MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 22/1979

Mathematical Problems in the Kinetic Theory of Gases

20.5. bis 26.5.1979

Die Tagung fand unter der Leitung von H.Neunzert (Kaiserslautern) und D.C.Pack (Glasgow) statt. Im Mittelpunkt des Interesses standen Fragen der Existenz und des qualitativen Verhaltens von Lösungen der Boltzmanngleichung, der Vlasov--Gleichungen und insbesondere auch von sog. Modellgleichungen. Daneben gab es auch Vorträge über allgemeine mathematische Methoden, die im Bereich der kinetischen Gastheorie Anwendungen finden wie auch über numerische Untersuchungen. Während die Herleitungs- und Existenzfragen für die Vlasov-Gleichung in gewissem Sinn nahezu abgeschlossen zu sein scheinen, sodaß sich das Interesse hier mehr auf Fragen des qualitativen Verhaltens der Lösungen verlagern wird, ist ein zeitlich globaler Existenzsatz für die Boltzmanngleichung selbst noch nicht in Sicht. Allerdings zeigte gerade diese Tagung, wie viele Anstrengungen hier aufgebracht wurden; die dabei entwickelte Methodenvielfalt stimmt daher doch optimistisch.

Auch die Untersuchung spezieller Lösungsalgorithmen (z.B. Chapman-Enskog), die Berechnung neuer expliziter Lösungen und Fragen der Formulierung von Randwertaufgaben waren Gegenstand verschiedener Vorträge.

Wie schon bei einer verwandten Tagung vor drei Jahren erwies sich die Zusammenarbeit von Wissenschaftlern verschiedener Disziplinen (Physik, Mechanik, Mathematik) als außerordentlich fruchtbar - sie ist in solchen Bereichen der angewandten Mathematik geradezu unabdingbare Voraussetzung für wissenschaftlichen Fortschritt+.

Die Tagung verlief in sehr harmonischer Atmosphäre, wozu die wie immer ausgezeichnete Betreuung der Teilnehmer durch das Personal des Instituts wesentlich beitrug.

+ Es sei noch vermerkt, daß nahezu die Hälfte aller Teilnehmer zum erstenmal in Oberwolfach war.

Vortragsauszüge

D.S.BUTLER:

Making the Chapman-Enskog expansion uniformly convergent

The Chapman-Enskog expansion for a solution of the Boltzmann equation can be obtained formally as an expansion in powers of the Knudsen number. Examination of the asymptotic behavier of the terms of the expansion for large molecular velocities shows that the expansion cannot be uniformly convergent, evenasymptotically, over the whole of the molecular velocity space. This mean that the derivation of the continuum equations such as the Navier-Stokes and Burnett equations from the Chapman-Enskog solution is invalid.

As a first step towards the construction of a uniformly valid Chapman-Enskog solution, the possible forms of the asymptotic behavier of the solutions of the Boltzmann equation for large molecular velocity are considered. Two types are identified - type (a)which involve very rapid changes in the distribution function and seem likely to arise mainly as local catastrophes and type (b) where changes are much slower. It is shown that asymptotically type (b) solutions must be spherically symmetric in velocity space and a consistency condition is derived for the leading term.

H.CABANNES:

Global solution of discrete Boltzmann equation

The general evolution equations of a gas with adiscrete repartition of velocities appear in a form, (1), called the discrete Boltzmann equation. The unknown functions are the densities N_i of molecules having velocities represented by a finite member of constant vectors $\vec{u}_i$:

(1) $$\frac{\partial N_i}{\partial t} + \vec{u}_i . \vec{\nabla} N_i = \frac{1}{2} \sum_{jkl} A_{ij}^{kl} (N_k N_l - N_i N_j)$$

(2) $$N_i(\vec{x},0) = N_{0i}(\vec{x}) \qquad (i=1,2 \ldots p)$$

The global existence of the initial value problem (1)-(2) has been proved first for certain models by Nishida and Mimura for small initial values, and by Tartar and Crandall for periodic, then for arbitrary but bounded initial values. We extnd the global existence theorem to a more complex model: three-dimensional model with 14 velocities related to the symmetries of the cube, for arbitrary initial values and for more complex boundary conditions: shock tube problem.

C.CERCIGNANI:

A nonlinear criticality problem in the kinetic theory of gases

If one lets a gas flow in a half space normal to an infinitely extended flat wall(the latter representing, e.g., a perforated plate or an evaporating surface), the following phenomenon is observed in both numerical and approximate analytical solutions of the Boltzmann equation with boundary conditions appropriate to the problem: a steady solution exists if and only if the Mach number tends to a nonsupersonic value at infinity ($M_\infty \leq 1$).
An interesting mathematical problem arises: to prove that a critical Mach number exists and that sucha number is exactly unity. The first part of this conjecture is proved, several remarks are made in order to make it plausible. In particular, a three-dimensional version of the problem (with a finite plate) is used to show that the one-dimensional solution can represent an actual situation just in a neighbourhood of the plate if the pressure downstream is sufficiently low. A discussion of this extended problem illuminates the simpler one.

R.J.COLE:

Complementary bivariational principles for linear problems involving non-self-adjoint operators

The Boltzmann equation in his exact form is highly non-linear. However, many flow features can be adequately by linearised model equations (such as the B.G.K.equation) which we may write as $f_o - Tf=0$, T being a linear operator from a Hilbert space into itself and f_o being a known function. In problems where it is required to know only a global quantity, for example the wall shear stress in Couette flow or the flow rate of a gas through an internal body, then often this quantity is related to a functional $\langle g_o,f \rangle$ where g_o is a known function and f is the solution of the linear equation. In the case in which T is a self-adjoint positive definite operator, there have been developed complementary bivariational principles that supply upper and lower bounds to $\langle g_o,f \rangle$. When T is not self-adjoint, new variational principles are needed. Application to Fredholm integral equations will be outlined.

S.M.DESHPANDE:

MONTE CARLO SIMULATION OF LOW DENSITY FLOW PAST A CONE USING RCN AND TC STRATEGY

The random collision number strategy (RCN) has been found to take very nearly the same computer time as the Time Counter (TC) strategy od Bird for the transitional flow past cone in spite of that the RCN strategy involvs summation over all molecular pairs in a cell. A modification in the RCN strategy has been suggested so as to make it linearly related to the number of molecules in a cell. This modified strategy also has been

found to require very nearly the same computer time as that required by the TC or the RCN strategies for the problems handled.

M.H.ERNST:

EXACT SOLUTIONS OF NON-LINEAR MODEL BOLTZMANN EQUATIONS

We discuss model Boltzmann equations, in a spatially homogeneous case, for the energy distribution function of a gas in two-dimensions. The model is defined in terms of a transition probability of the form $a(\varepsilon)\delta(\varepsilon-\varepsilon')$ or its discrete analogue, where ε and ε' are the total kinetic energies of the colliding particles resp. before and after a collision, and $a(\varepsilon)$ stillcan be chosen.

With the choice $a(\varepsilon)=\varepsilon^{1-d}$ one obtains a discrete or continuous models with an energy independent collision rate (simular to Maxwell molecules) for which special solutions have been obtained, which are extensions of the similarity solution, found by Bobylev, Krook and Wu (BKW-mode).

With the choice $a(\varepsilon)=1$ one obtains a two-dimensional model with a continuous or discrete range of energy states, which can be solved exactly for arbitrary initial distributions. One of the consequences is a non-uniform approach to the final Maxwellian distribution function, as will be illustrated by a typical example. Similar non-uniformities appear in Maxwell models. Finally a new similarity solution for three-dimensional Maxwell molecules with isotropic scattering is given. It is closely related to the Weierstrass elliptic function, and contains the BKW-mode as a special case.

R.GATIGNOL:

NUMERICAL STUDY OF UNSTEADY FLOWS DESCRIBED BY BOLTZMANN'S MODEL EQUATIONS

Using discrete kinetic theory, Boltzmann's equation is replaced by a system of nonlinear differential equations. Our purpose is to investigate the starting motion of the gas. As a result of particle collisions, i.e. of the nonlinear character of the equations, in a Couette flow, we point out the component v of the flow velocity normal to the plates developed after the initial time. Digital computations are made for simple models of gas (four or six velocities) and with diffuse boundary conditions. Numerical results are obtained for different values of the wall velocities and for different distances between the two plates. As time is increased, the maximum value of v oscillates from one plate to the other and approaches zero about some mean free times. For the far-away plates, the results are in good agreement with those found about Rayleigh problem by other methods.

By using the numerical scheme, we can deduce some results on existence and uniqueness of a solution of the problem; the solution existence is proved for a time interval of the order of some mean free time.

W.GREENBERG:

DISCRETE MODEL OF A BOLTZMANN GAS

The Boltzmann Equation is studied on a periodic lattice, in which the streaming term is replaced by a finite difference approximation. Utilizing analyticity properties of the free semigroup, it is possible to demonstrate that an iterative scheme for the corresponding integral equations on L^1 converges to a global solution of the differential equation for all initial values. By using positivity, an H-theorem, and energy conservation, the limit as the lattice spacing is reduced to zero is shown to exist weakly. These results are obtained for Maxwellian and non-Maxwellian gases.

J.HEJTMANEK:

WEAK SOLUTIONS OF THE CARLEMAN EQUATIONS

Hilbert 1912 and Chapman-Enskog 1916 tried to find a derivation of the hydrodynamic equation from the Boltzmann equation through a limiting process. Both derivations should be made mathematically rigorous. During the last years some hope grew that this could be preformed: 1) Nonlinear functional analysis and nonlinear semigroup theory have been in steady process. 2) For a very simple, but nevertheless very instructive model, which was proposed by Carleman 1931, mathematically exact solutions for the Cauchy problem were found, and the limiting process $\varepsilon \to 0$ for the semigroup has been studied, where is the mean free path.
The aim of this work together with H.Kaper, Argonne National Laboratory, is to use a method proposed by Godanov and Sultangazin, to prove that the solution of the Carleman equations approaches the solution of a hydrodynamic equation in a weak sense with the square root of the mean free path.

E.HORST:

ON THE EXISTENCE OF GLOBAL CLASSICAL SOLUTIONS OF THE INITIAL VALUE PROBLEM OF STELLAR DYNAMICS

The initial value problem of stellar dynamics is the following

$$\frac{\partial}{\partial t}\phi(t,x,v) + v\frac{\partial}{\partial x}\phi(t,x,v) + E(t,x)\frac{\partial}{\partial v}\phi(t,x,v) = 0, \quad \phi(0,.) = \varphi,$$

where $E(t,x) = \gamma \cdot \int |x-y|^N \phi(t,y,w)\,d(y,w)$, $t \in \mathbb{R}$, $x,v \in \mathbb{R}^N$ with $\gamma < 0$.

In plasma physics the same equation with $\gamma > 0$ is considered. The non-linearity E is rather nasty because of the singularity of the integral kernel $e(x) = \gamma \cdot x/|x|^N$. Therefore J.Batt in 1963 considered a mollified equation where e is replaced by a smooth e^ε which tends to e as $\varepsilon \to 0$. It can now be shown that in certain cases the solutions of the modified equation converge to a solution of the original problem.

(i) if N = 1,2 there exists a global solution, (ii) if N ≥3 there exists a local solution, (iii) if N = 3 and φ has rotational symmetry there exists a global solution, (iv) if N≥4 there exist counterexamples which show that not always a global solution exists.

R. ILLNER:

A GLOBAL EXISTENCE THEOREM FOR THE NONLINEAR VLASOV EQUATION

The initial value problem for the modified Vlasov equation with a mollification parameter $\delta > 0$, as introduced by Batt, has a unique global solution in the weak sense whenever $f_o \in L^1(\mathbb{R}^6)$ and $f_o \geq 0$ a.e. Assuming boundedness of f_o and boundedness of the kinetic energy uniformly in δ and t, it is shown that, as $\delta \to 0$, there are subsequences $\delta_n \to 0$ such that the corresponding solutions converge weakly in the measure-theoretical sense. The limits can be seen to be global weak solutions of initial value problem for Vlasov's equation, and these solutions are weakly continuous with respect to t. Boundedness of the kinetic energy is in the plasma physical case a simple consequence of energy conservation, in the stellar dynamic case, Horst has found a estimate for the potential energy that, together with energy conservation, entails boundedness of the kinetic energies. Hence the weak existence theorem is true in both cases.

E.A. JOHNSON:

GENERALIZED HYDRODYNAMICS AND MODEL EQUATIONS: LINSHAPE APPROXIMATIONS

Various approximation methods have been proposed for obtaining predictions from the Boltzmann equation in the kinetic regime (external scale comparable to mean free path in a gas). Here we investigate the effectiveness of several of these approximations by considering a simple model equation, whose properties are known exactly. We compare the corresponding exact number density fluctuation spectrum $S(k,\omega)$ with various approximate predictions, over a range of wavelengths. The approximations considered here are one proposed in the context of generalized hydrodynamics by Bixon, Dorfman, and Mo; and one resulting from an adaption of the perturbed eigenvalue expansion discussed by Foch and Ford.

R. KURTH:

RECURRENT PHASE FLOWS AND STELLAR DYNAMICS: PROBLEMS AND CONJECTURES

The differentiable phase-flows on a certain kind of compact subsets of the n-dimensional Euclidean space are considered. The set of recurrent flows includes a $\binom{n}{2}$- dimensional vectorspace; nevertheless, it is a nowheredense subset of the space of all flows, en-

dowed with an appropriate metric.
These results suggest analogous questions and conjectures about the qualitative behaviour of the global solutions to the initial-value problem of stellar dynamics, which is presented in a simplified formulation.

I. KUSCER:

GENERALIZED MAXWELL METHOD FOR SOLVING KINETIC BOUNDARY VALUE PROBLEMS

Maxwell's method for solving velocity-slip and temperature-jump problems is generalized to curved surfaces. For strong curvature one should not directly extrapolate the bulk distribution, but carry out one iteration of the integral version of the linearized Boltzmann equation. As an example, the heat flux exchanged with a cylindrical surface is evaluated for the case of perfect accommodation. The results are compared to those obtained by other authors by aid of variational methods.

H. LANGE:

ON A NON-LINEAR SCHRÖDINGER EQUATION

We consider a non-linear Schrödinger equation of the form

$$(\pm) \qquad i\frac{\partial\psi}{\partial t} = \Delta\Psi \pm v(\Psi)\Psi + \frac{\Psi}{|x|}$$

$(x \in \mathbb{R}^3 \setminus \{0\}; t \geq 0;\ v(\Psi)(x,t) = \int_{\mathbb{R}^3} |\Psi(y,t)|^2/|x-y|dy)$ which plays a role in quantum mechanics of many particles systems (e.g. in the theory of helium gases); (−) is a time-independent version of the well-known HARTREE-FOCK equation, whereas (+) was proposed by CHOQUARD to describe an electron trapped in his own hole. We can show by using appropriate energy estimates that the integrated form of (+) has global solutions in $H^1(\mathbb{R}^3) \cap H^2(\mathbb{R}^3)$ (which was shown by CHADAM & GLASSEY for (−)). Furthermore we can derive some facts on the asymptotical behaviour in time of solutions of (±) of the form

$$\lim_{t\to\infty} \int_{|x|\leq R} (|\Psi|^2 + |\nabla\Psi|^2)dx = 0$$

and some results concerning the non-existence of an adequate scattering theory for (±).

W. MAASS:

STABILITY PROPERTIES OF THE BOLTZMANN EQUATION

Starting with a discrete spatially homogeneous version of Boltzmann's equation a continuous (one-parameter) family of Lyapunov functionals is given which allows – for the positive solutions – the proof of 1) uniform equi-boundedness and 2) uniformly asymptotic stability in the large and integral stability of the equilibrium.
The structure of certain members of the above family leads to more detailed results about the structural stability of the equation (persistent perturbations). The generalization to the spatially inhomogeneous Boltzmann equation with appropriate boundary

conditions is mentioned.

J.MAYER:

DIFFUSION TYPE SOLUTIONS OF THE FOKKER-PLANCK EQUATION

A generalization of the usual diffusion theory derived from the Fokker-Planck equation is given. For any "free moment" w^m (symmetric tensor field of degree m with $\nabla\cdot w^m=0$) a Smoluchowsky type equation is prescribed: $(\partial_t+m)w^m+\nabla\cdot(K-\nabla)w^m=0$ (K= const). Then

$f_m(u,x,t):=\sum_{n=m}^{\infty}\frac{1}{n!}\binom{n}{m}\langle(K-\nabla)^{n-m}\nabla w^m(x,t),\phi^n(u)\rangle$ (ϕ^n tensorial Hermite function of order n) is an exact solution of the Fokker-Planck equation, if the series converges properly. Under usual conditions the higher free moments of diffusion type decay exponentially with time. This is the reason why one usually need consider only solutions of the Fokker-Planck equation which depend on density alone. But Fick's law can be violated if one prescribes suitable boundary conditions for the particle flow.

J.MIKA:

ASYMPTOTIC ANALYSIS OF THE CARLEMAN MODEL IN KINETIC THEORY

An important class of singularly perturbed evolution equations are equations of resonance type. For such equations two types of asymptotic expansions can be applied related to the Hilbert and Chapman-Enskog expansions introduced originally in the kinetic theory. Both expansions are analyzed for systems of ordinary differential equations and the results applied to the discretized form of the Carleman model.

A.PALCZEWSKI:

THE CAUCHY PROBLEM FOR THE NONLINEAR BOLTZMANN EQUATION

We consider spatially nonhomogeneous nonlinear Boltzmann equation. We assume repulsive intermolecular potential r^{-s} with $s\leq 4$ and angular cut-off. We look for a solution of the Cauchy problem for this equation in the space $L^1(\mathbb{R}^3,L^\infty(\mathbb{R}^3))$. It is proved that the unique solution exists in the time intervall $[0,T]$, where T is of the order of mean free time. We show also that for initial data which are non-negative the solution is non-negative too.

The proof is based on the analysis of the bilinear form of the equation. The main tool is a theory of evolution operators in Banach space.

R.RAUTMANN:

ON THE CONVERGENCE-RATE OF SOME NAVIER-STOKES APPROXIMATIONS

Let $\Omega\subset\mathbb{R}^3$ be a bounded open set with C_3-boundary. We approximate the local strong solution u of the Navier-Stokes initial-boundary value problem on Ω by means of a

Galerkin-ansatz on the basis of the eigenfunctions of the linear Stokes boundary value problem on Ω. The Galerkin-approximation u_k , $k = 1,2, \ldots,$ is written in terms of the first k eigenfunctions corresondeng to the first k eigenvalues $\lambda_1 \leq \ldots \leq \lambda_k$ of the Stokes problem.

In addition to a former result, which will appear in the "Internationale Schriftenreihe zur Numerischen Mathematik", the error-estimate

$$|\nabla(t,\cdot) - \nabla u_k(t,\cdot)|^2_{L^2(\Omega)} \leq \frac{f(t)}{\lambda_{k+1}}$$

holds on a (possibly small) time interval $[0,T]$, if the initial value u_0 of u belongs to $H_1 \cap H_2$. The continuous functions f can be constructed explicitly from $|u_0|_{H_2}$ and T, using a theorem of J.Heywood (1978) on local existence and boundedness of the strong Navier-Stokes solutions.

M.REED:

SINGULARITIES IN SEMI-LINEAR WAVES

The propagation of singularities for semi-linear wave equations is not always the same as the propagation for their linear parts. In particular, new singularities can be created in certain circumstances when singularities collide. There is now a general theory which describes this phenomenon. Although some of the details of the geometric structure remains to be investigated, the general theory explains when this production will occur. Classical wave equations (e.g. the Klein-Gordon equation with non-linear term) as well as the approximations to the Boltzmann equation for discrete velocities (Carleman, Broadwell, and Cabannes equations) are covered by the theory.

G.F.ROACH:

ON OPERATOR VALUED FUNCTIONS

Equations involving operator valued functions of a parameter $k \in \mathbb{C}$ present additional problems in the sense that many of the criteria which may be established to govern a constructive approach towards their solution will also depend on k. In particular, approximation techniques can often degenerate with k. One of the more natural ways in which such equation arise is when a boundary value problem is replaced by the equivalent problem of solving an associated boundary integral equation. A strategy for dealing with such problems is given for the paricular case of boundary integral equations associated with the Helmholtz equation. It is shown, by combining layer theoretic and approximate Green's function techniques, that it is possible to develope a method of solution which is free of any k-dependence for an arbitrary, preselected range of values of k.

M.SHINBROT:

ENTROPY CHANGE AND NO-SLIP CONDITION

The following surprising result is proved: within the framework of a simple class of molecular reflection laws, the usual, macroscopic no-slip boundary condition for fluids is equivalent to the condition of the non-decrease of entropy.

J.SCHRÖTER:

NORMAL SOLUTIONS OF THE FOKKER-PLANCK EQUATION

The Fokker-Planck equation (F.P.E) $\partial_t f + \nabla_x \cdot uf + \nabla_u \cdot Kf = \nabla_u \cdot (\nabla_u + u) f$ (1) is treated with the help of an ansatz $f = \sum_{n=o}^{\infty} \frac{1}{n!} \langle h^n, \phi^n \rangle$ (2). (ϕ^n is the n-th tensorial Hermite function).

Sufficient conditions are given which ensure that the series converges pointwise and in an appropriate Hilbert space L_o^2. The latter is an L^2 space with the weight function ϕ^{o-1}. With the help of (2) the F.P.E. is transformed in a sequence of Hermite transfer equations: $\nabla \cdot h^{n+1} = -(\partial_t + n) h^n - n(\nabla - K) v h^{n-1}$ (3). Each solution of (3) can be written in the form $h^n = \sum_{k=o}^{n} g_k^n (w^k)$, where g_k^n is an operator defined in a certain set of tensorfields w^k of degree k. The w^k are called free moments. Those solutions of (1) which depend only on a finite number of w^k, i.e. $f = G_j (w^o, \dots, w^j)$ (4) are called normal solution s of (1). It is shown that they exist. They are of special interest:

1. They allow the formulation of an initial and boundary value problem in terms of a few moments of f.
2. With the help of (4) the kinetic theory can be reduced of thermodynamics.

H.SPOHN:

DERIVATION OF KINETIC EQUATIONS FROM MICROSCOPIC DYNAMICS (THE EXAMPLE OF THE LORENTZ GAS)

The dynamics of a dilute gas is adequately described by the Boltzmann equation. More fundamentally, however, the gas is conceived as a system of many molecules with their dynamics governed by Newton's equations of motion. Therefore one should understand in what sense the former is related to the latter description. This is a problem with a long tradition in non-equilibrium statistical mechanics and has now reached a mathematically presentable form.

We discuss as an example the low density limit for the Lorentz gas. There a mechanical paricle moves through randomly placed fixed hard sheres od diameter ε and is specularly reflected upon hitting one of the spheres. The distribution of hard spheres is given by a probility measure μ^ε on the space of locally finite configurations with

(i) its correlation functions satisfy the bound $p_n^\varepsilon (q_1, \dots, q_n) \leq M \varepsilon^{-2n} z^n$

(ii) $\lim_{\varepsilon \to 0} \varepsilon^{\varepsilon}(q_1,\ldots,q_n) = \prod_{j=1}^{n} r(q_j)$ uniformly on compakt sets of $\mathbb{R}^{3n}$ with $r \in C(\mathbb{R}^6)$.

Let $X^{\varepsilon}(t)$ be the stochastic process describing the motion of the Lorentz particle starting at x. Then $X^{\varepsilon}(t)$ converges to $X(t)$ as $\varepsilon \to 0$ (the process measure converges weakly). X(t) is a Markov process with the linear transport equation as forward equation.

J.WICK:

ON MODEL EQUATIONS FOR SYSTEMS WITH A FINITE NUMBER OF PARTICLES

Am Beispiel des Straßenverkehrs wird für ein N-Teilchensystem zunächst das System der Bewegungsgleichungen für jedes Teilchen betrachtet. Diese System wird als zeitabhängiges, diskretes Maß interpretiert und untersucht, ob die zugehörige Bewegungsgleichung durch absolutstetige Maße derart gelöst werden kann, daß sich eine Approximation des diskreten Problems ergibt. Dabei wird untersucht, ob diese Interpretation für andere Systeme mit endlich vielen Teilchen übertragbar ist.

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