

MATHEMATISCHES FORSCHUNGSIINSTITUT OBERWOLFACH

Tagungsbericht 19/1979

Gruppentheorie

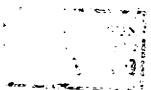
29.4 bis 5.5.1979

Die diesjährige Tagung über Gruppentheorie stand unter der Leitung der Professoren W. Gaschütz (Kiel), B. Huppert (Mainz) und J.E. Roseblade (Cambridge). Der Teilnehmerkreis umfaßte nahezu 50 Gruppentheoretiker, davon über ein Drittel aus dem Ausland.

Die Themen der 29 Vorträge stammten sowohl aus dem Gebiet der unendlichen als auch dem der endlichen Gruppen; hier waren die Darstellungstheorie bzw. Charaktertheorie sowie die Theorien der p-Gruppen, der auflösbaren Gruppen und der Verlagerung schwerpunktmäßig vertreten. Den Abschluß der Tagung bildete eine Problem-Sitzung.

Teilnehmer

- R. Baer (Zürich)
- R. Bieri (Freiburg)
- N. Blackburn (Manchester)
- D. Blessenohl (Kiel)
- A. Brandis (Heidelberg)
- R. Burkhard (Würzburg)
- Chat Yin Ho
- D. Collins (London)
- W. Deskins (Pittsburgh & London)
- K. Doerk (Mainz)
- S. Donkin (Coventry)
- G. Eigenthaler (Wien)



52

P. Förster (Mainz)  
T. Gagen (Sydney & Freiburg)  
W. Gaschütz (Kiel)  
B. Hartley (Manchester)  
H. Heineken (Würzburg)  
C. Hering (Tübingen)  
M. Herzog (Tel Aviv)  
B. Huppert (Mainz)  
O. Kegel (Freiburg)  
M. Klemm (Mainz)  
W. Knapp (Tübingen)  
H. Kurzweil (Erlangen)  
H. Laue (Kiel)  
R. Laue (Aachen)  
T. Lennox (Freiburg)  
A. Lichtman (Manchester)  
G. Michler (Essen)  
J. Neubüser (Aachen)  
P. Neumann (Oxford)  
H. Pahlings (Giessen)  
A. Reifart (Heidelberg)  
D. Robinson (Freiburg)  
K. Roggenkamp (Stuttgart)  
J. Roseblade (Cambridge)  
K.-U. Schaller (Kiel)  
P. Schmid (Tübingen)  
R. Schmidt (Kiel)  
U. Stammbach (Zürich)  
G. Stroth (Heidelberg)  
O. Tamaschke (Tübingen)  
F. Timmesfeld (Köln)  
B. Wehrfritz (London)  
H. Wielandt (Tübingen)  
W. Willemse (Mainz)  
J. Wilson (Cambridge)



VORTRAGSAUSZOGE

D.J.S. ROBINSON: Homology of hypercentral groups

Let  $G$  be a hypercentral group. It is shown that if  $H_n G := H_n(G, \mathbb{Z})$  is a torsion group, then  $G$  has finite Hirsch number  $h(G) \leq n - 1$ . This leads to a new interpretation of  $h(G)$  when  $G$  is hypercentral. Some related problems will be discussed.

B.A.F. WEHRFRITZ: Finitely generated soluble linear groups

We discussed necessary and sufficient conditions for a finitely generated abelian-by-polycyclic group  $G$  to be isomorphic to a linear group.  $G$  is isomorphic to a linear group of characteristic  $p > 0$  if and only if  $G$  is nilpotent-by-abelian-by-finite and has an abelian normal  $p$ -subgroup  $A$  with  $G/A$  polycyclic. If  $G$  is isomorphic to a linear group of characteristic 0, then  $G$  is nilpotent-by-abelian-by-finite and has a torsion-free abelian normal subgroup  $A$  with  $G/A$  polycyclic. The converse is false. What is required is an extra condition on  $A$  as  $G/A$ -module, the actual extension being irrelevant.

D.J. COLLINS: Aspherical groups

A group presentation is aspherical if there are no reduced spherical cancellation diagrams over the presentation. We seek to remedy some deficiencies in R.C. Lyndons original account of this concept and then to show that asphericity is preserved under various group constructions.

P.M. NEUMANN: Amoebic modules for soluble groups

The substance of the lecture was a description of a non-zero cyclic module  $V$ , over the group-ring of a certain finitely generated soluble group  $G$ , having the property that  $V \cong V \oplus V$ . The talk finished with commentary on this result, its context,



and some questions that it raises.

A.I. LICHTMAN: On Lie-algebras of free products of groups

Let  $G$  be an arbitrary group,  $R$  be a field and  $RG$  be the group algebra of  $G$  over  $R$ . We denote by  $L_p(G)$  the restricted  $p$ -algebra of  $G$ , which is associated with the  $N_p$ -series of the dimension subgroups of  $RG$ , when  $\text{char } R = p$ .

Our main result is the following

Theorem. Let  $F = F_1 * F_2$  be a free product of groups  $F_1$  and  $F_2$ . Then the restricted Lie- $p$ -algebra  $L_p(F)$  is isomorphic to the free Lie sum (in the category of restricted Lie- $p$ -algebras) of the restricted Lie-algebras  $L_p(F_1)$  and  $L_p(F_2)$ :  $L_p(F) \cong L_p(F_1) * L_p(F_2)$ .

B. HARTLEY: Powers of the augmentation ideal

Let  $\Delta(G)$  be the integral augmentation ideal of a group  $G$ , and  $D_n(G) = (1 + \Delta^n(G)) \cap G$ . Let  $K \trianglelefteq G$ . Some connections are given between the series  $\Delta^n(G)$  and  $\Delta^n(K)$ , and hence for the corresponding dimension subgroups, when  $G$  is nilpotent and  $G/K$  is torsion-free.

Theorem 1: There exists an integer-valued function  $m(n,c)$  such that if  $G$  has class  $c$ , then  $\Delta^{m(n,c)}(G) \cap \mathbb{Z}K \leq \Delta^n(K)$ , and

$$D_{m(n,c)}(G) \leq D_n(K).$$

Theorem 2: Suppose that  $G$  is nilpotent,  $K$  is the torsion subgroup of  $G$ , and let  $K_p$  be the Sylow  $p$ -subgroup of  $K$ . Then  $(\bigcap_{n=1}^{\infty} (G)^n) = (\bigcap_{n=1}^{\infty} \Delta(K)^n) \cdot \mathbb{Z}G$  provided that  $\bigcap D_n(G) = 1$  and  $G/K$  has no elements of infinite  $p$ -height whenever  $K_p \neq 1$ .

W. GASCHOTZ: Ein allgemeiner Sylow-Satz  
für endliche auflösbare Gruppen

Es sei  $P$  eine Teilmenge der Primzahlpotenzmenge,  $1 \notin P$ ,  $P'$  das Komplement von  $P$  in  $N$ . Ist  $S \trianglelefteq G$ , so werde  $S$   $P$ -Sylowgruppe von  $G$  genannt, wenn  $W \leq S \Rightarrow |S:W| \in P$   
und  $S \trianglelefteq K \trianglelefteq H \trianglelefteq G \Rightarrow |H:K| \in P'$  gelten.

Satz.  $P'$  sei multiplikativ abgeschlossen. Dann existieren in jeder endlichen auflösbaren Gruppe  $P$ -Sylowgruppen und alle diese sind konjugiert.



R. LAUE: Stabilitätsgruppen und Zentralreihen

Sei  $A$  eine Automorphismengruppe einer Gruppe  $G$ , die eine endliche Kette  $G = G_1 > G_2 > \dots > G_{n+1} = 1$  von Untergruppen von  $G$  stabilisiert. Dann ist nach P. Hall  $\text{cl}(A) \leq \frac{n(n-1)}{2}$  und, falls die  $G_i$  in  $G$  normal sind, sogar  $\text{cl}(A) \leq n - 1$ . Für endliche Gruppen  $G$  läßt sich stets eine Kette von Normalteileln finden, die von  $A$  stabilisiert wird. Alle solche Ketten kürzester Länge bilden einen Verband. Es folgt, daß für eine endliche  $p$ -Gruppe die Zentralreihen kürzester Länge einen Verband bilden. Jeder Zentralreihe läßt sich außerdem eine neue zuordnen, die die gleiche Länge besitzt. Dadurch erhält man eine Abbildung dieses Verbandes in sich, für die einige Eigenschaften untersucht wurden.

T.M. GAGEN: Some complete finite groups

A group is complete if  $Z(G) = 1$  and if  $\text{Out}(G) = 1$ . The following results were discussed.

Theorem 1 (Gagen and Robinson). Let  $G$  be a finite metabelian group such that  $\text{Out}(G) = 1$ . Then either  $|G| \leq 2$  or  $G$  is a direct sum of holomorphs of cyclic odd primary groups of different orders. Conversely, every such group has trivial outer automorphism group.

Theorem 2. Let  $G$  be a finite abelian-by-nilpotent group and suppose  $\text{Out}(G) = 1$ . Then either  $|G| \leq 2$  or  $G$  is a direct product of groups  $A_i X_i$ , where  $A_i$  is a homocyclic  $p$ -group for some odd prime  $p$  and  $A_i = A_j$  only if  $i = j$ , and  $X_i$  is a 2-Sylow normalizer of  $\text{Aut}(A_i)$ . Conversely, any such group is abelian-by-nilpotent and has trivial outer automorphism group.

Since in both the above cases  $Z(G) \neq 1$  implies  $|G| \leq 2$ , these theorems completely classify all complete metabelian, respectively abelian-by-nilpotent groups. All these groups manifestly have even order. The proof of Theorem 2 depends on the following lemmas which seem to be new.

Lemma 1. Let  $X$  be a nilpotent self normalizing subgroup of  $\text{GL}(n, q)$  where  $q = p^m$ . Then  $X$  is a 2-Sylow normalizer of  $\text{GL}(n, q)$  unless  $n > 1$  and  $p = 2$ ,  $m > 1$ . The groups  $\text{GL}(n, 2^m)$ ,  $n > 1$ ,  $m > 1$ , have no nilpotent self normalizing subgroups.

Lemma 2. Let  $X$  be a nilpotent self normalizing subgroup of  $\text{Aut}(A)$ , where  $A$  is an abelian  $p$ -group. Then  $X$  is a 2-Sylow normalizer, and every 2-Sylow normalizer is nilpotent.

8

The above two lemmas give examples of infinite classes of non-solvable groups which have a single conjugacy class of nilpotent self normalizing, i.e. Carter subgroups.

J. NEUBUSER: Raumgruppen und p-Gruppen – Gegenbeispiele zur class-breadth conjecture

Ch. Leedham-Green und M.F. Newman haben auf die Möglichkeit hingewiesen, aus einer Raumgruppe, deren Punktgruppe eine p-Gruppe ist, die auf dem Translationennormalteiler einreihig operiert, eine unendliche Seire von p-Gruppen konstanter Koklasse zu erhalten. Die Frage der Isomorphie dieser Serien wurde in einem Vortrag auf der vorjährigen Gruppentheorie-Tagung geklärt. Mittlerweile liegen weitere Ergebnisse vor, aus denen sich insbesondere Gegenbeispile zu der class-breadth conjecture ergeben.

G. MICHLER: The character tables of the simple groups of Ree type

By J. Walters theorem all finite simple groups with an abelian Sylow 2-subgroup are known up to the class of simple groups R(q) of Ree type.

In joint work with P. Landrock we obtained the following result:

Two simple groups of Ree type having the same order  $q^3(q^3 + 1)(q - 1)$ ,  $q = 3^{2n+1}$ ,  $n > 1$ , have the same character table. The values missing in H.N. Ward's character table (TAMS 121 (1966)) were stated.

W. WILLEMS: Some remarks on the projectives of a group algebra

Es werden Zusammenhänge zwischen den Projektiven einer Gruppe und denen eines Normalteilers bzw. einer Faktorgruppe studiert. Als Anwendung erhält man unmittelbar Sätze von Brauer/Michler über die Kerne von Blöcken. Weiterhin erhalten wir Abschließungseigenschaften für die Klasse  $P_0(p)$  aller endlichen Gruppen G, für die der projektive unzerlegbare FG-Modul ( $\text{char } F = p$ ) mit Kopf  $\cong 1$  die Dimension  $|G|_p$  hat.



M. HERZOG: On character tables

A finite group  $G$  is called a TI-group if the intersection of distinct Sylow 2-subgroups of  $G$  is equal to 1.

Theorem 1 (Chillag and Herzog). The TI-property can be read from the character table of  $G$ .

Theorem 2. Let  $G$  be a simple group. Then the property of having an abelian Sylow 2-subgroup can be read from the two columns of the character table of  $G$  corresponding to 1 and to the involutions.

B. HUPPERT: Darstellungen von Kranzprodukten

Sei  $K = G \wr H$  ein Kranzprodukt und  $F$  ein algebraisch abgeschlossener Körper von beliebiger Charakteristik. Es werden die irreduziblen  $FK$ -Moduln und ihre projektiven Hüllen beschrieben. Zur Berechnung der Cartan-Matrix reicht diese Beschreibung jedoch nur in sehr speziellen Fällen.

A. BRANDIS: Verlagerungssätze

Sei  $G$  eine endliche Gruppe,  $p$  ein Primteiler von  $|G|$ ,  $H$  eine Untergruppe von  $G$ , die eine  $p$ -Sylowgruppe enthält. Sei ferner  $O^P(G)$  der kleinste Normalteiler von  $G$  mit  $p$ -Faktorgruppe und  $F_G^p(H) = T = O^P(G) \cap H$ . Die behandelte Frage lautet: Wie kann man die Normalteiler von  $H$  zwischen  $T$  und  $H$  bestimmen?

Im Vortrag wird gezeigt, wie man durch Verfeinerung der Methoden aus der Arbeit des Vortragenden zum gleichen Thema (Math.Z. 166 (1979)) mit Hilfe der Abbildung (setze  $[T, H]O^P(H) = T_0$ ):

$$\Psi : H \rightarrow T/T_0 : \Psi(h) = \prod_{r, r' \in \Phi, r^h r' \in T} r^h r'^{-1} T_0$$

( $\Phi$  ein Repräsentantensystem von  $O^P(G)$  mod  $T_0$ )  
die klassischen Resultate Wielandts und P. Halls und ihre Verallgemeinerungen von Yoshida erhalten kann mit ausschließlich elementaren Methoden. Z.B.:

Satz. Sei  $Q$  schwach abgeschlossen in  $P$ ,  $P_0$  stark abgeschlossen in  $P$ ,  $H \geq N_G(Q)$ ,  $N_G(P_0)$ . Dann ist

$$T \in \langle [t, y; p-1] \in P_0, t \in P_0, y \in Q, x \in G \rangle_{T_0},$$

wobei  $[t, y; i] = [[t, y; i-1], y]$ .



W. KNAPP: Codes with prescribed permutation group

A program is proposed how to determine all linear codes  $C$  of length  $n$  over a field  $F$  admitting a given permutation group  $G$  acting transitively on the  $n$  coordinates.

The program is applied to determine all codes admitting the groups  $A_n$ ,  $S_n$ ,  $PSL(2,p)$ ,  $PGL(2,p)$ ,  $M_{11}$ ,  $M_{12}$ ,  $M_{22}$ ,  $\text{Aut}(M_{22})$ ,  $M_{23}$ ,  $M_{24}$ . As particular cases the following results hold:

Theorem. Let  $C$  be a non-trivial code over  $F$  of length  $n \geq 7$  and assume  $C$  admits  $A_n$ . Then  $C$  is isomorphic to the repetition code or to its dual.

Theorem. Let  $C$  be an extended QR-code over  $F$  of length  $p+1 \geq 8$ . Then the permutation group of  $C$  is properly contained in  $A_{p+1}$ . (This answers a question of Rasala (J. Algebra 42 (1976)) to the affirmative.)

This work was done jointly with P. Schmid.

H. WIELANDT: Maximal  $\pi$ -subgroups of composite groups

Let  $G$  be a finite group,  $G = G_0 \triangleright G_1 \triangleright \dots \triangleright G_l = 1$  a subnormal series of  $G$ , and  $G^\lambda := G_{\lambda-1}/G_\lambda$ . Subgroups  $A \leq G$  can be investigated by means of their "projections"  $A \rightarrow G := (A \cap G_{\lambda-1})G_\lambda/G_\lambda$ . In general they determine only the composition factors of  $A$ , but much more can be said if  $A$  is a maximal  $\pi$ -subgroup of  $G$ ,  $A \in M_\pi G$ , for a set  $\pi$  of prime numbers. Provided Schreier's conjecture is true within  $G$  they determine  $A$  up to conjugacy; more precisely:

(1) Let  $A, B \in M_\pi G$ . Assume  $A \rightarrow G^\lambda = B \rightarrow G^\lambda$  whenever  $G^\lambda$  happens to contain a subgroup which does not possess a nilpotent Hall  $\pi$ -subgroup. Then  $A$  and  $B$  are conjugate in their join. (This contains P. Hall's theory of the  $\pi$ -subgroups of a soluble group  $G$ : pick a chief series  $\{G^\lambda\}$ .)

An iterative procedure to determine  $M_\pi G$  can be based on

(2) Let  $S$  be a non-abelian simple subnormal subgroup of  $G$ . Then  $\{S \cap A \mid A \in M_\pi G\} = \{S \cap B \mid B \in M_\pi \hat{S}\}$  where  $\hat{S}$  denotes the automorphism group of  $S$  induced by  $N_G(S)$ .

The maximal  $\pi$ -subgroups behave badly with respect to homomorphisms:

(3) For each finite group  $H$  and for each set  $\pi$  of at least two prime numbers in which at least one prime number is missing there is a finite group  $G$  and an epimorphism  $\varphi: G \rightarrow H$  such that  $\varphi(M_\pi G) = 1$ .



consists of all  $\pi$ -subgroups of  $H$ . But:

(4) In each  $G$  one can construct  $A \in M_{\pi}G$  such that the map  $X \mapsto X \cap A$  is a lattice homomorphism of the subnormal lattice  $s_n G$  into  $s_n A$ .

G. EIGENTHALER: A Generalization of a Theorem of Gaschütz and its Application to Polynomial Permutations

Let  $A$  be a universal algebra,  $A_1$  a subset of  $A$ , and  $k$  a positive integer. Then  $U_k(A, A_1)$  denotes the group of permutations on  $A^k$  induced by polynomials on  $A$  with coefficients in  $A_1$ . (For details see Lausch-Nöbauer, Algebra of Polynomials, 1973.) The algebra  $A$  is called congruence uniform if, for any congruence  $\Theta$  on  $A$ , the blocks of the partition induced by  $\Theta$  are of the same cardinality. Suppose that  $A, B$  are finite algebras of a variety  $V$  with the property that every finite algebra of  $V$  is congruence uniform, and let  $\eta$  be a surjective homomorphism from  $A$  onto  $B$ , then  $\eta$  induces a surjective group homomorphism from  $U_k(A, A_1)$  onto  $U(B, \eta(A_1))$ . This theorem generalizes two results of Lausch-Nöbauer and is applicable in particular in case that  $V$  is a variety of groups with multiple operators. The proof of this theorem depends on a generalization of a theorem of Gaschütz on finite generating systems of groups. (See Gaschütz, Math. Nachr. 15 (1955)).

J.S. WILSON: Uncharacteristically simple groups

Prompted by the fact that, if  $G$  is a characteristically simple group, then every countable subset of  $G$  lies in a countable characteristically simple subgroup of  $G$ , P. Hall asked whether a group is necessarily characteristically simple if each countable subset lies in a characteristically simple subgroup. Examples of groups showing that the above question has a negative answer, and satisfying a variety of additional conditions, were discussed.

H. HEINEKEN: Finite nilpotent groups all of whose normal subgroups are characteristic

For odd  $p$  a way of constructing nilpotent groups ( $p$ -groups) of



class two is given. These groups are all products of two abelian groups, they have order  $p^{6t}$  (every  $t$  not smaller than 4 can be chosen for  $p \neq 3, 5$ ; for  $p = 5$  we have to exclude  $t = 4$  and for  $p = 3$  we have to take  $t = 1t^*$  with  $l \neq 1$  and  $t^*$  not smaller than 4). All automorphisms of these groups are central.

W.E. DESKINS: Some consequences of good behaviour  
in the Fitting subgroup

This is a continuation of work of Sastry and Deskins (J.Algebra 52 (1978)).

A subgroup  $H$  of a finite group  $G$  is  $\pi$ -quasinormal in  $G$  if  $H$  permutes with every Sylow subgroup of  $G$  (O. Kegel, Math.Z. 78 (1962)).

Theorem. Let  $G$  be solvable,  $G_2$  be quaternion-free. If each  $s$ -minimal subgroup of  $\text{Fit}(G)$  is  $\pi$ -quasinormal in  $G$  then  $G$  is supersolvable ( $s < \text{number of not-necessarily-distinct prime factors of } |\text{Fit}(G)|$ ).

Lemma 1. Let  $G$  be solvable,  $G_2$  quaternion-free. If each minimal subgroup of  $\text{Fit}(G)$  is  $\pi$ -quasinormal in  $G$ , then  $G$  is supersolvable.

This result is based on Corollary 3 of R. Laue (J.Algebra 52 (1978)).

Lemma 2. Let  $|\text{Fit}(G)| = p^n > p$ . Suppose each subgroup of  $\text{Fit}(G)$  of order  $p^s$ , some  $s$  between 1 and  $n$ , is  $\pi$ -quasinormal in  $G$ . Then  $G \triangleright P \geq 1$ ,  $P \leq \Phi(G) \cap \text{Fit}(G)_p$ , such that for  $\bar{G} = G/P$  there exists  $t < s$  for which every subgroup of  $\text{Fit}(\bar{G})$  of order  $p^t$  is  $\pi$ -quasinormal in  $\bar{G}$ .

These permit an inductive proof of the Theorem.

F. TIMMESFELD:  $p$ -groups of  $GF(p^n)$ -type

Let  $q = p^n$  and  $\mathfrak{Q}$  the set of special  $p$ -groups  $Q$  with  $|Z(Q)| = q$ . For  $x \in Q - Z(Q)$  let  $\Delta(x) = Z(C_Q(x))$ . Let  $\Delta(Q) = \{\Delta(x) \mid x \in Q - Z(Q)\}$  with  $|\Delta(x)| = q^2$  and  $\Delta(x) = \Delta(y)$  for each  $y \in \Delta(x) - Z(Q)\}, \Lambda(Q) = \{\Delta(x) \in \Delta(Q) \mid \Delta(x) \text{ is elementary abelian}\}$ .

We say  $Q \in \mathfrak{Q}$  is of  $GF(q)$ -type, if  $\Delta(x) \in \Delta(Q)$  for each  $x \in Q - Z(Q)$ .

Applications of the following theorem were discussed:

Theorem 1: Suppose  $Q \in \mathfrak{Q}$  and  $Q = \langle \Lambda(Q) \rangle$ . Then  $Q$  is of  $GF(q)$ -type.

Theorem 2. Suppose  $Q$  is of  $GF(q)$ -type and  $Q$  is generated by self centralizing elementary abelian subgroups. Then  $Q$  is isomorphic...



to the central product of Sylow 2-subgroups of  $L_3(q)$ .

H. KURZWEIL: Nicht auflösbare Automorphismengruppen  
auflösbarer Gruppen

Die endliche, auflösbare Gruppe  $G$  mit nilpotenter Länge  $f$  besitze eine Automorphismengruppe  $A$ , so daß  $|G|, |A| = 1$ . Sei  $S_1, \dots, S_f$  eine  $A$ -invariante Fittingkette von  $G$  und  $I = \{i \in \{1, \dots, f\} \mid C_{S_i}/\varphi(S_i)(A) = 1\}$ . Es wurden Voraussetzungen über  $A$  diskutiert, unter denen eine Schranke  $c$ , die allein von  $A$  abhängt, gefunden werden kann, so daß  $|I| \leq c$  gilt. Dies ist z.B. richtig unter den Voraussetzungen (+) und (++):

(+)  $A$  besitzt Normalteiler  $1 \neq N_1 \neq N_2 \neq A$ , so daß  $N_1$  und  $A/N_2$  auflösbar sind, während  $N_2/N_1$  ein direktes Produkt von einfachen Gruppen  $A_i$  ist, die eine auflösbare Untergruppe  $H_i$  besitzen mit folgender Eigenschaft: Für jedes  $p \in \pi(G)$  ist der Primkörper  $GF(p)$  Zerfällungskörper von  $l_{H_i}^{A_i}$ , und jeder irreduzible Bestandteil von  $l_{H_i}^{A_i}$  tritt mit Vielfachheit 1 auf.

(++) Ist  $p$  der kleinste Primteiler von  $|G|$ , so ist die Anzahl der minimalen Untergruppen von  $A$  kleiner als  $p$ . (Diese Voraussetzung muß noch abgeschwächt werden.)

G. STROTH: 2-Gruppen vom  $GF(q)$ -Typ

Der folgende Satz wurde bewiesen:

Sei  $Q$  eine spezielle 2-Gruppe mit  $|Z(Q)| = q = 2^n$ . Sei weiter für jede Involution  $x \in Q - Z(Q)$  stets  $Z(C_Q(x))$  elementar-abelsch von der Ordnung  $q^2$  und  $Z(C_Q(x)) = Z(C_Q(y))$  für alle  $y \in Z(C_Q(x)) - Z(Q)$ . Dann gilt eine der drei folgenden Aussagen:

(i)  $Z(Q) = \Omega_1(Q)$ ,  $|Q| \leq q^3$ ;

(ii)  $Q \cong D_q * \dots * D_q$ ;

(iii)  $Q \cong D_q * \dots * D_q * U_q$ .

Hierbei sind

$$D_q = \left\{ \begin{pmatrix} 1 & \\ a & 1 \\ b & c \\ & 1 \end{pmatrix} \mid a, b, c \in GF(q) \right\} \text{ und}$$

$$U_q = \left\{ \begin{pmatrix} 1 & \\ a & 1 \\ b & -\bar{a} \\ & 1 \end{pmatrix} \mid a, b \in GF(q^2), a\bar{a} + b + \bar{b} = 0, \bar{a} = a^q \right\}.$$



R. SCHMIDT: Subgroup lattices of groups generated by involutions

A projectivity  $\varphi$  of a group  $G$  is an isomorphism of the subgroup lattice of  $G$  onto the subgroup lattice of some group  $H$ . It is called 2-regular if  $|U^\varphi| = 2$  for all  $U \in G$  with  $|U| = 2$ ; it is called 2-singular otherwise.

Theorem 1. Let  $G$  be a group,  $\varphi$  a projectivity of  $G$ , and assume there exist involutions  $a$  and  $b$  in  $G$  such that  $|\langle a \rangle^\varphi| \neq |\langle b \rangle^\varphi|$ . Then  $G = S \times T$  where  $S$  is a P-group generated by involutions and for all  $t \in T$ ,  $s \in S$  we have that  $o(t)$  is finite and relatively prime to  $o(s)$ .

Corollary. If  $G$  contains a four-group as a subgroup, then every projectivity of  $G$  is 2-regular.

Theorem 2. Let  $G$  be a group and  $\varphi$  a projectivity of  $G$  such that  $|\langle a \rangle^\varphi| = |\langle b \rangle^\varphi|$  for all involutions  $a$  and  $b$  in  $G$ . If  $a \in G$  is an involution,  $\langle a \rangle^\varphi = \langle x \rangle$ , and  $U \in G$  such that  $U^a = U$ , then  $(U^\varphi)^x = U^\varphi$ .

Corollary. Let  $G$  be a group generated by involutions which is not a P-group. If  $N \not\in G$ , then  $N^\varphi \not\in G^\varphi$  for every projectivity  $\varphi$  of  $G$ .

S. DONKIN: Locally Finite Representations of Polycyclic Groups

Let  $G$  be a polycyclic-by-finite group and  $k$  a field of characteristic 0. We describe some features of the following theorem.

Theorem. If  $V$  is a finite dimensional  $kG$ -module then the injective hull of  $V$  is artinian.

Let  $J_0(G, k)$  be the Hopf algebra of finitary functions from  $G$  to  $k$ , where  $f: G \rightarrow k$  is said to be finitary if there exist  $f_1, \dots, f_n, f'_1, \dots, f'_n$  such that  $f(xy) = \sum f_i(x)f'_i(y)$  for all  $x, y \in G$ . The proof involves viewing a locally finite dimensional  $kG$ -module as a comodule for  $J_0(G, k)$ , specifically if  $V$  is finite dimensional we view  $E(V)$ , the injective hull of  $V$ , as a comodule for a finitely generated sub-Hopf-algebra of  $J_0(G, k)$ . The corresponding theorem in characteristic  $p$  was proved by I.M. Musson.

K. ROGGENKAMP: Einige ganzzahlige Gruppenringe

Sei  $G$  eine endliche Gruppe mit normaler  $p$ -Sylowuntergruppe  $P$ , so daß jede Untergruppe von  $P$   $G$ -invariant ist,  $\hat{\mathbb{Z}}_p$  und  $\hat{\mathbb{Q}}_p$  seien die  $p$ -adischen Vervollständigungen. Sei  $V$  ein irreduzibler  $\hat{\mathbb{Q}}_p G$ -Modul.



Dann ist der Verband der  $\widehat{\mathbb{Z}}_p$ -Gitter in V linear geordnet. Wenn e das zu V gehörige zentrale primitive Idempotent von  $\widehat{\mathbb{Q}}_p G$  ist, dann ist  $\widehat{\mathbb{Z}}_p G$  eine erbliche Ordnung.

CHAT YIN HO: Quadratic Action

Let V be a normal subgroup of a group X and A a subset of X. We say that A acts quadratically on V if  $[V, A, A] = 1$ . The case V is a finite dimensional vector space over a field of p elements and G is a subgroup of  $\text{Aut}(V)$  generated by quadratic elements is discussed. The connection with translation plane is mentioned. Finally, finite groups with TI-property for Sylow p-subgroups and have a faithful module over  $\text{GF}(p)$  such that an element of G acts quadratically are classified.

A. REIFART: Strongly irreducible representations  
of finite groups on finite projective planes

The following two theorems seem to be useful when considering finite groups acting strongly irreducible on a finite projective plane and containing perspectivities.

Theorem 1. Let P be a p-group ( $p \neq 3$ ) containing perspectivities. Then  $Z(P)$  contains perspectivities.

Theorem 2. Let P be a 3-group containing perspectivities. Then P contains a perspectivity  $\alpha$  such that  $|P:C_P(\alpha)| \leq 3$ .

In a joint paper with G. Stroth we have shown that most of the sporadic simple groups cannot act strongly irreducible on a finite projective plane when containing perspectivities.

P. Förster (Mainz)

