MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 3 / 1980

MODELLTHEORIE

13.1. bis 19.1.1980

Die diesjährige Modelltheorie-Tagung in Oberwolfach wurde von Herrn Prof. S. Shelah (Jerusalem), Prof. E.-J. Thiele (Berlin) und Prof. M. Ziegler (Bonn) geleitet.

In einer Reihe von Vorträgen wurden intensiv und weiterführend Themen aus den Arbeiten von S. Shelah ("Classification Theory and the Number of Non-Isomorphic Models") und B. Poizat ("Théories stables") behandelt.

Zudem wurden in weiteren Vorträgen interessante und neue Beiträge zu verschiedenen Fragestellungen gebracht.

VORTRAGSAUSZÜGE

G. HESSE:

Heirs and stability

The talk was based upon the introductory article of S. EHRSAM in THÉORIES STABLES (B. POIZAT) 1978. We introduced the notion of heir of a type and gave the following definition:

DEFINITION: $p \in S(M)$ ist called stable iff every extension of p on any model $N > M$ is definable.

A theory T is called stable iff every type on any model of T is stable.

($p \in S(M)$ is definable iff for every $N > M$ p has exactly one heir on N).

THEOREM: Suppose $p \in S(M)$. Then the following are equivalent:

i) p stable;

ii) for every $N > M$ there are at most $|N|^{|T|}$ extensions of p on N;

iii) there is $\lambda \geq |M| + |T|$ s.t. for every $N > M$, $|N| = \lambda$, p has $\leq \lambda$ extensions on N.

COROLLARY: The following are equivalent:

i) T stable;

ii) T stable in every $\lambda^{|T|}$;

iii) T stable in at least one $\lambda \geq |T|$.

J. SAFFE:

Coheirs, Saturated Models and Superstability

In this talk the notion of a coheir of a type is introduced, which is some kind of dual to the notion of heir, as the type $t(B, M \cup A)$ is coheir of $t(B, M)$ iff $t(A, M \cup B)$ is heir of $t(A, M)$. It is shown that for a stable type these two notion coincide, from which we have some kind of symmetry. That helps to prove the following theorem: Let T be a theory in a countable language, that is stable in $\lambda \geq \omega$. Than there exists a saturated model of T in λ. In the second part the notion of rank of types is investigated; it is proved that the explicitly defined rank RU is the least possible one. A theory T is superstable iff all types are ranked by RU iff T is stable in all cardinalities bigger then $2^{|T|}$. From that it follows that for a countable theory exactly one of the following holds:

1) T unstable in all $\lambda \geq \omega$.

2) T stable in λ iff $\lambda = \lambda^{\omega}$.

3) T stable in λ iff $\lambda \geq 2^{\omega}$.

4) T stable in λ for all $\lambda \geq \omega$.

G. CHERLIN:

Relatively Diophantine Predicates

For $X \subseteq \mathbb{N}$, let D(X) denote the class of sets which are diophantine relative to X (that is, existentially definable in $\mathfrak{N}_X = \langle \mathbb{N}; +, \cdot, X \rangle$). My student M. Weiss has investigated the status of the following assertion, which for $X = \emptyset$ is the solution of Hilbert's 10^{th} problem:

(*) D(X) is the class of sets recursively enumerable in X.

As is to be expected, there are sets X for which (*) fails; even recursively enumerable counterexamples to (*) exist.

A consequence of (*), with model-theoretic consequences due to J.Hirschfeld, is the following:

(**) There is a D(X)-simple set S (that is: $S \varepsilon D(X)$, $\mathbb{N} - S$ is infinite, and $\mathbb{N} - S$ contains no infinite set in D(X)). An unexpected result (with a simple proof) is that (**) holds for arbitrary X. It is then easy to extend the earlier results of Hirschfeld correspondingly.

H. LEISS:

The fundamental odering and forking types on sets

Orderings $\leq, \leq_M$ between arbitrary complete types over sets were defined, s.th. for $M \subset A$, $p \varepsilon S(M)$, $q \varepsilon S(A)$ extending p, and M a model, $p \leq_M q$ iff q is an heir of p on A. It was proved that for stable types p on models, $p \leq_M p$ iff $p \leq q$. For stable theories, it was shown that types on sets always have $\leq$-maximal extensions to types on models and that these form a unique equivalence class, with respect to the equivalence induced by $\leq$, independent of the model. We said $p \varepsilon S(A)$ does not fork over $B \subset A$, if these equivalence classes are the same for p and $p \upharpoonright B$, thereby generalizing the notion of heir to types on sets. Some properties of nonforking types were shown, the symmetry lemma and a characterization theorem for nonforking types with some applications.

M. ZIEGLER:

Forking II (siehe Vortrag von H.Leiss): T stable

We proved some properties of non-forking extensions:

1) Let N(A,B) the set of nonforking extensions of types from B to types on A: The restriktion map $N(A,B) \to S(B)$ is open.
2) If q_1, q_2 do not fork over A, $q_1 \upharpoonright A = q_2 \upharpoonright A$, then there is a finite equivallence relation E, defined over A s.t. $q_1(x) \wedge q_2(y) \vdash \neg E(x,y)$ (The finite equivalence relation theorem)

3) T is superstable iff for all p there is a finite $A \subset \text{dom } p$ s.t. p does not fork over A.

A.R.D. MATHIAS:

The consistency of a partition relation

As set-theoretical relief from a week of otherwise unalleviated model theory, the background to the following conjecture was described:
If θ is the limit of $\aleph_1$ measurable cardinals then in the least inner model of ZF closed under countable sequences, $\forall\lambda < \theta \forall k < \omega \quad \theta \to (\omega k)^{\omega k}_{2^\lambda}$.
Sadly, the proof of one lemma is an intended demonstration of that assertion sprong a leak shortly before the lecture, so that its truth at the time of writing remains a mystery.

C.U. JENSEN und H. LENZING:

Applications of Model theory to representations of finite dimensional algebras

A finite dimensional algebra R over a field K is said to be of finite representation type, if R has only finitely many nonisomorphic indecomposable modules. $\text{Ind}_t(R)$ denotes the set of isomorphism classes of indecomposable R-modules of K-dimension t.
Theorem 1: For fixed d, there are universal bounds c_t such that for any d-dimensional algebra R over an algebraically closed field K either $\text{Ind}_t(R)$ is finite or $|\text{Ind}_t(R)| = |K|$.
Theorem 2: The corresponding result is true for algebras of finite representation type over arbitrary infinite fields.
Theorem 3: The class of d-dimensional algebras of finite representation type over algebraically closed fields of fixed characteristic p is finitely axiomatizable.
Theorem 1 uses Steinitz's theorem on algebraically closed fields, Theorem 2 a theorem of Nazarova-Rojter. Theorem 3 is basically equivalent to a theorem of Gabriel.

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L.v.d.DRIES:

Some Model Theory of PAC-fields

A PAC-field is a field K such that every absolutely irreducible variety defined over K has K-rational points.

Examples:

(1) Infinite algebraic extensions of, and non-trivial ultraproducts of finite fields (follows from "R.H. for curves over finite fields").

(2) Let $e \in \mathbb{N}$; then for almost all $(\sigma_1,\ldots,\sigma_e)$ in the compact group $G(\mathbb{Q})^e = \mathrm{Gal}(\tilde{\mathbb{Q}}|\mathbb{Q})^e$ its fixed field is PAC, and $\langle\sigma_1,\ldots,\sigma_e\rangle$ is free as a profinite group on $\sigma_1,\ldots,\sigma_e$. (Ax, Jarden, Kiehne)

(3) Let F be a set of irreducible polynomials in $\mathbb{Q}[X]$, and let T_F be the class of fields $K \supset \mathbb{Q}$ over which all $f \in F$ remain irreducible. Then the existentially closed members of T_F are exactly those which are PAC and have no proper algebraic extension in T_F. (v.d.D.)

In each case one obtains decidability for the corresponding class of PAC-fields (where in example (3) F is restricted to finite sets). For an exhaustive classification of PAC-fields up to elementary equivalence one would first of all need a similar classification of their absolute Galois groups.

Fact: These Galois groups are exactly the profinite groups of cohomological dimension ≤ 1, or equivalently, the projective profinite groups. Some partial results are known on the classification of projective profinite groups (Mel'nikov).

Some other results on pseudo-finite fields (perfect PAC-fields with absolute Galois group $\hat{\mathbb{Z}}$), due to v.d.Dries and Macintyre:

(1) If $K \subset \tilde{\mathbb{Q}}$ and $G(K)$ is procyclic and K is not PAC, then there exist pseudo-finite fields $L_1, L_2 \supset K$ s.t. K is algebraically closed in L_1 and L_2, $\mathrm{trdeg}_K L_1 = 1$, $\mathrm{trdeg}_K L_2 = \aleph_0$ and L_2 has no PAC-subfields of finite transcendence degree over $\mathbb{Q}$.

(2) A non-separably closed PAC-field cannot be henselian (w.r.t. a non-trivial valuation).

V. WEISPFENNING:

Prime and minimal model extensions

The notions of a prime extension and a minimal extension of a set to a model of a given theory T is motivated by the concept of algebraic closure for fields. The talk presents some results on the existence, uniqueness and characterization of prime and minimal extensions, based on Shelah's "Classification theory" and the Poizat seminar notes. The technical tools required are isolated types, atomic extensions, and constructions of extension sets of a given set, in particular the rearrangement lemma for constructions. The principal theorems proved are Ressayre's uniqueness theorem for models constructed over a set, and the fact that for countable T, every set has a prime extension for T iff for every set A the isolated types are dense in $S^1(A)$.

J.T. BALDWIN:

Dimension Theory for Models of Stable Theories

We surveyed Shelah's notion of dimension for models of stable theories. This involved introducing the concepts of stationary, regular types and the relations of orthogonality and parallelism between types. We emphasized that both orthogonality and parallelism were congruences with respect to the equivalence relation of orthogonality. Define $p \mid\vdash_K q$ if K is a class of models (e.g. all models $\aleph_0$-sat. models), ε-saturated models (cf. below)) $p \cup q \in S(M)$ and $M \in K$ then every $N \in K$ containing M which realizes p also realizes q. We showed for stationary regular types $p \Vdash_{\varepsilon\text{-sat}} q \wedge q \Vdash_{\varepsilon\text{-sat}} p$ iff p not orthogonal to q. This shows compulsion is a property of parallelism types. (M is ε-sat. if any stationary type over a finite subset of M is realized in M. This notion is also called $F^a_{\aleph_0}$-sat and $\aleph_\varepsilon$-sat. since it is slightly stronger than $\aleph_0$-sat.)
Various other results from chapter 5 of Shelah: Classification theory and the number of non-isomorphic models were mentioned.

S. SHELAH:

On the spectrum problem

We present a work on the number of non-isomorphic models of T or a function of the cardinality $I(\lambda,T)$. We replace the class of models by the class of $\aleph_\varepsilon$-saturated models, which is still general enough (e.g. for $\aleph_0$-categorical T, there is no difference) but more easily accessible. We note first that for unsuperstable T, $I_\varepsilon(\aleph_\alpha,T) = 2^{\aleph_\alpha}$(the ε- for $\aleph_\varepsilon$-saturated, a slight strengthening of $\aleph_0$-saturated). Then we find that some superstable T have a hidden order property called the dop (dimensional order property), and they too have the maximal number of non-isomorphic models. If T is superstable without the dop, every $\aleph_\varepsilon$-saturated model of T is $\aleph_\varepsilon$-prime on $\bigcup_{\zeta\in S} M_\zeta$. $S \subseteq {}^{\omega>}\lambda$, where $\langle M_\zeta : \zeta \in S\rangle$ is an indiscernible tree (i.e. if $(M_\zeta, \bigcup\{M_\nu : \neg\, \zeta \leqslant \nu\})$ does not fork over $M_{\zeta|H(\zeta)-1}$. If the tree is not well-founded we call T deep and $I_\varepsilon(\aleph_\alpha,T) = 2^{\aleph_\alpha}$ otherwise we get a bound.

J. SCHMID:

Model companions for theories of distributive p-algebras

Let $B_0 \subseteq B_1 \subseteq \dots \subseteq B_n \subseteq \dots \subseteq B_\omega$ be the canonical sequence of equational classes of distributive lattices with pseudocomplementation (p-algebras, for short) and $T_i = Th(B_i)$ their elementary theories $(0 \leqslant i \leqslant \omega)$. S. Burris (1975) proved that the model companions T_i^* exist for all i and that they are $\aleph_0$-categorical. Effective descriptions are known for T_0* (folklore = theory of atomless Boolean algebras) and for T_1* (see Schmitt, The model-completion of Stone algebras, Ann.Sci.Univ.Clermont Ser.Math.13(1976),135-155). The purpose of the present talk was to provide explicitly axiomatizations of the remaining T_i*'s. These are shown to be complete; moreover T_i* is a model completion exactly if i = 0,1,2 or ω.

D. GIORGETTA und J.A. MAKOWSKY:

Classification theory for the non-elementary case

We reported results from two yet unpublished manuscripts by S. Shelah with the above title. We gave a definition of an abstract elementary class K with a substructure relation $<_K$ and Löwenheim-Tarski-number $\lambda(K)$. We show that every such class is $PC_{\lambda(K)}$, i.e. a projective class of $\vdash_{\lambda(K)+\omega}$. Then we concentrated mostly on $\lambda(K) = \omega$. For arbitrary $\lambda > \lambda(K)$ it was shown that if $I(\lambda^+,K) < 2^{\lambda(K)^+}$ (the number of isomorphism types of K in λ^+) then there is amalgamation in K_λ (the structures of K of card $\leqslant \lambda$), provided there is a limit model in K_λ. A limit model is the abstract version of a saturated model. Then we showed that (for $\lambda(K) = \omega$), if $I(\aleph_1,K) < 2^{\aleph_1}$, then $I(\aleph_2,K) \neq 0$. For this we had to prove a series of lemmas exploiting two techniques:

(i) Use almost disjoint stationary sets to get non-isomorphic models.

(ii) Use Lopez-Escobar theorems (on not-definability of well-order in $L_{\aleph\omega}$ to get non-standard models of occuring model theoretic situation, which overcome technical difficulties.

The techniques were presented in detail in a series of seven lectures held by D.Giorgetta, J.Makowsky.

U. FELGNER:

The number of non-isomorphic models of unstable and unsuperstable theories

A proof of the following theorem of S. Shelah was presented: "If T is an unstable theory, then for all cardinals $\lambda > |T| + \aleph_0$, T has exactly 2^λ non-isomorphic models of power λ". To prove this, let $\aleph_\alpha$ be a regular cardinal such that $|T| < \aleph_\alpha \leqslant \lambda$. For each $A \subseteq \aleph_\alpha$ a linear ordering I^A and a T-model $\mathfrak{M}(I^A)$ "generated" by I^A is de-

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fined.. If the symmetric difference A Δ B is stationary in $\aleph_\alpha$, then $\mathfrak{M}(I^A)$ and $\mathfrak{M}(I^B)$ are nonisomorphic T-models of power λ. Since each regular cardinal $\aleph_\alpha$ splits into $\aleph_\alpha$ pairwise disjoint stationary sets (Solovay), it follows that T has at least $2^{\aleph_\alpha}$ non-isomorphic models of power λ. Thus, if $\lambda = \aleph_\alpha$ is regular, then we are done. If λ is singular, then socalled contradictory orders are introduced in order to get the required number of models. The above construction was applied to the $\aleph_0$-categorical (hence complete) unstable theory of extraspecial p-groups of exponent p(p a fixed prime) and their presentations. We also discussed a generalization to unsuperstable theories.

P.H. SCHMITT:

More about prime model extensions

In addition to the existence proof for prime model extensions in ω-stable theories proved before we show: If T is $\aleph_1$-categorical countable, then for infinite A, there exist minimal prime model extensions. If T is in addition not $\aleph_0$-categorical, the same is true for finite A. Using the uniqueness theorem for constructed prime model extensions and the technique of forcing it was proved: If T is stable, countable and has prime model extensions for every A, then these are unique upto isomorphism over A. (We have to add the assumption of existence of prime model extensions since there are even superstable theories which do not satisfy this.) For ω-stable theories we have the following characterization: $A \subseteq M$ is a prime model extension over A iff M is atomic over A and M contains only countable sets of indiscernibles over A. In general (even for superstable theories) the implication from right to left is not true. We proved that the other implication is true if T is superstable and prime model extensions always exist.

B. POIZAT:

Theories with or without the property of independance

Thm 1: Let T be a complete theory, M a model of T, X a subset of $S_1(M)$, of cardinality c greater than the one of T. If T does not have the independance property, then the cardinality of the topological closure of X is at most 2^c; if T has the independance property, then this cardinality can reach 2^{2^c}.

Thm 2: If T is unstable, but without the independance property, then there is an infinite sequence of elements (no tuples) which is order-indiscernible and not totally indiscernible; there is a formula with parameters which orders an infinite set of elements.

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