Mathematisches Forschungsinstitut Oberwolfach

Tagungsbericht 20|1979

Mathematische Optimierung

6.5. bis 12.5. 1979

Leitung: Heinz König (Saarbrücken)
Bernhard Korte (Bonn)
Klaus Ritter (Stuttgart)

Das relativ große Interesse an dieser Tagung wird durch insgesamt 58 Teilnehmer aus 10 Ländern dokumentiert. 14 der 58 Teilnehmer kamen von außerhalb Europas.

Von den 38 Vorträgen beschäftigte sich ein Teil mit Fragestellungen der diskreten Optimierung. Hier wurde insbesondere über neuere Ergebnisse der Polyedertheorie, Adjazenzcharakterisierung, Facettialstrukturen, Algorithmen für Matroid Eigenschaften, Scheduling-Probleme, Dualitätsfragestellungen und Komplexitätsprobleme berichtet. Einen weiteren Schwerpunkt bilden Optimalitätsbedingungen höherer Ordnung und deren Verallgemeinerung für abstrakte Optimierungsprobleme. Außerdem wurde über Komplementaritätsprobleme, parametrische Optimierung und variable metric-Verfahren sowie über Anwendungen und numerische Aspekte von Optimierungsverfahren berichtet. Daneben wurden in ad-hoc-Diskussionen am Abend offene Probleme und Hypothesen diskutiert.

Es wird ein Sonderheft der Zeitschrift "Mathematical Programming" erscheinen, in dem Vorträge der Tagung thematisch zusammengefaßt veröffentlicht werden.

Die Tagungsteilnehmer danken besonders herzlich dem Direktor des Mathematischen Forschungsinstituts, Herrn Professor Dr. M. Barner und seinen Mitarbeitern für die ausgezeichnete Betreuung.

Es folgen eine Liste der Teilnehmer und Kurzfassungen der Vorträge in alphabetischer Reihenfolge sowie eine vollständige Adressenliste der Teilnehmer.

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Teilnehmer

A. Bachem, Bonn
E. Balas, Pittsburgh (USA)
M. L. Balinski, New Haven (USA)
M. J. Beckmann, München
P. Bod, Budapest, Ungarn
G. Bol, Karlsruhe
J. Bräuninger, Stuttgart
B. Brosowski, Frankfurt
R. Bulirsch, München
R. E. Burkhard, Köln
L. Collatz, Hamburg
R. W. Cottle, Bonn, Stanford (USA)
W. Dinkelbach, Saarbrücken
U. Eckhardt, Hamburg
W. Eichhorn, Karlsruhe
J. Fischer, Stuttgart
F. R. Giles, Bonn, Lexington (USA)
K. Glashoff, Hamburg
B. Gollan, Würzburg
M. v. Golitschek, Würzburg
M. Grötschel, Bonn
P. L. Hammer, Waterloo (Kanada)
D. Hausmann, Bonn
R. Henn, Karlsruhe
J. Jahn, Darmstadt
R. G. Jeroslow, Atlanta (USA)
E. L. Johnson, Yorktown Heights (USA)
P. Kall, Zürich (Schweiz)
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S. Schaible, Köln
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W. Vogel, Bonn
H. Werner, Münster
Ph. Wolfe, Yorktown Heights (USA)
L. Wolsey, London (England)
P. Young, Laxenburg (Österreich)
J. Zowe, Würzburg

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Vortragsauszüge

A. Bachem:

Adjacency of faces and polyhedral polarity

If P is a polyhedron and $F(P)$ its face lattice with supremum "$\vee$" and infimum "$\wedge$" we define faces $F_1, F_2 \in F(P)$ to be adjacent if $\dim(F_1 \wedge F_2) = \max\{\dim(F_1), \dim(F_2)\} + 1$ and $\dim(F_1 \wedge F_2) = \min\{\dim(F_1), \dim(F_2)\} - 1$. We report about a couple of results (worked out jointly with M. Grötschel) characterizing adjacent faces. One of the main results is the following

Theorem.

Let P be a polyhedron and let $F_1, F_2 \in F(P)$ be two distinct faces of P with $\dim(F_1) = \dim(F_2)$. In case P is not a polytope we assume further $F_1 \wedge F_2 \neq \emptyset$. Then F_1 and F_2 are adjacent iff there exist no faces $F_3, F_4 \in F(P)\setminus\{F_1, F_2\}$ with $\dim(F_3) = \dim(F_4) = \dim(F_1)$ such that $F_3 \vee F_4 = F_1 \vee F_2$ and $F_3 \wedge F_4 = F_1 \wedge F_2$.

Other results include characterizations using a description of P such as $P = P(A,b) = \{x \in \mathbb{R}^n \mid Ax \leq b\}$ or $P = \text{conv}(V) + \text{cone}(E)$. In particular we show that the adjacency relation is invariant under polarity relations such as the (α,β)-polarity defined by $S^{\alpha,\beta} = \{y \in \mathbb{R}^n \mid \beta yx \geq \alpha\beta \ \forall\, x \in S\}$ for $S \subseteq \mathbb{R}^n$ and $\alpha\beta \leq 0$ $(\alpha \in \{-1, 0, 1\}, \beta \in \{-1,1\})$.

E. Balas:

A Restricted Lagrangean Approach to the Travelling Salesman Problem

This talk is based on a joint paper with Nicos Christofides of Imperial College of Science and Technology, London. We discuss a branch and bound method for the travelling salesman problem (TSP) based on arc-premia/penalties. The approach uses a new Lagrangean relaxation of TSP and a restricted Lagrangean problem based on it, with constraints on the multipliers. The role of the constraints is to ensure that at every stage the Lagrangean function defines a spanning subgraph such that a tour in this subgraph which satisfies a complementarity condition is optimal. Several polynomially bounded procedures are given for generating valid inequalities that can be taken into the

Lagrangean function with a positive multiplier without violating these constraints, so as to strengthen the current lower bound. Upper bounds are also generated by a polynomial-time procedure. When the bound strengthening procedures are exshausted without matching the upper with the lower bound, we branch according to some new rules based on disjunctions from conditional bounds. Computational experience on randomly generated asymetric problems with up to 325 cities indicates that the procedure is a substantial improvement over earlier techniques both in terms of the number of order generated and the time used.

M.L. Balinski and H.P. Young

Apportionment: A Problem in Fair Divisions

We wish to allocate an integer $h \geq 0$ proportionally to s rational numbers $(p_1,\ldots,p_s) = \underset{\sim}{p} > \underset{\sim}{0}$ in integers $(a_1,\ldots,a_s) = \underset{\sim}{a} > \underset{\sim}{0}$, $\Sigma a_i = h$. A real-world example is a parliament having h seats where $p_1,\ldots,p_s$ are the populations of s states (or vote tctals of s parties). Apportionment methods must obey certain fundamental properties to be "fair". An essiantal one is "population monotonicity": if populations change and state i's increases relative to j's, then in apportioning h seats i should not now receive less and j more than before. Subject to niceties, any population monotone method must be a "divisor method": there is some real-valued function $d(a) \geq 0$ on integers $0 \leq a \leq h$, monotone increasing in a, such that $\underset{\sim}{a}$ is an apportionment of h for $\underset{\sim}{p}$ iff $\Sigma a_i = h$ and $\min_{i:a_i>0} p_i/d(a_i-i) \geq \max_j p_i/d(a_j)$. A second essential property is that a method be "unbias d": over all problems small states should not tend to be favored over large states, and vice versa. The unique unbiased divisor method has $d(a) = a + 1/2$, a method first proposed by Daniel Webster in 1832.

M. Beckmann

On Continous Models of Transportation and Location

The classical continuum model of transportation

Min $\iint k|\varphi|dxdy$

subject to $\operatorname{div}\varphi = q$ (where $\iint q dxdy = 0$)

$\varphi_n = 0$ on the boundary

(k transportation cut, φ flow vector, q local net surplus)

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with solution

$$k\frac{\varphi}{|\varphi|} = \text{grad } \lambda \quad \text{when} \quad \varphi \neq 0.$$

$$k \geq |\text{grad } \lambda| \quad \text{when} \quad \varphi = 0$$

is generalized to consider the following economic sceneries:

1.) the local net surplus depends on price $q = q(\lambda, x, y)$

2.) production of agricultural products is subject to a constraint on land availability
$\text{div}\varphi \leq a$ or $\text{div} \sum_j \text{div}\varphi_j \leq a$

3.) Production may be similarly constrained by local labor availability

4.) two conditons may be considered, one produced by land only, one produced by labor only

5.) a budget constraint may be imposed in the local consumption vector, and

6.) an arbitrary number of commodities produced for both land and labor may be considered.

P. Bod

On some invariant properties of the solutions of certain linear planning models against price transformations.

Two different implementations of a multiperiodic linear planning model will be considered. The first (M1) is operating with constant prices; the second (M2) with current prices. It will be shown that:

-primal feasibility is invariant against arbitrary price transformations;

-optimality is invariant against price transformations included by shadow prices.

J. Bräuninger

An effective method for linear programming with triangular matrices

A modification of the projection method for linear programming is presented. This modification determines the step direction by solving two triangular systems of linear equations. The triangular matrix is updated in each step by deleting a row and adding a new one whose elments were already computed for the step-size determination. Thus there is no real computational effort in the matrix-updating.

The size of the triangular systems depends on how many of the active constraints have become active after the constraint that's going to become inactive. The worst case, i.e. if the oldest active constraint becomes inactive, the computational effort in solving the triangular systems correspond to that of the matrix-updating in the projection method, whereas in all other cases the effort is reduced. This reduction can be very high.

B. Brosowski

Zur parametrischen Optimierung

Sei X ein lokalkonvexer Raum über $\mathbb{R}$, $F: X \to \mathbb{R}$ ein sublineares Funktional und V, P nichtlineare Teilmengen von X. Für jeden Parameter $x \in P$ werden mit P_x die Lösungen $v_o \in V$ des Minimierungsproblems

$$F(v_o - x) = E_x := \inf_{v \in V} F(v - x)$$

bezeichnet. Auf diese Weise wird eine Abblidung $P: X \to P \circ \tau(v)$ definiert. Es wurde bewiesen

Satz. Ist V und $\overline{\text{conV}}$ kompakt und die Abbildung P oberhalbstetig und konvexwertig, so gibt es für jedes $x \in X$ mindestens einen Minimalpunkt $v_o \in P_x$, der der folgenden Bedingung genügt:

$$\forall_{v \in V} \quad \min_{x^* \in \mathrm{Ep}\,\delta F(v_o - x)} x^*(v_o - v) \leq 0.$$

E. Burkard

Admissible transformation and their application to min cost flows in graphoids

Let E be a finite set and $(H, *, \leq)$ be an ordered commutative semigroup with reduction rule $a \leq b \Rightarrow \exists c \in H!\ a * c = b$. The costs for a feasible solution $S = \{e_1, \dots, e_k\} \subseteq E$ of a combinatorial optimization problem defined by $c(S) = c(e_1) * \dots * c(e_k)$. A transformation of the cost coefficients $c \rightsquigarrow \bar{c}$ is called admissable, if $\exists \beta \in H$ that for all feasible solutions S: $c(s) = \beta * \bar{c}(s)$ holds. For these and general admissible transformations of the form $c(s) * \alpha(s) = \bar{c}(s) * \beta(s)$ with side conditions on $\alpha(s)$, $\beta(s)$ two optimality criteria are shown and their

use for solving some combinatorial optimization problems is outlined.

Admissible transformation can also be used to solve the following min cost flow problem in graphoids: Let $(E, \mathcal{Z})$ and $(E, \mathcal{C})$ be a pair of dual regular matroids with circuit set $\mathcal{Z}$ and cocircuit set $\mathcal{C}$. Every $Z \in \mathcal{Z}$ and $C \in \mathcal{C}$ can be partitioned in Z^+, Z^- resp. C^+, C^- with $|Z^+ \cap C^+| + |Z^- \cap C^-| = |Z^+ \cap C^-| + |Z^- \cap C^+|$. A mapping $f: E \to \mathbb{R}$ is a flow in the digraphoid $(E, \mathcal{Z}, \mathcal{C})$, if it fulfills capacity constraints $0 \leq f(e) \leq k(e)$ $\forall e \in E$ and $\forall c \in \mathcal{C}$: $f(C^+) = f(C^-)$. The costs of a flow are given by $[f \,\square\, c] := (f(e_1) \,\square\, c(e_1)) * \ldots * (f(e_n) \,\square\, c(e_n))]$, where $\square : \mathbb{R} \times H \to H$ is an outer composition, compatible with $(H, *, \leq)$. If E can be partitioned in E_1 E_2 with $f(e_1) = k(e)$ $\forall e \in E_1$, $c(e) = 0$ $\forall e \in E_1$ and if there exists a finite maximal flow, then admissible transformation can be used to solve the problem of finding a maximal digraph flow with minimal costs. The form of corresponding admissible transformations is specified and an algorithm is outlined which allows to find a min cost flow in a finite number of steps. Transportation problems and minimal circuit problems in matroids form examples for the general problem stated above.

L. Collatz

Remarks on recent applications of optimization methods to boundary value problems

Eine für viele Probleme in Natur- und anderen Wissenschaften wichtige Anwendung der Optimierungstheorie besteht in der numerischen Behandlung von Gleichungen, in allgemeiner Form von

$$Tu = \theta.$$

Dabei ist u gesuchtes Element eines Banachraumes, (z. B. ein Vektor, eine Funktion, ein System von Funktionen u.a.), T ein gegebener linearer oder nichtlinearer Operator und θ das Nullelement. (T kann z.B. ein gewöhnlicher partieller Differentialoperator, ein Integraloperator sein). Man sucht ein Näherungselement v, (z.B. eine von Parametern a abhängende Funktion $v(x_1,\ldots x_n, a_1,\ldots a_p)$ mit

$$-\delta_1 \leq Tv \leq \delta_2 \quad \delta_1 \geq 0, \quad \delta_2 \geq 0, \quad \delta_1 + \delta_2 \overset{!}{=} \text{Minimum.}$$

Die Optimierungsmethoden liefern in vielen Fällen ein Verfahren zum Auffinden brauchbarer Näherungen v, und falls Monotoniesätze gelten, sind diese Methoden oft die einzigen Verfahren, die eine garantierbare numerische Schranke für den Fehler liefern. Das wird an Randwertaufgaben mit Singularitäten, mit freien Rändern und anderen Problemen gezeigt.

W. Cottle

Approaches to the solution of quadratic programs over transportation polytopes

We consider four approaches to the problem of minimizing a strictly convex separable quadratic function of many variables subject to the constraints of a capacitated transportation problem. This problem is motivated by the need to estimate input-output matrices with prescribed row and column sums.
The methods considered are: linear approximation, block succession over-relaxation, fixed point computation, and parametric linear complementarity. The convergence of the second method depends on the compactness of the level sets of a certain quadratic function. A theorem stating five equivalent conditions for such compactness will be given.

A conditional study of these (and other) methods is planned for the future.

U. Eckhardt

A Nonconvex Minimization Problem

Given a potential energy functional of the form

$$\Sigma\varphi_{ij}(r_{ij}) - \Sigma P_j x_j$$

(x_j are unknown positions, $r_{ij} = ||x_i - x_j||$, $||.||$ = Euclidean distance, P_j are exterior forces acting on the system in x_j), one wants to characterize absolute minima of this functional. This problem arises in optimal

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location theory, in the theory of cable nets, in structural mechanics of geometrically nonlinear elastic trusses, in geodesy and in solid state physics. For a special case (Thomas system) and under the assumption of symmetry of the system, this problem can be completely solved when the potentials fulfill some requirements.

J. Fischer

Eine Anwendung der nichtlinearen Optimierung auf nichtparametrische Maximum-Likelihood-Schätzung von Wahrscheinlichkeitsdichten (An application of nonlinear programming to nonparametric maximum likelihood estimation of probability densities)

Zu den grundlegenden Problemen in der Statistik gehört die Bestimmung der unbekannten Verteilung einer Zufallsvariablen aufgrund von beobachteten Realisierungen. In diesem Vortrag wird eine Maximum-Likelihood-Schätzung der Dichtefunktionen als Lösung eines Optimierungsproblems in einem geeigneten Hilbertraum eingeführt. Neben Existenz- und Eindeutigkeitsaussagen wird - durch Einführung eines parameterabhängigen Optimierungsproblems - eine Charakterisierung der optimalen Lösung hergeleitet, die dann Anhaltspunkte für die numerische Lösung des Problems bietet.

R. Giles

Adjacency on the Postman Polyhedron

Let $G = (V, E)$ be a loopless, undirected graph and $C \subseteq V$ have even cardinality. A postman set is a subset $J \subseteq E$ such that for every node $v \in V$, the number of edges of J incident to v is odd if and only if $v \in C$. The postman polyhedron $P(G)$ is the sum of the convex hull of all incidence vectors of postman sets and the nonnegative orthant $\mathbb{R}_+^E$. We give a simple characterization of adjacency for vertices of $P(G)$. An upper bound on the distance between two vertices, and hence the diameter of $P(G)$, is given.

K. Glashoff

On simplicial methods for optimization problems

We discuss the application of simplicial methods to optimization of functions. It is shown how these methods can be used without computation of gradients by means of piecewise quadratic interpolation of the objective function. Numerical results are discussed.

M. v. Golitschek

Ein Verfahren zur Skalierung von Matrizen und seine Anwendung in der Graphentheorie

Um eine (m,n)-Matrix $B = (b_{ij})$ zu skalieren, müssen wir positive Zahlen u_i $(i = 1,\dots,m)$ und v_j $(j = 1,\dots,n)$ finden, die das Minimierungsproblem

$$\text{(I)} \qquad \inf_{u_i,v_j} \frac{\max_{i,j} |b_{ij}\, u_i\, v_j|}{\min_{(i,j)\in E} |b_{ij}\, u_i\, v_j|}, \quad E := \{(i,j) : b_{ij} \neq 0\}.$$

lösen. Setzen wir $x_i := -\log u_i$, $y_j := -\log v_j$, $a_{ij} := \log |b_{ij}|$, so können wir (I) überführen in das Minimierungsproblem

$$\text{(II)} \qquad \inf_{x_i,y_j} \max_{(i,j)\in E} |a_{ij} - x_i - y_j|.$$

Ist $G = (V, E_o)$ ein Digraph, dessen Kanten $e \in E_o$ Gewichte $f(e)$ zugeordnet sind, so definieren wir den Mittelwert $M(c)$ eines gerichteten Zyklus $c = e_1, e_2,\dots,e_k$ In G durch

$$M(c) := \frac{1}{k} \sum_{r=1}^{k} f(e_r).$$

Im 1. Teil des Vortrages wird gezeigt, wie die Aufgabe, im Graphen G gerichtete Zyklen mit kleinstem Mittelwert zu finden, in ein Problem der Form (II) übergeführt werden kann. Im 2. Teil stellen wir einen neuen, schnellen Algorithmus zur Lösung des Problems (II) vor, der ebenso effektiv zum Auffinden von Zyklen mit kleinstem Mittelwert verwendet werden kann.

B. Gollan

Higher Order Necessary Conditions for an Abstract Optimization Problem

Higher order necessary optimality are given for an abstract optimization problem in a Banach space with a finite number of equality or inequalitiy constraints. The main result is a multiplier rule of the Fritz John type involving higher order Fréchet derivatives, which generalizes a second order conditions from Hestenes. The approach is straightforward and does not require any constraint qualification. Also it allows for an immediate application to new results in perturbation theory. A general theorem and an example demonstrate, how known sensitivity results can be sharpened considerably by using these higher order necessary conditions.

M. Grötschel

Hypotraceablefacets of the asymmetric travelling salesman polytope

A digraph $G = (V,E)$ is called hypotraceable if it is not traceable (i.e. does not contain a hamiltonian path) but $G-v$ is traceable for all $v \in V$. We first show that such digraphs of order n exist if and only if $n \geq 7$. It is easily seen that, given a hypotraceable digraph $G = (V,E)$ of order n, the inequality $x(E) \leq n-2$ is valid with respect to the (monotone) asymmetric travelling salesman polytope $\tilde{P}^n_T$ (i.e. the convex hull of all incidence vectors of hamiltonian circuits and subsets of these in a complete digraph of order n). We then prove that certain maximal hypotraceable digraphs induce facets of $\tilde{P}^n_T$. Since it is very difficult to check whether a given digraph is hypotraceable these hypotraceable inequalities constitute a rather complicated class of facets of $\tilde{P}^n_T$ These results indicate that for LP-approaches to solve the ATSP, at least when they are based merely on facetial cutting planes, a convergence proof can hardly be obtained.

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P. L. Hammer and B. Simeone

Quasimonotone quadratic 0-1 optimization and bistellar graphs

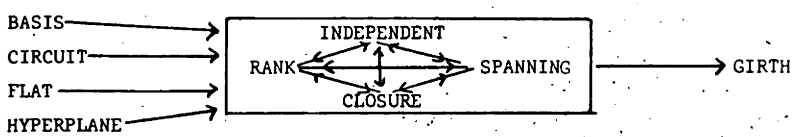
The maximization of a quadratic pseudoboolean function in positive form with the properties that

1) Each complemented variable appears only once
2) No quadratic term involves two complemented variables is equivalent to the problem of finding a weighted maximum stable set in a graph such that its edges can be covered by stars so that no vertex is incident to more than three stars. Such graphs (bistellar graphs) have a very simple structure and they are seen to be closely related to injective graphs: however, it can be shown that the maximum stable set problem for such graphs is NP-complete.

D. Hausmann

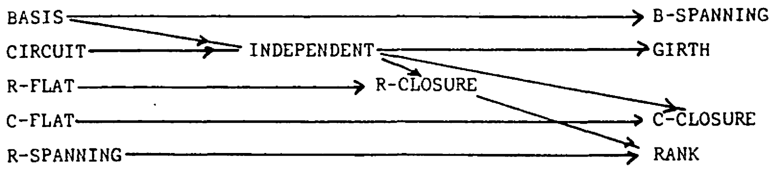
Computational Relations between Various Definitions of Matroids

The class of matroids on a finite ground set E is a well known combinatorial structure which can be defined by a lot of different, but equivalent concepts. Here it is shown that these concepts are only theoretically, but not computationally equivalent. We say that a concept A is polynomially reducible to a concept B and indicate it by an arc from A to B iff given the possibility to check the B-property of a subset E in unit time, then, for any $S \subseteq E$, it is possible to check the A-property of S in polynomial time (polynomial in $|E|$). Our main results can be presented in form of the following digraph:



Some of these results have also been obtained by G. Robinson and D. Welsh.

For the defining concepts of general independence systems we get similar results:



(Here all arcs resulting from transitions have been omitted.) The main tool in the formal derivation of all these results is the concept of an oracle algorithm developed by Hausmann and Korte.

R. Jeroslow

The Limiting Lagrangean

The consistent convex program in R^n, with possibly infinitely many constraints as given by

$$\text{(CP)} \qquad \begin{aligned} &\inf f_o(x) \\ \text{subject to } &f_h(x) \leq 0,\ h \in H \\ \text{and } &x \in K \end{aligned}$$

is conventionally treated by the usual lagrangean, and of course there can be duality gaps even when all f_h are closed mappings and $|H|$ is finite. Fairly recently it was discovered that the duality gap can almost always be closed by perturbing the objective function by a linear term, and letting the size of the perturbation go to zero. I.e.

$$\text{(LL)} \qquad \lim_{\theta \to 0^+} \sup_{\lambda \geq 0} \inf_{x \in K} \{f_o(x) + \theta(wx + w_1) + \sum_{h \in H} \lambda_h f_h(x)\} = v(P)$$

holds for suitable $w \in R^n$, $w_1 \in R$, where $v(P)$ is the value of the primal (also w and w_1 can be found in most cases), under hypotheses far weaker than a Slater point.

E. Johnson

On the generality of the Subadditive Characterization of Facets

Gomory's characterization of facets of the group problem and Araoz's characterization for certain semigroups are shown to apply to a very general situation where only closure of the addition operation is assumed.

B. Korte

Good and Bad Matroid Properties

Matroids as considerably rich combinatorial structures gave rise to a lot of nice properties. This paper deals with the question whether these properties are computationally easy or hard, i.e. whether there exist a polynomial algorithm for these properties or not. As well known, some matroid properties are easy, like finding the maximum weighted bases (Greedy), 3-connectivity, k-disjoint-bases-property. We are able to state a general theorem about matroidal structures relative to the number of automorphisms. This enables us to proof that "almost all" matroid properties (i.e. uniformity, Tutte-connectivity, girth, transversality, Crapo-invariant, Tutte polynomial, duality orientability, representability, etc.) are hard insofar that they have within a certain framework of oracle algorithms exponential computational complexity.

V. Kovacevic-Vujcic

Some computational aspects of nonlinear programming methods

Two algorithms for solving linearly constrained nonlinear programming methods are presented. The basic difference between the two algorithms is in the antizigzagging requirements, i.e. in the policy with respect to active constraints. Algorithms are presented in general form and they apply to the whole class of feasible directions methods. Some numerical results are provided.

E. L. Lawler

Preemptive Scheduling

Among the more interesting developments in scheduling theory in recent years have been results concerning the preemptive scheduling of parallel processors. We describe a simple and efficient algorithm (based on the ideas of T. Gonzaley and S. Sahni) for constructing a preemptive schedule of minimum length for processors with time-varying speeds. This algorithm can be generalized to solve a variety of other problems. Moreover, it provides the basis for a result concerning the minimization of the number of date jobs. Namely for m processors and n jobs, this can be accomplished in $O(n^4)$ time for $m = 2$, and $O(n^{3(m-1)})$ time for $m \geq 3$.

F. Lempio

Some Remarks on the Existence and Boundedness of Lagrangean Multipliers

Given real Banach spaces X and Ω and a convex function $f: X \times \Omega \to \overline{\mathbb{R}}$ the family

(P_ω) Minimize $f(., w)$ on X! $(\omega \in \Omega)$

of convex optimization problems is considered.
Necessary conditions and sufficient conditions for the subdifferential $\partial\varphi(o)$ of the corresponding minimal value function φ at 0 to be nonempty and bounded are given.
These results immediately yield necessary conditions and sufficient conditions for the set of Lagrangean multipliers in nonconvex differentiable optimization to be nonempty and bounded.

H. Maurer

Sufficient optimality conditions in optimization and optimal control

Consider the follwing nonlinear programming problem

(P) minimize $f(x)$ subject to $g(x) \in K$,

where $f: X \to \mathbb{R}$, $g: X \to Y$ are twice

differentiable, X, Y are Banach spaces and $K \subseteq Y$ is a closed convex cone. Optimal control problems with state constraints are a special case of (P). It turns out that the existing proof of the second order sufficient conditions for optimal control problems is incomplete. Moreover, the standard sufficient conditions for the optimization problem (P) are not applicable to optimal control problems. In order to fill this gap between optimization and optimal control we derive modified second order sufficient conditions for (P) which can be applied to optimal control problems.

L. McLinden

Monotone Complementarity Problems

A number of results are announced for complementarity problems associated with maximal monotone multifunctions. Existence, stability, and generic uniqueness of solutions are covered. A parametrized family of approximate complementarity problems is introduced, upon which a general solution procedure can be based. A variety of applications is sketched.

R. R. Meyer

Two-Segment Seperable Programming and Extensions to the Non-seperable Case

A new iterative programming method for convex optimization is described. The method differs significantly from existing separable programming techniques in that it employs a piecewise-linear approximation of at most two segments for each objective term at each objective term at each iteration. The optimal values of the approximating problems can be shown to converge to the optimal value of the original problem. At each iteration a feasible solution as well as a lower bound for the optimal value of the original problem are generated, so that the algorithm may be terminated after a finite number of iterations with a feasible solution whose objective value lies within a prespecified optimal tolerance. Computational experience with this method on a variety of

problems (incuding a model of an actual water supply system that involves about 500 variables and 500 constraints) has shown it to be very efficient and rapidly convergent. An extension of the method to non-seperable problems in n variables employs piecewise-linear objective function approximations determined by the objective values at (n+1) points at each iteration.

K. Neumann

<u>Optimizaiton Problems in cost planning by means of special stochastic activity networks</u>

Stochastic activity networks all of whose nodes have exclusive-or entrance and stochastic exit (so-called STEOR networks) can be associated with Markov renewal processes. Minimizing the expected cost of a project to which a STEOR network is assigned leads to an optimal control problem or to a stochastic dynamic programming problem. The control problem can be solved with the aid of gradient methods in Hilbert spaces. As concerns dynamic programming, the minimum cost function and a corresponding optimal policy may be determined by solving a fixed-point equation. To compute tha (unique) fixed point, the method of "successive approximations" and a "policy improvement" routine can be used.

M. W. Padberg

<u>Solving Large-Scale Travelling Salesman Problems to Optimality</u>

Recent results concerning the characterization of the symmetric travelling salesman problem by way of linear inequalities are reviewed. We then discuss how these results were used in a computational study. Ten large-scale symmetric travelling salesman problems were investigated computationally. In all cases the optimum solution was found and proven to be optimal. The largest sample problem is a 318-city problem involving the (exact) optimization over 50,403 zero-one variables. We also comment on how to use existing software packages such as IBM's MPSX-MIP/370 in the solution of hard combinatorial optimization problems.

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K. Ritter

Rates of superlinear convergence of a class of variable metric methods

This paper considers a class of variable metric methods of unconstrained minimization. Without requiring exact line searches each algorithm in this class converges globally and superlinearly. Various results on the rate of the superlinear convergence are obtained.

S. Rolewicz

On conditions warrantying -convexity and -subdifferentability of primal functionals

Let X, Y be two metric spaces. Let f(x) be a real valued function defined on X. Let Γ be a multifunction defined on Y with values in 2^X. We consider the following optimization problems

(1) $f(x) \to \inf, \quad x \in \Gamma y_o$

S. Dolecki and S. Kurcyusz (SIAM Jour. of Control 16(1978), pp 277-300) have invented a notion of lagrangian for the problems (1). They also have shown that the equivalence of the probelm (1) and the corresponding Lagrange problems are strongly dependent on properties of so called primal functional

(2) $\overline{f\Gamma}(y) \overset{df}{=} \inf \{f(x) : x \in \Gamma y\}$

The most important properties are Φ-convexity and Φ-subdifferentiability. In the talk conditions warrantying Φ-convexity and Φ-subdifferentiability for different classes of functions Φ will be presented.

S. Schaible

Generalized convex quadratic functions - a unified approach

It is shown that all criteria for quasiconvex and pseudoconvex quadratic functions known so far can be derived from a characterization earlier proved by the author. In case of arbitrary convex sets of R^n we derive Ferland's criteria. In addition to his representation of the maximal domain quasiconvexity we obtain

also new ones. In the special case of the nonnegative orthant the criteria of Cottle, Ferland and Martos are proved which these authors derived by means of positive subdefinite matrices. Our proofs do not make use of this concept. Instead we get all criteria by specializing the general characterization of quasiconvex functions. Apart from unifying known characterizations of generalized convex quadratic functions we shall present also new criteria including ones for strictly pseudoconvex functions.

C. P. Schnorr

Combinatorial problems which are hard in the average

We start from the simple observations:

(A) Suppose that there is a fast transformation of the satisfiability to a sparse set, then there is an everywhere fast decision procedure for satisfiability.

(B) Suppose there is a decision procedure for satisfiability which is fast except some sparse set of inputs, then satisfiability can be decided fast everywhere.

The simple proofs that underly these observations hold for the quite general class of self-reducible probelms:

Def Let $C \subset \{0,1\}^*$ then $\varphi: \{0,1\}^* \to \mathrm{pot}\{0,1\}^*$ is a self-transformation of C if (1) $x \in C \Leftrightarrow \varphi(x) \cap B \neq \emptyset$ (2) $\overset{\infty}{\forall} x: \max\{|y| : y \in \varphi(x)\} < |x|$.

With appropriate notions of fast and sparse statements (A), (B) hold for all decision problems that admit a self-transformation. Typical examples are: all known NP-complete problems. various types of isomorphism problems, including the graph-isomorpism problem.

L. E. Trotter

Integer Rounding Properties for Branching Optimization Problems

The lecture is taken from the manuscript "Integer Rounding for Polymatroid and Branching Optimization Problems" by S. Baum and L. E. Trotter, Jr. (Report No. 78120-OR, Institut für Ökonometrie und Operations Research, Universität Bonn).

Suppose M is a nonnegative matrix and w is a nonnegative vector, each with rational entries, and define

$$r^*(w) = \max\{1\cdot y : yM \leq w,\ y \geq 0\},\quad z^*(w) = \max\{1\cdot y : yM \leq w,\ y \geq 0,\ y \text{ integral}\}.$$

Then M has the integer round-down (IRD) property if, for all integral $w \geq 0$, $\lfloor r^*(w) \rfloor = z^*(w)$. Similarly, if $\lceil r_*(w) \rceil = z_*(w)$ for all integegral $w \geq 0$, where $r_*(w) = \min\{1\cdot y : yM \geq w,\ y \geq 0\}$, $z_*(w) = \min\{1\cdot y : yM \geq w,\ y \geq 0,\ y \text{ integral}\}$, then integer round-up (IRU) holds for M.

A branching in a directed graph is a subgraph which contains no (undirected) cycles and whose edges are directed toward different vertices. We show that IRU holds for matrix M whose row are incidence vectors of the edge sets of maximal branchings in a digraph and that when the rows of M are incidence vectors of the edge sets of maximum cardinality branchings in a digraph, then IRD holds for M and IRU holds for M provided each edge is in some maximum cardinality branching. Our proofs rely on Edmonds' theorem for edge-disjoint branchings.

L. A. Wolsey

Integer and Nonconvex Duality: Decomposition With Price Functions

We examine first the standard resource and price decomposition algorithms of Benders and Dantzig-Wolfe can be applied to general mathematical programs, using the duality theory based on price functions developed recently. This leads to a very general Benders or "projection" algorithm which produces an "implementable" algorithm for specific structural non-convex problems, e.g. an apparently new algorithm for bilinear programs, and the algorithm of Balas for quadratic mixed integer programs, as well as a "theoretical" decomposition algorithm for pure integer programs.

J. Zowe

First and second order necessary optimality conditions for extremum problems in topological vector spaces

An abstract optimization problem in infinite-dimensional spaces is studied. From a general extremality condition necessary first and second order conditions are derived. Under differentiability assumptions and a constraint qualification these conditions reduce to classical results.

Berichterstatter: Achim Bachem, Bonn

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