MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 10/1980

Mathematische Spieltheorie

2.3. bis 8.3.1980

The purpose of this meeting was to assemble game theorists from Europe, the United States, and Israel in order to stimulate a discussion of current research mainstreams.

No survey talks were scheduled since it was assumed that the participants were all specialists in the field. However, apart from the regularly scheduled talks (in general three in the morning and three in the afternoon) there were evening sessions and discussion meetings organized by the participants on an informal basis.

Also no particular emphasis was put on distinguishing between topics which were of more or less purely mathematical interest and more applicable versions of Game Theory. However, there was an afternoon session concentrating on applications and experiments in the field of Game Theory.

Three areas of the theory were particularly of interest. First, Cooperative Theory, Bargaining Theories and related topics; second, the area of repeated games, stochastic games and multistage games; and third, attempts to link Cooperative Theory (the characteristic function form) with Non-Cooperative Theory (mainly in the strategic form). In the first topic discussion centered around the presentation of new bargaining theories, on the application of solution concepts (like the continuous Shapley value) to cost-sharing problems, and on relations to Mathematical Economics and Equilibrium Theory. The second topic was dealing with existence problems of values, in particular for stochastic and repeated games. In the third area it was mainly attempted to explain cooperative solution concepts by means of non-cooperative bargaining procedures.

The organizer is particularly thankful to the Mathematisches Forschungsinstitut and to the VW-Foundation for supporting the conference generously. Otherwise it would have been impossible to concentrate the core of game theorists at one place for such a delightable and successful meeting.

Programm

Monday, March 3rd

	Chairman: D. Schmeidler
9.15 - 10.00	S. Zamir, Jerusalem: Repeated games with transcendental values
10.15 - 11.00	G. Owen, Bogota: The role of information in cooperative games
11.30 - 12.15	T.E.S. Raghavan, Chicago: Algorithms for stochastic games when one player controls the law of motion
	Chairman: E. Kalai
15.15 - 16.00	W. F. Richter, Bielefeld: Bargaining theory and progressive taxation
16.15 - 17.00	L. J. Billera, Cornell: A homogeneity property for nonatomic games and a representation property for the core
17.15 - 18.15	Y. Tauman, Jerusalem: Demand compatible equitable cost-sharing prices

Tuesday, March 4th

	Chairman: R. Selten
9.15 - 10.00	R. J. Weber, Evanston: Strategies and information: Equilibrium of games of incomplete informations in normal form
10.15 - 11.00	Y. Kanai, Rehovot: The Brouwer fixed point theorem as the fundamental theorem of calculus
11.30 - 12.15	P. Dubey, Yale: Efficiency properties of strategic market games: An axiomatic approach

Chairman: M. Maschler

15.30 - 16.15	W. F. Lucas, Cornell: On games with no stable sets and empty cores
16.30 - 17.15	E. Kohlberg, Jerusalem: Multistage games and nonexpansive mappings
17.30 - 18.15	A. Neyman, Jerusalem: Stochastic games

Wednesday, March 5th

Chairman: J. C. Harsanyi

9.00 - 9.40	R. Selten, Bielefeld: An equilibrium point interpretation of the Shapley-value
9.50 - 10.30	T. Ichiishi, Pittsburgh: The t-core of a game without sidepayments

Chairman: W. F. Lucas

10.40 - 11.20	W. Güth, Köln: Game theoretical analysis of wage bargaining in a simple business cycle model
11.30 - 12.10	M. Mares, Praha: Domination of imputations in general coalition games
13.30 - ?	everybody, everywhere: hiking adventures, sightseeing tours, idling, gambling in Baden-Baden

Thursday, March 6th

Chairman: Y. Kannai

9.15 - 10.00	E. W. Zachow, Münster: Nuclear equilibrium points
10.15 - 11.00	E. Kalai, Evanston: Preplay negotiations
11.30 - 12.15	L. S. Shapley, Santa Monica: Nonatomic games in strategic form

Chairman: G. Owen

15.30 - 16.15 J. C. Harsanyi, Berkeley:
Outcome models and stability models in noncooperative analysis of cooperative games

16.30 - 17.15 M. Maschler, Jerusalem:
New properties of the nucleolus

17.30 - 18.15 N. Megiddo, Tel-Aviv:
Repeated games of incomplete information with revealed payoff

Friday, March 7th

Chairman:

9.15 - 10.00 S. H. Tijs, Nijmegen:
Existence of values for arbitration games

10.15 - 11.00 S. Hart, Jerusalem:
Endogenous coalition formation

11.30 - 12.15 J. Łoś, Warszawa:
Information in games

Chairman:

15.30 - 16.15 R. Avenhaus, Karlsruhe:
Application of game theoretical methods in data verfication

16.30 - 17.15 W. Albers, Bielefeld:
Some conceptual ideas based on experimental results

17.30 - 18.15 D. Schmeidler, Tel-Aviv:
Application of non-cooperative games to markets and allocation schemes

Vortragsauszüge

Some Conceptional Ideas based on Experimental Results

Wulf Albers, University of Bielefeld, Bielefeld, Germany

The findings given are first results of a sequence of experiments with games of apex type with free communication.

Bargaining in these games was usually done exactly up to multiples of a prominence step Δ , as for example $\Delta = 5$ or 10 in a game with total payoff of 100.

Sometimes the players formed blocs, i. e., coalitions which behave as one player and distribute their common share equally. The formation of blocs is a strategical instrument to improve the payoff of players outside a formed coalition. It urges the players within the coalition to choose their payoffs distribution in a way which is adequate for both alternatives: to the formation of a bloc (or blocs) and to the case that a bloc is not formed.

Primary bloc structures were considered, which are formed (or imagined in the minds of the players) before the bargaining procedure begins. These bloc structures are described by the fact that players form a bloc if thereby all of them improve their expected payoff.

The stability of primary bloc structures could be explained by the (first order) assumption that players outside the primary bloc structure may give a proposal and then the players decide independently if they will leave the primary structure. The temptation to leave can be measured by the payoff improvement devided by the prominence value Δ .

Application of Game Theoretical Methods to Data Verification Problems

Rudolf Avenhaus, Kernforschungszentrum Karlsruhe, Karlsruhe, Germany

Safeguards procedures based on the principle of data verification are analyzed under the assumption that false accusations cannot be excluded. A number of classes of material are considered which contain different numbers of batches with different material properties. The data of these batches, which are reported to a safeguards authority, shall be verified by an inspector of this authority with the help of independent measurements on a random sampling basis under the assumption that a part of these data may be falsified intentionally.

If the possibility of false accusations did not exist, this conflict situation could be modelled by a two person zero sum game; this would have the advantage that optimal strategies (e. g. sample series) would not depend on the payoff parameters the values of which frequently can hardly be estimated. However, because the measurement errors may cause false alarms, this conflict situation has to be modelled as a general non-cooperative two person game. In case of false alarm both parties have the same interest of classifying this alarm.

It is the purpose of the presentation to show under which conditions optimal verification strategies can be determined which are independent of the values of the payoff parameters and furthermore, which quantities to be determined (e. g. inspection costs) do require the estimation of the values of these parameters.

Dynamic Games and Dynamic Programming

E.E.C. van Damme, Eindhoven University, Eindhoven, The Netherlands

Consider a dynamic system, which is observed simultaneously by two players at discrete time points. At each point of observation both players take an action and the system moves to another state (depending on the previous state and the actions taken). In each period (= time between two observation points) costs are incurred by both players, depending on the state at the beginning of the period and on the actions taken. We assume binding agreements are not possible and also that both players are only interested in minimizing their own total costs. So we have a noncooperative dynamic game and the Nash equilibrium point (= e.p.) is an appropriate solution concept. At each observation point a player can base his action on the state of that moment (Markovstrategy) or on all states that have occured up to and including that moment (history dependent strategy). In a few examples we will show that both players can obtain a considerable better performance by using history dependent strategies. In particular we will show that there exists an e.p. which cannot be found by dynamic programming and which is for both players better than any e.p. in Markov strategies (which can be found by dynamic programming). Although only some examples are given, it will become clear that the phenomenon is present in a wide class of dynamic games.

Finiteness and Inefficiency of Nash Equilibria

P. Dubey, The Hebrew University, Jerusalem, Israel

We explore the finiteness, efficiency and strongness of Nash Equilibria (N.E.) of finite-player strategic games. Let

$N = \{1,\dots,n\}$ = the players set, $n \geq 2$.

For $i \in N$, $S^i = \{ x \in \mathbb{R}_+^{k(i)} : \sum x_j = 1 \}$, $k(i) \geq 2$.

$S = S^1 x \dots x S^n$

$S' = \{x = (x^1,\dots,x^n) : x^i$ is a vertex of S^i for at least one $i\}$

$S'' = \{x = (x^1,\dots,x^n) : x^i$ is not a vertex of S^i for at most one $i\}$

$\mathcal{U}$ = vector space of all C^2 functions from S to the reals. Impose the C^2-norm on $\mathcal{U}$, i. e., for $u \in \mathcal{U}$, $\|u\| = \sup\{\|u(x)\|, \|Du(x)\|, \|D^2(x)\| : x \in S\}$.

The space of games is $(\mathcal{U})^n$. For $u = (u^1,\dots,u^n) \in (\mathcal{U})^n$, interpret u^i to be the payoff function of player i. Finally define $\Upsilon(u) \subset \mathcal{E}(u) \subset \eta(u) \subset S$ by: $\eta(u)$ = the set of N.E. of u, $\mathcal{E}(u)$ = the set of efficient N.E. of u, $\Upsilon(u)$ = the set of strong N.E. of u.

Theorem 1. There is an open dense set O of $(\mathcal{U})^n$ such that, if $u \in O$,

(a) $\eta(u)$ is a finite set

(b) $\mathcal{E}(u) \subset S'$

(c) $\Upsilon(u) \subset S''$

This is a typical example of the kind of result that can be proved. For instance, if the S^i were taken to be spheres, then $S' = \emptyset$ (the empty set), hence (b) and (c) may be replaced by "$\mathcal{E}(u) = \emptyset$". In general let S^i be "stratified" sets and define vertices of S^i to be vertices in its "triangulation". Then again Theorem 1 holds.

Besides $(\mathcal{U})^n$ we also analyse "multi-matrix" games, i. e. mixed extensions of finite pure-strategy games as defined by Nash. Here, if player i has $k(i)$ pure strategies, then the space of games is the Euclidian space $\mathbb{R}^{kn}$, with $k = k(1)x\dots xk(n)$.

Theorem 2. Same as Theorem 1, but with $(\mathcal{U})^n$ replaced by $\mathbb{R}^{kn}$ and S'' replaced by $S^* = \{x = (x^1,\dots,x^n) : x^i \text{ is a vertex of } S^i \text{ for each } i\}$.

A third example comes from certain kinds of "strategic market games" (in which case inefficiency is generic).

Efficiency Properties of Strategic Market Games: an Axiomatic Approach

P. Dubey, A. Mas-Colell, M. Shubik, The Hebrew University, Jerusalem, Israel

We investigate conditions under which an abstractly given market game will have the property that, if there is a continuum of traders, then every non-cooperative equilibrium is Walrasian. In other words, we look for a general axiomatization of Cournot's wellknown result. Besides some convexity, continuity, and non-degeneracy hypotheses, the crucial assumptions are: anonymity (i. e. the names of traders are irrelevant to the market) and aggregation (i. e. the net trade received by a trader depends only on his own action and the mean action of all traders). It is also shown that the same axioms do not guarantee efficiency if there is only a finite number of traders. Some examples are discussed and a notion of strict noncooperative equilibrium for anonymous games is introduced.

Endogenous Coalition Formation

Sergiu Hart (and Mordecai Kurz), The Hebrew University, Jerusalem, Israel

The examination of some recent examples of the application of the λ-transfer value indicates that, in some instances, group(s) of players may form a "coalition" (or, "syndicate", "union", "negotiating unit"). By this we mean that in all situations, either the whole group is "in" or it is all "out". Their purpose is to increase their share in the devision of the total payoff (to the grand coalition), and not just to guarantee their worth.

This endogenous theory of coalition formation consists of two steps: First, given a coalition structure (i. e., a partition of the set of players into disjoint coalitions), define a "value" - the"expected utility" of a player when playing the game with that coalition structure. Second, based on the above value, find those stable coalition structures - i. e., those which no players want to change.

The value is defined either constructively (taking into account both bargaining between coalitions and inside each coalition), or axiomatically; it coincides with Owen's value for games with "a priori unions". The stability concept used,is that of a strong equilibrium (where coalitions, not only individuals, can deviate).

A number of games and classes of games is then analysed, and, in each case, the stable coalition structures are found; this includes all 3-player games. symmetric 4-player games, "apex" games, Bott games, games with non-negative dividends. Moreover, an example (computer generated, randomly) shows that there is no universal existence theorem for stable coalition structures.

Preplay Negotiations

by Ehud Kalai, Northwestern University, Evanston, Ill., USA

Given a finite normal form game with m players, we define a formal process of involving n preplays leading up to a final play of the game. Each step of preplay consists of possible modifications of the strategies produced by the previous preplay and generate strategies for the next preplay. During a preplay iteration each player is allowed to change his previous strategy at most once. But the process is defined in such a way that a player always has the chance to react to other players changes. After all the preplays have been completed, the payoffs are determined by the underlying game with the strategies resulting from the final preplay. We are interested in the payoffs resulting from the entire preplay game. We assume that the players will play a Selten Perfect Nash equilibrium. We use backwards induction technique in order to compute the equilibrium payoffs. This technique is justified by the subgame perfectness property of these equilibrium points. It is shown that in Prisoner's Dilemma type games the only equilibrium payoffs are the cooperative ones. Bottle-of-the-sexes type games have the equilibrium payoffs converge to the Pareto set of payoffs as the number of preplays increases.

The Brouwer Fixed Point Theorem as the Fundamental Theorem of Calculus

Yakar Kannai, Rehovot, Israel

The Brouwer Fixed Point Theorem follows in a wellknown manner from the differentiable no-retraction theorem, which states that there exists no C^2-function $f : B \to S$ such that $f(x) = x$ for all $x \in S$. Here B is the unit ball in $\mathbb{R}^n$, and S is the unit sphere. Here we prove the non-existence of f via advanced calculus.

Set $f = (f_1,\dots,f_n)$. Let $J(f)$ denote the Jacobian determinant of f. Then $J(f) = \sum_{i=1}^{n} (-1)^{i+1} \frac{\partial f_1}{\partial x_i} E_i(x)$, where $E_i(x)$ is the determinant obtained from $J(f)$ by striking out the first column and the i-th row. In our case $J(f) = 0$. Hence

$$0 = \int_B J(f)dx = \int_B \sum_{i=1}^{n} (-1)^{i+1} \frac{\partial f_1}{\partial x_i} E_i dx$$

$$= \int_S \sum_{i=1}^{n} (-1)^{i+1} f_1 E_i x_i d\sigma + \int_B \sum_{i=1}^{n} (-1)^{i} f_1 \frac{\partial E_i}{\partial x_i} dx .$$

Here we have integrated by parts using Gauss' divergence theorem. It follows from the equality of the second order mixed derivatives that $\sum_{i=1}^{n} (-1)^{i} \frac{\partial E_i}{\partial x_i}$ vanishes identically (Jacobi 1847). Recalling that $f_i \equiv x_i$ on S and writing $\sum_{i=1}^{n} (-1)^{i+1} E_i x_i$ as a determinant, we see that $\sum_{n=1}^{n} (-1)^{i+1} E_i x_i = x_1$ on S. Hence

$$\int_B J(f)dx = \int_S f_1 x_1 d\sigma = \int_S x_1^2 \, d\sigma \neq 0,$$ a contradiction, q. e. d.

Non-Expanisve Mappings and Repeated Games

by (Abraham Neyman and) Elon Kohlberg , The Hebrew University, Jerusalem, Israel

We prove the following theorem on non-expansive mapping $T : C \to C$, where C is a closed convex subset of a normal space, and T satisfies $\| Tx - Ty \| \leq \| x-y \| \quad \forall \quad x,y \in C$.

(i) $\forall \; x \in C$, $\| \frac{T^n x}{n} \|$ converges to $\inf_{x \in C} \| Tx - x \|$

(ii) $\exists \; f \in$ dual space, $\| f \| = 1$, s.t $\bigwedge_{\forall x \in C} f(\frac{T^n x}{n}) \to \inf_{x \in C} \| Tx-x \|$.

(iii) Given $x_0 \in C$, the f in (ii) can be so chosen that

$$f(T^n_{x_0} - x_0) \geq n \inf_{x \in C} \| Tx-x \| .$$

We note that $\frac{T^n x}{n}$ may not converge. In fact, if F is a separate face of the unit ball, a non-expansive mapping T may be constructed in the accumulation points of $\left\{ \frac{T^n p}{n} \right\}_n$ cover F .

Still, in the game theory contexts, both in repeated games of incomplete information and in stochastic games, there are additional special features which allow us to conclude that $\frac{T^n x}{n}$ converges. In those contexts, the meaning of $\frac{T^n x}{n}$ is the value-per-play in an n-times repeated game.

There are two major open problems in repeated game theory: Does the value-per-play converge in stochastic games with an infinite state space ? ... in repeated games of incomplete information on both sides?

One might hope that our theorem on non-expansive mappings will help in solving these problems.

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Information in Games

Jerzy Łoś, Polish Acadamy of Sciences, Warsaw, Poland

The simple information system for an n-person game in normal form is defined and it is shown that by suitable iterations of such systems every game can be converted into a game with absolutely dominant strategies for all players (a strategy is absolutely dominant if the payoff for it does not depend on strategies of other players and it equals max over all possible values of the payoff function).

Games with No Solutions and Empty Core

by William F. Lucas, Cornell University, Ithaca, N. Y., USA

In 1944 J. v. Neumann and O. Morgenstern introduced a theory of solutions (also called stable sets) for n-person cooperative games in characteristic function form. The main question concerning their model has been whether a solution exists for every such game. In 1968 [Bull. of the Amer. Math. Soc.] W. Lucas gave a ten-person game with no solution. In this talk he presented 14-person games which have no solutions and also have empty cores. The core is another solution concept in the multiperson games, namely the imputations which are maximal with respect to a certain "domination" relation.

Domination of Imputations in General Coalition Games

Milan Mareš, ÚTIA-ČSAV, Prague, Czechoslovakia

A general coalition game is a pair (I,V) where I is a non-empty and finite set of players, and V is a mapping from 2^I into the class of subsets of R^I, such that for any coalition $K \in 2^I$ is:

- $V(K)$ is closed,
- if $x \in V(K)$, $y \in R^I$, $x_i \geq y_i$ for all $i \in K$, then $y \in V(K)$,
- $V(K) \neq \emptyset$ and $V(K) = R^I$ iff $K = \emptyset$.

A partition $\mathcal{K}$ of the set I into disjoint coalitions is called a coalition structure; vectors from R^I are imputations.

If $K \in 2^I$, $x,y \in R^I$, then $x \operatorname{dom}_K y$ iff $x_i \geq y_i$ for all $i \in K$ and $x_j > y_j$ for some $j \in K$. Let us denote for any $K \in 2^I$ $V^*(K) = \{y \in R^I : \text{there is no } x \in V(K) \text{ such that } x \operatorname{dom}_K y\}$.

Further, if $\mathcal{K}$ is a coalition structure then we denote

$$V(\mathcal{K}) = \bigcap_{K \in \mathcal{K}} V(K), \text{ and } V = \bigcup_{\mathcal{K} \text{ is coal.str.}} V(\mathcal{K}).$$

An imputation $x \in R^I$ is strongly stable iff

aa) $x \in V(\mathcal{K})$ for some coalition structure $\mathcal{K}$,
ab) $x \in V^*(K)$ for all coalitions $K \in 2^I$.

The set of all strongly stable coalitions is denoted by S^*.

A von-Neumann-Morgenstern solution S is such a class of imputations for which

ba) $S \subset V$
bb) if $K \in 2^I$, $x,y \in V(K) \cap S$ then never $x \operatorname{dom}_K y$,
bc) if $z \in V-S$ then there exists $K \in 2^I$ and $x \in S \cap V(K)$ such that $z \in V(K)$ and $x \operatorname{dom}_K z$.

A necessary and sufficient solution is given under which for a coalition $K \in 2^I$ is $V(K) \cap V^*(K) \neq \emptyset$.

A few more special classes of games fulfilling the sufficient part of this condition are presented. It is shown that if S^* is non-empty and equal to the von-Neumann solution then it is also the unique von-Neumann solution of the given game.

New Properties of the Nucleolus

Michael Maschler and LLoyd S. Shapley, The Hebrew University, Jerusalem, Israel

The nucleolus of a cooperative game $(N;v)$ with sidepayments was introduced by Schmeidler in 1969. Since then, it attracted the attention of several scientists. Nonetheless, in spite of its many attractive properties, one does not have at present a satisfactory system of axioms which defines the nucleolus as a solution concept having an intuitive meaning. It seems that some further basis properties of the nucleolus are needed in order to establish such a system. We shall offer here two properties of the pre-nucleolus, i. e., the nucleolus for the set $X \equiv \{(x \in \mathbb{R}^N \mid x(N) = v(N)\}$, which have an intuitive appeal and may serve as axioms.

Let $x \in X$ and let S be an arbitrary coalition, $S \notin \{\phi,N\}$. Define the reduced game $(S;w)$ by the relations

$w(S) = x(S)$

$w(\phi) = 0$

$w(R) = \text{Max}\ [v(R \cup Q) - x(Q)]$, $R \subseteq S$, $R\{\phi,N\}$.

Theorem 1: (the reduced game property) If x is the pre-nucleolus point of $(N;v)$ and x^S is the restriction of x to the members S , then x^S is the pre-nucleolus point of $(S;w)$.

The reduced game property expresses the fact that players who "believe" in the pre-nucleolus will not find themselves "in contradiction" when they examine the allocation of payoffs within a coalition S .

Theorem 2: (positive and limited response to growth of wealth) Let $(N;v')$ be derived from $(N;v)$ by changing $v(S)$ to $v'(S) = v(S) + a$, $a > 0$, for a fixed coalition S , $S \notin \{\phi,N\}$, leaving the rest of the worths of v unchanged. Let x and x' be the pre-nucleolus points of $(N;v)$ and of $(N;v')$ respectively. Under these conditions,

$0 \leq x'(S) - x(S) < a$.

This property states that if a coalition becomes wealthier by an amount a, its members receive more in the pre-nucleolus, but not more than a ; in fact - strictly less than a . During a bargain which leads to the pre-nucleolus, part of a should go to the other players. It is well known that the pre-nucleolus does not have the monotonicity property. Theorem 2 can be regarded as a partial "rehabilitation". It can be sharpened in several ways at the expense of somewhat weakening its intuitive appeal.

Repeated Games of Incomplete Information with Revealed Payoffs

by Nimrod Megiddo, Tel-Aviv University, Tel-Aviv, Israel

An (rxc)-matrix G is repeatedly played as a two-person zero-sum game. Player I (rows) knows only the number r , while player II (columns) may know everything. Payoffs are revealed after each play, and there is perfect recall. Strategy choices are not revealed.

A strategy for I is constructed which guarantees him that his sequence payoff will satisfy

$$\text{Prob}\left\{\liminf_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} x_i \geq v(G)\right\} = 1$$

The strategy is based on alternating between "experimental" plays (testing new strategies for G) and "compensational" plays of previously tested strategies to cancel losses due to the "experiments".

Stochastic Games have a Value

by J. F. Mertens and A. Neyman, The Hebrew University, Jerusalem, Israel

A stochastic game is played in stages. At each stage, the game is in one of finitely many states and each of the players observes the current state z_t and chooses one of finitely many actions. The pair of actions at stage t, together with z_t, determines the payoff x_t to be made by player II to player I at stage t, and the probability used by the referee to select the next state. All the referee's choices are made independently of the past. A player's strategy is a specification of a probability distribution over his actions at each stage conditional on the current state and the history of the game up to that stage. Any pair of strategies, σ of player I and τ of player II, induces together with z_0, a probability distribution on the stream $(x_1, x_2, \ldots)$ of payoffs. The definition of a value depends on how the players evaluate a distribution of streams of payoffs. Shapley (1953) proved that the λ-discounted game, i. e., the game with "evaluation" $E(\sum_{t=1}^{\infty} \lambda(1-\lambda)^{t-1} x_t)$ for $0 < \lambda < 1$, has a value and that both players have optimal stationary strategies. v_λ^i denotes the value of the λ-discounted game with initial state i, and σ_λ denote the stationary optimal strategy of player I in the λ-discounted game. Using Tarski's principle for real closed fields, Bewley and Kohlberg (1976). proved that both v_λ^i and σ_λ have a convergent expansion in fractional powers of λ, and that the limit v_∞^i of v_λ^i as $\lambda \to 0$ exists. The question as to whether or not the undiscounted stochastic games, i. e., the games with "evaluation" $E(\liminf_{N\to\infty} \bar{x})$ where $\bar{x}_N = \frac{1}{N} \sum_{t=1} x_t$, always have a value, was open for many years. Gilette (1958) proved the existence of the value in two cases: first when all games have perfect information and also in the so called cyclic case. Blackwell and Ferguson (1968) found in a particular example ("The Big Match") a strategy that would prove to be basic for further generalizations.

Our main result is that the undiscounted stochastic games always have a value. We have

Main Theorem: For every stochastic game and for every $\varepsilon > 0$ there exist strategies σ_ε of player I and τ_ε of player II, and $N > 0$ such that for every strategies τ of player II and σ of player I

Rational Repeated Games with Transcendental Values

J. F. Mertens and S. Zamir, The Hebrew University, Jerusalem, Israel

Repeated Games with Incomplete Information and Stochastic Games are two classes of multistage games in which the payoff functions depend on the state of nature which may be one of a given set of states. However, while in Stochastic Games the state is changing from stage to stage but it is always known to all players, in Repeated Games with incomplete information the state is chosen at random at the beginning of the game and is fixed for all stages but it is unknown to the players. In such games what is changing along the game is the information of the players about the state of nature.

It turns out that this difference between the two classes of games is well reflected in their mathematical properties: When dealing with two-person zero-sum multistage games one can define two notions of value;

(1) the value v_∞ of the infinite stage game,

(2) the asymptotic value $v = \lim_{n\to\infty} v_n$, where v_n is the value of the game consisting of n stages. Each of v_∞ and v may or may not exist.

Exept for very special classes of games, v_∞ does not exist in general for Repeated Games with Incomplete Information (Aumann and Maschler, Mertens and Zamir). On the other hand it was proved recently (Mertens and Neyman, Monash) that v_∞ does always exist for Stochastic Games.

The second difference between the two types of games is one concerning the asymptotic value v . This was proved to exist both for Stochastic Games (Bewley and Kohlberg) and for a large class of Repeated Games with Incomplete Information (Aumann and Maschler, Mertens and Zamir). However, the existence of v for Stochastic Games holds for any real closed field. In particular this implies that if the parameters of a stochastic game are all rational numbers then v is an algebraic extension of the rational numbers. We show in this work that this is not true for Repeated Games with Incomplete Information. We do this by giving examples of games with rational parameters but transcendental asymptotic value.

(1) $\varepsilon + E_{\sigma_\varepsilon, \tau}(\liminf_{n\to\infty} \bar{x}_n) \geq v_\infty \geq -\varepsilon + E_{\sigma, \tau_\varepsilon}(\limsup_{n\to\infty} \bar{x}_n)$

(2) for every $n > N$,

$$\varepsilon + E_{\sigma_\varepsilon, \tau}(\bar{x}_n) \geq v_\infty \geq -\varepsilon + E_{\sigma, \tau_\varepsilon}(\bar{x}_n) .$$

An Austrian Approach to the Role of Information in Cooperative Games/ Market Situations

by Guillermo Owen (and S.C. Littlechild), University of the Andes, Bogota, Colombia

The role of information in cooperative games and market situations is analysed by assuming the existence of several disjoint markets, together with a finite number of entrêpreneurs whose role is limited to arbitrage operations between markets. Under certain reasonable conditions, it is shown that, if all markets are linked by entrêpreneurs, then the market prices for a given commodity will converge to some average, common limiting value. It is also shown that, if the links between markets are established in a stochastic manner, and if the probabilities of establishing links are proportional to price differentials, then all markets will, with probability 1, eventually be linked.

Algorithms for stochastic games when one player controls the law of motion

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Stochastic games are infinite move games where in each move players play one of the finitely many matrix games called the states of the stochastic game. The matrix game they play next time is completely determined by a transition probability matrix on states that depends only on the current state and the current choices of the players. Shapley introduced these games and showed that if the payoff for the infinite move game is taken to be the expected discounted payoff at a fixed discount rate β, $0 \leq \beta < 1$, then the players can play optimally by choosing their pure strategies at each state according to a fixed probability distribution that depends only on the state they are in and not on how they reached that state. Such strategies are called stationary strategies. In general with rational entries with rational discount factor and with all transition probabilities rational the value of the game for some starting state could be irrational. From the point of view of solving these games in finitely many arithmetic steps, when one player, say player II, alone could influence the law of motion by his choices, the data and the values of the stochastic game lie in the same ordered subfield of the reals. Further the components of a pair of good stationary strategies for the two players also lie in the same ordered field and the problem can be solved by linear programming. When the payoff is the long run average payoff per play, again for the games, where player II controls the law of motion, the above results on orderfield property for the data and good stationary strategies and values hold. From the point of view of computing the value for these games, the following results are seen.

Consider a matrix game for each starting state whose i-th row corresponds to the choice of the pure stationary strategy σ_i of the stochastic game by player I and whose j-th column corresponds to the choice of the pure stationary strategy ν_j of the stochastic game by player II. Let the payoff corresponding to the i-th row j-th column be the limiting average income in the stochastic game, when σ_i and ν_j are used by the players starting at state s. Then the value of this matrix game is the same as the value of the stochastic game. Further there is a common optimal strategy for player I for all these matrix games which in turn yields an optimal stationary strategy for player I for the stochastic game. If the substochastic game $\Gamma_{s^*,k}$ where column k is deleted in state s^*, has the value $\nu(s^*)$ at state s^* and if $\nu(s^*)$ coincides with the value of the original stochastic game, then the two games have the same value at all states. Thus by a series of such reductions a new game can be constructed where all columns at all states are essential for player II in any optimal stationary strategy - such games can be solved for good stationary strategies of player II by solving the game separately for each irreducible components of optimal stationary stretegies of II that are uniquely defined by any other completely mixed stationary strategy. At such components the problem is reduced to a problem in linear inequalities. At transient states of such games there is only one column and thus one gets an optimal stationary strategy for the game

Bargaining Theory and Progressive Taxation

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The normative justification of progressive taxation is an open problem in Public Finance. In the language of Game Theory it might be stated as follows:

Tax payer $i \in N = \{1,\dots,n\}$ with income y_i derives utility from income after taxes $y_i - t_i$ according to $u_i : \mathbb{R}_+ \to \mathbb{R}$ (where $u_i' > 0$, $u_i'' < 0$). Denote $y := (y_1 \dots, y_n)$, $u := (u_1,\dots,u_n)$. For fixed $g > 0$ the set of feasible distributions of tax shares t is given by

$$X := \{t \in \mathbb{R}^n \mid \Sigma t_i = g\ ,\ 0 \le t \le y\}\ .$$

Let $t^* : \{(u,y)\} \to X$ be such that

$$1 > \frac{\partial t_i^*}{\partial y_i} > \frac{t_i^*}{y_i} \qquad (1)$$

whenever t^* is an interior solution $(0 < t^* < y)$.

The latter inequality defines progressive tax schedules.

Problem: Designate and axiomatize t^*!

By transformation invariance, w.l.o.g., $u(0) = 0$,

$$\Sigma u_i(y_i)/U_i(y_i - t_i) \to \min\ (t \in X)$$

satisfies (1) but lacks an axiomatic justification.

$$\| (u_i(y_i)-u_i(y_i-t_i))_{i\in N} \| \rightarrow \min$$

can be axiomatically justified (theorem!) but - as other well-established solution concepts in bargaining theory - violates (1).

Approximately Efficient Equilibria and Perfect Equilibria of a Strategic Outcome Function

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Let $(U_t)_{t\in T}$, $(W_t)_{t\in T}$ be a pure exchange economy with l commodities. The strategic outcome function $f : S \xrightarrow{T} (R^l)^T$ is defined as follows:
$S = R^l_+ \times R^l$; for $\underset{\sim}{s} = (a_t,b_t)_{t\in T}$, $k \leq l$ and $p \in R^l_+$ define
$\varphi^k_t(p^k) = b^k_t - a^k_t p^k$, $\bar{\varphi}^k(p^k) = \sum_{t\in T} \varphi^k_t(p^k)$ and similary
$\bar{b}^k$ and $\bar{a}^k$. Set $\overset{*}{p}^k = \bar{b}^k / \bar{a}^k$ then

$$f^k_t(\underset{\sim}{s}) = \varphi^k_t(\overset{*}{p}^k) = b^k_t - a^k_t\, \bar{b}^k / \bar{a}^k \quad \text{for all } k \text{ if } \sum_{k=1}^{l} \varphi^k_t(\overset{*}{p}^k)\overset{*}{p}^k \leq 0 .$$

Otherwise $f_t(\underset{\sim}{s}) = -w_t$.

Results:

a) For any list of positive $(a^*_t)_{t\in T}$ there is a list $(b^*_t)_{t\in T}$ s. t. $\underset{\sim}{s}^* = (a_t,b_t)_{t\in T}$ is a Nash Equilibrium.

b) For $(a^*_t)_{t\in T}$ uniformly bounded away from zero and "large" $\#T$ N.E. is approximately efficient.

c) If $\underset{\sim}{s}^*$ is a Perfect Equilibrium then a^*_t is the tangent to excess demand function at the equilibrium. There exists a Perfect Equilibrium.

d) There are $(a^*_t)_{t\in T}$ such that the corresponding N.E. is approximately efficient.

An Equilibrium Point Interpretation of the Shapley Value

by Reinhard Selten, Universität Bielefeld, Bielefeld, Germany

Myerson's generalization of the Shapley value for characteristic functions with communication graphs is used in order to construct a non-cooperative bargaining game model which yields the Shapley value as an equilibrium point with certain additional properties. Bargaining begins with the full communication graph; every player has an agent for every link in the graph. The two agents of a link bargain for the payoff difference between the two players. Conflict results in the removal of links where conflict is reached.

Subgame consistency, truncation consistency, symmetry and conflict avoidance are the properties which characterize a unique equilibrium point whose payoff is the Shapley value.

Game Theoretical Analysis of Wage Bargaining in a Simple Business Cycle Model

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Apart from the fact that the functional distribution of income is explicitey considered, the business cycle model considered is of the multiplier-accelerator type. We assume a given distribution of income for the case of conflict. The collective behaviour is specified by a a wage bargaining function which yields a bargaining result for every past history of any of the infinitely many periods t. This result can be either the choice of conflict in t or a wage level yielding more than the conflict income for both parties. We specify four intuitively desirable properties which the wage bargaining function for the dynamic game with short run cooperation should satisfy. It is shown that these four properties define a unique wage bargaining function. Furthermore, we analyse the economic development implied by this unique wage bargaining function.

Nonatomic games in strategic form: Abstract

Lloyd Shapley, The Hebrew University, Jerusalem, Israel

A problem, both technical and conceptual, may arise when one attempts to express the outcome or payoffs of a nonatomic game as a function of the players' strategies. The idealized model of the independent decision-maker, free to choose any strategy he pleases from a designated set, may be in conflict with the technical demands of integrability. If an arbitrary, possibly nonmeasurable set of producers (say) happens to decide to send their output to market, then what quantity of goods will be in stock when the customers arrive to buy?

A game-theoretical resolution of the difficulty will be sketched. Let the players make collective decisions, for the sake of integrability, but only in very small coalitions. By passing to the limit, a "most nearly noncooperative" mode of play is definded, dependent, however, on the adoption of some cooperative solution rule for the embedded group-decision games. Techniques for carrying out this extension of the original strategic form will be sketched, making use of the space BD of set funtions of bounded deviation, and a number of examples will be given.

Demand Compatible Equitable Cost Sharing Prices

by(L. Mirman and)Y. Tauman, CORE, Louvain-la-Neuve, Belgium

We propose a new approach for equitable cost-sharing pricing based upon the Shapley value for non-atomic games. The basic idea behind this work stems from Billera, Heath and Raanan who used the proposed prices to set telephone billing rates for a large institution.

It is shown in the paper that the proposed price mechanism can be justified on economic terms since it is uniquely determined by a set of axioms involving only cost functions and quantities consumed and not any notion of game theory. However taking into account the utilities of the consumers one can prove the existence of an equilibrium under this price mechanism for a general class of cost functions. This approach has the advantage of not involving any interpersonal comparisons of utilities and it can be applicable in the cases without information about the private demands of consumers when total demand is known.

Let J^m be the set of all functions F defined on E^m_+ continuously differentiable there and satisfying $F(0) = 0$. Each F represents a cost function and m the number of consumption goods.

Definition. A price mechanism is a functional $\underline{P}$ which associates with each m , each $F \in J^m$ and each $\alpha \neq 0$ in E^m_+ a vector $\underline{P}(F,\alpha)$ in E^m .

A reasonable cost sharing price mechanism should satisfy to our opinion the following axioms:

Axiom 1. Cost sharing. For every F and every α

$$\alpha \cdot \underline{P}(F,\alpha) = F(\alpha)$$

Axiom 2. Additivity. If F and G are in J^m then for each $\alpha \in E^m_+$

$$\underline{P}(F+G,\alpha) = \underline{P}(F,\alpha) + \underline{P}(G,\alpha)$$

Axiom 3. Positivity. If $F \in J^m$ is nondecreasing then for each α

$$\underline{P}(F,\alpha) \geq 0$$

Axiom 4. Dummy Commodity. Let $F \in J^m$. If

$$F(x_1,\ldots,x_{i-1},x_i,x_{i+1},\ldots,x_m) = F(x_1,\ldots,x_{i-1},0,x_{i+1},\ldots,x_m)$$

then

$$\underline{P}_i(F,\alpha) = 0 \; .$$

($\underline{P}_i(F,\alpha)$ is the i-th coordinate of $\underline{P}(F,\alpha)$).

Axiom 5. Consistency. Let $F \in J^m$ and let $G \in J^1$. If

$$F(x_1,\dots,x_m) = G(\sum_{i=1}^{m} x_i)$$

then for each i , $1 \leq i \leq m$, and for each $\alpha \in E_+^m$

$$\underline{P}_i(F,\alpha) = \underline{P}(G, \sum_{i=1}^{m} \alpha_i)$$

Axiom 6. Rescaling. Let F be in J^m. Let $\lambda_1,\dots,\lambda_m$ be m positive real numbers. Let G be the function in J^m defined by

$$G(x_1,\dots,x_m) = F(\lambda_1 x_1,\dots,\lambda_m x_m) \; .$$

Then for each $\alpha \in E_+^m$ and each i , $1 \leq i \leq m$,

$$\underline{P}_i(G,\alpha) = \lambda_i \underline{P}_i(F,(\lambda_1\alpha_1,\dots,\lambda_m\alpha_m)).$$

Theorem 1. There exists a unique price mechanism $P(\cdot,\cdot)$ which obeys axioms 1 - 6 . This price mechanism is given by the formula

$$\underline{P}_i(F,\alpha) = \int_0^1 \frac{\partial F}{\partial x_i}(t\alpha)\, dt \qquad , i = 1,\dots,m \; .$$

This prices are called the Aumann-Shapley prices.

Consider now the following economic model:

(1) There are m goods and l consumers.

(2) For each i , $1 \leq i \leq l$, $C^i \subseteq E_+^m$ is the consumption set of the consumer i .

(3) For each i , $1 \leq i \leq l$, $u^i : C^i \rightarrow E^1$ is the utility function of consumer i .

(4) Consumer i is endowed with an amount of money a_i .

(5) There is one producer who can produce each vector in a set $M \subseteq E_+^m$.

(6) There is a function $F : E_+^m \rightarrow E^1$. For every $\alpha \in M$ $F(\alpha)$ is the cost in terms of money of producing α .

Definition. A pair $(P,(\alpha^1,\dots,\alpha^l))$ where $\alpha^i \in C^i$ and $p \in E_+^m$ is an equilibrium if

(1) For each i α^i is the maximum point of the set $u^i(B_i(p))$ where

$$B_i(p) = \{x \in C^i \mid p \cdot x \leq a_i\} \cap M \; .$$

(2) The cost is shared by the consumers, i. e. if

$$\alpha = \sum_{i=1}^{l} \alpha^i \quad \text{then} \quad p\alpha = F(\alpha) .$$

Theorem 2. Under the following assumptions an equilibrium $(p,(\alpha^1,\ldots,\alpha^l))$ exists with the following property:

$$p = \underline{P}(F,\alpha) \quad , \quad \alpha = \sum_{i=1}^{l} \alpha^i .$$

(i. e. p is the Aumann-Shapley prices associated with F and the total demand α) .

Assumptions:

(1) For every i $a_i > 0$.

(2) u^i is continuous, quasi-concave and non-decreasing in the week sense.

(3) C^i is a box of the form $C^i = \overset{m}{\underset{j=1}{\mathsf{X}}} [0,c_j^i]$, $c_j^i > 0$.

(4) $F(0) = 0$ (no fixed cost).

(5) The set M is a box of the form $\overset{m}{\underset{i=1}{\mathsf{X}}} [0,m_i]$, $m_i > 0$.

(6) There is a positive b s. t. if $\frac{\partial F}{\partial x_j}(\alpha)$ exists then $\frac{\partial F}{\partial x_j}(\alpha) \leq b$.

(7) F is continuous and non-decreasing on M and for every $\alpha \in M$ the line segment $\{t\alpha\}_{0 \leq t \leq 1}$ contains only a finite number of points in which F is not cont. diff.

Remark

(1) This work was extended by us to the general equilibrium theory.

(2) Application to this theory to the transportation problems and to linear programming in general was done by D. Samet, I. Zang and Y. Tauman.

(3) L. Billera and D. Heath from Cornell have independently characterized the same price mechanism by a set of axioms very similar to the one we use.

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Values for Arbitration Games

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Two-person games in normal form are considered, where the players may use correlated strategies and where the problem arises, which Pareto optimal point in the payoff region to choose. We suppose that the players solve this problem with the aid of an arbitration function, which is continuous and profitable, and for which the original of each Pareto point is a convex set. Then the existence of values and defensive ε-optimal strategies is discussed. Existence theorems are derived, using families of suitable dummy zero-sum games. The derived existence theorems contain all known existence results as special cases.

Distributional Strategies in Games of Incomplete Information

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In a game with incomplete information, each player receives a signal (his "type"), and subsequently chooses an action; the types and actions of the n players determine each player's payoff. A player's strategy can be represented by the joint distribution of his signals and actions. Conditioning on types yields a classical behavioural strategy.

Types are "near" one another if they lead to similar beliefs about payoffs and about the types of others. A topology on types induces topologies of weak convergence on the spaces of distributional strategies and on the space of information structures (joint distributions on types).

These ideas can be used to establish the existence of Nash equilibrium points in a variety of settings. Furthermore, the set of equilibrium points of a game can often be shown to vary upper-semicontinuously with the information structure of the game. Finally, the distributional approach yields new techniques for the explicit computation of equilibrium strategies. A typical existence theorem is stated below.

$1,2,\dots,n$	players
$A_1,\dots,A_n$	compact B rel spaces of actions; $A = A_1 x \dots x A_n$
$T_1,\dots,T_n$	topological measurable spaces of types; $T = T_1 x \dots x T_n$
η	probability measure on T ; marginal distributions $\eta_1,\dots,\eta_n$
$u_1,\dots,u_n$	bounded continuous payoff functions on AxT

Definition. A distributional strategy μ_i for player i is a probability measure on $A_i x T_i$ satisfying $\mu_i(A_i x B) = \eta_i(B)$ for all measurable $B \subset T_i$.

Theorem. Assume that η is a tight probability measure. Further assume that $\eta \ll \eta_1 x \ldots x \eta_n$, and that $\frac{d\eta}{d(\eta_1 x \ldots x \eta_n)}(t) = f(t)$ is continuous a. e. Then the game with information structure η has an equilibrium point $\mu = (\mu_1, \ldots, \mu_n)$ in distributional strategies.

Nuclear Equilibrium Points

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In noncooperative, finite n-person games in normal form it is possible that the mixed extension possesses several equilibrium points which are neither interchangeable nor equivalent. To determine a solution in such games, Harsanyi (1975) has developed the tracing procedure. Because there are some objections against this procedure (e. g. Bjerring (1978)) a new solution concept is introduced. With reference to Schmeidler's definition of the nucleolus one firstly assumes fictively that the players may come by cooperation and bargaining to the decision which equilibrium point should be realized. Then, with respect to this bargaining solution, the excesses of the different equilibrium points are computed and those equilibria with the lexicographic minimal excesses are selected to be be prenuclear equilibrium points. In a second stage of the selection procedure a hierarchical rearrangement of the excesses of prenuclear equilibrium points is performed and then those prenuclear equilibria with lexicographic minimal hierarchical excesses are selected to be nuclear equilibrium points. It is proved that always nuclear equilibria exist and that their payoff vector is uniquely determined up to a permutation of its components. Finally the nuclear equilibrium points in "battle of sexes" games are computed.

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