MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 14/1980

Distributionen und partielle Differentialgleichungen

30.3. bis 5.4.1980

Die Tagung fand unter der Leitung von Herrn Professor Dr. J. Wloka (Kiel) und Herrn Professor Dr. Z. Zielezny (Buffalo, N.Y.) statt. An der Tagung nahmen 39 Mathematiker teil, von denen 16 aus dem Ausland kamen (aus Belgien, Iran, Japan, Jugoslavien, Kanada, Österreich, Polen, Schweden und den USA). Die insgesamt 29 Vorträge behandelten Themen aus den Gebieten Distributionen, partielle Differentialgleichungen und folgenden verwandten Gebieten: Differentialgeometrie, Funktionalanalysis, insbesondere in Funktionenräumen, Katastrophentheorie, Operatortheorie, unendlichdimensionale Holomorphie. Die in den Vorträgen angesprochenen Themen wurden in zahlreichen privaten Diskussionen weitergeführt. Die angenehme und anregende Atmosphäre des Forschungsinstituts trug zum harmonischen Verlauf der Tagung wesentlich bei.

Vortragsauszüge

W. ABRAMCZUK:

A problem in the theory of systems of pde's with constant coefficients

Formulation of the problem: given polynomials in n variables,

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$P_1,\dots,P_j$ which for technical reasons are assumed to be homogeneous and all of the same degree N , and a distribution with compact support f , to solve the equation

(1) $\quad P_1(D)u_1 + \dots + P_j(D)u_j = f$

with u_j N steps more regular than f in some Sobolev space meaning and the support of u_j contained in the convex hull of the support of f (which is assumed to have non-empty interior).

Results: the trivially necessary condition obtained by taking the Fourier-Laplace transform of (1) is also sufficient provided the polynomials have no common complex non-trivial zero.

E. ALBRECHT:

Local operators on spaces of ultradifferentiable functions

Let (M_p) be a sequence in $(0,\infty)$ satisfying the conditions (M.0) - (M.3) of Komatsu. For an open set $\Omega\subset\mathbb{R}^n$ let $D^{(M_p)}(\Omega)$ and $D^{\{M_p\}}(\Omega)$ be the corresponding spaces of ultradifferentiable functions (in the sence of Beurling and Roumieu). It is proved that every local linear operator

$T : D^{(M_p)}(\Omega)$ (resp. $D^{\{M_p\}}(\Omega)) \longrightarrow L^1_{loc}(\Omega)$

(i.e. supp $T\varphi \subset$ supp φ for all φ) is automatical continuous. Local operators $T : D^{(M_p)}(\Omega) \longrightarrow D^{(M_p)}(\Omega)'$ have some continuity properties too, while the situation in the Roumieu case $T : D^{\{M_p\}}(\Omega) \longrightarrow D^{\{M_p\}}(\Omega)'$ is worse. In a second part we give representations of continuous local operators as ultradifferentiable operators. The results have been obtained in joint work with M. Neumann (Essen).

G. BENGEL:

Formale und konvergente Lösungen partieller Differentialgleichungen

Es wurde über eine gemeinsame Arbeit mit R. Gérard (Strasbourg) berichtet. Wir betrachten partielle Differentialoperatoren der Gestalt $P(x, x\frac{\partial}{\partial x}) = \sum_{|\alpha| \leq m} a_\alpha(x)\,(x\frac{\partial}{\partial x})^\alpha$ und die Gleichung $Pu = f$, wobei die Koeffizienten a_α und f in einer Umgebung von 0 reell (oder komplex) analytisch sind. Sei $P_0(\xi) = \sum_{|\alpha| \leq m} a_\alpha(0)\,\xi^\alpha$.

Unter der Voraussetzung (*) $|P_0(n)| \geq C\,|n|^m$ für alle $n = (n_1, \ldots, n_N)$ mit $n_i \geqq 0$ wird gezeigt, dass jede formale Potenzreihe, die die Gleichung $Pu = f$ löst, automatisch konvergiert. Dieses Ergebnis lässt sich auch auf die nichtlineare Gleichung $Pu = F(x,u)$ ausdehnen. Ein Gegenbeispiel zeigt, dass auf die Voraussetzung (*) im allgemeinen nicht verzichtet werden kann.

K.D. BIERSTEDT, joint work with R. Meise (Düsseldorf) and W.H. Summers (Fayetteville, Ark.):

A projective description of weighted inductive limits

For a decreasing sequence $\mathcal{V} = \{v_n\}_{n \in \mathbb{N}}$ of upper semicontinuous functions v_n on a locally compact space X such that $\inf\{v_n(x);\ x \in K\} > 0$ for each compact subset K in X, let us denote by $\mathcal{V}_0C(X)$ the inductive limit of the Banach spaces $(n = 1, 2, \ldots)$

$C(v_n)_0(X) = \{f \in C(X);\ v_n f \text{ vanishes at infinity}\}$,

$$p_{v_n}(f) = \sup_{x \in X} v_n(x)|f(x)|.$$

Similarly, for $X \subset \mathbb{C}^{\mathbb{N}}$ open, $H(v_n)_o(X)$ denotes the Banach subspace of all holomorphic functions in $C(v_n)_o(X)$ and $\mathcal{V}_o H(X) = \operatorname{ind}_{n \to} H(v_n)_o(X)$.

With the sequence $\mathcal{V}$, we associate the system

$\overline{V} = \{\overline{v} \geq 0$ u.s.c. on X ; $\frac{\overline{v}}{v_n}$ bounded for each $n \in \mathbb{N}\}$ and put

$C\overline{V}_o(X)$ (resp. $H\overline{V}_o(X)$) $= \{f \in C(X)$ (resp. $H(X)$); $\overline{v}f$ vanishes at infinity for each $\overline{v} \in \overline{V}\}$,

endowed with the system $(p_{\overline{v}})_{\overline{v} \in \overline{V}}$ of seminorms.

Theorem 1. $C\overline{V}_o(X)$ is the completion of $\mathcal{V}_o C(X)$.

Theorem 2. If all functions v_n are continuous, then the following conditions are equivalent:

(1) $\mathcal{V}_o C(X)$ is complete, and hence $\mathcal{V}_o C(X) = C\overline{V}_o(X)$.

(2) $\mathcal{V}_o C(X)$ is a regular inductive limit.

(3) $\forall n \in \mathbb{N}\ \exists m \geq n : \forall k \geq m\ \forall \varepsilon > 0\ \exists \delta(k,\varepsilon) > 0$:

$$\frac{v_m(x)}{v_n(x)} \geq \varepsilon \implies \frac{v_k(x)}{v_n(x)} \geq \delta(k, \varepsilon).$$

Theorem 3. If $\forall n \in \mathbb{N}\ \exists m > n$: v_m/v_n vanishes at infinity, then $\mathcal{V}_o H(X) = H\overline{V}_o(X)$ holds (algebraically and topologically).

Theorem 1 is still valid in a much more general setting (i.e., for decreasing sequences $\mathcal{V}$ of systems V_n of "weights" and for Banach space valued functions). Theorem 3 had already been presented, with a somewhat different approach, at the functional analysis meeting at Oberwolfach in 1977.

In the talk, we also discussed the case of σ-growth conditions: there we obtain the algebraic equality $\mathcal{V}C(X) = C\overline{V}(X)$ whenever X is σ-compact, but an example of Köthe from the theory of echelon spaces shows that the topologies of the two spaces are different in general.

E. BINZ:

Geometrie auf Einbettungsmannigfaltigkeiten

Sei M eine kompakte orientierte und N eine beliebige parakompakte C^∞-Mannigfaltigkeit. Die Menge aller C^∞-Einbettungen von M in N heisse E(M,N). Diese ist offen in der C^∞-Fréchetmannigfaltigkeit $C^\infty(M,N)$ aller C^∞-Abbildungen von M nach N . Die Gruppe DiffM aller C^∞-Diffeomorphismen von M operiert fixpunktfrei auf E(M,N). Über dem Quotienten E(M,N)/DiffM einer C^∞-Fréchetmannigfaltigkeit ist E(M,N) ein Prinzipalbündel mit DiffM als typischer Faser. Sei $N = \mathbb{R}^n$ und $\dim M = n-1$. Auf $E(M,\mathbb{R}^n)$ werden zwei schwache Riemann'sche Metriken G und $\tilde{G}$ betrachtet. Diese sind wie folgt definiert: Sei $\langle\ \rangle$ ein festes Skalarprodukt auf $\mathbb{R}^n$. Für $i \in E(M,\mathbb{R}^n)$ bezeichne $\mu(i)$ die Riemann'sche Volumenform von $i^*\langle\ \rangle$.

Weiter sei $\rho: E(M,\mathbb{R}^n) \longrightarrow C^\infty(M,\mathbb{R})$ eine C^∞-Abbildung, die nur positive Funktionen als Werte annimmt und die Kontinuitätsgleichung $D\rho(i)(h)\cdot\mu(i) = -\rho(i)\cdot D\mu(i)(h)$ für alle $h \in C^\infty(M,\mathbb{R}^n)$ erfülle.

Für zwei Tangentialvektoren $h,k \in C^\infty(M,\mathbb{R}^n)$ an i sind G und $\tilde{G}$ wie folgt definiert:

$$G(i)(h,k) = \int_M (h,k)\, d\mu(i)$$

und

$$\tilde{G}(i)(h,k) = \int_M \rho(i)\, \langle h,k\rangle\, d\mu(i)\ .$$

Für beide Metriken werden die Riemann'schen Sprays berechnet. Derjenige von $\tilde{G}(i)$ ist trivial.

Eulers Gleichung für eine inkompressible Flüssigkeit wurde in diesem Zusammenhang diskutiert.

P. DIEROLF and S. DIEROLF:

Topological properties of the space $\mathcal{B}(\mathbb{R}^n)$ of L. Schwartz

(The lecture was given by the first named author).

The space $\mathring{\mathcal{B}}(\mathbb{R}^n) := \{\varphi \in C^\infty(\mathbb{R}^n);\ \partial^\alpha\varphi \in c_o(\mathbb{R}^n)$ for all $\alpha \in \mathbb{N}_o^n\}$ provided with the obvious Fréchet-Space topology $\mathcal{T}_o^\infty$, its dual $\mathring{\mathcal{B}}(\mathbb{R}^n)'$ and its bidual $\mathring{\mathcal{B}}(\mathbb{R}^n)'' = \mathcal{B}(\mathbb{R}^n) :=$ $\{\varphi \in C^\infty(\mathbb{R}^n);\ \partial^\alpha\varphi \in L^\infty(\mathbb{R}^n)$ for all $\alpha \in \mathbb{N}_o^n\}$ are important for the theory of convolution of distributions. Several locally convex topologies γ (different from the strong topology $\beta(\mathring{\mathcal{B}}'', \mathring{\mathcal{B}}'))$ were suggested which make $(\mathcal{B}(\mathbb{R}^n), \gamma)$ a space of distributions. We show that all these topologies coincide with the Mackey-topology $\tau(\mathcal{B}, \mathcal{B}')$. In proving this result we determine the $\sigma(\mathring{\mathcal{B}}', \mathcal{B})$ - compact sets; we show that $(\mathring{\mathcal{B}}(\mathbb{R}^n)', \sigma(\mathring{\mathcal{B}}', \mathcal{B}))$ is sequentially complete and that the LB-space $(\mathring{\mathcal{B}}(\mathbb{R}^n)', \beta(\mathring{\mathcal{B}}', \mathring{\mathcal{B}}))$ has a strong retraction property: A $\sigma(\mathring{\mathcal{B}}', \mathcal{B})$ - convergent sequence is already norm-convergent in some Banach space of the inductive sequence which generates $\beta(\mathring{\mathcal{B}}', \mathring{\mathcal{B}})$. Finally, we show that the topology $\tau(\mathcal{B}, \mathring{\mathcal{B}}')$ is an Orlicz-Pettis-topology. Therefore the space $(\mathcal{B}(\mathbb{R}^n), \tau(\mathcal{B}, \mathring{\mathcal{B}}'))$ - although it is not quasi-barrelled - is a suitable domain space for the measurable-graph-theorems which were recently proved by H. Pfister.

G.F.D. DUFF:

The principle of stationary phase for distributions and the elementary catastrophes

The asymptotic expansion of singularities of wave functions can be performed with integrals

$$\int f[S(\eta)]\ a(\eta)d\eta$$

where f is a distribution singular at the origin, $S(\eta)$ is a phase function and η a dual parameter set. By means of fractional integrals the complete expansion can be calculated explicitly provided the transformation to standard form of the phase function as a Thom catastrophe is known. This reduces the local representation problem for wave functions to an equivalence problem in catastrophe theory.

V. EBERHARDT:

Kompakte Operatoren zwischen lokalkonvexen Räumen

Der folgende Satz kann als gemeinsame Verallgemeinerung von Ergebnissen von Goldberg, Kruse (Proc. AMS 13, 1962) und von Valdivia (Ann.Inst.Fourier 25, 1975) interpretiert werden:

Satz. Für unendlichdimensionale separierte lokalkonvexe Räume X,Y sind äquivalent:

(i) Es gibt eine injektive, nukleare lineare Abbildung X ⟶ Y mit dichtem Bild.

(ii) X' enthält eine $\sigma(X',X)$-totale gleichstetige Folge, und in Y gibt es eine absolutkonvexe, schwach kompakte, totale, separable Teilmenge.

Anschliessend wurde eine vereinfachte Version von Valdivia's Beweis der Nicht-B_r-Vollständigkeit von $D(\Omega)$ (und anderer Räume) erläutert, in welcher das obige Resultat zur Anwendung kommt.

B. GRAMSCH:

Path components of special sets of operators

In connection with papers of S. Hayes, J.L. Taylor and A. Douady on a complex Banachalgebra $\mathfrak{B}$ with e a complete metric ρ is defined with respect to two appropriate norm ideals $\mathfrak{J},\dot{\mathfrak{J}}$ of $\mathfrak{B}$: $a,b\in\mathfrak{B}$

$$\rho(a,b) = \inf\left\{\sum_{\text{finite}} \|x_j\| + \|y_j\| : b = (\prod_j \exp x_j)a(\prod_j \exp y_j);\ x_j\in\mathfrak{J},\ y_j\in\dot{\mathfrak{J}}\right\}.$$

On the set $\mathfrak{R} = \{a\in\mathfrak{B} : \exists b\in\mathfrak{B} \text{ with } aba = a\}$ of regular elements of $\mathfrak{B}$ the topology induced by ρ coincides with the topology γ of Douady (1966) ($\mathfrak{B} = \mathcal{L}(E)$, $\gamma(a,b) = \|a-b\| + \text{gap}(\ker a,\ker b)$) introduced on $\mathfrak{R}$, if we put $\mathfrak{J} = \dot{\mathfrak{J}} = \mathfrak{B}$. Essential is the existence of local analytic cross sections s: $(\mathfrak{R},\gamma) \longrightarrow \mathfrak{B}^{-1}\times\mathfrak{B}^{-1}$. From this "follows" that the connected components of $(\mathfrak{R},\gamma)$ are Banach-analytic homogeneous spaces. From this one derives an Oka principle for the "set" $(\mathfrak{R},\gamma)$. A number of extensions of this result are possible varying $\mathfrak{J}$ and $\dot{\mathfrak{J}}$ in a suitable way. This can be applied to special classes of pseudo-differential operators (generalized singular integral operators) and also to the corresponding operator functions.

O. v. GRUDZINSKI:

Solvability of convolution equations in spaces of distributions of finite order

For spaces of functions and distributions which do not grow faster than a given logarithmically convex increasing function, the surjectivity of convolution operators is characterized by suitable "slowly decreasing" conditions on their Fourier transforms. Using function theoretical methods and a refined version of a construction of slowly decreasing entire functions (which is due to Ehrenpreis and Malliavin) one obtains precise results about problems of the following kind: let X and Y be any two of the spaces above, and let $\mathcal{C}$ be a class of convolutors which act surjectively on X ; under which conditions do the convolutors of $\mathcal{C}$ operate surjectively on Y as well?

S. HANSEN:

Lower bounds of pseudo-differential operators

To establish lower bounds for a pseudo-differential operator a(x,D) of order m means to find for a given gain $\varkappa > 0$ conditions on the symbol which imply an estimate (w.r.t. Sobolevnorm)

$$\mathrm{Re}(a(x,D)u,u) \geq -c \, \|u\|^2_{(m-\varkappa)/2} \, , \quad u \in \mathcal{S}(\mathbb{R}^n).$$

For nonnegative symbols this estimate was shown to hold with $\varkappa = 2$ by Fefferman and Phong. If the symbol satisfies the weaker condition of Melin $a(x,\xi) + \mathrm{Tr}^+ a''(x,\xi) \geq 0$ then this estimate holds with $\varkappa = 6/5$. Building on earlier work of Melin this was shown by Hörmander. With the aim of bringing these two theorems closer together a sketch of the main ideas involved in their proofs is given.

J. KISYŃSKI:

The argument of a Lagrange subspace and the Maslov index

The Maslov index of a path in a Lagrange submanifold of a symplectic space occurs in some problems concerning asymptotic solutions of linear PDE with a large parameter. This index is closely related to the Maslov-Arnold 1-cocycle on the Lagrange Grassmannian L. Although $\pi_1(L) \sim \mathbb{Z}$, the analytical formulas for the M-A cocycle, proved independently by several authors, are not very simple. The lecture shows that all these formulas are simple consequences of one theorem about differences between values of primitive functions of the Arnold differential 1-form in common points of various contractible subsets of L .

H. KOMATSU:

The local theory of ultradifferentiable manifolds

Assume that M_p is a sequence of positive numbers, satisfying the conditions: (M.o) $M_0 = M_1 = 1$, (M. 4)' $(M_q/q!)^{1/(q-1)} \leq H\,(M_p/p!)^{1/(p-1)}$, $2 \leq q \leq p$, (and (M.3)" $(M_p/p!)^{1/(p-1)} \longrightarrow \infty$ as $p \longrightarrow \infty$ in the case of (M_p)). Denote by * either $\{M_p\}$ or (M_p). Then we have:

Theorem 1. The composite of ultradifferentiable mappings of class * is ultradifferentiable of class *.

Theorem 2. If an ultradifferentiable mapping of class * has a non-vanishing Jacobian, then the local inverse is also ultradifferentiable of class *.

Theorem 3. If $f(t,x) : (-T,T)\times U \longrightarrow \mathbb{R}^n$, $U\subset\mathbb{R}^n$, is an ultradifferentiable mapping of class * , then the solution $x = \varphi(t,y)$ of the initial value problem $dx/dt = f(t,x)$, $x|_{t=0} = y$ is ultradifferentiable of class * in (t,y).

M. LANGENBRUCH

Representation of distributions (of finite order) as boundary values and the topology of D' .

Let P(D) be a hypoelliptic PDO on $\mathbb{R}^{N+1}$ with constant coefficients and define $\mathcal{N}_{D_F}$ (and $\mathcal{N}_D$) to be the nullsolutions on $\mathbb{R}^{N+1}\setminus\mathbb{R}^N$ of P(D) which are (locally) slowly growing with respect to $\mathbb{R}^N$. $f\in\mathcal{N}_D (f\in\mathcal{N}_{D_F})$ iff f has a distribution (of finite order) as boundary value. By the aid of a representation of the dual of $\mathcal{N}_D$ (and $\mathcal{N}_{D_F}$) it is proved, that the topologies of $\mathcal{N}_D$ and $\mathcal{N}_{D_F}$ are induced by $D(\mathbb{R}^{N+1})'_b$. This has various topological consequences, especially: $\mathcal{N}_{D_F}$ is a topological sequential dense subspace of $\mathcal{N}_D$.

O. LIESS:

Uniqueness theorems for solutions of linear partial differential operators

Let $p = D_t^m + \sum_{|\alpha|+j\leq m} a_{\alpha j}(x,t)D_x D_t$ be a linear p.d.o. with real analytic coeff. defined near $0\in R^{n+1}$. It follows from the Cauchy-Kowalevsky theorem that for every $\delta > 0$ there is $\delta' > 0$ such that for every $u\in\mathcal{A}'((x,t)\in C^{n+1}$; $|(x,t)| < \delta')$ there are

unique $v_j \in \mathcal{A}'(x \in C^n \; ; \; |x| < \delta)$ and $w \in \mathcal{A}'((x,t) \in C^{n+1} \; ;$ $|(x,t)| < \delta)$ such that $u = \sum v_j \otimes D_t^j \, \delta_t + {}^t p \, w$.
($\mathcal{A}'$, the analytic functionals, ${}^t p$ the adjoint of p). One can write down explicit formulas which give good approximations of the map $u \longrightarrow (v_j, w)$. In fact, if $\sum q_j(x,t,\zeta,\tau)$ is (for $|\tau| > c|\zeta|$; c "great") a formal analytic symbol s.t. ${}^t p \circ \sum q_j \sim 1$ in the symbol algebra, then $\min_k | \hat{w}(\zeta,\tau) -$

$$- \frac{1}{2\pi i} \, u \int_{|\sigma| = M(\zeta,\tau)} e^{-i(x,\zeta) - it(\tau+\sigma)} \, \frac{1}{\sigma} \sum_{j<k} q_j(x,t,\zeta,\tau+\sigma) \, d\sigma) | \leq c \, \|\hat{u}\|$$

, for $(\zeta,\tau) \in R^{n+1}$ (in fact in s suitable neighbourhood), $|\tau| \leq c'|\zeta|$, $c = c(c',p)$, M great.
Using this, or similar formulas related with the Goursat problem, one can prove that the Cauchy problem for p has a microlocal nature (this follows also from the "water melon" theorem), and uniqueness theorems for the Goursat problem.

G. LUMER:

Diffusion equation and related distribution methods

In studying evolution problems in sup-norm setting (see: (1) G. Lumer, Annales Inst. Fourier t. 25(1975) fasc.3 et 4, p. 409-446; (2) ——— "Evolution equations ...", Linear Spaces and Approximation, ISNM Vol.40 (1978), p. 547-558; (3) - "Connecting of local operators and evolution equations on networks", to appear in Springer Lect.Notes in Math., Proceedings of the Danish-French Colloquium on Potential Theory, Copenhagen may 1979), beside the standard distribution and Sobolev spaces methods, certain special techniques have proved very useful (we do not know

exactly what of these techniques is or is not known, but in any case the applications are new and definitely useful):
In comparing sup-norm problems with L^2-variational problems, special cases of the following result are used:"Let Ω be an open set in R^n, $f \in W^{m,p}_{loc}(\Omega)$, $g \in W^{m',p'}_{loc}(\Omega)$, (m,m' integers ≥ 0 $p \geq 1$, $\frac{1}{p} + \frac{1}{p'} = 1$, $W^{m,p}$ the usual Sobolev spaces). Then for any multiindex $\alpha, |\alpha| \geq 0$, the Leibniz formula holds for $D^{\alpha}(fg)$ (say $= \sum c_{\beta\gamma} D^{\beta}fD^{\gamma}g$, with all products $D^{\beta}fD^{\gamma}g$ - products of perhaps functions with distributions - are well defined) provided $m + m' \geq |\alpha|$" .
In working with diffusion equations on networks (see (3)), and constructing barriers, we prove some special results (needed for networks, about "subsolutions" which do not belong to a standard Sobolev space, i.e. results of the type "Au $\leq$ 0 ==> u is A-superharmonic", say A a 2nd order, one-variable, unif. elliptic operator, with continous coefficients, u having derivatives up to order 2 (distributional sense) which are at worst measures, Au as a measure is ≤ 0 ; (we do it for u continuous, with sectionally continuous pointwise derivatives, but other variants are of interest).

R. MEISE:

Entire functions on spaces of distributions

If E is a locally convex space over $\mathbb{C}$, then $(H(E),\tau_0)$ denotes the space of entire functions on E , endowed with the compact open topology τ_0 .The bornological topology associated with τ_0 is denoted by $\tau_{0,bor}$. The proof of the following thèorem was outlined:

<u>Theorem.</u> a) $(H(\mathcal{D}_b'),\tau_{o,bor}) \cong \mathcal{D}$

b) $(H(\mathcal{D}^{(p^{ps})}(\mathbb{R})_b'),\tau_{o,bor}) \cong \bigoplus_{\mathbb{N}} \Lambda_\infty(((\ln(n+1))^{1+\frac{1}{s}})_{n\in\mathbb{N}})$ for $s > 1$.

The proof relies on a representation theorem of Vogt and the following two results.

<u>Proposition.</u> $(H(s_b'),\tau_o) \cong s$.

<u>Theorem.</u> Let α be an exponent sequence with $\sup_{n\in\mathbb{N}} \frac{\ln(n+1)}{\alpha_n} < \infty$ for which there exists $k\in\mathbb{N}$ such that $1 < \inf_{n\in\mathbb{N}} \frac{\alpha_{kn}}{\alpha_n} \leq \sup \frac{\alpha_{kn}}{\alpha_n} < \infty$.

Then $(H(\Lambda_\infty(\alpha)_b'),\tau_o)$ is isomorphic to $\Lambda_\infty(\beta(\alpha))$, where $\beta(\alpha) = ((\ln(n+1))\,\alpha_{[\ln(n+1)]})_{n\in\mathbb{N}}$. The isomorphism is given by an appropriate (simultaneous) arrangement of Taylor coefficients of the functions in $H(\Lambda_\infty(\alpha)_b')$.

(joint work with M. Börgens (Düsseldorf) and D. Vogt (Wuppertal))

N. ORTNER:

<u>Faltung von Distributionen und euklidischen Pseudofunktionen</u>

Ausgehend von einer allgemeinen Definition der Faltung zweier Distributionen untersuchen wir die Frage

(1) nach Kriterien der Faltbarkeit und

(2) nach effektiver Berechnung der Faltung

einer Distribution mit einer euklidischen Pseudofunktion.

Hilfsmittel zur Beantwortung von (1) und (2) sind

(1) ein Darstellungssatz (für die Faltung),

(2) ein Fourieraustauschsatz und

(3) ein Satz über die "Verschiebung der Differentiation".

Wir erhalten als Antworten:

(1) $T \in \mathcal{D}'$, $\lambda \in \mathbb{C}$, Pf r^{λ} und T faltbar <=> $(1+r^2)^{\mathrm{Re}\,\lambda/2}\, T \in \mathcal{D}'_{L^1}$.

(2) $\lambda, \nu \in \mathbb{C}$. Pf r^{λ} und Pf r^{ν} faltbar <=> $\mathrm{Re}(\lambda+\nu) < -n$.

(3) $\lambda, \nu \in \mathbb{C} \setminus \{-2l / l \in \mathbb{N}_0\}$: R_{λ} und R_{ν} faltbar <=> $\mathrm{Re}(\lambda+\nu) < n$.

(4) $\lambda, \nu \in \mathbb{C} \setminus \{-2l-1 / l \in \mathbb{N}_0\}$: N_{λ} und N_{ν} faltbar <=> $\mathrm{Re}(\lambda+\nu) < n$.

Dabei sind R_{λ} die Rieszschen Kerne und N_{λ} ihre Gradienten.

Beispiele für die Beantwortung von Frage (2) sind

(5) $\mathrm{Re}(\lambda+\nu) < n$: $R_{\lambda} * R_{\nu} = R_{\lambda+\nu}$.

(6) $\mathrm{Re}(\lambda+\nu) < n$: $N_{\lambda} * N_{\nu} = - R_{\lambda+\nu}$.

E. PAP:

<u>A sequential theory of some spaces of generalized functions and applications on partial differential equations</u>

Let $\{\psi_s^i\}$ $i = 1,\dots,q$ be a complete orthonormal set in the space $L^2(I_i)$ and $R_i : A_i \longrightarrow A_i$ (A_i is the linear span of $\{\psi_s^i\}$) linear operator such that $R_i \psi_s^i = \lambda_{s,i} \psi_s^i$ $s=0,1,\dots$ $(i=1,\dots,q)$ and $|\lambda_{s,i}| \to \infty$ as $s \to \infty$. In q-dimensional case we introduce $\{\psi_n(x)\} = \{\psi^1_{\nu_1}(\xi_1)\dots\psi^q_{\nu_q}(\xi_q)\}$ as an orthonormal complete set of functions in the space $L^2(I_1 x \dots x I_q)$ and $R = R_1 \dots R_q$. R, $\{\psi_n\}_{n \in P^q}$ and $\{\lambda_n^1\}_{n \in P^q}$ generate with the notion of R-fundamental sequences the space of generalized functions U' , whose elements have orthonormal expansions. Convergence is defined sequentially. We examine on such spaces some special linear operators and semigroups of such operators. The obtained results can be used for simply solving of some evolution equations. We examine also some convergence problems in the space U' .

H.J. PETZSCHE:

Distributions and ultradistributions as boundary values of holomorphic functions

The talk was a report on some results which were obtained in collaboration with D. Vogt (Wuppertal). In the first part a new definition of the boundary values of holomorphic functions which define ultradistributions is given. It depends on the following result.

Theorem: If φ is an ultradifferentiable test function of class (M_p) (M_p has to fulfill certain conditions) on $\mathbb{R}^N$ then there exists $\psi \in \mathcal{D}(\mathbb{C}^N)$ with $\psi|_{\mathbb{R}^N} = \varphi$ and $\bar{\partial}\psi$ vanishes on $\complement \mathbb{C}^N \setminus (\mathbb{C}\setminus\mathbb{R})^N$ with all derivatives.

This can be proved by putting $\psi(x + iy) = \sum_{\alpha} \frac{\varphi^{(\alpha)}(x)}{\alpha!} (iy)^{\alpha} \, \rho_{\alpha}(y)$

where ρ_{α} are chosen appropriately in order to ensure convergence of the series in $\mathcal{D}(\mathbb{C}^N)$. The argument allows to give a new and direct proof of Komatsu's theorem on the extension of Whitney fields and can also be generalized to the case of a totally real submanifold of a Stein manifold. Using Taylor's remainder formula it can be shown that $|\bar{\partial}\psi(x + iy)|$ decreases with a certain speed if $y \longrightarrow 0$. Using Stoke's theorem one gets

$$\langle Tg, \varphi \rangle = \lim_{\varepsilon \searrow 0} \int_{\mathbb{R}^N} \varphi(x) \sum_{\sigma} \operatorname{sign}\sigma \, g(x + i\varepsilon\sigma)\,dx =$$

$$= \lim_{\varepsilon \searrow 0} \int_{\mathbb{C}^N} \frac{\partial^N}{\partial \bar{z}_1 \dots \partial \bar{z}_N} \psi(t) \sum_{\sigma} \operatorname{sign}\sigma \, g(x + i\varepsilon\sigma)\, d\bar{z} \wedge dz \, .$$

If the holomorphic function g is such that the integrands are uniformly bounded then the integrals converge.

In the second part it was shown that any distribution or ultradistribution on any open subset of $\mathbb{R}^N$ or any totally real submanifold of a Stein manifold is the boundary value of a function holomorphic in a suitable set with a prescribed increasing rate. The proof proceeds in two steps: First it is shown that such a representation can be given locally and what ker T locally looks like. In the socond step the first Czech-Cohomologie group with values in the kernel sheaf of T is proved to vanish. This can be done by means of the following result.

Theorem: Let $V \subset \mathbb{C}^N \times \mathbb{R}^M$ be open and suppose $\forall x \in \mathbb{R}^M$ $V_x := \{z \in \mathbb{C}^N \mid (z,x) \in V\}$ to be a domain of holomorphy (not necessarily connected) or empty. Let $\mathcal{E}$ be a locally finite covering of V with cubes. Then $H^k(\mathcal{E}; \mathcal{H}_N \otimes \mathcal{D}_M^{s'}) = 0 \ \forall k \geq 1$.

Here $\mathcal{H}_N \otimes \mathcal{D}_M^{s'}$ stands for the sheaf of ultradistributions of Gevrey class with exponents or distributions on $\mathbb{C}^N \times \mathbb{R}^M$ which are holomorphic in the complex variables.

D. PRZEWORSKA-ROLEWICZ:

Green's formula for right invertible operators

A Green's formula is derived for polynomials in right invertible operators with operator coefficients. This formula yields to several questions connected with the duality of operators. In particular, one can define adjoint and self-adjoint operators and boundary value problems (with respect to a bilinear form appearing in the formula under consideration) and show the natural duality between right and left invertible operators. Applications to differential and difference operators are obtained in this way.

M. SCHOTTENLOHER:

Applications of infinite dimensional holomorphy

In this talk it is indicated how infinite dimensional holomorphy can possibly be applied to problems in Analysis. In order to do this there are sketched a number of examples which are divided into 3 sections.

1. Differential Equations in infinitely many variables: Cauchy-Kowalewsky theorem in real Banach spaces; convolution operators on spaces of entire functions; $\bar{\partial}$-problem for pseudoconvex domains in DFN-spaces.

2. Differential Equations in n variables: Compact holomorphic operators; bifurcation of solutions of $T(\lambda,u) = 0$, λ a parameter, for holomorphic T and D_2T a Fredholm operator.

3. Holomorphic functional calculus on lmc algebras. Within the 3. section it is proven for a commutative Fréchet algebra A over $\mathbb{C}$ that every homomorphism $A \longrightarrow \mathbb{C}$ is continuous if and only if this holds for the special algebra $\Gamma = \mathcal{O}(s')$ of holomorphic functions on the sequence space $s' \subset \mathbb{C}^N$. Moreover, with the use of a generalization of Cartan's Nullstellensatz it is shown that every homomorphism $\Gamma \longrightarrow \mathbb{C}$ is continuous if and only if $\mathcal{S}\Gamma$, the space of continuous homomorphisms on Γ, is closed as a subset of $\mathbb{C}^\Gamma$.

M. SHAFII-MOUSAVI:

Functional dimension of solution space of differential operators

Let P(D) be a differential operator on R^n with constant coefficients and E the space of all continuous solutions, u , of $P(D)u = 0$. We endow the solution space E with the topology of uniform convergence on compact subsets of R^n. When P(D) is d-hypoelliptic, for $d = (d_1,\dots,d_n)$ where $d_j > 0$, Kōmura proved that the functional dimension (in the sence of Kolmogorove-Tihomirove) of E, dfE, is $\leq 1 + (d_1 + \dots + d_n)$. We prove:

Theorem A: P(D) is d-hypoelliptic iff $dfE = 1 + (d_1 + \dots + d_n) - \min d_j$.

Theorem B: Let P(x,D) be an operator of constant strength with C^∞ coefficients on Ω. Then for every $x_0 \in \Omega$ there exists a neighborhood $\omega \subset \Omega$ of x_0 such that $df\{u \in L_2(\omega): P(x,D)u = 0\} = df\{u \in L_2(\omega): P(x_0,D)u = 0\}$.

Theorem C: Let P(x,D) be a formally d-hypoelliptic differential operator with C^∞ coefficients on Ω. Then for every $x_0 \in \Omega$ there exists a neighborhood ω, such that $df\{u \in C(\omega) : P(x,D)u = 0\} = 1 + (d_1 + \dots + d_n) - \min d_j$.

Corollary: An operator P(x,D) of constant strength with analytic coefficients on Ω is elliptic iff for every $x_0 \in \Omega$, there exists a neighborhood of x_0 such that $df\{u \in C(\omega) : P(x,D)u = 0\} = n$.

F. SOMMEN*:

Representations of distributions and analytic functionals by hypercomplex functions

Let m and n be natural numbers, $n \geq m$, let A be a Clifford Algebra constructed over an n-dimensional real quadratic vector space with basis $\{e_1,\dots,e_n\}$ and let $\Omega \subset R^{m+1}$ be an open set. Then $f \in C_1(\Omega; A)$ is called monogenic in Ω when $Df = 0$ where $D = \sum_{i=0}^{m} e_i \frac{\partial}{\partial x_i}$ is a generalized Cauchy-Riemann operator.

A function $H(x,y)$, $(x,y) \in R^{m+1} \setminus (R^{m+1} \setminus \{0\})$ has been constructed, which is monogenic in $x \in R^{m+1}$ and in $y \in R^{m+1} \setminus \{0\}$ and which generalizes the holomorphic function $\frac{1}{z}e^{u/z}$, $(u,z) \in C\times(C \setminus \{0\})$.

Furthermore, let T be a left linear analytic functional in $R^{m+1} \setminus \{0\}$; then the transform

$$\sigma_+(T)(y) = \langle T_x, H(x,y) \rangle$$
$$\sigma_-(T)(x) = \langle T_y, H(x,y) \rangle \qquad \text{is studied.}$$

For m = 1 we obtain the transform $\sigma_+ T(z) = \langle T_u, \frac{1}{z} e^{\frac{u}{z}} \rangle$,

$$\sigma_- T(u) = \langle T_z, \frac{1}{z} e^{\frac{u}{z}} \rangle$$

which is related to the Fourier-Borel transform. This transform will be applied to characterize distributions on the unit sphere in R^{m+1} .

(*Aspirant N.F.W.O.)

A. TAKAČI:

On the asymptotic behaviour of distributions

Definition: The distribution $f(x) \in \mathcal{K}_\mu'^{+}$ has μ-asymptotic behaviour (μ-a.b.) of order k iff the tempered distribution g(x) defined by $\langle g, \psi \rangle \triangleq \langle f, \psi/\mu \rangle$, $\psi(x) \in \mathcal{S}$, has quasi-asymptotic behaviour (q.a.b.) of order $k \in R$.

Here $\mathcal{K}_\mu \triangleq \{\varphi \in C^\infty : \sup_{x \in R, 0 \le j \le n} (1+|x|^2)^{n/2} |\mu(x) \cdot \varphi^{(j)}(x)| < \infty, n=0,1,2,.\}$

and $\mathcal{K}_\mu'$ its dual space, $\mu(x)$ a smooth function which is nonzero and satisfies $|\mu^{(n)}(x)| \le C_n |\mu(x)|$ for some

$C_n > 0$, $n = 0,1,2,\ldots$; the q.a.b. was defined by Drožinov and Ziviyalov (Mat.Sbornik, tom 102,pp. 372-390 (1977)).

We have the following

Theorem 1: Let the distribution $f(x) \in \mathcal{K}'^{+}_{\mu}$ has μ-a.b. of order k , then the functional $x^m f(x)$ $(\in \mathcal{K}'^{+}_{\mu})$ has μ-a.b. of order k+m , m > 0 . For $\mu(x) \triangleq e^{ax}$, $a \in R$, one can show that each element from $\mathcal{K}'^{+}_{e^{ax}}$ has the distributional Laplace transform in the half plane Re s > a . We have

Theorem 2: Suppose $f(x) \in \mathcal{K}'^{+}_{\mu}$ has μ-a.b. of order k . Then: $\mathcal{L}\{f\}(s) \sim C/(s-a)^{k+1}$ as $s \to a$ staying on a line $L_{a,w} \triangleq \{s \in C : \text{Re } s > a , \arg (s-a) = w\}$, with $0 \leq |w| < \pi/2$. ("Final-value" Abelian theorem)

V. WROBEL:

Operators on tensorproducts of locally convex spaces

Let E and F denote two complex locally convex spaces, and let $E \hat{\otimes}_{\alpha} F$ denote the completion of the tensor product $E \otimes_{\alpha} F$ with respect to some tensornorm topology $\varepsilon \leq \alpha \leq \pi$. For $T \in L(E)$, $S \in L(F)$ and a polynomial P conditions were considered for the spectral mapping theorems

$$(*) \qquad P(\sigma_{cl}(T), \sigma_{cl}(S)) = \sigma_{cl}(P(T\hat{\otimes}I, I\hat{\otimes}S))$$

$$(**) \qquad P(\sigma(T), \sigma(S)) = \sigma(P(T\hat{\otimes}I, I\hat{\otimes}S))$$

to hold (σ_{cl} resp. σ denoting the "classical" spectrum resp. the Waelbroeck-spectrum). Under mild assumptions upon $E\hat{\otimes}_{\alpha}F$ it turns out that the inclusion "$\subset$" in (**) is true, whereas the reverse inclusion fails even for "nice" spaces E and F . For the purpose of illustration an abstract Cauchy problem in a locally convex space was considered.

J. VOIGT (joint work with P. Dierolf):

Calculation of the bidual of some function spaces; integrable distributions

For $\Omega = \mathring{\Omega} \subset \mathbb{R}^n$, we consider a class of function spaces H, $\mathcal{D}(\Omega) \subset H \subset \mathcal{E}(\Omega)$, whose bidual H" can be calculated as a subspace of $\mathcal{E}(\Omega)$. This class is defined by four properties.
The result is used to obtain the bidual of $\mathring{\mathcal{B}}(\Omega) := \{f \in \mathcal{E}(\Omega);\ \partial^\alpha f \in C_o(\Omega)\ (\alpha \in \mathbb{N}_o^n)\}$, $\mathring{\mathcal{B}}(\Omega)'' = \check{\mathcal{B}}(\Omega) := \{f \in \mathcal{E}(\Omega);\ \partial^\alpha f$ bounded, $\partial^\alpha f|_{\partial\Omega} = 0\ (\alpha \in \mathbb{N}_o^n)\}$. Let $r(x) := \operatorname{dist}(x, \complement\Omega)$ if $\Omega \neq \mathbb{R}^n$, $r(x) := 1 + |x|$ if $\Omega = \mathbb{R}^n$ $(x \in \Omega)$. Besides a known property of Ω, for $\mathring{\mathcal{B}}(\Omega)$ to be reflexive, we have the characterization: $\mathring{\mathcal{B}}(\Omega)$ is nuclear if and only if there exists $p \in (0,\infty)$ such that $r(\cdot) \in L^p(\Omega)$.
As a second example we give the bidual of $\mathring{\mathcal{B}}_i(\Omega) := \{f \in \mathcal{E}(\Omega);\ r(\cdot)^{|\alpha|}\partial^\alpha f \in C_o(\Omega)\ (\alpha \in \mathbb{N}_o^n)\}$, $\mathring{\mathcal{B}}_i(\Omega)'' = \mathcal{B}_i(\Omega) := \{f \in \mathcal{E}(\Omega);\ r(\cdot)^{|\alpha|}\partial^\alpha f$ bounded $(\alpha \in \mathbb{N}_o^n)\}$. Since $1 \in \mathring{\mathcal{B}}_i(\Omega)''$, the elements of $\mathring{\mathcal{B}}_i(\Omega)'$ are "integrable distributions". By the fact that $\mathring{\mathcal{B}}(\mathbb{R}^n)' \subsetneqq \mathring{\mathcal{B}}_i(\mathbb{R}^n)'$ holds we obtain extended notions of integrability of a distribution and convolvability for two distributions.

S. ZAIDMAN:

Pseudo-differential operators in Sobolev spaces

If $\mathcal{A}(x,D)$ is a pseudo-differential operator in a certain general class and if t is a given real number, for any $\varepsilon > 0$ there is a constant $C_{\varepsilon,s}$ such that

$$|(\mathcal{A}(x,D)U)(v)| \leq C_{\varepsilon,s} \, \|U\|_{H^{-s-1}} \, \|v\|_{H^s}$$

for all $U \in H^t$, $v \in \mathcal{S}$, $\mathrm{dist}(\mathrm{supp}\,U, \mathrm{supp}\,v) \geq \varepsilon$, $s \geq -t-1$.

Z. ZIELEZNY:

<u>Regularity and growth of solutions of convolution equations</u>

Let K_p', $p > 1$ be the space of distributions growing no faster than a power of $e^{|x|^p}$. Further, let Γ_p^d, $d = (d_1,\dots,d_n)$, $d_j \geq 1$, be the space of functions $f \in C^\infty(R^n)$ such that

$$\sup_{x \in R, \alpha} \frac{|D^\alpha f(x)| e^{-a|x|^p}}{\alpha_1^{\alpha_1 d_1} \cdots \alpha_n^{\alpha_n d_n} A^{|\alpha|}} < \infty ,$$

for some a, A depending on f.

A convolution operator S in K_p' is d-hypoelliptic in K_p' if every solution $u \in K_p'$ of the equation

$$S * u = v$$

is in Γ_p^d, whenever $v \in \Gamma_p^d$. Necessary and sufficient conditions on the Fourier transform $\hat{S}$ of S are given in order that S be d-hypoelliptic in K_p'.

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