MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 8/1980

Funktionentheorie

17.2. bis 23.2.1980

Die Funktionentheorietagung in deren Mittelpunkt Funktionen einer Veränderlichen stehen, fand in diesem Jahr vom 17. bis 23. Februar im Mathematischen Forschungsinstitut in Oberwolfach statt. Die Leitung hatten G.Frank (Dortmund), Ch. Pommerenke (Berlin) und K.Strebel (Zürich) übernommen. Der Tagung wohnten 51 Teilnehmer, darunter 27 aus dem Ausland bei.

Die diesjährige Tagung hatte wieder den Charakter einer internationalen Tagung. Die Vorträge behandelten im wesentlichen die beiden Themen "Ganze Funktionen" und "Riemannsche Flächen". Durch die Beschränkung auf diese beiden Gebiete und durch die Anwesenheit insbesondere vieler ausländischer Spezialisten wurde ein reger Gedankenaustausch möglich.

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Vortragsauszüge

J.M. ANDERSON: Meromorphe Funktionen ohne Defektwerte

Für eine meromorphe Funktion $f(z)$ ist die sphärische Ableitung $\rho(f)$ durch

$$\rho(f) = \frac{|f'(z)|}{1+|f(z)|^2} \qquad \text{definiert.}$$

(wobei unter "meromorphe" "meromorphe in der ganzen Ebene $\mathbb{C}$" zu verstehen ist)

Satz: Es sei $f(z)$ meromorph von der Ordnung null mit $\delta(\infty,f) > 0$. Dann gilt:

$$\limsup_{r\to\infty} \frac{r\rho(r,f)}{T(r)} > 0$$

Korollar: Es sei $f(z)$ meromorph mit $\rho(f(z)) = O(\frac{\log r}{r})$ $r \to \infty$. (*) Dann besitzt $f(z)$ keine Defektwerte, und die Bedingung (*) ist bestmöglich.

I.N. BAKER: Entire Functions with Linearly Distributed Values

The only entire transcendental functions such that $\forall w\ \exists$ straight-line $l(w)$ on which all solutions of $f(z) = w$ lie are $Ae^{Bz}+C$, A,B,C constant, $B \neq 0$.
Recent extensions of this result: we get almost the same result if we assume the above only for three values w_1,w_2,w_3 not collinear in the plane, or even two w_1,w_2 such that then in addition $l_1(w) \neq l_2(w)$.
We may also consider the case when the roots of $f(z) = w$ lie on a system of k $(<\infty)$ straight lines.

H. BEGEHR: Klassische Randwertprobleme für partielle Differentialgleichungen im Komplexen

Die klassischen Randwertprobleme der Funktionentheorie dienen als Ausgangspunkt zur Lösung von zwei nichtlinearen Randwertproblemen für quasilineare Gleichungen. Sei Γ eine glatte einfach geschlossene Kurve in $\mathbb{C}$. Dann sind unter geeigneten Voraussetzngen und Normierungen folgende Probleme eindeutig lösbar:

Problem I	Problem II
$w_{\bar z} = h(z,w)$ in D $(\partial D = \Gamma)$	$w_{\bar z} = H(z,w)$ in $\mathbb{C} - \Gamma$
$\mathrm{Re}(e^{i\tau}w) = \psi(z,w)$ auf Γ	$w^{+} = w^{-} + F(z,w^{+},w^{-})$ auf Γ

Die Beweise fußen auf einer Einbettungsmethode, die darin besteht, daß überall auf den rechten Seiten ein Faktor t eingeführt wird, so daß für $t = 0$ einfache Randbedingungen für analytische Funktionen auftreten. Für $t > 0$ werden die nichtlinearen rechten Seiten nach Newtons Methode in lineare

Funktionen überführt und ein Approximationsbeweis gegeben.- Die Methode wurde erstmals von W. Wendland angewandt und geht auf H. Wacker zurück. Die Ergebnisse wurden zusammen mit G.C. Hsiao und G.N. Hile erhalten.
Eine Ausdehnung auf Gleichungen der Form $w_{\bar{z}} = H(z,w,w_z)$ als Verallgemeinerung der Beltramischen Gleichung erscheint naheliegend.

J.G. CLUNIE: A sharp result for Picard sets

A set $E \subseteq \mathbb{C}$ is a Picard set if any transcendental entire function restricted to $\mathbb{C} \setminus E$ takes every (finite) value infinitely often with at most one exeption.

Theorem. Let (a_n), $a_i \neq 0$, be a sequence such that $\frac{|a_{n+1}|}{|a_n|} \geq q > 1 \, (n \geq 1)$ and $D_n = \{|z-a_n|<\rho_n\}$, where $\log \frac{1}{\rho_n} > K \frac{(\log|a_n|)^2}{\log q}$ with $K > \frac{1}{2}$. Then $E = \bigcup_1^\infty D_n$ is a Picard set.

Baker and Liverpool have proved the above result with $K > \frac{1}{2}$ replaced by $K > \frac{4(q+1)}{q-1}$ and they have shown that in general E is not a Picard set if $K < \frac{1}{2}$. This latter result has been extended by Toppila to $K = \frac{1}{2}$. Hence the above theorem is sharp.

Our proof is direct and shows that if E is not a Picard set then $K \leq \frac{1}{2}$ by considering in a number of separate cases the possible zero distribution of a function f(z) whose zeros and ones ultimately all lie in E. Otherwise it uses only a zero of the fundamental results of complex function theory.

D. DRASIN: Quasiconformal mappings and meromorphic functions

By composing with quasi-conformal mappings with small dilatation, we convert meromorphic function of finite order λ having (*) $\sum\delta(a) = 2$ to one having $\delta(0) = \delta(\infty) = 1$.
As an application we show that under the hypothesis (*) necessarily each $\delta(a)$ is a rational number whose denominator is not large. Remarks are also made concerning the sharpness of Nevanlinna's second fundamental theorem.

C. EARLE: Degeneration of Riemann surfaces and Kleinian groups

When the conformal structure of a Riemann surface is varied by pinching some nonintersecting simple loops, these surfaces will "degenerate" to a singular surface. The degenerating surfaces can be represented by Kleinian groups that also converge to a limiting group.
In joint work, Albert Marden and the speaker describe the degenerating surfaces by natural geometric parameters that are closely related to some of these Kleinian groups.

M. ESSEN, D. SHEA: Conjugate functions in the unit disk and the class L log L

Let F be in the Nevanlinna class in the unit disk U, let N(r,w,F) be the counting function for the value w of F and let $T = \delta U$ be the unit circle. Then $N(1,w,F) = \lim_{r\to 1-} N(r,w,F)$ exists and $N(w) = \limsup_{\zeta\to w} N(1,\zeta,F)$ is subharmonic in $\mathbb{C}\setminus\{F(0)\}$.
Zygmund has proved that if $F \in H^1(U)$ and Re $F \geq 0$, then Re $F \in L \log L$. Conversely, Re $F \in L \log L$ implies that $F \in H^1(U)$.

Theorem. Let $F \in H^1(U)$. The following two conditions are equivalent .

(1) $\int_{-\infty}^{\infty} N(u+iv) \log^+|v|dv < \infty$ for some $u \in \mathbb{R}$.

(2) Re $F \in L \log L$.

Remark (1) is equivalent to

(1') $\int_{-\infty}^{\infty} N(u+iv) \log^+|v|dv < \infty$ for all $u \in \mathbb{R}$.

Corollary 1 Let $F \in H^1(U)$. Assume that $N(w)$ vanishes on two curves Γ_1 and Γ_2 such that

i) $\Gamma_1 \subset \{(u,v) = v \geq c|u|^\alpha\}$,

$\Gamma_2 \subset \{(u,v) = v \leq -c|u|^\alpha\}$,

where $c > 0$ and $\alpha > 1$ are constants.

ii) Γ_1 and Γ_2 go out to infinity. Then Re $F \in L \log L$.

Corollary 2. Let $F \in L^1(T)$, let g be the symmetric, decreasing rearrangement of f and let $\tilde{f}$ and $\tilde{g}$ be the corresponding conjugate funtions with mean zero. Then $\tilde{g} \in L^1(T)$ if and only if $f \in L \log L$.

Corollary 3. Assume that $F \in \mathfrak{X}^1(U)$ and that $F(U) \subset \{w = \text{Re } w \neq 0\} \cup \{w = iv = v \in \cup I_k\}$ where $I_k = (a_k-d_k, a_k+d_k)$ are nonoverlapping intervalls, $k \in \underline{Z}$. If $a_k = 2^{|k|}$ sign k, $d_k = a_k/|k|^q$, $k \in \underline{Z}\setminus\{0\}$, then Re $F \in L \log L$ if $q > 1$.

D. GAIER: Funktionentheoretische Beweise von High-Indices Theoremen

Some Tauberian gap theorems for logarithmic summability are proved using complex analysis methods. Let Σa_k have partial sums s_n, and let $L(x) = \sum_{k=0}^{\infty} \frac{s_k x^{k+1}}{k+1} \Big/ \log \frac{1}{1-x}$ $(0 < x < 1)$.

Theorem: If Σa_k has Hadamard gaps, then

(✱) $|L(x)-s| \leq A(1-x)^{\alpha} \Big/ \log \frac{1}{1-x}$ for $0 < x < 1$ implies

$|s_n-s| \leq AC(\alpha,q)n^{-\alpha}$ for $n = 1,2,\ldots$ if $\alpha \neq -1$, and

$|s_n-s| \leq AC(\alpha,q)n^{-\alpha} \log(n+1)$ if $\alpha = -1$.

There is a similar o-Theorem. If the log is dropped in (✱), the factor log(n+1) appears in the conclusions. As a corollary we obtain the equivalence of the methods L,l for series with Hadamard gaps, and a new proof of the high indices Theorem for L-summability follows (Krishnan, 1979). In the proof some new results for functions in H^p and a "Two-Term-Theorem" appear.

W. HARVEY : Kleinian Groups

There are fruitful inter-relations between the theories of discrete hyperbolic groups in dimensions two (the Fuchsian case) and three (the Kleinian case). Study of the Teichmüller spaces provides many instances of this, and there is much evidence that new insights are to be gained hereby into the geometry of three dimensional manifolds of finite hyperbolic volume.

Perhaps the most precise analogy between the two theories occurs in consideration of the range of possible volumes and the convergence of discrete groups in the sense of the Hausdorff topology on closed subsets of the real and complex groups of Möbius transformations.

Theorem: (Thurston,Jørgensen,Gromov) The set of volumes for Kleinian 3-manifolds is well ordered, and at most finitely many manifolds can have a given volume. The volume function is continuous.

One can furthermore give a precise unified description of the convergence of groups in topological and group theoretic terms.

W.K. HAYMAN : Wertverteilung von meromorphen Funktionen in einem Winkel

Es sei μ ein positives, im Endlichen beschränktes Mass,

$$S : \alpha < \arg z < \beta$$

ein Winkel, $n(r,\alpha,\beta)$ die Totalmenge von μ in $S \cap |z| < r$ und

$$k(\alpha,\beta) = \overline{\lim_{r\to\infty}} \frac{\log n(r,\alpha,\beta)}{\log r}$$

die Ordnung von μ in S. Offenbar wächst k mit β und $-\alpha$. Wir definieren ferner die innere Ordnung

$$k_o(\alpha,\beta) = \lim_{\varepsilon\to 0^+} k(\alpha+\varepsilon,\beta-\varepsilon).$$

Wenn f in S meromorph ist, schreiben wir $k_o(a,S)$ wenn μ die Anzahlfunktion der Wurzeln von f = a ist und $k_o(S)$ wenn μ das Flächenmass des Bildes von f auf der Riemann-Kugel ist.

Satz: Es gilt $k_o(a,S) = k_o(S)$ ausser für höchstens zwei a für die $k_o(a) < k_o$ sein kann und eine Menge V für die $k_o < k_o(a) \leq \frac{\pi}{\alpha-\beta}$ möglich ist.

V kann recht genau charakterisiert werden und ist im Sinn des Hausdorff Masses sehr klein.

S. HELLERSTEIN: The Zeros of Successive Derivatives of Meromorphic Functions Which Have Only Real Zeros.

In 1914 and 1915 G.Polya and A. Wiman presented several conjectures relating to the location of the zeros of successive derivatives of entire functions which have only real zeros. A survey of progress on these conjectures is given, which includes recent work by the speaker and Jack Williamson. Some new problems arising from these investigations are presented.

W. HENNEKEMPER: Über Ungleichungen zwischen $N(r,f)$, $N(r,\frac{1}{f})$ und $N(r,\frac{1}{(f^n)^{(k)}-c})$

Wir zeigen die Ungleichungen

$$N(r,\tfrac{1}{f}) \le O(\bar{N}(r,f)) + O(\bar{N}(r,\frac{1}{(f^{k+1})^{(k)}-c})) + S(r,f) \quad k \in \mathbb{N}, c \in \mathbb{C}\setminus\{0\}$$

$$\bar{N}(r,f) \le O(N_1(r,f)) + O(\bar{N}(r,\frac{1}{(f^{k+2})^{(k)}-c})) + S(r,f), \quad k \in \mathbb{N}, c \in \mathbb{C}\setminus\{0\},$$

$$\bar{N}(r,\tfrac{1}{f}) \le O(N(r,f)) + O(N(r,\frac{f^{\nu}}{(f^{\nu+k})^{(k)}})) + S(r,\tfrac{f'}{f}) + O(\log r)$$

$$k,\nu \in \mathbb{N},\ k \ge 2$$

und ziehen daraus Folgerungen über die Werteverteilung von $(f^{k+1})^{(k)}$ und $(f^{k+2})^{(k)}$.

J.A. JENKINS: On the representation and compactification of Riemann surfaces.

Any Riemann surface can be obtained by the following process. Let Δ denote the open unit disc, C is circumference. To Δ adjoin pairs of open arcs on C of equal angular measure, all such arcs being disjoint. Identify points of such an associated pair isometrically in opposite senses of description on C. This involutory association is denoted by T. If α_k, $k = 1,2,\ldots,2l$, are certain of these arcs such that (writing $\alpha_{2l+1} = \alpha_1$)

(i) α_{2i-1}, α_{2i} have a common endpoint Q_i, $i = 1,\ldots,l$,

(ii) $\alpha_{2i+1} = T\alpha_{2i}$, $i = 1,\ldots,l$,

we identify all Q_i, $i = 1,\ldots,l$, and call the corresponding element a cycle point. The elements of R consist of points of Δ, elements consisting of pairs of identified points on associated boundary arcs and certain cycle points. The local uniformizing parameters are chosen so that Δ is conformally embedded in R and the arcs in R obtained from pairs of associated boundary arcs are (open) analytic arcs.

Performing possible further identifications on the points of C not included in the above arcs one obtains a compactification of R, which is more articulated than the Kerékjártó-Stoilow compactification and in many ways seems the most natural one for Function Theoretic purposes.

Y. LO: Angular Distribution Theory of Entire Functions

Principally the following theorem has been proved.

If $f(z)$ is an entire function of order λ $(0 < \lambda < \infty)$, then there exists a ray B: $\arg z = \theta_0$ $(0 \le \theta_0 < 2\pi)$ such that: For any positive number ε , every pair of finite complex numbers α, β $(\beta \neq 0)$ and every pair of positive integers k, l satifying $\frac{2}{k} + \frac{1}{l} < 1$, we have

$$\limsup_{r\to\infty} \frac{\log\{n_{k)}(r,\theta_0,\varepsilon,f=\alpha)+n_{l)}(r,\theta_0,\varepsilon,f'=\beta)\}}{\log r} = \lambda$$

where $n_{k)}(r,\theta_0,\varepsilon,f=\alpha)$ denotes the number of zeros of order less than k of $f(z)-\alpha$ in the region $(|z| \le r) \cap (|\arg z-\theta_0| \le \varepsilon)$, and $n_{l)}(r,\theta_0,\varepsilon,f'=\beta)$ has a similar meaning.

H.H. MARTENS: Mappings of Closed Riemann Surfaces

Hurwitz showed that an automorphism of a closed Riemann surface of genus $g \geq 2$ is completely determined by the homomorphism it induces on the first homology group. This result receives a very natural proof when examined in the light of the universal mapping property of the Jacobian variety, and admits an immediate generalization.
Let $X \to X'$ and $X \to X''$ be given maps of X onto surfaces X' and X'' of the same genus. Assume that, with respect to given bases for homology, the induced homomorphisms of the first integral homology groups are the same (i.e. expressed by the same matrix). Then there is an isomorphism $X' \to X''$ inducing the natural identification of homology and commuting with the given maps. Thus the induced homomorphism of homology determines the map $X \to X'$ in the strong sense that the conformal stucture of X' is determined.

H.A. MASUR: Quadratic Differentials

I will be concerned with properties of quadratic differentials on compact Riemann surfaces, particularly their trajectory structures. I will discuss the related notion of a measured foliation and the result of Hubbard and Masur that on any Riemann surface, a quadratic differential is determined by the topological and metric properties of its horizontal trajectories. On the other hand there exist examples of families of quadratic differentials each with dense trajectories such that the trajectory structures of the members of the family are topologically (but not metrically) the same. We show that these examples are rather infrequent that is, there is a natural measure on the space of quadratic differentials such that "almost all" are determined up to scalar multiples by the topological strucure of their horizontal trajectories.

G.P. MEYER: Zur Wertverteilung von Exponentialpolynomen und ihrer Quotienten

Es werden die Pólyaschen Exponentialpolynome

$$f(z) = \sum_{k=1}^{n} p_k(z)\, e^{a_k(z)} \qquad (p_k, a_k \text{ Polynome})$$

und ihre Quotienten $w(z) = f(z)/g(z)$ (f,g Exponentialpolynome) unter dem Gesichtspunkt der Nevanlinnaschen Wertverteilungslehre untersucht. Ist eine sehr allgemeine Exponenten- und Koeffizientenbedingung erfüllt, so hat man einfache geometrische Formeln für die charakteristischen Größen $T(r,w)$, $\delta(\alpha,w)$, $\theta(\alpha,w)$, $\Theta(\alpha,w)$ und Φ_e. Eine wesentliche Rolle spielt die Abschätzung der gemeinsamen Nullstellen zweier Exponentialpolynome. Weiter wird mit Hilfe einer von P. Erdös gegebenen Abschätzung der Lösungszahl einer gewissen diophantischen Ungleichung gezeigt, daß der Quotient zweier Exponentialpolynome einer algebraischen Differentialgleichung genügt.

J. MILES: Some remarks concerning the Fourier series of the logarithm of the modulus of a meromorphic function

A number results concerning the distribution of values of meromorphic functions f have been obtained by studying the Fourier series of $\log |f(re^{i\theta})|$, $0 \le \theta \le 2\pi$. A survey of these results is presented, including the work of Edrei and Fuchs, Rubel and Taylor, and Miles and Shea. Emphasis is placed on results concerning quotient representations of meromorphic functions, bounds for the ratio $(N(r,0) + N(r,\infty))/m_2(r,f)$ where $m_2(r,f)$ denotes the L^2 norm of $\log |f(re^{i\theta})|$, and bounds on the ratio $N(r,0)/T(r,f)$ for entire f of infinite order with zeros on a finite number of rays through the origin. Certain open problems in value distribution theory are indicated to which these techniques may be applicable.

P.J. NICHOLS: Dirichlet Regions for Fuchsian Groups

Let G be a fuchsian group in the unit disk Δ . With the non euklidean metric ρ in Δ and for $a \in \Delta$ not a fixed point, we write

$$D_a = \{z \in \Delta: \rho(z,a) < \rho(V(z),a) \text{ all } V \in G,\ V \neq I\}$$

the Dirichlet region for G centered at a. We set

$$E_a = \{\xi \in \partial\Delta : V(\xi) \subset \bar{D}_a \text{ for some } V \in G\} .$$

A point $\xi \in \partial\Delta$ is a horocyclic limit point if one (and hence all) G-orbits enter every horocycle at ξ . We write $\xi \in H$ in this case. We show that

$$\partial\Delta = H \cup E_a \cup g_a \quad \text{for any } a \in \Delta$$

where g_a is the set of Garnett points recently introduced by Sullivan in his study of the ergodic properties of discrete groups. A result of Pommerenke shows that $\lambda^m(g_a) = 0$. We give an example to show that in general $g_a \neq \emptyset$ and that $E_a \neq E_b$.

For $\xi \in \partial\Delta$ we construct a region $D_\xi \subset \Delta$ which for $\xi \notin H \cup (\bigcup_{a\in\Delta} g_a)$ turns out to be a fundamental region for G and generalizes the classical Ford construction.

E. PESCHL: Beiträge zu ungelösten Problemen der konformen Abbildungen

Betrachtet wird die Aufgabe, die Kreisscheibe $|z| < 1$ auf das Innere eines sphärisch konvexen Kreisbogenpolygons $\mathfrak{P}^{(w)}$ konform abzubilden. Da die Gruppe der sphärischen Isometrien 3 reelle Parameter, die allgemeine Möbiusebene dagegen deren 6 enthält, kann man erwarten, daß man hierbei mit einer Differentialinvarianten 2. Ordnung (anstelle der Schwarzschen Derivierten) auskommt. Dies ist der Grundgedanke der folgenden Behandlung.

Seien $ds = \frac{dz}{1-z\bar{z}}$, $d\sigma = \frac{dw}{1+w\bar{w}}$ die Linienelemente in der z-Ebene ($|z| < 1$), bzw. in der w-Ebene (Riemannsche Zahlenkugel). Hier gibt es als absolute Differentialinvarianten niedrigster Ordnungen:

$$\alpha = \lg \frac{d\sigma}{ds} = \lg \left(\frac{(1-z\bar{z})}{1+w\bar{w}} |w'|\right), \quad u = \left(\frac{w''}{w'} - \frac{2\bar{w}w'}{1+w\bar{w}}\right) z+1.$$

$w = w(z)$ bildet genau dann die Kreisscheibe $|z| < 1$ auf einen sphärisch konvexen Bereich $\mathfrak{G}^{(w)}$ mit stetiger sphärischer Randkrümmung ab, wenn Re $u \geq 0$ in $|z| < 1$. Für u gilt

$$\underset{(z)}{\delta}{}_2(u) = (1-z\bar{z})^2 u_{z\bar{z}} = -2\, e^{2\alpha} u, \text{ oder}$$
$$\underset{(w)}{\delta}{}_2(u) + 2u = (1+w\bar{w})^2 u_{w\bar{w}} + 2u = -e^{-2\alpha} \underset{(w)}{\partial}{}_2 u + 2u = 0.$$

Nach einem allgemeinen Darstellungssatz*) gibt es für jede Lösung u dieser linearhomogenen Differentialgleichung $\underset{(w)}{\delta}{}_2 w + 2u = 0$ (in $\mathfrak{G}^{(w)}$) eine in $\mathfrak{G}^{(w)}$ holomorphe Erzeugende g(w) mit

$$g(w) = -\frac{1}{2}(1+w\bar{w})^2 u_{\bar{w}} = [w'z]_{z = z(w)}, \quad \frac{d}{dw}\, g(w) = \left[\frac{w''}{w'}\, z+1\right]_{z = z(w)}$$

sodaß die Darstellung gilt $u = -2\, \frac{\bar{w}}{1+w\bar{w}}\, g(w) + \frac{d}{dw} g(w)$. Liegt die Ecke (mit Innenwinkel $\alpha\pi$) in $w_o = 0$, so hat man für das "zugehörige" Zweieck $z(w) = \operatorname{tgh}\left(\frac{1}{2\alpha} \lg w\right)$, $g(w) = \alpha w \sinh\left(\frac{1}{\alpha} \lg w\right)$ $(:= \hat{g}_o(w))$ und $u = -\frac{2\bar{w}}{1+w\bar{w}}\; \alpha w \sinh\left(\frac{1}{\alpha} \lg w\right) + \frac{1}{w} \sinh\left(\frac{1}{\alpha} \lg w\right) + \cosh\left(\frac{1}{\alpha} \lg w\right)$.

Bei allgemeiner Lage in w_ν hat man $z(w) = \varepsilon_\nu \operatorname{tgh}\left(\frac{1}{2\alpha_\nu} \lg L_\nu(w)\right)$ mit $|\varepsilon_\nu| = 1$, $L_\nu(w) = \mu_\nu \frac{w-w_\nu}{1+\bar{w}_\nu w}$, $|\mu_\nu| = 1$ und nach einem allgemeinen Transformationsgesetz*) für die Erzeugende :

$$g_\nu(w) = [\hat{g}_o(w)]_{w = L_\nu(w)}\; \frac{\bar{\mu}_\nu (1+w\bar{w}_\nu)^2}{1+w_o\bar{w}_o} \text{ und daher}$$

$$u_\nu = -\frac{2\bar{w}}{1+w\bar{w}}\, g_\nu(w) + \frac{d}{dw} g_\nu(w).$$

Somit erhält man für die Erzeugende

$$G(w) = \sum_{\nu=1}^{n} g_\nu(w) = \sum_{\nu=1}^{n} [\hat{g}_o(w)]_{w=L_\nu(w)} \cdot \frac{\bar{\mu}_\nu(1+w\bar{w}_\nu)^2}{1+w_o\bar{w}_o}$$

die dazugehörige Lösung

$$U = \sum_{\nu}^{n} u_\nu = -\frac{2\bar{w}}{1+w\bar{w}} \sum_{\nu=1}^{n} g_o(w) + \sum_{\nu=1}^{n} \frac{d}{dw} g_\nu(w)$$

$$= -\frac{2\bar{w}}{1+w\bar{w}} G(w) + \frac{d}{dw} G(w) \quad \text{mit } G(w) = \sum_{\nu=1}^{n} g_\nu(w) =$$

$$= \sum_{\nu=1}^{n} [\hat{g}_o(w)]_{w} = L_\nu(w) \cdot \frac{\mu_\nu(1+w\bar{w}_\nu)^2}{1+w_o\bar{w}_o}$$

$$= -\frac{1}{2}(1+w\bar{w})^2 U_{\bar{w}} = \frac{z(w)}{z'(w)}, \text{ hieraus kommt:}$$

$$z = \exp \int \frac{dw}{G(w)} .$$

Analog läßt sich der Fall der w-Ebene ($|w| < 1$) behandeln, wobei der Bereichsrand ganz im Inneren des Einheitskreises bleiben soll. Der Wegfäll dieser Forderung bedarf einer gesonderten Betrachtung. Weitere Untersuchungen betreffen die nichtkonvexen Polygone.

*) K.W. Bauer und E. Peschl, Ein allgemeiner Entwicklungssatz für die Lösungen der Differentialgleichung $(1+\varepsilon z\bar{z})^2 w_{z\bar{z}} + \varepsilon n(n+1)w = 0$ in der Nähe isolierter Singularitäten. Sitz Ber.d.Bayer. Akademie d. Wiss. 8.10.65.

F. PITTNAUER: <u>Zur Theorie der asymptotischen Entwicklungen ganzer Funktionen</u>

Zusammen mit H. Wyrwich zeigen wir, daß es ganze Funktionen gibt, welche in endlich vielen Winkelräumen asymptotische Potenzreihenentwicklungen um den Punkt Unendlich besitzen und zusätzlich von minimaler Wachstumsordnung wie von vorgeschriebenem Normaltyp sind.

Zum Beweis stützen wir uns auf eine von HANDELSMAN und LEW (1969) angegebene Verallgemeinerung des WATSONschen Lemmas für Integraltransformationen, deren Kern die WRIGHT-Funktion enthält, sowie auf die bekannte Buckelmethode.

Unser Satz verfeinert einschlägige Ergebnisse von POLYA (1922) und von RITT (1916) und steht darüber hinaus zu einem Ergebnis von HAYMAN (1969) über ganze Funktionen mit vorgebenen asymptotischen Werten längs endlich vieler lediglich stückweise rektifizierbaren JORDAN-Kurven in Beziehung.

E. REICH: Quasikonforme Abbildungen der punktierten Ebene

Sei S eine Menge von ≥ 3 Punkten der komplexen Ebene $\mathbb{C}$ ohne Häufungspunkte, $\Omega = \mathbb{C} \setminus S$. Für $K > 1$, sei $F_K(z) = Kx+iy$, $(z=x+iy)$, die affine Streckung, $\Omega' = F_K(\Omega)$. Vortragender und K. Strebel betrachten folgende Probleme : Unter allen quasikonformen Abbildungen von Ω auf Ω' die zu F_K homotop sind, ist F_K extremal ? Wenn ja, ist F_K eindeutig extremal ? Die Fragen werden teilweise für verschiedene Typen von Mengen S beantwortet, unter Anwendung "geometrischer" sowohl wie "analytischer" Methoden. Für Letzere sind Abschätzungen des Funktionals

$$H(S) = \sup_{\varphi} \frac{\left|\iint_{\Omega} \varphi(z)\, dxdy\right|}{\|\varphi\|}$$

relevant, wo φ über die Klasse der in Ω holomorphen Funktionen φ, $\|\varphi\| = \iint_{\Omega} |\varphi(z)|\, dxdy < \infty$, variiert.

B. RODIN: External length and conformal mapping

Suppose the Riemann mapping function $f: R \to S$ of a simply connected region R onto a standard region S makes a given prime end w_o of R correspond to the boundary point z_o of S. Several unsolved classical questions regarding the behavior of f near w_o are subsumed under the general problem of finding an asymptotic representation of f in terms of geomètric data of R. If S is normalized to be the parallel strip $\{z | 0 < \mathrm{Im}\, z < 1\}$ with z_o at $+\infty$, such a representation takes the form $f(w) = [...w...] + o(1)$ where the term in brackets involves only the geometry of R, and where $o(1) \to 0$ for a given mode of approach of $w \to w_o$.

This general problem is attacked in several stages. First an asymptotic representation in terms of extremal length is obtained. When it is applied to several special cases, new and improved estimates of f(w), as $w \to w_o$, are obtained. Applications to the classical problems will be treated in the talk of Professor S.E. Warschawski.

G. SCHMIEDER: Über eine Klasse von Trinomen

Komplexe Trinome der Form $z+az^k+bz^n$ zeigen in verschiedener Hinsicht (Schlichtheitsfragen, Nullstelleneigenschaften) unterschiedliches Verhalten je nachdem, ob k-1 Teiler von n-1 ist oder nicht. Dabei zeichnet sich die Klasse der Trinome mit $k-1 \nmid n-1$ durch interessante Eigenschaften aus. So ist für solche Trinome die lokale und globale Schlichtheit im Einheitskreis äquivalent. Weiter gilt für solche Trinome : Ist $z+az^k+bz^n$ schlicht im Einheitskreis und $0 \leq s,t \leq 1$, so ist auch $z+asz^k+btz^n$ schlicht in D.

Aus einer im Beweis benötigten Koeffizientenabschätzung läßt sich eine Aussage über Nullstellen gewinnen: Sei $f(z) = 1+az^l+bz^m$ mit $l \nmid m$, $a,b \in \mathbb{C}$. Ist r_o die positive Nullstelle von $|b|r^m+|a|r^l-1$ und r_1 die positive Nullstelle von $|b|r^m+r^l|a|\cos\frac{\pi}{q} -1$,

<u>Satz 2</u> Ist g eine ganze Funktion mit negativen Nullstellen vom Geschlecht Null, der Ordnung ρ und der unteren Ordnung $\lambda < \rho$, so gilt

$$\overline{\lim_{r\to\infty}} \frac{\log|g(re^{i\theta})|}{\log M(r)} \geq \cos \theta\lambda .$$

<u>Satz 3</u> Ist g ein kanonisches Produkt mit negativen Nullstellen der nichtganzzahligen Ordnung ρ und dem Geschlecht q, so gilt

$$\overline{\lim_{r\to\infty}} \frac{\log|g(-r)|}{\log|g(r)|} \geq \begin{cases} \cos \pi\rho, & q \text{ gerade} \\ 1, & q \text{ ungerade} \end{cases} .$$

S.E. WARSCHAWSKI: <u>Asymptotic representations of conformal maps of strip domains</u>

Let R be a simply connected region in the w plane with an accessible boundary point w_∞ at $w = \infty$ along a curve L. Let $f : R \to S$ be a one-to-one conformal map of R onto the parallel strip $S = \{z \mid 0 < \text{Im } z < 1\}$ with $\text{Re } f(w) \to +\infty$ as $w \to w_\infty$. Under certain "regularity" assumptions on ∂R one has asymptotic representations, $f(w) = [\ldots w \ldots] + o(1)$, where the expression in the brackets involves only the geometry of R and $o(1) \to 0$ as $w \to w_\infty$. In the present paper asymptotic representations for f are obtained without such boundary regularity conditions, by comparing R with a region R_0 which has w_∞ as an accessible boundary point along the same curve L and whose mapping $f_0 : R_0 \to S$ is similarly normalized. We derive necessary and sufficient proximity conditions on ∂R and ∂R_0 near w_∞ such that $f = f_0 + c + o(1)$ (c is a real constant) as $w \to w_\infty$ in $R \cap R_0$. Thus asymptotic representations of f_0 apply also to f. Various applications are given, among these a necessary and sufficient condition or the existence of the Visser-Ostrowski limit.

J. WILLIAMSON:

Suppose f is a real entire function (i.e., real on the real axis) such that f and f' have only real zeros. Let F = 1/f and note that F' has only real zeros since f' does. We consider f" and F", and investigate the relationship between the number of non-real zeros of f" and the number of real zeros of F". For example, if f is of finite order, then f" (F") has at most a finite number of non-real (real) zeros. In fact, the number of non-real zeros of f" = the number of real zeros of F".
However if f is of infinite order this duality breaks down since f" will have an infinite number of non-real zeros while F" need not have any real zeros. The latter result can hold even when f has infinitely many zeros.

Important corollaries to these results are obtained as well as other results on the number of non-real zeros of F".

J. WINKLER: On the minimal growth of the spherical derivative of meromorphic functions

Lehto and Virtanen introduced in 1957 the spherical derivative of meromorphic functions as an natural characteristic for the behaviour of meromorphic functions near an essential singularity. In 1958 Lehto got the following result (that had to be modified in one point as Gavrilov pointed out in 1965 and the speaker had shown by examples in 1964)

If for a meromorphic function f(z), a point sequence $z_1, z_2, z_3 \ldots \to \infty$ and a real valued function $h(|z|) = O(|z|)$

$$\lim_{\gamma\to\infty} h(|z_\gamma|) \frac{|f'(z_\gamma)|}{1+|f(z_\gamma)|^2} = \infty$$

then f(z) takes all values with at most two exeptions in each subsequence of the discs ($\varepsilon > 0$ arbitrary) $C_\gamma = \{z \mid |z-z_\gamma| \le \varepsilon h(|z_\gamma|)\}$

If conversely $f(z)$ takes in each subsequence of the discs C_γ each values with at most two exceptions, then there exists a sequence of points z'_γ with $|z'_\gamma - R_\gamma| = o(h(|z_\gamma|))$ and

$$\lim_{r\to\infty} h(|z_\gamma|) \frac{|f'(z_\gamma)|}{1+|f(z_\gamma)|^2} = \infty.$$

From this result it is of very high interest for the value-distribution to get small functions $a(|z_\gamma|)$. Letho proved 1958

$$\overline{\lim_{|z|\to\infty}} |z| \frac{|f'(z)|}{1+|f(z)|^2} \geq \frac{1}{2}$$

and

(*) $$\overline{\lim_{|z|\to\infty}} |z| \frac{|f'(z)|}{1+|f(z)|^2} = \infty \text{ if } f(z) \text{ is entire.}$$

For entire functions Clunie and Hayman gave in 1966 a striking improvement of (*). They proved for entire functions

(**) $$\lim_{|z|\to\infty} \frac{|z|}{\log M(r,f)} \frac{|f'(z)|}{1+|f(z)|^2} > 0,$$

where $M(r,f)$ is the maximum modulus of $f(z)$ on $|z| = r$. The question is whether there is an result similar to (**) true for meromorphic functions. In the lecture it will be shown for meromorphic functions that

$$\overline{\lim_{|z|\to\infty}} \frac{|z|}{h(|z|)} h(|z|) \frac{|f'(z)|}{1+|f(z)|^2} = \infty$$

for each real valued function $h(|z|) \underset{z\to\infty}{\to} \infty$ and

$$n(|z|) = |n(|z|),f) - n(|z|,\tfrac{1}{f})| \text{ with}$$

$$\text{Max } \{n(|z|,f), n(|z|,\tfrac{1}{f})\} = O(n(|z|))$$

and $n(|z| + h^2(|z|) \frac{|z|}{h(|z|)}) - n(|z|) = O(\frac{n(|z|)}{\log^2 n(|z|)})$

S.A. WOLPERT: The Fenchel Nielsen Twist Deformation

A deformation of a Riemann surface R is a modification of the conformal structure. An example, the Fenchel Nielsen twist, is given by cutting R along a simple closed Poincaré geodesic. The two sides are seperated, rotated relative to each other, and glued. The Poincaré metric is preserved off the cut locus. A quasiconformal map between surfaces also prescribes a deformation. Our investigation began with constructing a quasiconformal map which induces the twist deformation. The quasiconformal automorphism of the upper half plane H

$$w(z) = \begin{cases} z & 0 < \arg z < \theta_1, \\ e^{\varepsilon(\theta-\theta_1)}z & \theta_1 < \arg z = 0 < \theta_2 \\ e^{\varepsilon(\theta_2-\theta_1)}z & \theta_2 < \arg z < \pi \end{cases}$$

is the prototype.
The net effect of w is to slide the left hyperbolic half plane relative to the right hyperbolic half plane.
Let H be the universal cover of a compact Riemann surface R.
An automorphic $\tilde{w}$ can be constructed from the above map w.
Given an automorphic Beltrami differential $\mu(z)d\bar{z}/dz$ the equation

$$\begin{cases} w^\mu_{\bar{z}} = \mu w^\mu_z & \text{in } H \\ w^\mu_{\bar{z}} = 0 & \text{in } L \end{cases}$$

has a solution. Assume Γ is the uniformization group of R thus $R \simeq H/\Gamma$. The association of the surface $w^\mu(H)/w^\mu \circ \Gamma \circ (w^\mu)^{-1}$ to the quadratic differential $\{w^\mu, z\}$ in L ($\{,\}$ denotes the Schwarzian derivative) is the Bers embedding of Teichmüller space. Assume Γ has been conjugated so that the imaginary axis is the axis of an element $A \in \Gamma$. Our calculations yield

Theorem 1: Let $\langle A \rangle$ be the cyclic subgroup of Γ fixing the imaginary axis. The first variation of the Bers embedding for the Fenchel Nielsen twist deformation along the imaginary axis is

$$\frac{1}{2\pi i} \sum_{B \in \langle A \rangle \backslash \Gamma} (B'(z)/B(z))^2$$

where the sum is over the left cosets of $\langle A \rangle$ in Γ.

We shall describe two new results which are consequences of this calculation. Given γ_1, γ_2 geodesics on R let $l(\gamma_1)$, $l(\gamma_2)$ denote their lengths. If the surface is deformed the lengths of geodesics necessarily change.

Theorem 2: (Reciprocity of Fenchel Nielsen twists).

Denote by $\frac{\partial}{\partial \text{ twist } \gamma_j}$ the derivative with respect to the twist deformation about γ_j, $j = 1,2$. Then

$$\frac{\partial l(\gamma_1)}{\partial \text{twist } \gamma_2} = \frac{\partial l(\gamma_2)}{\partial \text{ twist } \gamma_1}.$$

Given a simple closed geodesic γ there is an associated Poincaré series Θ_γ as in Theorem 1. On a surface of genus g the maximal number of disjoint closed simple geodesics is 3g-3. Let $\gamma_1, \ldots, \gamma_{3g-3}$ be any such collection on R.

Theorem 3. $\Theta_{\gamma_1}, \ldots, \Theta_{\gamma_{3g-3}}$ span the space of quadratic differentials on R.

Our methods also yield variational formulas for other geometric invariants.

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