MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 21/1979

Kommutative Algebra und algebraische Geometrie

13. 5. bis 19. 5. 1979

Die Tagung stand unter der Leitung der Herren E. Kunz (Regensburg), H. J. Nastold (Münster und L. Szpiro (Paris).

Wie die hohe Teilnehmerzahl erwarten ließ, ergab sich ein reichlich ausgefülltes Tagungsprogramm, das noch neben den eigentlichen Vorträgen über neue Forschungsergebnisse durch extracurriculare Veranstaltungen der Herren Mumford und Russell ergänzt wurde.

Im Vordergrund des Interesses standen Fragen der lokalen, kommutativen Algebra, ihre Anwendungen in der algebraischen Geometrie, Fragen der algebraischen Geometrie selber und Fragen, die sich aus beiden Gebieten gemeinsam ergeben. Als einzelne Themen sind besonders zu erwähnen: Homologische Fragen über lokale Ringe, hyperelliptische Jacobische Varietäten, Fragen der affinen Geometrie, Deformationen von Singularitäten unter verschiedenen Gesichtspunkten, Residuen und Dualität, Klassifikation 3-dimensionaler algebraischer Varietäten im Sinne von Kodaira und Ueno.

Das Interesse an der Tagung zeigte sich nicht zuletzt auch an der großen Zahl von ausländischen Gästen: etwa die Hälfte der Tagungsteilnehmer, davon 7 aus den USA, 2 aus Kanada, 1 aus Japan, 5 aus Skandinavien und 3 aus dem Ostblock.
Besonderes Gewicht erhielt diese Tagung durch die aktive Mitwirkung von D. Mumford, J. Lipman, M. Hochster, H. Matsumura, J.-L. Verdier u.a.

Vortragsauszüge

G.-M. GREUEL

Deformations of Complex Curve Singularities

The purpose of the talk was to explain how to use the dualising module ω of a reduced complex curve singularity (X_o,x_o) in order to obtain topological and analytic results about (flat) deformations of (X_o,x_o). These results were partly obtained together with R. Buchweitz.

If Ω denotes the module of Kähler differentials on (X_o,x_o)

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then there is a canonical map $d : \mathcal{O}_{X_o,x_o} \longrightarrow \Omega \longrightarrow \omega$ and we define the Milnor number $\mu(X_o,x_o) = \dim_{\mathbb{C}}(w/d\mathcal{O}_{X_o,x_o})$ which generalizes the wellknown Milnor number for plane curves (or complete intersections), and has the properties

1) $\mu = 2\delta - r + 1$, $\delta = \dim_{\mathbb{C}} \overline{\mathcal{O}}/\mathcal{O}$, $r = \#$ branches of (X_o,x_o)

2) If $f : X \longrightarrow D$ is a "good" representative of (X_o,x_o), $X_t = f^{-1}(t)$

   then $\mu(X_o) - \sum_{x \in \mathrm{Sing}(X_t)} \mu(X_t,x) = \dim_{\mathbb{C}} H^1(X_t,\mathbb{C})$

3) If $X_o \subseteq \mathbb{P}^N$ is a complete reduced curve then $2\chi_{an}(X_o) = \chi_{top}(X_o) - \mu(X_o)$ where χ_{an} (resp χ_{top}) denotes the analytic (resp. topol.) Eulercharacteristic. Among other applications one also gets a simple proof that certain curve singularities found by Mumford and Pinkham are not smoothable.

D. LAKSOV

Pfaffian schemes

Let a be an integer, k a field and $R = k[\ldots,x_{ij},\ldots]_{1 \le i < j \le a}$ the polynomial ring in x_{ij}. Denote
$$X = \begin{pmatrix} 0 & \cdots & x_{ij} \\ & \ddots & \\ -x_{ij} & \cdots & 0 \end{pmatrix}$$
the alternating $a \times a$ matrix formed from these variables, and for $c \le \frac{a}{2}$ denote $Pf_{2c}(X)$ the ideal in R generated by the Pfaffians of X of order c.

Theorem: $R/Pf_{2c}(X)$ is a Gorenstein domain of pure codimension $(a-2c+2)(a-2c+1)/2$ in R. The singular locus of this ring is defined by the ideal $Pf_{2c-2}(X)$.

We then deduce that every pfaffian scheme $Pf_{2c}(A)$ of an alter-

nating matrix A in affine space $\mathbb{A}^n$ can be globally deformed, into a scheme $X \subseteq \mathbb{A}^n$ with a stratification $X = X_{2c} \supseteq X_{2c-2} \supseteq \dots \supseteq X_2$ such that X_{2i-2} is the singular locus of X_{2i} and the codimension of X_{2i} in $\mathbb{A}^n$ is $(a-2i+2)(a-2i+1)/2$ for all i.

R.-O. BUCHWEITZ

Linking and Deformations of Cohen-Macaulay singularities

Main theme: If (X,x) is the germ of an isol., red., C-M. singularity, which singularities can occur in the base of the semi-universal deformation?

1) It is shown, that there is a) a canonical reduction to the case of dim 0, b) every artinian algebra of length $n+1$ is deformation of $A_n = \mathbb{C}\{x,\dots,x_n\}/m^2$, so principally it suffices to know the deformation theory of A_n.

2) Often the singularities are of a given "type": determinantial, Pfaffian (= Gorenstein in codim 3), it is shown that all deformations are of the same type iff it is so for the first-order deformations, iff the generic singularity V is rigid and the module $I_V/I_V^2 \otimes \omega_V$ is Cohen-Macaulay. These "very" rigid singularities characterize for example (by spezialisation) all 0-dim, unobstructed, smoothable singularities. If $\underline{x} \subset \mathcal{O}_V$ is a max. reg. sequence, then

$$(*) \quad \lg (I/I^2 \otimes \omega_V \otimes \mathcal{O}_V/\underline{x}) - \operatorname{rg} I/I^2 . \lg \mathcal{O}_V/\underline{x} = 0 \text{ iff } I/I^2 \otimes \omega_V$$

is CM.

3) It is shown, that the bases of the semi-universal deformation of two linked singularities are the same up to smooth factors.

Furthermore the left side of (*) is an invariant of the linking class. So all rigid singularities linked to an complete intersection fulfill 2) and one obtains immediately known results concerning CM-sing of codim 2, Gorenstein of codim 3 etc.

E. VIEHWEG

Classification theory of algebraic threefolds

In this talk a classification table for algebraic threefolds (regular and projective/$\mathbb{C}$) was given and it was shown that the conjectures $C'_{3,1}$ and $C'_{3,2}$ of Iitaka are true.

Conjecture $C'_{n,m}$ (Iitaka): Let $f: V \longrightarrow W$ be a surjective morphism of regular projective verieties, having a regular, connected general fibre V_w, let $n = \dim V$ $m = \dim W$, Kodairadim $K(V_w) \geq 0$, then

$$K(V, \omega_V \otimes f^* \omega_W^{-1}) \geq \text{Max} \{K(V_w), \text{Var}(f)\}$$

K. P. RUSSELL

Problems in the biregular geometry of affine space

The problem is to clarify epimorphisms $k^{[n]} \longrightarrow k^{[m]}$, where $m \leq n$ integers and $k^{[r]} = k[X_1, \dots, X_r]$, up to automorphisms of $k^{[r]}$.

Theorem 1 (Sathaye-Russell) Let k be a locally factorial Krull domain and $F \in k^{[2]}$ s. th. $k^{[2]}/F = k^{[1]}$. Then $k^{[2]} = k[F]^{[1]}$.

Theorem 2 (Gemong) Let k be a field and $F \in k^{[2]}$ a curve with one place at ∞, res. rational over k. Then

1) $F-t$, t transcendantal over k, is a curve with one place at ∞ res. purely inseparable over $k(t)$.

2) $F-\lambda$, $\lambda \in k$, is a curve with one place at ∞ for almost all λ. This is true for all λ if char $k = 0$, but not in general.

Theorem 3 (Russell-Sathaye) Let k be a field, $A = k^{[2]}$, $F \in A[T] \setminus A$ s. th. $A \subsetneq A[T]/F = B \simeq k^{[2]}$, then $A[T] = k[F]^{[2]}$ in the following cases

1) $F = bT+a$; $a,b \in A$

2) F is a Galois equation, i.e. if $G = \mathrm{Aut}_A B$ then $qt(A) = qt(B^G)$

3) $F = a_o+a_1T+ \dots +a_rT^r$, $a_i \in A$, $GCD(a_1,\dots,a_r) \neq 1$

K. P. RUSSELL

Cancellation for $\mathbb{A}^2$ (extracurricular)

A report on the proof of Fujita-Miyanishi-Sugie of

Theorem 1 Let k be a perfect field, A a k-algebra s.th. $A^{[1]} = k^{[3]}$, then $A \simeq k^{[2]}$

This follows from

Theorem 2 Let $A \subsetneq k^{[2]}$ be f.g., regular and factorial. Then $A \simeq k^{[2]}$ if $k^{[2]}$ is separable over $qt(A)$. This follows from

Theorem 3 Let V be a non singular affine surface over k with Kodaira-dimension (in the sense of Iitaka) $-\infty$. Then V contains an open $U \simeq C \times \mathbb{A}^1$, where C is a curve.

A. V. GERAMITA

Principal Ideals and Smooth Curves

Let k be an algebraically closed field, A a regular domain which

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is a f.g. k-algebra, dim A = d, V = spec A. When is it true that every closed point of V is an ideal-theoretic complete intersection? Classically known to be true for V = Spec $(k[x_1,\dots,x_n])$ and more recently when V = Spec (A[x]) (Davis-Geramita).

d = 1: Theorem (Cunnea) A is a P.I.D. $\Leftrightarrow$ genus V = 0

$g > 0$: Let V be presented as that $\bar{V}$ is a smooth projective curve, $\bar{V} \setminus V = \{P_1,\dots,P_t\}$ and embed $\bar{V}$ into Jacobian by $Q \longrightarrow Q-P_1$. Let Γ = subgroup generated by $\{P_i - P_1 \mid 1 \leq i \leq t\}$

Prop. The complete intersections points are precisely the points of $\Gamma \cap V$.

Conjecture (Geramita-Weibel) If char k = 0 and $\bar{V} \leq \mathbb{P}^n(k)$ is a smooth curve of genus ≥ 2 and A = coordinate ring of the affine curve described by $x_n \neq 0$, then A has only finitely many principal maximal ideals.

Theorem (Bombieri) This conjecture is equivalent to the Mordell conjecture.

D. MUMFORD

Hyperelliptic Jacobian Varieties

If $s^2 = \prod_{i=1}^{2g+1} (t-a_i) = f(t)$ is a hyperelliptic curve C, then the affine piece Jac - Θ of the Jacobian of C can be embedded in $\mathbb{A}^{2g}$ and is a complete intersection. This idea goes back to Jacobi, and was rediscovered by Mac Kean and van Moerbeke.

In fact

$$\text{Jac} - \Theta = \left\{ \text{set of divisors } D = \sum_1^g R_i \text{ on } C, \text{ s.th. } R_i \neq R_j \text{ if } i \neq j \text{ and } R_i \neq \infty \right\}$$

and each such divisor D can be described as $V(X(t), s-Y(t))$ for unique monic polynomial $X(t)$ of degree g, polynomial $Y(t)$ of degree $g-1$. Moreover X and Y define such a D iff $X \mid f-Y^2$. Thus if $X(t) = t^g + X_1 t^{g-1} + \ldots + X_g$, $Y(t) = Y_1 t^{g-1} + \ldots + Y_g$, we can embed

$$\begin{aligned} \text{Jac} - \Theta &\hookrightarrow \mathbb{A}^{2g} \\ \text{Divisor Class } D &\longmapsto (X_1, \ldots, X_g, Y_1, \ldots, Y_g) \end{aligned}$$

and the image is given by the g polynomials in X_i, Y_j, being zero, which are the coefficients of the remainder when $f-Y^2$ is divided by X.

Letting $f-Y^2 = X \cdot Z$ ($Z(t)$ monic of degree $g+1$), we can embed $\text{Jac} - \Theta$ in $\mathbb{A}^{3g+1}$ by X_i, Y_i, Z_i. Then since $f = XZ + Y^2$ we find

$$\mathbb{A}^{3g+1} = \coprod_{\text{all } f} (\text{Jac} - \Theta)$$

In this case, we can find g "universal" polynomial vector fields on $\mathbb{A}^{3g+1}$ which are tangent to each Jacobian and restrict to the invariant vector fields here. The rest of the talk dealt with making the embedding explicit by theta functions, constructing a solution of the KdV equation from these functions and using this embedding to characterize hyperelliptic Jacobians.

J. BINGENER

Representationcriteria for analytic functors

In algebraic geometry M. Artin gave very general representation-

criteria for functors, which he used among other things to prove the existence of Hilbert- and Picard-module spaces.

We show, that results can also be obtained in the analytic case. The main step is to replace the conditions of Artin concerning the compatibility with inductive and projective limits by one single condition. As an application we show the

<u>Theorem</u>: Let $X \xrightarrow{f} S$ be a proper flat holomorphic map s. th. $f_*(\mathcal{O}_X) \longrightarrow (X_s, \mathcal{O}_{X_s})$ is surjective for all $s \in S$. Then the relative Picard functor $\mathrm{Pic}_{X/S}$ is representable by a (not necessarily separated) complex space.

O. A. LAUDAL

<u>Tri-secant Lemmas</u>

Let X be any closed subscheme of $\mathbb{P}^N_k$, $k = \bar{k}$, char $k = 0$. Pick a pair of closed subschemes, H and H_o of $\mathrm{Hilb}_{\mathbb{P}^N}$ and consider the set $I = \{Y \in H \mid Z = X \cap Y \in H_o\}$. I is a subscheme of H. If H is the component of Y and H_o the component of Z in $\mathrm{Hilb}_{\mathbb{P}^N}$ we may compute the completion of the local ring of I at Y. Consider the diagramm

$$H^{q-1}(X,N_X) \xrightarrow{k_q} H^{q-1}(X,N_X \otimes O_Z) \xleftarrow{l_q} H^{q-1}(Z,N_Z) \xrightarrow{m_q} H^{q-1}(Y,N_Y \otimes O_Z) \xleftarrow{n_q} H^{q-1}(Y,N_Y)$$

and suppose Y and Z are loc. complete intersections, and put

$$A^1 = \ker(l_1, m_1 - n_1) \subseteq H^o(Z,N_Z) + H^o(Y,N_Y)$$

$$A^2 = \mathrm{coker}(l_1, m_1 - n_1) \oplus \ker(l_2, m_2 - n_2)$$

then exists a morphism of compl. local k-algebras

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$0 : T^2 = \mathrm{Sym}_k(A^{2*})^\wedge \longrightarrow T^1 = \mathrm{Sym}_k(A^{1*})^\wedge$ s.t. $0(\underline{m}_{T^2}) \subseteq \underline{m}^2_{T^1}$ and $\hat{O}_{I,Y} \simeq T^1 \otimes_{T^2} k$. In particular $\dim_Y I \geq \dim A^1 - \dim A^2$ and I is non-singular iff σ is trivial.

If $H = \mathrm{Grass}\,(2,N+1)$ and H_o is the subscheme $\mathrm{Hilb}^r_{\mathbb{P}^N}$, parametrizing the finite closed subschemes of length r, then $I = \mathrm{Sec}_r(X)$ is the scheme of r-secants. Our results imply the classical dimension formulas for $\mathrm{Sec}_r(X)$, and a new proof of the tri-secant lemma. When: $N = 3$, $\dim X = 1$, H the subscheme parametrizing the plane curves of degree n, and $H_o = \mathrm{Hilb}^r_{\mathbb{P}^N}$, our main theorem has the following corollary: Suppose the generic hyperplane section of an irreducible reduced curve X contains a finite subset Z sitting on a plane curve of degree n, but on no more than 3 curves of degree n+1, then X is contained in a surface of degree n.

J. LIPMAN

Residues, Differentials and Dualizing Sheaves

Let ω be a dualizing sheaf on a d-dimensional irreducible projective algebraic variety V over a perfect field k. Let $\Omega^d_{V/k} = \Omega$ be the sheaf of Kähler differential d forms. There is a canonical homomorphism $\Psi : \Omega \longrightarrow \omega$ whose restriction to the smooth part of V is an isomorphism. Thus ω can be canonically realized as a subsheaf of the meromorphic differentials on V, coinciding with Ω at any smooth point.

The existence of Ψ requires some generalization of the theory of residues on curves (possibly singular) to higher dimensional

varieties. Grothendieck deduces such a generalization from his global duality theory. Using a definition due to Cartier, we outlined an elementary, a priori local, approach to residues, and we use this to get the existence of $\wp$ (in the spirit of a paper of Kunz, where CM.-varieties were treated).

G. HORROCKS

Vector bundles on the punctured spectrum of a local ring

Let A be a regular local ring of dimension n and Y = punc A. Let E be a Y-bundle of even rank $2r$ and $n = 2k+1$ be odd. Put $\gamma(E) = \dim H^k(Y, \wedge^r E) \bmod 2$. Then E extending to a local ring of dimension $n+1$ implies that $\gamma(E) = 0$. Moreover if we look at a bundle $\mathcal{E}$ on $\mathbb{P}^{n-1}$ and take any torsionfree extension to $\mathbb{P}^n$, then $\gamma(E) = \sum_{i=1}^{r} \gamma(E_i)$, where E is the bundle obtained by lifting $\mathcal{E}$ to the puncuted cone over $\mathbb{P}^{n-1}$ and localizing, and E_i are the isolated singularities of $\mathcal{E}$.

Let Q be the ring A divided by the homothties of E factoring through free modules. Put $q = \min\{p \mid m^p Q = 0\}$, m the maximal ideal of Q. q has some of the properties of the degree of the divisor of jumping lines in the projective case. The annihilator J of Q has the following property: if $y_1,\dots,y_n$ is a system of parameters in J and $H^d(Y,E)$ is the last non vanishing finite length cohomology group of E, then a homomorphism of $A/(y_1,\dots,y_n)$ into $H^d(Y,E)$ can be lifted to the d-th syzygy of the ideal into E. This enables us to hope to find good lower bounds for rank E in terms of n and q.

H. LINDEL

Projective modules over polynomial rings

Quillen's and Suslin's results justify to conjecture that for arbitrary regular local rings A all the f.g. projective modules P over a polynomial extension $R = A[T_1,\ldots,T_n]$ are free. The following results were shown:

Theorem 1 Let C be a k-algebra of finite type, k a perfect field and $A = C_{\mathfrak{p}}$ a localisation of C with respect to a prime ideal $\mathfrak{p}$. Then the f.g. projective $A[T_1,\ldots,T_m]$-modules are free.

Theorem 2 Let A be a regular local ring with coefficient field k and P a f.g. projective $A[T_1,\ldots,T_m]$ module. Let B be a subring of A that is dense with respect to the $m(A)$-adic topology in A and s.th. A is faithfully flat over B. If $\operatorname{rg} P \geq \dim A$, then P is extended from $B[T_1,\ldots,T_m]$.

M. HOCHSTER

Recent progress on homological conjectures on local rings

It was shown that the direct cummand conjecture implies the "usual" consequences of the existence of big Cohen-Macauley modules in all characteristics, including mixed characteristic. The implication: direct summand conjecture $\Rightarrow$ new intersection conjecture, is proved via the intermediary of a new homological conjecture, the "canonical element" conjecture.

S. GRECO

Quasi coherent sheaves over Henselian schemes
(joint work with R. Strano)

An affine henselian scheme is a ringed space constructed from a

Hensel couple $(A,\mathfrak{a})$ with the same procedure used for formal schemes but with henselization in place of completion. Thus the henselian spectrum sph $(A,\mathfrak{a})$ of the henselian couple $(A,\mathfrak{a})$ is $(X,\mathcal{O}_X)$, where $X = \text{spec}\,(A/\mathfrak{a})$ and $\mathcal{O}_X\,(D(f)\cap X) = {}^hA_f$ (= henselization of ${}^hA$ with respect to $\mathfrak{a}\,{}^hA_f$). For any A-module M one can define $\tilde{M}(D(f)\cap X) = {}^hA_f \otimes_A M$. Both $\mathcal{O}_X$ and $\tilde{M}$ are quasicoherent sheaves.

Theorem: Let $X = \text{sp h}(A,\mathfrak{a})$. Then every quasicoherent sheaf $\mathcal{F}$ on X is of the form $\tilde{M}$, where $M = \Gamma(X,\mathcal{F})$.

Corollary 1 For any quasi coherent sheaf $\mathcal{F}$ on X one has $H^1(X,\mathcal{F}) = 0$.

Corollary 2: If $f : X \longrightarrow Y$ is an integral mophism of henselian schemes and Y is affine, then X is affine.

P. BERTHELOT

A survey of crystalline Dieudonné theory

The aim of the lecture was to outline a generalization of Dieudonné theory for commutative finite p-group schemes or p-divisible groups over an arbitrary base scheme of characteristic $p>0$, from a joint work with L. Breen (Rennes) and W. Messing (Irvine). Let $\Sigma = \text{Spec}(\mathbb{Z}_p)$, and S be a scheme on which p is nilpotent; to any abelian sheaf G on the site of S-Schemes we associate an abelian sheaf $\underline{G}$ on the big crystalline site CRIS(S/Σ), defined by $\Gamma((U,T,\delta),\underline{G}) = G(U)$ for any object (U,T,δ) in CRIS(S/Σ). Extending a construction of Grothendieck and Messing, we define

$$\Delta(G) = T_{\mathfrak{I}}\,\mathbb{R}\,\mathcal{H}om_{S/\Sigma}(\underline{G},\mathcal{O}_{S/\Sigma})$$

$$\mathbb{D}(G) = \mathcal{E}xt^1_{S/\Sigma}(\underline{G},\mathcal{O}_{S/\Sigma})$$

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where $Hom_{S/\Sigma}$ is the sheaf of homomorphisms in the category of abelian sheaves on $CRIS(S/\Sigma)$ and $Ext^i_{S/\Sigma}$ its derived functors. Using these terms some results of classical Dieudonné theory were generalized.

J.-L. VERDIER

Local Euler numbers

Let X be an analytic space and $x \in X$ a point. Three algebraic descriptions of the local Euler number $Eu(X,x)$ are given:

Let $\tilde{X} \longrightarrow X$ be the Nash transform of X, Y the fiber over x, D the divisor in the blown-up variety of $\tilde{X}$ along Y, above Y, N the normal bundle of D, $\tilde{T}$ the Nash tangent bundle over $\tilde{X}$ and $\tilde{T}'$ its lifting to D.

Theorem 1 $Eu(X,x) = c_{n-1}(\tilde{T}'-N) \cap [D]$

Let $X \xrightarrow{p} \mathbb{A}^2$ be a generic projection, $\Gamma' \subset X_{reg}$ the diamation locus of p, Γ its closure in X. Let H be a generic hyperplane going through x.

Theorem 2 $Eu(X,x) = m(\Gamma,x) + (-1)^{\dim X} Eu(X \cap H,x)$, where $m(\Gamma,x)$ is the multiplicity.

A third description is given involving the inverse of a map $Eu : C_J \longrightarrow F_J$, where J is a Whitney stratafication of X, F_J the functions on X constant on the strata, C_J the free group generated by the closures of the strata.

Theorem 1 is due to Conzalez and Verdier, theorem 2 to Lé and Teissier and the third description to Dubson.

L. SZPIRO

On the rigidity theorem of Arakelov and Parsin

We extended the quoted theorem to char $p > 0$ in the following form:

Let $g \geq 2$ be a number, C a smooth projective curve over a field k, S a finite number of points of C. Then the set of projective, relatively minimal morphisms $X \xrightarrow{f} C$ from a smooth surface X to C such that

a) genus of the fibers = g

b) fibers are semistable

c) the fibers are smooth over C-S

d) the Kodaira-Spencer class of f is not zero

is a finite set.

H. FLENNER

Deformations of holomorphic mappings

Let $f_o : X_o \longrightarrow Y_o$ be a holomorphic mapping between complex spaces. The two following deformationtheories were considered.

1) A deformation of X_o/Y_o is a deformation $X \longrightarrow S$ of X_o and a map $f : X \longrightarrow Y_o$ with $f/X_o = Y_o$. 2) A deformation of f_o is a deformation $X \longrightarrow S$ of X_o, a deformation $Y \longrightarrow S$ of Y and a S-map $f : X \longrightarrow Y$ with $f/X_o = Y_o$. The following statements were true: Is $f_o : X_o \longrightarrow Y_o$ a holomorphic mapping germ then there exists a semiuniversal deformation of X_o/Y_o resp. of f_o under suitable finiteness conditions on certain tangent functors. Is $f_o : X_o \longrightarrow Y_o$ a holomorphic mapping between complex spaces, where X_o (and Y_o) is (are) compact, then there exists a semiuniversal deformation of X_o/Y_o (resp. f_o).

H. MATSUMURA

Dimension of formal fibers

Let $A \longrightarrow B$ be a homomorphism of noetherian rings. For $\mathfrak{p} \in \operatorname{spec} A$, let $K(\mathfrak{p})$ denote the residue field of $A_{\mathfrak{p}}$. Then the fiber of B over $\mathfrak{p}$ is $B \otimes K(\mathfrak{p})$.

Theorem 1 If B is catenary and if going-down holds for $A \longrightarrow B$, then $\dim B \otimes K(\mathfrak{p}) \geq \dim B \otimes K(\mathfrak{p}')$ for $\mathfrak{p} \subset \mathfrak{p}'$.

Let (A,m) be a noetherian local domain of dim $n > 1$, and let K be the quotient field of A. The generic formal fibre $\hat{A} \otimes K$ has the largest dimension among the formal fibers, and certainly $\dim B \otimes K \leq n-1$.

Theorem 2 If A is geometric (i.e. $A = S_{\mathfrak{p}}$ where S is of finite type over a field), then $\dim \hat{A} \otimes K \leq n-1$.

Theorem 3 We have $\dim \hat{A} \otimes K \leq n-2$ in the following cases

(α) $\exists$ ideal $I \neq 0$ of A s.th., A is I-adically complete or

(β) $\exists$ a subring A_o of A s.th. A_o is a complete local ring with $\operatorname{rad} A_o = m_o = m \cap A_o$, $[A/m : A_o/m_o] < \infty$ and $0 < \dim A_o < \dim A$.

The surprising fact is that we cannot have any better estimate.

Example: $A = k[x_1]_{(x_1)}[[x_2,\dots,x_n]]$

or $A = k[[x_1,\dots,x_{n-1}]][x_n]_{(x_1,\dots,x_n)}$.

In both cases we have $\dim \hat{A} \otimes K = n-2$

C. ROTTHAUS

Completions of semilocal excellent rings

Theorem: Let A be a semilocal, noetherian ring, I an ideal contained in the Jacobsonradical of A. Assume A to be

complete and the formal fibers of A/I to be geometrically regular. Then the formal fibers of A are geometrically regular.

H.-B. FOXBY

Complexes of modules and relative K groups

For an additive functor $S : \underline{\underline{P}} \longrightarrow \underline{\underline{P}}'$ between additive categories a bounded complex $P.$, where $P_m \in \text{ob}\,\underline{\underline{P}}$, is said to be an S-complex if $SP.$ is contractible. Write $K_o(A,S) = F/R$ where F is the free abelian group on the isomorphism classes of bounded complexes $(P.)$ of f.g. projective A-modules which are also S-complexes and where R is generated by the elements $(P.)-(P.')-(P.'')$ whenever there is a degreewise split exact sequence $0 \longrightarrow P.' \longrightarrow P. \longrightarrow P'' \longrightarrow 0$; and by the classes $(P.)$ where P is contractable. Then there is a group homomorphism $W : K_o(A,S) \longrightarrow K_o(S)$ ($K_o(S)$ the relative K-group of Bass). Under certain assumptions a more detailed description of the map W is given, which leads to a formular for $\det : K_o(A,S) \longrightarrow \text{Pic}(A,S)$. A similar description is also given for the Whitehead torsion.

M. KNEBUSCH

Signatures and real closures of algebraic varieties

Let X be a divisorial scheme. A signature on X is a ring homomorphism $\sigma : W(X) \longrightarrow \mathbb{Z}$, where $W(X)$ denotes the Wittring of X. To each signatur σ of X there exists a "real closure" uniquely determined up to an isomorphism. It is the maximal profinite unramified covering S of X, on which σ can be extended. Denoting by $\tilde{X}$ the universal covering of X we have $[\tilde{X} : S] \leq 2$. If

$\Gamma(X,\mathcal{O}_X^*)$ contains a rational prime number p, then $[\tilde{X} : S] = 2$ and thus S is characterized by an involution $\alpha \in \pi_1(X)$. In this case, if X is a spectrum of a semilocal ring, we have a one to one correspondence between signatures and conjugacy classes. Bijectivity seems to hold also for arbitrary curves (proved if X is affine and regular). In the case X = spec k, k a field, our theory is identical with Artin-Schreier's theory.

L. AVRAMOV

On the convergence radius of Poincaré series of local rings

Let (A,m,k) be a local noetherian ring, $P_A(t) = \sum b_i t^i$ be its Poincaré series. The growth of the Betti-numbers b_i is reflected in the value of r_A, the convergence radius of the (formal) power series $P_A(t)$. Golod and Gulliksen noticed that if $P_A(t)$ is rational, then either $r_A < 1$ or A is a complete intersection. We study an invariant of A, the Lie Algebra L^A of primitive elements of the Hopf algebra $\mathrm{Ext}_A(k,k)$ with the classical Yoneda product. If either (i) edim A - depth A $3 \leq$ or (ii) edim A - depth A = 4 and A Gorenstein, then L^A contains non abelian free graded subalgebra and from this we obtain that under the condition (i) or (ii) the following statements are equivalent.

1) A is a complete intersection
2) $r_A > \frac{\sqrt{2}}{2}$
3) $\mathrm{Ext}_A(k,k)$ is a noetherian k-Algebra
4) $L_i = 0$ for some $i \equiv 0 \bmod (6)$.

E. D. DAVIS

Prime Sequence Questions

It is known that a complete intersection prime ideal is generated by a regular sequence; and moreover, in the case of local rings, this generating sequence is necessarily a prime sequence, i.e. a regular sequence with initial intervals generating prime ideals. Ohm formulated the Prime Sequence Question: Is every complete inter- section prime ideal generated by a prime sequence? The answer is "no" in general (Heitmann). However it is of interest to investigate this question in "natural" situations, e.g. the answer is "yes" for homogeneous prime ideals in graded rings over fields. For affine domains over an infinite field the following theorem was proved:

Theorem: Let I be a proper ideal in an affine domain over an infinite field, then I has a minimal basis B, i.e. a basis of least possible cardinality with the property:

$C \subseteq B$ and $|C| <$ depth I $\Rightarrow$ C generates a prime ideal.

W. V. VASCONCELOS

The homology of $I/I^2, \Omega_k(S)$ and $Der_k(S,S)$

Let I be an ideal in a noetherian local ring R and consider the conjecture (C): If I has finite projective dim. over R and I/I^2 has finite projective dim. over R/I, then I is generated by a regular sequence.

(C) is true if $pd_{R/I}(I/I^2) = 0,1$, or I CM of codim 2, I Gorenstein of codim 3, or I an almost complete intersection. Attempts to construct an example backwards, that is by picking an S-module M of finite proj. dimension and taking R=symmetric

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algebra of M/S and I=augmentation ideal of R, will always fail if S contains a field.

When R is a polynomial ring over a perfect field k and $S = R/I$, $\Omega_k(S)$ and $Der_k(S,S)$ show similar rigidity. Thus, in the case of $\Omega_k(S)$, one has the homological triviality above (i.e. I is locally a complete intersection, if $pd_S \Omega_k(S) < \infty$) in some cases. Despite the risks involved in view of the Zariski-Lipman's conjecture, it might still be that $Der_k(S,S)$ is also homologically trivial.

M. HERRMANN

Characterizations of complete intersections
(joint work with U. Orbanz)

We try to characterize c.i. ideals in C.M.-ring by a certain equimultiplicity condition. (This was also independently done by Vogel and Achilles, but their proof is more complicated than ours). The idea is to describe the behaviour of R along $\mathfrak{a}$ by the multiplicity-symbol $e(\underline{x}, gr^n_{\mathfrak{a}} R)$, where $\underline{x}$ is a multiplicity-system for $R/\mathfrak{a}$. From that one can deduce a result of Cowsik-Nori, and a strengthing of that by Waldi and the result of Vogel and Achilles. With our methods we also obtain a proof for the fact that the dimension of the fibers of a blowing up $Bl_{\mathfrak{p}}(R)$ of a quasiunmixed ring R along a prime $\mathfrak{p}$ with $R/\mathfrak{p}$ regular is constant iff $e_o(R) = e_o(R_{\mathfrak{p}})$. If R is quasimunmixed, 2-dimensional with $|R/m| = \infty$, then the morphism $Bl_{\mathfrak{p}}(R) \longrightarrow spec\ R$ is finite if one has "equimultiplicity" $e_o(R) = e_o(R_{\mathfrak{p}})$.

M. BORATYNSKI

Generating ideals up to radical

Let R be a commutative (noetherian) ring with $\dim R = d > 0$ and let $I \subset R$ be an ideal. Then there exists a projective R-module P of rank d and an exakt sequence $P \longrightarrow J \longrightarrow 0$ with an ideal $J \subsetneq R$ s. th. $\operatorname{rad} I = \operatorname{rad} J$.

This result is a global version of the existence of parameters theorem in the local case. The proof follows from the two Lemmas.

Lemma 1 Let $I \subseteq R$ be an ideal, then there exists an ideal $J \subseteq R$ with $\operatorname{rad} I = \operatorname{rad} J$ and the minimal number of generators of $J/J^2 = \mu(J/J^2) \leq \dim R$.

Lemma 2 Let $I \subseteq R$ be an ideal with $\mu(I/I^2) \leq n$, $n \geq 1$. Then there exists a projective R-module P of rank P and an exact sequence $P \longrightarrow J \longrightarrow 0$, where $J \subsetneq R$ is an ideal with $\operatorname{rad} I = \operatorname{rad} J$.

M. BRODMANN

Arithmetic depth of blowing up

Let (R,m) be a local ring of a pure-dimensional algebraic variety of dimension d. Let $I \subseteq R$ be an ideal minimally generated by elements and put $h = \operatorname{ht}(I)$. Consider

$$X = \operatorname{Proj}\,(R]I[:= \bigoplus_{n \geq 0} I^n)$$

put $m]I[= m \oplus I \oplus I^2 \oplus \ldots$, and denote

$$\mathcal{A} = R]I[/ m]I[$$

which describes the vertex of the affine cone over X.

Theorem (Valla) R C.M., $\mu = h \Rightarrow \mathcal{A}$ C.M.

We generalize this in two directions

Theorem 1 Let R be CM, $\mu \leq h+1$ and I is generically a complete intersection, then

(i) depth $(\mathcal{A}) \geq \min(d+1, \mu+1+\dim R/I)$, if $h > 0$

(ii) $d \geq$ depth $(\mathcal{A}) = \min(d, \mu+1+\dim R/I)$, if $h = 0$

Theorem 2 Let $\mu = h$, put $e = \sup\{n / H^i_m(R)$ f.g. for $i < n\}$

There is a ν s.th. whenever $x_1, \ldots, x_\mu \in m^\nu$, $\mu \leq e$

$x_1, \ldots, x_\mu$ R_p regular for all $p \in \operatorname{spec} R$, then

(i) $H^i_{m]I[}(R]I[) := H^i = 0$ for $i \leq$ depth R

(ii) H^i f.g. for $i \leq \mu$.

G. ALMKVIST

Decompositions, Invariants and Reciprocity Theorems in Characteristic p

Let k be a field of characteristic p and G a group with p elements. Let $V_1, \ldots, V_p$ be the indecomposable $k[G]$-modules of dimension $1, \ldots, p$. Set $\tilde{R}_G = R_G/(V_p)$ where R_G is the representatinon ring. If $\sigma(V_{n+1}) = \sum_{r=0}^{\infty} \widetilde{S^r V_{n+1}} t$ in $\tilde{R}_G[[t]]$ then we have the reciprocity theorem

$$\sigma_{1/t}(V_{n+1}) = \begin{cases} -t^{n+1}\sigma_t(V_{n+1}) & \text{if } n \text{ is even} \\ -t^{n+1}V_{p-1}\sigma_t(V_{n+1}) & \text{if } n \text{ is odd.} \end{cases}$$

Similar results are obtained for the Hilbert series of invariants.

K. LANGMANN

Etale Extensions of excellent and japanese rings

Let A be a noetherian regular ring with $A \supseteq \mathbb{Q}$, $B \supset A$ noetherian, B integral over A and for every maximal ideal $m \subset A$ let mB be

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reduced, then, if A is japanese, then so is B and in this case for primes $p \subset A$ and $q \subset B$ $(A/p)_{q \cap A}$ is normal iff $(B/pB)_q$ is normal. It follows if $p = \bigcap_{i \in I} p_i$ with p_i prime, then $pB = \bigcap_{i \in I} p_i B$. If A is excellent, then so is B and $(A/p)_{q \cap A}$ is regular iff $(B/pB)_q$ is regular. Conversely if B is excellent then A is excellent iff A is japanese or the condition above for the normal locus of A/p and B/pB holds. An example for the situation of this theorem are étale extensions B of A.

Berichterstatter: M. Steurich

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