MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 12/1980

Optimierung und optimale Steuerungen

16. 3. bis 22. 3. 1980

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Das große Interesse am Arbeitsgebiet der Optimierung und der optimalen Steuerung wird durch die insgesamt 55 Teilnehmer aus neun Ländern, darunter zehn Mathematiker aus Frankreich, bestätigt. Neben den 41 Vorträgen, die einen Überblick über den neuesten Stand der Forschung gaben, bot sich unmittelbar im Anschluß und am Rande des offiziellen Programms reichlich Gelegenheit zu angeregten und fruchtbaren Diskussionen. Aufgrund der vielen gemeinsamen Interessensgebiete wurde sowohl von den französischen als auch von den deutschen Kollegen angeregt, einen wechselseitigen Erfahrungsaustausch dieser Art auch in Zukunft regelmäßig fortzusetzen.

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Von den insgesamt 41 Vorträgen beschäftigten sich 22 mit Fragestellungen zum Thema "Optimierung". Dabei wurden in 10 Referaten neuere Verfahren und Vergleiche einzelner Optimierungsprogramme vorgestellt; bei den 12 Vorträgen über theoretische Resultate lagen Schwerpunkte im Bereich der konvexen Analysis und der Optimalitätsbedingungen höherer Ordnung. Unter den 19 Vorträgen zum Thema "Optimale Steuerung" beschäftigten sich zehn mit theoretischen Ergebnissen und neuen formalen Zugängen, in vier Vorträgen wurden Verfahren präsentiert. Über Anwendungen der Kontrolltheorie bei der optimalen Steuerung von Flugzeugen und Raketen sowie bei der Heizung mit Solarenergie wurde in fünf Referaten berichtet.

Von allen Teilnehmern wurden die ausgezeichnete Arbeitsatmosphäre des Instituts ebenso gelobt wie die hervorragende Betreuung durch die Mitarbeiter. Besonderer Dank gebührt an dieser Stelle dem Direktor des Mathematischen Forschungsinstituts, Herrn Professor Dr. M. Barner.

Proceedings der Tagung werden beim Springer-Verlag in der Reihe "Lecture Notes in Control and Information Sciences" erscheinen.

Vortragsauszüge

M. ATTEIA:

Quasi-convex Duality

We give a new notion of duality map for quasi-convex functions and study algebraic and topological properties of this duality map.

H.G. BOCK:

A Multiple Shooting Algorithm for Parameter Identification in Systems of Nonlinear Differential Equations

One of the most important problems in the control of dynamical processes appears to be the identification of parameters p of a model differential equation

$$\dot{z} = f(t,z,p)$$

from a sufficiently large number of measurements of functions of the variables z describing the process

$$g_i = g(t_i, z(t_i,p),p).$$

The times t_i where the measurements are taken may be given either explicitly or implicitly as functions of the dependent variable.
We treat this problem in a rather general formulation as an overdetermined nonlinear multipoint boundary value problem.
An effective numerical algorithm based on the multiple shooting technique is presented. The nonlinear approximation problem with nonlinear constraints appearing in this scheme is solved by an underrelaxed Gauß-Newton-Method.
The performance of the algorithm is demonstrated by the problem of identifying the constants of reaction in the process of denitrogenisation of Pyridin.
Extensions to other types of boundary value techniques as well as polyhedral norm approximation are stated.

M. BROKATE:

Necessary Optimality Conditions for Differential Games

We report work on the following task:
Within the framework of abstract maximum principles, develop necessary conditions for deterministic nonlinear differential games which can be used for numerical computation of the optimal trajectory.
We present a technique, which copes with singular surfaces, and apply it to the bang-bang-bang surfaces of Isaacs illustrating it by an example. We conclude with remarks on the state-of-the-art of numerical algorithms and on some efforts of the author using Bulirsch's algorithm for multiple shooting.

F. COLONIUS:

Necessary Optimality Conditions for Systems with Time Delay

This talk is concerned with necessary optimality conditions for time delay systems with function space end condition. The relaxed problem in the sense of Warga is introduced and as a first step, a theorem with Lagrange multiplier in the dual space of L_∞ is obtained. Then, using the notion of "regular reachability", a pointwise, global maximum principle is formulated. Finally, regular reachability is investigated for linear relaxed systems. It is claimed that - in a certain sense - regularity is generic for these systems.

J.P. CROUZEIX:

Some Results on Differentiability of Quasiconvex Functions

Little has been written about the differentiability of order 1 of quasiconvex functions on R^n. Still, quasiconvex functions have properties rather similar to these of convex functions. The aim of the talk is to give some new results about this subject with particular focus on existence conditions.

U. ECKHARDT:

A Method for L_1-Approximation

An iterative method is presented for solving linear L_1-approximation problems. This method is widely used in practice. A convergence proof is given which is motivated by a paper of Weiszfeld (1937) on optimal location problems. The convergence of the method is rather slow. It can be used, however, in many practical circumstances since it can be easily programmed and yields relatively quickly results of moderate accuracy.

I. EKELAND:

Duality Methods in the Hamiltonian Formalism

Problems in optimal control can be brought back to Hamilton's equations:

$$(-\dot{p},\dot{x}) \in \partial H(t,x,p)$$

A new method is given to find global solutions of these equations. This method requires the Hamiltonian H to be convex, and relies on extremizing the integral

$$\int_0^T \{-\dot{x}p + H^*(t,x,p)\}dt ,$$

where H^* is the Fenchel transform of H with respect to both variables x and p together.

K. GLASHOFF:

Variation diminishing operators and the finite bang-bang-principle for parabolic control problems

We consider the following control problem:
$y_t = y_{xx} (x \in [0,1],\ t > 0)$, $y(x,0) = 0\ (x \in [0,1])$, $y_x(0,t) = 0$, $y_x(1,t) + \alpha y(1,t) = u(t)\ (t > 0)$, where it is required to

minimize

$$\max_{x\in[0,1]} |y(u;x,T) - z(x)|$$

subject to $|u(t)| \leq 1$ a.e. (z given in $C[0,1]$). We give a new proof for the theorem that the optimal control $\bar{u}$ satisfies $|\bar{u}(t)| = 1$ a.e. and alternates finitely many times between $+1$ and -1.

B. GOLLAN:

Perturbation results for non-differentiable optimization problems

Optimization problems with a finite number of equality and inequality constraints are treated. All functions involved are assumed locally Lipschitz. Upper bounds for the directional derivatives of the optimal value function are obtained. No regularity assumptions are made, and thus both stable and unstable situations are covered.

M. GRÖTSCHEL:

On the relations between "Separation" and "Optimization"

It is well-known that separation theorems can often elegantly be used to prove duality-type theorems and vice versa. Therefore, these two types of theorems are in a sense equivalent. We address ourselves to the algorithmic content of such proof relations, namely the question whether efficient algorithms for separating a point from a set can be used to optimize efficiently over the set and vice versa.
We say that $(K;n,a,r,R)$ is a convex body if $n \geq 1$, $K \subseteq R^n$ is a convex set, $a \in K$ and $S(a,r) \subseteq K \subseteq S(a,R)$ holds where $0 < r < R$. The separation problem for $K \subseteq R^n$ is the following: Given $y \in R^n$, $\varepsilon > 0$ then decide whether $d(y,K) \leq \varepsilon$ or find $c \in R^n$ such that $cy + \varepsilon \geq cx$ for all $x \in K$. The optimization problem for K is: Given $c \in R^n$ and $\varepsilon > 0$,

find $y \in R^n$ such that $d(y,K) \leq \epsilon$ and $cy + \epsilon > cx$ for all $x \in K$. Using results of Khachian and Shor on the exponential convergence of the ellipsoid method we prove: Let $\mathcal{K}$ be a class of convex bodies, then there is a polynomial algorithm to solve the separation problem for the members of $\mathcal{K}$ if and only if there is a polynomial algorithm to solve the optimization problem for the members of $\mathcal{K}$.

J. GWINNER:

On optimality conditions in nondifferentiable infinite programming

Clarke's definition of the generalized directional derivative for locally Lipschitz functions in normed spaces is extended for functions that are only defined on subsets of an arbitrary topological vector space. We deal with infinite programs of the following form

$$\min f(x) \text{ , } x \in A \text{ , } g_t(x) \leq 0 \quad t \in T \text{ ! } ,$$

where T is a compact metric space, and more generally

$$\min f(x) \text{ , } x \in A \text{ , } \mathcal{G}(x) \cap \mathcal{M} \neq \emptyset \text{ ! } ,$$

where $\mathcal{G}$ is a set-valued mapping. We describe methods that lead from the local optimality via convex approximations to convex inequality systems in order to obtain optimality conditions of F.John and Kuhn-Tucker type.

W. HACKBUSCH:

Numerical solution of parabolic boundary control problems

A multi-grid method is proposed for solving a discretized parabolic boundary control problem. The state is defined by a linear or nonlinear parabolic differential equation. The state is controlled by virtue of the boundary condition. Constraints of the control are possible. The cost function should be quadratic or at least differentiable. The optimal control can be characterized by an equation of the form $u=Ku+f$ (linear case) or $u=h(u)$ (nonlinear case). Discretizations of such equations can be solved by the multi-grid iteration of the second kind. This is a very fast iteration. Its rate of convergence tends to zero when the step size approaches zero.

J.-B. HIRIART-URRUTY:

OPTIMALITY CONDITIONS FOR DISCRETE NORM-APPROXIMATION PROBLEMS

In the recent years, a lot of work has been devoted to deriving necessary conditions for optimality in nondifferentiable programming. In that context, Clarke's generalized gradient has been proved to be a powerful tool from the analysis viewpoint as well as from the optimization viewpoint. There is a field where the minimization of nondifferentiable functions typically occur, that is of best-approximation problems. As examples of such situations, we consider some discrete nonlinear norm-approximation problems like

$$\text{minimize} \quad \sum_i |b_i(x)| \quad \text{over} \quad S \subset X \qquad (P_1)$$

$$\text{minimize} \quad \max_i |b_i(x)| \quad \text{over} \quad S \qquad (P_\infty) \quad .$$

Still recently, necessary conditions for optimality have been derived for such problems, through different approaches, provided that the involved functions f_i are C^1 (WATSON 1978, EL ATTAR et AL. 1979, CHARALAMBOUS 1979). Our aim in this paper is two-fold:

(i) first, to present in an unified fashion the optimality conditions for discrete nonlinear norm-approximation problems dealing with C^1 functions.

(ii) secondly, to treat the l^1 - approximation problem in the broader context where the involved functions are merely locally Lipschitz.

W.van HONSTEDE:

Numerical Experience with Methods to Solve Semi-Infinite Problems by Reduction to Finite Ones

Given second-order conditions which are sufficient for a point to be an optimal solution of the generalnonlinear semi-infinite problem, it is possible to reduce the problem by one with finite number of constraints. To solve these nonlinear programming problems a lot of methods are available.
We have chosen Wilson's method due to the local equivalence with Newton's method. [Newton applied to the first-order equality-conditions].
To globalize the convergence properties some strategies are discussed and numerical results are given.

P. HUARD:

On feasible quasi-Newton methods for nonlinearly constrained problems

Several quasi-Newton methods, like Han's method, have been proposed for solving nonlinearly constrained mathematical programs. Each step consists in the resolution of a linearly constrained quadratic program, which gives for the convex case an unfeasible current solution. If we want to modify this solution at each step, in such a way to re-enter in the feasible domain, it is possible to use the finite procedure proposed in [1], using a bidimensional grid. With this sort of perturbation, it is very easy to prove the global convergence. We discuss here on the possibility to re-enter in the domain, with a procedure saving both global convergence and super-linear local convergence.

[1] P. Huard "Implementation of gradient methods by tangential discretization, JOTA, Vol.28, No.4 (1979), 483-499

J.L. de JONG:

Optimization in soaring, an example of the fruitful application of some simple optimization concepts

Three different problems encountered in the sport of soaring will be discussed: First, as an introduction, the classical

"MacCready-problem", which is concerned with the determination of the best cruise speeds in between columns of rising air under cumulus clouds, will be reviewed. Next a new solution concept will be presented for the "optimal dolphin flight" problem. This is the problem of the best (varying) speed through regions with varying vertical atmospheric velocities. Finally, some new ideas will be discussed which make new solutions possible to the "optimal zigzagging problem", which is the problem of whether, and if yes, how to make use of favorable regions with upwards directed atmospheric velocities which are present aside of the track to be flown.

F. KAPPEL:

Approximation von hereditären Kontrollproblemen

Differentialgleichungssysteme mit Zeitverzögerung können durch Differentialgleichungssysteme hoher Ordnung approximiert werden. Man gewinnt solche approximierende Systeme mit Hilfe von Approximationsresultaten über C_o-Halbgruppen (Trotter-Kato-Theorem). Man kann nun Kontrollprobleme für Systeme mit Zeitverzögerung numerisch lösen, indem man das gegebene System durch ein gewöhnliches Differentialgleichungssystem ersetzt und dann das Kontrollproblem für dieses approximierende System löst.

H.W. KNOBLOCH:

Notwendige Bedingungen höherer Ordnung in der Theorie der optimalen Steuerungen

Betrachtet werden Steuerungsprobleme, die durch deterministische gewöhnliche Differentialgleichungen mit klassischem Zielfunktional in Integralform gegeben sind. Unter notwendigen Bedingungen höherer Ordnung versteht man notwendige Bedingungen, die entlang einer optimalen Lösung gelten und unabhängig vom Pontryaginschen Maximumprinzip sind. Für die Diskussion sogenannter singulärer Extremalen sind diese Bedingungen vor allem unter dem Aspekt der Anwendungen von Bedeutung, da man mit ihrer Hilfe unter Umständen explizite Steuergesetze finden kann. Der Vortrag dient vor allem dem Ziel, die Vielzahl der in der ingenieurwissenschaftlichen Literatur vorkommenden Kriterien für singuläre Extremalen zu systematisieren und die methodischen Zugänge zu analysieren. Dabei wird die Rolle der algebraischen Theorie nicht-linearer Systeme hervorgehoben; in der Tat ist für die Anwendung der Bedingungen höherer Ordnung die Kenntnis einer Multiplikationstafel - im Sinne der Lie-Multiplikation - entscheidender als die Wahl der Hilfsmittel aus der allgemeinen Optimierung.

M. KOHLMANN:

PROBLEMS WITH EXISTENCE IN NONLINEAR PARTIALLY OBSERVED OPTIMAL STOCHASTIC CONTROL

The problem of existence in partially observed control (poc) has been outstanding until today for nonlinear stochastic optimal control problems. The difficulties are shortly described which arise when one tries to carry some of the well known methods of deriving existence results in completely observable control (coc) to the poc case: Easy examples show that the usual convexity and closure conditions do not fit into the general framework of poc.

Two different kinds of approaches to existence results for the poc problem are presented:

In the (Markov) diffusion case, the uniqueness of the solution of the martingale problem associated with the control problem allows to give existence results for problems with the following information patterns:

(i) Fix a sequence of times $0=t_1< \dots < t_N = 1$ and at time t allow observation of some rational approximation of some components of the state at times t_j before t and some components of the current state.

(ii) Again fix some sequence of times as in (i) and the information available to the controller is given at time t by some noisy observation of some components of the state times t_j before t and some components of the current state (common work with R. J. Elliott).
In the general case of non-Markovian Itô-processes a tightness condition on the set of admissible controls allows to use SKOROKHOD's imbedding and convergence theorem to derive convergence of an imbedded sequence associated with a minimizing sequence of admissible controls in some strong sense. From the limit function of this "imbedded" sequence an optimal control is constructed for the poc problem.

P. Kosmol

On Continuity of Parametric Problems

Let C be a topological space and (f_n) a sequence of functions $C \to R$ converging pointwise to $f: C \to R$. A description of the set of accumulation points of sequences (x_n) of minimal solutions x_n of f_n on C is valuable e.g. for questions of numerical stability and perburbation methods. Under reasonable circumstances the accumulation points turn out to be solutions of certain two-stage problems. Generalization to variational inequalities are given.

W. KRABS:

Controllability and Time-Minimal Controllability in the View of Optimization

Controllability of linear systems within a given time can be phrased as solvability of a linear operator equation. If in addition minimum norm solutions are looked for, then a convex optimization problem is obtained and the question of controllability turns out to be the question of feasibility of this problem. This can be characterized in terms of the corresponding dual problem having a finite extremal value. From this characterization one easily obtains a sufficient condition for complete controllability (which is also necessary) being equivalent to the solvability of the linear operator equation for all right-hand sides.
If the time varies, the controllability is expressed by a time-dependent family of operator equations being solvable. If controllability with norm-restricted controls is possible for some time $T > 0$, then the essential condition that guarantees a weak bang-bang-principle for time-minimal controls is the complete controllability of the system for all times in the interval $(0,T]$.

D. KRAFT:

Auswahl von Minimierungsverfahren für parametrische optimale Steuerungsprobleme

Parametrische optimale Steuerungsprobleme sind Steuerungsprobleme, bei denen die Lösungsstruktur der Steuerfunktion

a priori einer bestimmten Funktionenklasse zugeordnet wird (z.B. Polynomspline oder Exponentialspline), die durch eine endliche Anzahl von Parametern charakterisiert wird. Das resultierende Nichtlineare Programm wird mit Verfahren, die auf der Lagrange-Funktion des Programms basieren, gelöst. Die Effizienz dieser Verfahren ist in starkem Maße abhängig von der Berechnung der Gradienten.
Anhand eines komplexen realistischen Flugbahnoptimierungsproblems werden Auswahlkriterien für nichtlineare Programmierungsverfahren für alternative Gradientenerzeugung gegeben.

P.J. LAURENT:

An algorithm for the computation of spline functions with inequality constraints

Generally speaking, the definition of (interpolating or smoothing) spline functions satisfying a minimization property involves a semi-inner product and n linear functionals. In the case where a semi-reproducing kernel (concerning both this semi-inner product and these linear functionals) can be explicitly given, the numerical determination of the spline is reduced to the resolution of a linear algebraic system of dimension $n+q$ (where q is the dimension of the null space of the semi-inner product). The problem becomes more difficult in the case where there is a (finite or infinite) number of linear inequality constraints: for example, the condition that the solution must be positive on a given domain. We propose here an iterative algorithm for solving this type of problem, requiring, at each iteration, the resolution of one or two linear algebraic systems of dimension $n+q+1$ or $n+q+2$. More generally this algorithm can be used for the minimization of a strongly convex function subject to inequality constraints.

C. LEMARECHAL:

A view of line-searches

Most algorithms for numerical optimization define the iterate x_{k+1} from x_k in two steps: Compute first a direction d_k, then a stepsize t_k. The line-search problem is that of finding t_k so as to obtain an efficient minimization method. The line-search is a subalgorithm, to be executed at each iteration k, and characterized by interpolation formulae and stopping criterion.
Here we consider only the question of stopping criterion, which is certainly the most important ingredient of a line-search. We insist on a specific such criterion (in fact defined in 1967 by Wolfe), which requires from the stepsize two properties:

-that the function sufficiently decreases from x_k to x_{k+1}.

-that its derivative sufficiently increases from x_k to x_{k+1}.

We recall a theorem related to convergence properties of the descent method; we give a flow chart for a possible implementation of the resulting line-search, and we prove its consistency, both in the continuously differentiable case and in the non-smooth case.

F.LEMPIO/J.ZOWE:

Higher order necessary and sufficient optimality conditions

Consider the general infinite optimization problem

(P) Minimize $f(x)$ s.t. $x \in X$, $g(x) \in -K$, $h(x) = 0$.

Here X denotes an arbitrary set, Y a real normed space, K a convex cone in Y (with non-empty interior for the necessary conditions), Z a real Banach space, $f: X \to R$, $g: X \to Y$, $h: X \to Z$ mappings.
In the first part of the lecture convex approximations of order n for the image set $\{(f(x) - f(x_o), g(x), h(x): x \in X\}$ are defined, which generalize well-known notions of first order approximations

for the finite-dimensional case. By means of such convex approximations of order n a general multiplier rule of order n is given.

In the second part of the lecture this multiplier rule is applied to the Fréchet differentiable case and various higher order necessary optimality conditions are obtained which have been published recently in the literature. These conditions are very weak, since no regularity assumption is needed, the multipliers depend on certain critical directions and the multiplier corresponding to the objective function may be zero. Therefore it seems that the necessary conditions are far from being sufficient. That this is not the case is shown in the third part of the lecture.

Only replacing positive semidefiniteness by positive definiteness we obtain a sufficient condition for problems with finite dimensional space of origin X and for problems with finitely many real inequalities. A third theorem gives sufficient conditions for arbitrary X and Y but makes the strong assumption that the hypothesis is satisfied with the same multiplier for all critical directions.

U. MACKENROTH:

Strong duality, weak duality and penalization for state constrained parabolic control problems

For a state constrained parabolic control problem (P) weak duality ($\inf(P) = \sup(D)$) and strong duality ($\inf(P) = \max(D)$) are considered. Moreover, the control problem is replaced by a penalized one (P_ε) (which has no state constraints). Theorems are given which describe the connexion between the primal problem, the weak and the strong dual problem and the dual of the penalized problem. In a certain manner, the latter may be interpreted as a "regularization" of the weak dual problem. Many of the results may also be obtained for much more general parabolic control problems, e.g. for problems where the objective functional is of the form $f(y(T)) + \int_0^T L(t,y(t),u(t))dt$ (with an extended real valued normal convex integrand L) and where the control appears on the boundary: $\alpha y_{|\Sigma} + \frac{\partial y}{\partial n_A} = u$.

K. MALANOWSKI:

Finite Difference Approximation to Optimal Control Problems

Finite difference approximation to optimal control problems for systems described by ordinary differential equations subject to state and control constraints is discussed.
After B.S. Mordukhovich sufficient conditions of convergence of such approximations in terms of the values of the cost functional are formulated.
The particular case of nonlinear equations with control appearing linearly, convex cost functional and convex constraints satisfying some regularity conditions is analysed using Lagrange formalism. The regularity of primal and dual optimal variables are investigated and the rate of convergence of approximations is estimated.
In the case of linear equations it is shown that both primal and dual approximating variables are convergent to optimal ones and the rate of convergence is estimated.

E. NURMINSKI:

Decomposition of Large-Scale Problems Based on a Nondifferentiable Optimization Approach

The nondifferentiable optimization approach provides a number of ways for decomposition of large-scale problems. Examples of such methods are given and numerical experience is discovered.

H.J. OBERLE:

Optimale Steuerung von Heizung und Kühlung eines Sonnenhauses

Ein numerisches Verfahren zur Behandlung von Zweipunkt-Randwertproblemen mit Schaltfunktionen wird vorgestellt. Derartige Probleme treten als notwendige Bedingungen bei Steuerungsproblemen mit "singulären" Teilstücken und bei Problemen der optimalen Steuerung mit Ungleichsbeschränkung für die Zustandsgröße auf.

Das vorgeschlagene numerische Verfahren ist eine Modifikation der Mehrzielmethode, welches die Schaltbedingungen als innere Bedingungen eines Mehrpunktrandwertproblems behandelt.
Als Anwendung dieses Verfahrens werden Modelle zur optimalen Steuerung von Heizung und Kühlung eines Hauses betrachtet.
Die Steuerung von Heizung und Kühlung ist unter Verwendung von konventioneller und solarer Energie so vorzunehmen, daß zum einen die Innentemperatur optimiert wird und zum anderen der Verbrauch an zusätzlicher konventioneller Energie minimiert wird.

J.-P. PENOT:

On the existence of Lagrange multipliers in nonlinear programming in Banach spaces

We focus our attention to the special features of the Banach space case of the following nonlinear programming problem:

$$\text{(P) minimize } f(x) \text{ subject to } x \in A = B \cap g^{-1}(C)$$

where B is a subset of a Banach space X and C is a subset (taken to be a closed convex cone here for simplicity) of a Banach space Y and $g: X \to Y$. If $a \in A$ is a local solution of (P) and f (resp. g) is differentiable at a (resp. strictly differentiable at a) then we wish to write the usual necessary condition

$$0 \in f'(a) + N_a A$$

where $N_a A$ is the normal cone to A at a under the following form

$$\text{(L)} \qquad 0 \in f'(a) + y' \circ g'(a) + N_a B$$

where $y' \in Y'$, positive on $P = -C$, is a "Lagrange multiplier". This question reduces to two subproblems: find sufficient conditions in order that

$$\text{(K-T)} \qquad T_a A = T_a B \cap g'(a)^{-1}(T_{g(a)} C)$$

for the usual tangent cones, and, for $M = T_a B$, $N = T_{g(a)} C$, $u = g'(a)$

$$(N) \quad [M \cap u^{-1}(N)]' = M' + u'(N')$$

where M'(resp.N') is the cone opposite to the polar cone of M(resp.N). It happens that both conditions (N) and (K-T) are valid under the following general regularity condition:

$$(R) \quad N + u(M) = Y .$$

This condition differs slightly from the regularity condition used by S. Robinson, Kurcyusz and Zowe and others as we use here the usual tangent cones instead of the radial tangent cones. We put in light the connection of condition (R) with the familiar transversality condition of differential topology. Moreover we relate this condition to conditions ensuring formulas of the type $\partial(f+g)(a) = \partial f(a) + \partial g(a)$ for f and g tangentially convex (but not convex) with values in $\bar{\mathbb{R}}$.

H.J. PESCH:

Optimale Flugbahnkorrektur von Raumfahrzeugen unter Dichteschwankungen der Atmosphäre

Die Berechnung (fast) optimaler Flugbahnkorrekturen von Raumfahrzeugen auf deren Bordrechner kann in Echtzeit-Rechnung nur mit Hilfe sehr schneller Algorithmen erfolgen. Dazu werden die notwendigen Bedingungen eines gestörten Problems der optimalen Steuerung linearisiert. Dies führt auf ein lineares Zwei-Punkt Randwertproblem, das - bei entsprechender Vorarbeit - an Bord über die Lösung eines linearen Gleichungssystems gelöst werden kann. Eine einmalige Integration der Bewegungsgleichungen und Auflösung einiger linearer Gleichungssysteme liefert dann den neuen, (fast) optimalen Steuerungsverlauf für den Restflug.
Es werden Steuerbarkeitsbereiche für die Bahnkoordinaten eines Space-Shuttle-Raumgleiters und erste numerische Ergebnisse für den Einfluß von Luftdichteschwankungen auf die Flugbahn eines Apollo-Raumfahrzeugs gezeigt.

G. PIERRA:

Some applications of decomposition methods in product space

It is classical when a problem of optimization is posed in a product space to search for a result of that problem by mean of simpler problems which have been posed in each of the space. It is proposed here to show how one such technique can be employed when the problem is not naturally posed in a product space.
For a problem posed in a space, one associate systematically an equivalent problem posed in a cartesiar product space. That new problem can then be decomposed, and the decomposition contains the following:

- a decomposition of operator
- a separation in the presence of constraints.

The following applications of the method will be shown

- uncovering of the projection on a convex sets intersection
- quadratic programming
- fractional steps method.

St.M. ROBINSON:

The second-order sufficient condition and weak stability in nonlinear programming

We present some new techniques for stability analysis in nonlinear programming, using the second-order sufficient condition and the hypothesis of constraint regularity. The techniques are applied to obtain stability theorems under what appear to be the weakest hypotheses known.

R.T. ROCKAFELLAR:

Existence of Optimal Arcs in Central Problems over Infinite Intervals

For a half open interval $[0,T)$, with T finite or $+\infty$, we consider problem of minimizing the functional

$$\Phi(x) = l(x(o)) + \int_{o}^{T} L(t,x(t), \dot{x}(t))dt$$

over all $x \in A[0,T)$, the space of absolutely continuous functions $x:[0,T) \to R^n$. Here l and L have $+\infty$ as a possible value, as does Φ, so the problem covers many types of constraints in implicit manner. We identify growth conditions on the Hamiltonian $H(t,x,p)$ associated with L which ensure the existence of an optimal arc x. Technically speaking, the situation is much harder to deal with than for a bounded interval $[0,T]$.

E. SACHS:

Bang-bang principles for nonlinear control problems

Nonlinear control approximation problems with bounded controls are considered. Requiring generalizations of controllability and normality for the adjoint of the linearized operator, we obtain various bang-bang-principles. For the numerical solutions, a version of the Osborne-Watson-algorithm using a Broyden-update for the derivative is proposed. Local convergence results require a local strong uniqueness property of optimal points of the linearized problems. For the proof of normality and uniqueness the role of variation-diminishing operators is discussed. An application to a boundary control problem of parabolic type with nonlinear boundary condition is given.

S. SCHAIBLE:

Convexifiability of Pseudoconvex C^2-Functions

Criteria are presented that characterize functions which are convex transformable by a suitable strictly increasing function.

We concentrate on twice continuously differentiable pseudoconvex and strictly pseudoconvex functions, and derive criteria which are both necessary and sufficient for these functions to be convex transformable. We study convexifiability of these functions with regard to every compact convex subset of an open convex set.

K. SCHITTKOWSKI:

Organization, test, and performance of nonlinear optimization codes

The lecture gives a survey about an extensive comparative study of programs for solving constrained nonlinear optimization problems of the form

$$\min f(x)$$

$$x \in R^n: \quad \begin{array}{ll} g_j(x) = 0 , & j=1,\ldots,m_1 \\ g_j(x) \geq 0 , & j=m_1+1,\ldots,m . \end{array}$$

In particular, the following topics are discussed:

1. Technical information and data about optimization programs.
2. Definition and evaluation of performance criteria.
3. Construction of test examples.
4. Development of a gradual performance evaluation.

Some numerical results and conclusions are presented obtained by 370 test runs which had to be passed by each of the 26 submitted optimization programs.

E. SPEDICATO:

On the use of slack variables in nonlinear programming

The use of slack variables to reduce inequalities into equalities in nonlinear programming is considered in the framework of diagonalized Quasi-Newton multiplier methods. We show that no essential increase in dimensionality is obtained and that the resulting equations are well defined around a Kuhn-Tucker point. Some numerical experience is presented.

P. SPELLUCCI:

Ein global und lokal superlinear konvergentes Verfahren für die allgemeine nichtlineare Optimierungsaufgabe

Es wird ein Verfahren zur Lösung der allgemeinen nichtlinearen Optimierungsaufgabe beschrieben. Dabei wird die Aufgabestellung in eine unrestringierte Minimierungsaufgabe mit einer exakten (nichtdifferenzierbaren) Penaltyfunktion überführt. Der Penaltyparameter wird im Verfahren mitbestimmt. Für die exakte Penaltyfunktion wird in jedem Iterationsschritt eine Abstiegsrichtung erzeugt, längs der ein "hinreichender Abstieg" im Sinne des Kuhn-Tucker-Kriteriums erster Ordnung möglich ist. Im Gegensatz zu den hierfür bereits bekannten Verfahren wird die Abstiegsrichtung aus einem linearen Gleichungssystem bestimmt. Dabei geht eine "Näherung" für die Hessematrix der Lagrangefunktion des Problems ein, die nur der Bedingung der positiven Definitheit im Tangentialraum der als bindend betrachteten Restriktionen unterliegt. Diese Bedingung kann algorithmisch leicht erfüllt werden. Als bindend werden alle verletzten und nahezu verletzten Ungleichungsrestriktionen (und natürlich alle Gleichungsrestriktionen) betrachtet. Eine wirksame und leicht zu realisierende Inaktivierungsstrategie für bindende Restriktionen wird angegeben. Die Schrittlänge längs der Abstiegsrichtung wird mit Hilfe eines Abstiegstests nach Armijo festgelegt. Unter schwachen Bedingungen konvergiert jede so erzeugte Folge gegen einen Kuhn-Tucker-Punkt des Problems. Falls dort die hinreichende Bedingung 2. Ordnung mit striktem komplementärem Schlupf erfüllt ist, falls konsistente Approximationen der Hessematrix der Lagrangefunktion verwendet werden und die Abstiegsrichtung um einen Korrekturterm zweiter Ordnung modifiziert wird, ist die Konvergenz Q-superlinear, und zwar sowohl in den primalen wie den dualen Variablen des Problems.

G. SZEFER:

Optimierungsprobleme der Elastomechanik als Aufgaben der optimalen Steuerung

In der Arbeit wird die allgemeine Optimierungsaufgabe von elastischen Systemen in abstrakter Form vorgestellt und diskutiert. Wesentliche mechanische Restriktionen führen zur Beschränkung der Zustandsvariablen. Zwei Klassen von Steuerungsproblemen werden in Einzelheiten diskutiert:
1/Optimierung von Bögen, die zu einem System hoher Ordnung mit Einzelparametern führt u. 2/ Optimierung von Platten, die ein Steuerungssystem mit verteilten Parametern bildet. Zur Lösung der ersten Aufgabe wird das Maximum-Prinzip von Pontryagin effektiv genutzt. Das zweite Problem wird mit Hilfe der Approximationstheorie in Sobolevschen Räumen gelöst.
Soweit wie möglich werden Existenz und Dualität betont; jedenfalls wird die Konstruktion der Lösung gegeben.

K.H. WELL:

On line approximation in solving real world optimization problems

"Real world" optimal control problems (OCP's) are sometimes characterized by differential equations that are partially given by tabular data. Before attempting to solve the OCP these data must be approximated by sufficiently differentiable functions. Solving the approximation problem (AP) can become difficult and time consuming for multidimensional tables.
To avoid solving the AP "on line" approximation for the two- and threedimensional cases using linear functions is suggested and the implementation into a conjugate gradient-restoration algorithm is described. Specifically, to handle the discontinuities of the adjoint differential equations a special initial value solver with a dependent variable stop is

used such that the integration is stopped at each data point. Results are given for the minimum time to climb trajectory of an aircraft.

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