$ is a vectorspace with a non degenerate symmetric or alternating form and V_i

will be provided with a form of opposite sign than V_{i-1}. Now Z as a subvariety of $\mathrm{Hom}(V_o,V_1) \times \mathrm{Hom}(V_1,V_2) \times \dots$ is defined by the equations

$$D_oD_o^* = D_1^*D_1 \ , \ D_1D_1^* = D_2^*D_2 \ , \ \dots$$

Again Z is a complete intersection having the given conjugacy class $\overline{C_X}$ as a quotient, but in general Z is not normal. Nevertheless this construction implies the following result:

Proposition: $\overline{C_X}$ is normal if and only if it is normal in its codimension 2 classes.

The problem of codimension 2 singularities is dealt with in the following talk.

H. Kraft: Singularities in closures of conjugacy classes

The nilpotent conjugacy classes in sl_n, so_n or sp_n (with respect to SL_n, O_n or Sp_n) are given by partitions of n, i. e. by Young diagrams η. We denote by C_η the corresponding conjugacy class. Now for sl_n we have the following result:

Proposition: Let $C_\sigma \subset \overline{C_\eta}$ and assume that the first r rows and the first s columns of η and σ coincide. Denote by η', σ' the diagrams obtained by removing these rows and columns. Then the singularity of $\overline{C_\eta}$ in C_σ is smoothly equivalent to the singularity of $\overline{C_{\eta'}}$ in $C_{\sigma'}$.

Corollary: If $C_\sigma \subset \overline{C_\eta}$ is of codimension 2 the singularity of $\overline{C_\eta}$ in C_σ is simple of type A_j.

For the other classical Lie algebras one has a result similar to the proposition which restricts the study of codimension 2 singularities to 5 cases. One case corresponds up to smooth equivalence to the closure of a non connected conjugacy class. Hence in this case which will be called exceptional in the following (and can easily be read of the Young diagram) we have a non normal closure. More precisely:

Theorem: Let C be a nilpotent conjugacy class in so_n or sp_n. Then $\overline{C}$ is normal if and only if C is not exceptional; in this case the codimension 2 singularities of $\overline{C}$ are all simple of type A_j or D_j. Furthermore all closures of conjugacy classes are seminormal. (Seminormal means that every homeomorphic regular map $X \longrightarrow \overline{C}$ is an isomorphism.)

The smallest examples of (connected) conjugacy classes with non normal closures are $C_{(3311)} \subset sp_8$ and $C_{(44221)} \subset so_{13}$.

J. E. Humphreys: Weyl Modules

This talk is a survey of results and open questions about Weyl modules for a semisimple algebraic group over an algebraically closed field of prime characteristic. To each dominant weight corresponds a Weyl module $V(\lambda)$, with formal character given by Weyl's formula. The following aspects will be discussed: (1) characterization of $V(\lambda)$ by its universal property; (2) determination of the composition factors of $V(\lambda)$: strong linkage and translation principles, generic patterns (due to Jantzen); (3) submodule structure of $V(\lambda)$: filtrations, minimal submodules;

(4) homomorphisms and tensorproducts of Weyl modules. Connections with Andersen's work on sheaf cohomology are emphasized.

L. Scott: <u>On the structure of representations in characteristic p</u>

Let G be a connected affine algebraic group over an algebraically closed field of characteristic p, and let B be a Borel subgroup. The structure (meaning at least some kind of filtration)of the common representations one studies is not understood, even for type A_2. As a last resort one might think about actually building some modules, perhaps using the following and related results:

<u>Theorem</u>(CPS): A rational B - module V extends to a rational G - module iff V extends to every minimal parabolic subgroup Q containing B.

In the spirit of thinking of less drastic approaches, certain non-reduced groups between B and G are discussed (the group schemes BG_r or PG_r with P a parabolic subgroup containing B and G_r the kernel of the r-th power of the Frobenius endomorphism) and a tensorproduct-theorem is proved for their irreducible representations. A method is sketched for determining the P - module filtrations arising in Kempf's proof of the vanishing theorem, and it is suggested that perhaps the algebraic geometry will not save us from some work, such as taking a detailed look at structure theoretic questions.

W. Haboush: <u>Representations of rings of differential operators on semisimple groups</u>

Let G be semisimple and simply connected over an algebraically closed field of characteristic p > o. Let P be the weight lattice

and let R be the root lattice. Let $\hat{P}$ (resp. $\hat{R}$) denote the p-adic completion of P (resp R). Let $D_{G/k}$ denote the invariant differential operators on G and let $Z_G \subset D_{G/k}$ denote its center. Let $D^{(v)}$ denote the dual of the kernel of v-th order Frobenius. Let τ be the character function of the Steinberg module $(p-1)\cdot\rho$, $\rho = \frac{1}{2}\sum$ positive roots. I spoke about my attempts to determine the image of Z_G in the locally constant functions from $\hat{P}$ to k and especially about my proof of the following:

<u>Theorem:</u> The image of Z_G in the functions on $\hat{P}$ is the set of $\tilde{W}$ invariant functions where

$$\tilde{W} = \{\delta \in \mathrm{Aut}_{\hat{Z}_p}(\hat{P}) \mid \delta(x) - w(x) \in p\hat{R},\ w \in \text{Weylgroup of } G\}$$

and $\tilde{W}$ acts on the functions by

$$w \circ f(\lambda) = f(w(\lambda + \rho) - \rho).$$

I described how to construct central operators by lifting through Frobenius.

W. H. Hesselink: <u>Desingularizations of varieties of nullforms</u>

Let G be a reductive group acting on an affine variety V. Assume that V is non-singular at a G - invariant base point $*$. We consider the nullform scheme $N(V) = \{v \in V \mid f(v) = f(*), \forall f \in A(V)^G\}$. If $v \in N(V)$, a procedure of Kempf and Rousseau gives us a class $\Lambda(v)$ of fractional powers of one-parameter subgroups $\lambda: GL(1) \longrightarrow G$ with $\lim_{t\to 0} \lambda(t)v = *$. This yields a stratification of N(V) in the following way. Put $x \sim y$ if $\Lambda(x) = \Lambda(gy)$ for some $g \in G$. The

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equivalence classes are called strata. N(V) is a finite disjoint union of strata. The strata are G-invariant, locally closed and irreducible. If Y is a stratum with closure Z we construct a vector bundle E over a homogeneous space G/P and a proper surjective morphism $\tau: E \longrightarrow Z$ such that τ induces a bijection $\tau': \tau^{-1}Y \longrightarrow Y$. If char(k) = o, then τ' is an isomorphism, so that Y is a non-singular rational variety and that $\tau: E \longrightarrow Z$ is a desingularization. If V = $\underline{g}$, the Lie algebra of G, then the strata are the orbits and the desingularization was known.

D. N. Verma: A review of representation-theoretic problems arising from Old Invariant Theory

The 19th century invariant theory (together with the theory of S- functions, with which it then overlapped only partially) may be regarded as a precursor of the representation theory of connected reductive groups. From those times we have inherited a basic working tool of today's Lie-theorists, viz. the representation theory of SL_2; however, we claim that the old literature contains still "untranslated" surprisingly novel facts in the SL_2-theory. This is illustrated by our discussion of " the transvectant solution" (versus Wigner's 3 - j coefficients) for the strong Clebsch-Gordan problem, and of the formal differential operators on the Verma module for SL_2 dating from 1895 in Elliott's work on "perpetuants".

It is remarked that while the Cayley - Sylvester Functional Equation for the "Poincare series" describing the decomposition of the symmetric algebra of an SL_2-module, possesses an easy extension to arbitrary reductive group (via Weyl's character formula), even the SL_2 functional equation for the r-1 dimensional irreducible module remains unsolved except for small r.

R. W. Richardson: <u>Commuting varieties of semisimple Liealgebras and algebraic groups</u>

Let $\underline{g}$ be a semisimple Lie algebra over an algebraically closed field k of characteristic zero. Let $C(\underline{g}) = \{(x,y) \in \underline{g} \times \underline{g} \mid [x,y] = 0\}$. Let $C_1(\underline{g})$ be the set of all pairs $(x,y) \in \underline{g} \times \underline{g}$ such that there exists a Cartan subalgebra of $\underline{g}$ containing both x and y. Then $C_1(\underline{g})$ is dense in $C(\underline{g})$; in particular $C(\underline{g})$ is irreducible. A similar result holds for simply connected semisimple algebraic groups over k. Analogous results hold for semisimple real Lie algebras and algebraic groups.

V. Lakshmibai: <u>Geometry of G/P</u>

Let G be a semisimple simply connected Chevalley group over a field k; let R be the root system of G relative to a maximal torus T and W the Weyl group. Given a fundamental weight ω, let P denote the associated maximal parabolic subgroup of G. For $w \in W/W_P$ let X(w) denote the Schubert subvariety of G/P associated to w. Call ω to be of classical type if $|(\omega,\alpha^*)| \leq 2$, $\alpha \in R$. Given $\tau, \varphi \in W/W_P$, call (τ,φ) to be an admissible pair, if either $\tau = \varphi$ or there exists a chain $X(\tau_0) = X(\tau) \supset X(\tau_1) \supset \ldots \supset X(\tau_m) = X(\varphi)$ such that $X(\tau_i)$ is a codim 1 Schubert subvariety of $X(\tau_{i-1})$ occuring with multiplicity 2 in the hyperplane section of $X(\tau_i)$, $1 \leq i \leq m$. We have the following

<u>Theorem</u>: There exists a basis $\{P_{\tau,\varphi}\}$ of $H^0(G/P,L)$ (L denotes the ample generator of Pic(G/P) indexed by admissible pairs such that:

(i) $P_{\tau,\varphi}$ is a weight vector of weight $-\frac{1}{2}(\tau(\omega) + \varphi(\omega))$

(ii) for $w \in W/W_P$ $P_{\tau,\varphi}|_{X(w)} \neq 0$ if and only if $w \geq \tau$ (i.e. $X(w) \supseteq X(\tau)$)

(iii) $\{P_{\tau,\varphi} \mid w \geq \tau\}$ is abasis of $H^0(X(w),L)$.

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A monomial $P_{\tau_1 \cdot \varphi_1} \cdots\cdots\cdots P_{\tau_r \cdot \varphi_r}$ is said to be standard on X(w) if $w \geq \tau_1 \geq \varphi_1 \geq \ldots \geq \varphi_r$.

(iv) distinct standard monomials of degree r on X(w) form a basis of $H^0(X(w), L^r)$.

More generally, the above theorem extends to the case of G/Q and its Schubert varieties, where Q is the intersection of maximal parabolic subgroups of classical type.

H. H. Andersen: <u>A G-equivariant proof of the vanishing theorem for dominant line bundles on G/B</u>

Let G be a connected algebraic group over a field k, and denote by B a Borel subgroup. A line bundle on G/B is called dominant if it allows a non-trivial global section. Under a weak restriction on the characteristic of k we give a G-equivariant proof of the theorem (first proved in complete generality by G. Kempf) which says that all higher cohomology groups of dominant line bundles on G/B vanish. The basic ingredient in our proof is the linkage principle for cohomological representations.

G. Harder: <u>Arithmetic properties of Eisenstein cohomology classes</u>

For a congruence subgroup $\Gamma \subset PGL_2(\mathbb{Z}[i])$ we discussed the restriction map $H^1(\Gamma\backslash\bar{X},R) \longrightarrow H^1(\partial(\Gamma\backslash\bar{X}),R)$, where X is the corresponding symmetric space and $R \subset \mathbb{C}$ is a coefficient ring. We used the process of constructing cohomology classes by starting from a class at the boundary and associating to it an Eisenstein series which is obtained by a transcendental process. The resulting Eisenstein classes still have some arithmetic properties and we discussed some of these properties. Especially the relationship to special values of L-series was discussed.

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J. Schwermer: Cohomology of $SL_n(\mathbb{Z})$ and Eisenstein series

Let Γ be a torsionfree congruence subgroup of $SL_n/\mathbb{Q}$, denote by $H^*(\Gamma,E)$ the cohomology of Γ with values in a finite dimensional irreducible rational representation (ϱ,E), computed by the complex of Γ - invariant smooth E - valued forms on the associated symmetric space $X = SO(n)\backslash SL_n(\mathbb{R})$. In the talk we discussed the general attempt to describe the cohomology of Γ at infinity with the help of Langlands' theory of Eisenstein series. We computed the cohomology of the various faces of the boundary $\partial(\bar{X}/\Gamma)$ of the Borel - Serre compactification $\bar{X}/\Gamma$ of X/Γ and considered the natural restriction

$$r: H^*(\Gamma,E) = H^*(\bar{X}/\Gamma,E) \longrightarrow H^*(\partial_B,E)$$

to the cohomology of the faces of maximal codimension. We associated to a class $\varphi \in H^*(\partial_B,E)$ an Eisenstein series $E(\varphi,\Lambda)$, depending on a complex parameter Λ. Evaluated at a special point Λ_o, $E(\varphi,\Lambda_o)$ can be interpreted as a form on X/Γ. By this method we obtain harmonic closed forms on X/Γ by taking analytic continuation of such Eisenstein series or residues at a pole. This yields $\dim \operatorname{im} r \geq \frac{1}{|W|} \dim H^*(\partial_B,E)$ resp. describes in part the structure of im r.
The proof involves representation-theoretical methods. There are also results in the case of a congruence subgroup of SL_n/k, k a totally real algebraic number field.

J. Rohlfs: A Lefschetznumber for $SL_n(\mathbb{Z})$

Suppose that Γ is a torsionfree subgroup of finite index in $SL_n(\mathbb{Z})$ stable under the involution $\tau: SL_n \longrightarrow SL_n$ given by

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$A \longrightarrow {}^tA^{-1}$. Then τ induces maps $\tau^i: H^i(\Gamma,\mathbb{Q}) \longrightarrow H^i(\Gamma,\mathbb{Q})$ and the Lefschetznumber $L(\tau,\Gamma) = \sum_{i=0}^{\infty} (-1)^i \operatorname{tr} \tau^i$ is defined. Let $d_1,\dots,d_k$ with $k = \left[\frac{n}{2}\right]$ be the degrees of the basic invariant polynomials for $SO(n)$. Define for $m \in \mathbb{N}$ $c(m) = m^{\frac{n(n-1)}{2}} \prod_{i=1}^{k} \prod_{p|m} (1-p^{-d_i})$

<u>Theorem:</u> If $\Gamma = \Gamma(m)$ is the full congruence subgroup mod m, if $n \geq 3$, and if $\sigma = 0$ if n is even and $\sigma = 1$ if n is odd, then the following holds:

$$L(\tau,\Gamma(m)) = 2^{-k\cdot\sigma} c(m) \prod_{i=1}^{k} \zeta(1-d_i) \quad .$$

Here ζ denotes the Riemannian zeta-function.

R.Hotta: <u>On a conjecture of Lusztig</u>

For the variety $\mathcal{B}_A$ of all Borel subgroups of a connected reductive algebraic group whose Lie subalgebras contain a fixed nilpotent A, Springer constructed a representation of the Weyl group on the cohomology space $H^*(\mathcal{B}_A)$. The top degree homology $H_{2n}(\mathcal{B}_A)$ ($n = \dim \mathcal{B}_A$) has a basis consisting of the fundamental cycles corresponding to irreducible components. Lusztig has conjectured a certain explicit form of matrix realization of this Springer representation under this bais. The precise statement of this form and several points towards aproof of this conjecture are given in the lecture.

T. Shoji: <u>On the Springer representations of Weyl groups</u>

Let G be a connected reductive algebraic group defined over a finite field, A a nilpotent element of its Lie algebra. Springer defined representations of the Weyl group W of G in the l - adic cohomology of the variety of Borel subgroups whose Lie algebra contains A.

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Let $C_G(A)$ be the quotient of the centralizer of A in G by its identity component. Then $C_G(A) \times W$ acts on it, and Springer showed that all the irreducible representations of W are parametrized as isotypic subspaces of $C_G(A)$ in the top cohomology for each A. We now consider about the identification of these representations. In the case of $G = GL_n$, it is nothing but the highest term of the Green polynomials of GL_n, and is well known. In this talk, we deal with the case of classical groups and Chevalley groups of type F_4. The proof is done by induction on the rank of W, considering the restriction to some Weyl subgroup of a parabolic subgroup.

P. Slodowy: Monodromy representations of Weyl groups

Let g be a semi simple complex Lie algebra, G its adjoint group, T a maximal torus, W the corresponding Weyl group, $\mathcal{B}$ the set of all Borel subgroups of G. If x is a nilpotent element of g we denote by $\mathcal{B}_x$ the set $\{B \in \mathcal{B} \mid x \in \text{Lie } B\}$. Let C(x) be the component group $Z_G(x)/Z_G(x)^o$ of the centralizer of x. Springer defined representations of $C(x) \times W$ on the cohomology $H^*(\mathcal{B}_x)$ of $\mathcal{B}_x$ (with coefficients in $\mathbb{Q}$).

In our talk we will give a different construction for actions of $C(x) \times W$ on $H^*(\mathcal{B}_x, \mathbb{Q})$ which is inspired by the construction of monodromy representations in singularity theory (Brieskorn, Milnor, Pham,....)

We will show that both constructions yield the same result for $G = SL_n$ and for important cases in the other types.

As an application we give an interpretation of a conjecture of Lusztig by means of an analogue of the Picard-Lefschetz - formula, relating the Weyl group actions to certain intersection numbers.

J. C. Jantzen: Darstellungen halbeinfacher Gruppen und ihrer Frobenius -Kerne

Es seien G eine halbeinfache, einfach zusammenhängende algebraische Gruppe über einem algebraisch abgeschlossenen Körper der Charakeristik $p > o$ und G_n der n - te Frobenius - Kern von G. Bekanntlich sind die einfachen G_n-Moduln auch G-Moduln; zumindest für $p \geq 2h-2$ (wo h = Coxeter-Zahl von G) trifft dies auch auf die projektiven (= injektiven) G_n-Moduln zu (Ballard). Genauer gilt (immer für $p \geq 2h-2$): Man nehme die Kategorie $\mathcal{M}(n)$ der (endlich-dimensionalen) G-Moduln M, so daß für alle Gewichte μ von M (relativ eines maximalen Torus von G) und alle Wurzeln α gilt: $|\langle\mu,\alpha^\vee\rangle| < 2p^n(h-1)$. Ist Q nun ein G-Modul, der als G_n-Modul injektive Hülle eines einfachen Moduls M ist, so gehört Q zu $\mathcal{M}(n)$ und ist darin die injektive (und projektive) Hülle von M. Weiter kann man zeigen, daß Q eine Filtration durch Weyl-Moduln zuläßt, wobei ein Weyl-Modul V so oft vorkommt, wie M als G-Kompositionsfaktor in V. Ähnliche Filtrationen findet man in Moduln der Gestalt $V^{(Fr^n)} \otimes Q$ mit einem Weyl-Modul V und Q wie eben. Andererseits findet man für Weyl-Moduln, zumindest mit "sehr regulärem" höchstem Gewicht, eine Filtration durch Moduln der Form $V^{(Fr^n)} \otimes M$ mit Weyl-Moduln V und auch für G_n

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einfachen G-Moduln M. Diese Filtration spiegelt G_nT - Kompositionsketten von "Baby-Verma-Moduln" wider und kann benutzt werden, um Bilder spezieller Homomorphismen zwischen Weyl - Moduln zu berechnen.

B. Parshall: Cohomology of algebraic groups

We discussed the rational cohomology groups $H^*(G,V)$ when V is a rational module for a semisimple group G split over GF(p). Two themes were treated: The first (developed in the Cline - Parshall - Scott - van der Kallen Paper Inv. Math. 39) deals with the relationship between the rational cohomology and the Eilenberg-MacLane cohomology of the finite groups G(q). The second concerns the relationship between the groups $H^*(G,V)$ and the cohomology of the infinitesimal subgroups, results of Cline - Parshall - Scott. Finally, we discussed the PIM's of the infinitesimal subgroups and ended with a quick proof that the G-injectives are direct limits of these.

J. Soto Andrade: Shintani transforms and the character theory of $SL_n(\mathbb{F}_q)$

Let G be a connected linear algebraic group defined over $k = \mathbb{F}_q$ and F its Frobenius endomorphism. The map Sh is defined on conjugacy classes of G(k) as follows: For each class C we choose a representative of the form $h^{-1}F(h)$, with $h \in G(\bar{k})$ (Lang's theorem) and we put Sh(C) = conjugacy class of $F(h)h^{-1}$ in G(k). For central functions φ on G(k) we put $Sh(\varphi) = \varphi \circ Sh^{-1}$. If G

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has connected centralizer, then Sh(C) = C. In particular Sh = Id for Gl_n.

For G = SL_n, call singular those irreducible characters of $SL_n(k)$ which are not restrictions of irreducible characters of $GL_n(k)$. Then Sh fixes every non-singular character of $SL_n(k)$ and for prime n, $n \mid q-1$, the singular characters can be naturally parametrized as χ_ω^x $(x \in \nu_n, \omega \in \hat{\mu}_n)$, where $\nu_n = k^*/(k^*)^n$, $\hat{\mu}_n$ = character group of $\mu_n = \{t \in k^* \mid t^n = 1\}$, so that for the corresponding Mellin transforms $\hat{\chi}_\omega^\eta = \sum_{x \in \nu_n} \eta(x)\chi_\omega^x$ $(\eta \in \hat{\nu}_n, \omega \in \hat{\mu}_n)$ one has $Sh(\hat{\chi}_\omega^\eta) = \hat{\chi}_{\omega\tilde{\eta}}^{\eta}$, with $\tilde{\eta} = \eta \circ V$ where V is the canonical transfer isomorphism from μ_n to ν_n.

N. Spaltenstein: <u>Induction for unipotent classes</u>

Let G be a connected reductive algebraic group defined over an algebraically closed field. Let X^G be the set of unipotent classes of G. Let P be a parabolic subgroup of G and let L be a Levi factor of P. Induction: $X^L \longrightarrow X^G$ associates to $C \in X^L$ the unique $C' \in X^G$ containing a dense open subset of CU_P. One can define also in X^G a subset $\tilde{X}^G$ with a decreasing involution. Let s be a semisimple element of G^* (the dual group of G), and let $M = (Z_{G^*}(s)^o)^*$. Using the order structure of $\tilde{X}^M$, X^G, we can define a map $\tilde{X}^M \longrightarrow X^G$ which behaves like induction and $X^G = \bigcup_M \text{image}(\tilde{X}^M)$ (when the characteristic is good).

K. Pommerening: <u>The nilpotent classes in good characteristic</u>

Let G be a semisimple algebraic group over an algebraically closed field k. Let $\underline{g}$ be the Lie algebra of G. Let the

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characteristic of k be good for G.

Classification theorem: There is a canonical bijective correspondence between the set of nilpotent Ad(G)-orbits in $\underline{g}$ and the set of conjugacy classes of pairs (H,Q) where H is a Levi-type subgroup of G and Q is a distinguished parabolic subgroup of H.

In other words, I can remove the restriction of the characteristic in the Bala-Carter classification. The main step is the proof of the following property:

(U) Let $\underline{g}$ be graded by a one-parameter subgroup $\lambda : G_m \longrightarrow G$; $\underline{g} = \bigoplus_{i \in \mathbb{Z}} \underline{g}_i$ where $\underline{g}_i = \{ x \in \underline{g} \mid \lambda(c) \cdot x = c^i x \text{ for all } c \in G_m \}$. Let $G_o = Z_G(\lambda(G_m))$, and let G^o_{ox} be unipotent for some $x \in \underline{g}_i$ $(i \neq o)$. Then G_{ox} is finite.

G.I. Lehrer: Generalized Hecke rings

The Harish-Chandra principle for a reductive group G over a finite field k shows that in order to study the complex representations of G(k), one needs to (i) construct discrete series representations (ii) decompose representations of the form $\mathrm{Ind}_{P(k)}^{G(k)} D$, where D is discrete series on a k-Levi subgroup M(k) of the parabolic subgroup P(k). Here we are concerned with (ii).

Theorem (Howlett-Lehrer): $E = \mathrm{End}_{G(k)}(\mathrm{Ind}_{P(k)}^{G(k)}(D)) \cong \mathbb{C}W(D)_\mu$, where W(D) is the ramification group of D ($\subset W(k)$), and μ is a 2-cocycle of W(D).

The cocycle μ arises when one extends D to a projective representation $\tilde{D}$ of $\tilde{M} = \langle M(k), n(w) \mid w \in W(D) \rangle$. Using $\tilde{D}$, one quickly finds a $\mathbb{C}$-linear basis of E. To determine its multiplication

table, one studies the structure of W(D), and finds that W(D) is a semidirect product W(D) = R(D)·C(D), where R(D) is a (large) normal reflection subgroup; the relations then become (for $w \in W(D)$, $t \in C(D)$, v a fundamental reflection in R(D)):

$$B(w)\ B(t) = \mu(w,t)\ B(wt)$$

$$B(w)\ B(v) = \begin{cases} B(wv) & \text{if a condition on positive roots holds} \\ p_v B(wv) + (p_v - 1)\ B(w) & \text{otherwise} . \end{cases}$$

Similar relations hold for left multiplications. Here p_v is a $\frac{1}{2}$ - integral power of p = char(k) related to a certain induced representation.

The cocycle μ is trivial on R(D) and so is really a cocycle of C(D). It is trivial when D is p - regular (in Gelfand-Graev). Known results on generic degrees should make a complete parametrization of representations (with dimensions and some character values) quite feasible now, except for type E_8.

B. Weisfeiler: <u>One - dimensional group schemes</u>

Two theorems were stated and the proof of the first one was indicated:

1. If A is an integrally closed domain and G is a smooth connected group scheme over A whose generic fiber is a rational curve then there excists an algebra B/A which is a projective rank 2 A-module such that $G \cong (R_{B/A}\mathbb{G}_m)/\mathbb{G}_m$.

2. If A is a domain, B = A + Ah, $h^2 = bh$, $b \in A$, $b \neq 0$, and $G = (R_{B/A}\mathbb{G}_m)/\mathbb{G}_m$ then $\mathrm{Ext}(G,\mathbb{G}_a) = A[b^{-1}]/A$ if $A \supseteq \mathbb{Q}$ and $= A[F]/A[F]b^{p-1}$ if $A \supseteq \mathbb{F}_p$. Here A[F] is the ring of polynomials in Frobenius.

The proof of 1. uses Neron Blow up.

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G. Rousseau: Vector buildings

Let G be a reductive group, then the rational vectorial building is $I(G) = Y(G) \times \mathbb{N}^{*}/\sim$ where $(\lambda, n) \sim (\lambda', n')$ iff $\lambda^{n} = \gamma \cdot \lambda'^{n'} \cdot \gamma^{-1}$ for a γ in the parabolic subgroup defined by the one-parameter subgroup λ. The apartments of this building are euclidean vector spaces. Its properties are used to prove that an unstable vector in a representation of G determines a class of one-parameter subgroups of G (namely a point of I(G)·). Hence, using Galois - descent, the Hilbert - Mumford criterion of instability is true on a perfect field. Galois - descent and buildings may also be used to give geometric proofs of the main results of Borel - Tits, on the structure of reductive groups, for example the conjugacy of the maximal K-split tori.

P. Gérardin: Completions of buildings and compactifications of symmetric spaces

1) Let G be a reductive connected group over a field k. Let Par G be its set of parabolic subgroups defined over k; let Dyn G the set of vertices of the relative Dynkin diagram of G; call Typ G the set of subsets of Dyn G. We have a map (coming from the inclusion Dyn (P/Rad P) $\subset$ Dyn G) Par G $\longrightarrow$ Dyn G. On the cone $\mathbb{R}_{+}^{\mathrm{Dyn}\ G}$ define a map to Typ G by $x \longmapsto$ the $\alpha \in$ Dyn G where $x(\alpha) = 0$. Then let $I(G) = \mathrm{Par}\ G \times_{\mathrm{Typ}\ G} \mathbb{R}_{+}^{\mathrm{Dyn}\ G}$, with the "left" topology on Par G: a subset F is closed if $P \in F$, $Q \supset P \Rightarrow Q \in F$. The group G(k) acts on Par G by conjugation, hence on I(G).

Proposition: I(G) is the vector building of the adjoint group of G.

Let $\overline{\mathbb{R}}_+$ be $[0,\infty]$ and $\overline{\text{Par } G}$ the set of $(P,Q) \in \text{Par } G \times \text{Par } G$ with $P \subset Q$, endowed with the topology induced by the left topology on the left factor Par G, and right on the right. We have a map $\overline{\mathbb{R}}_+^{\text{Dyn } G} \longrightarrow \text{Typ } G \times \text{Typ } G$ by $x \longmapsto (\alpha$ where $x(\alpha) = 0$, α where $x(\alpha) < \infty)$. Define $\overline{I(G)} = \overline{\text{Par } G} \times_{\text{Typ } G \times \text{Typ } G} \overline{\mathbb{R}}_+^{\text{Dyn } G}$, and $\overline{I(G)}_Q$ as the inverse image of Q by the second projection on Par G × Par G.

<u>Theorem 1</u> – $\overline{I(G)}_Q = I(Q/\text{Rad } Q)$

– I(G) is an open dense subset of $\overline{I(G)}$

– The $\overline{I(G)}_Q$ are the metric components of this "completion" of I(G)

– The apartments of I(G) have closure in $\overline{I(G)}$ the compact polyhedron defined by its Weyl chambers.

2) $k = \mathbb{R}$, S(G) the set of maximal compact subgroups of $G(\mathbb{R})$. Let $\overline{S(G)}$ be the quotient set of S(G) x Par G by $(s,P) \sim (s',P')$ iff $P = P'$ and the supports of the affine facets defined by (s,P) and (s',P') are conjugate under $U_P(\mathbb{R})$; put a topology on $\overline{S(G)}$ from this affine facets. Then, if $\overline{S(G)}_Q$ is the image of S(G) x Q:

<u>Theorem 2</u>(Satake) – $\overline{S(G)} = \coprod S(Q/\text{Rad } Q)$ is a compact $G(\mathbb{R})$-space

– S(G) is open dense

– the $\overline{S(G)}_Q$ are the metric components of this compactification

– the apartments of S(G) have the same closure as in theorem 1

3) Other completions and compactifications can be defined: Borel-Serre, Oshima, for any type $\tau \subset \text{Dyn } G$, there are τ-completions and compactifications. Applications: representations of groups G(k) (k finite or local field) with $\overline{I(G)}$, cohomology of arithmetic groups (Borel-Serre), differential operators on S(G) (Oshima).

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