

Math. Forschungsinstitut  
Oberwolfach  
E 20 101684

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 16/1980

Fastringe und Fastkörper

13.4. bis 19.4.1980

Die Vorträge und Diskussionen dieser Tagung behandelten vielfältige Themen aus der Theorie der Fastringe und Fastkörper. Folgende Themenkreise seien genannt:

- Distributiv erzeugte Fastringe und Verallgemeinerungen.
- Fastringe von Polynomen und formalen Potenzreihen.
- Spezielle Fragen der Fastkörpertheorie (Konstruktion von FK, Dicksonsche FK mit Primzahlrang, Galoistheorie).
- Fastringe von Gruppenabbildungen und Fastringe stetiger Funktionen.
- Planare Fastringe, Steinersche Neoringe, Beziehungen zur Geometrie.

Die Leitung hatte G. Betsch (Tübingen). -

Gleichzeitig mit dieser Tagung fand im Mathematischen Forschungsinstitut Oberwolfach eine Tagung über "Finite Geometries" statt (vgl. Tagungsbericht 17/1980). Es gab sehr anregende Diskussionen zwischen Geometern und Fastring-Theoretikern. Die Programme der beiden Tagungen wurden zeitlich so aufeinander abgestimmt, daß die Teilnehmer auch einen erheblichen Teil der Vorträge der jeweils anderen Tagung mithören konnten.



Vortragsauszüge

H. E. BELL:

On Algebraic and Periodic Near-rings.

For rings  $R$ , a theorem of Chacron asserts the equivalence of the following properties:

- (A)  $R$  is periodic - that is, for each  $x \in R$ , there exist distinct positive integers  $m, n$  for which  $x^m = x^n$ .
- (B)  $R$  is algebraic - that is, for each  $x \in R$ , there is a polynomial  $p(x)$  with integer coefficients, and an integer  $n \geq 1$ , such that  $x^n = x^{n+1}p(x)$ .

Property (B) has several possible extensions to near-rings, and I conjecture that none of these imply (A). However, with some additional commutativity hypotheses on  $N$ , certain variants of (B) do imply (A), as the following theorems indicate.

Theorem 1. Let  $N$  be a d.g. near-ring such that for each  $x \in N$ , there exists an integer  $n \geq 1$  and an element  $p(x)$  of the subnear-ring generated by  $x$ , for which  $x^n = x^n p(x)$ . If the nilpotent elements of  $N$  are multiplicatively central, then  $N$  is periodic and commutative.

Theorem 2. Let  $N$  be a near-ring with 1. Suppose that for each  $x, y \in N$  there exists an element  $p(x)$  which is a sum of positive powers of  $x$  and negatives of such, for which  $xy = yxp(x)$ . Then  $N$  is periodic, and  $(N, +)$  is abelian.

G. BETSCH:

Some results on near-rings of group mappings.

Let  $(\Gamma, +)$  be a (not necessarily commutative) group,  $\Gamma \neq \{0\}$ , and let  $G$  be a group of automorphisms of  $\Gamma$ , written as left operators of  $\Gamma$ . We may assign to the pair  $(G, \Gamma)$  two near-rings of group mappings:

- I. The right near-ring  $L(G)$  distributively generated by  $G$  and acting on  $\Gamma$  from the left;
- II. The left near-ring  $M_G(\Gamma)$  consisting of all mappings

$n: \Gamma \longrightarrow \Gamma : \gamma \longrightarrow \gamma n$  with the properties

i)  $0n = 0$ , ii)  $(g\gamma)n = g(\gamma n)$  for all  $g \in G$  and all  $\gamma \in \Gamma$ .

$M_G(\Gamma)$  acts on  $\Gamma$  from the right.

We confine our study mainly to  $M := M_G(\Gamma)$ .

Assume there exists an element  $\gamma \in \Gamma$  such that  $\gamma M = \Gamma$ .

Then the following results hold:

1. The semigroup of all distributive elements of  $M$  coincides with  $\text{Hom}_G(\Gamma, \Gamma) = \text{Hom}_{L(G)}(\Gamma, \Gamma)$ .

2. Let  $N$  be a subnear-ring of  $M$ , and define  $\bar{G} := \text{Aut}_N \Gamma$  and  $\bar{N} := M_{\bar{G}}(\Gamma)$ . Obviously,  $G \leq \bar{G} \leq \text{Aut } \Gamma$  and  $N \hookrightarrow \bar{N} \hookrightarrow M$ .

Now, if  $N$  is dense in  $M$  with respect to Jacobson's finite topology, then  $G = \bar{G}$  and consequently  $\bar{N} = M$ .

J. R. CLAY:

Tactical configurations having  $p$  affine planes sharing a pencil of lines.

A non-trivial group  $\Phi$  of fixed point free automorphisms of a finite group  $(N, +)$  is all one needs to apply G. Ferrero's method of constructing a planar near-ring  $(N, +, \cdot)$ . If  $\mathcal{B} = \{Na + b \mid 0 \neq a, a, b \in N\}$ , then  $(N, \mathcal{B}, \epsilon)$  is always a tactical configuration. H. Karzel noticed that  $\Phi = \{1, -1\}$  acting on  $(Z_9, +)$  in this way actually produces a tactical configuration  $(Z_9, \mathcal{B}, \epsilon)$  with three substructures  $(Z_9, \mathcal{X}_i, \epsilon)$ ,  $i = 1, 2, 3$ , each of which is an affine plane, and any two sharing  $Z_9/3Z_9$  as a pencil of lines.

Actually if  $(N, +) = (Z_p^2, +)$ ,  $p$  an odd prime, then the units  $U(Z_p^2)$  has a subgroup of order  $p-1$  which defines a  $\Phi$ , fixed point free, and of order  $p-1$ . The corresponding planar near-ring  $(N, +, \cdot)$  gives a tactical configuration  $(N, \mathcal{B}, \epsilon)$  and each  $\mathcal{X}_i = Z_p^2/pZ_p^2 \cup \{Na_j + kp + (i-1) \mid 1 \leq j \leq p, 0 \leq k \leq p-1\}$  makes  $(Z_p^2, \mathcal{X}_i, \epsilon)$  an affine plane. Also,  $i \neq t$  implies

$$\mathcal{X}_i \cap \mathcal{X}_t = Z_p^2/pZ_p^2.$$

Note: Elements of  $\mathcal{B}$  of the form  $Na$  are:

$$pZ_p^2, Na_1, Na_2, \dots, Na_p.$$

C. FERRERO COTTI:

Near-rings with involutions, planar near-rings, and near-rings with particular ideals.

1. An involution  $j$  on a near-ring (n.r.)  $N$  is an automorphism of  $N^+$  such that  $j^2 = i$  and  $j(xy) = j(y)j(x)$ . The involutions of a n.r.  $N$  are studied and constructed, using the canonical homomorphisms  $N \rightarrow N/A(N)$ . The theory also uses  $m$ -homomorphisms of Jordan and of Lie and the center (anticenter) of  $N$ .

2. Pilz asked for an example of a planar near-ring with a non-planar homomorphic image. We obtain the desired example. Typical auxiliary results:

- Every proper homomorphic image of a finite planar n.r. is planar.
- Every proper homomorphic image of a planar ring is planar.
- Let  $N$  be a planar n.r. and  $I$  one of its proper ideals. The n.r.  $N' = N/I$  is planar iff there are no elements  $x \in A_j(N) \setminus I$  such that  $ax = bx + i$ , where  $a \not\equiv b$  and  $i \in I$ .

3. We study the n.r. in which every proper ideal is maximal. In the non-trivial cases, such a n.r. admits at most two proper ideals. A simple n.r. in which the proper right ideals are maximal has at most two proper right ideals.

For n.r. in which every proper ideal is prime, a typical result is: Let each proper ideal of the n.r.  $N$  be prime. Then  $N$  has exactly two minimal ideals iff  $N$  is a semiprime non-prime n.r. - Another interesting result from the works of Ferrero-Cotti, Suppa, Pellegrini, G. Rinaldi, F. Rinaldi, is the following

Theorem. Let  $N$  be a zero-symmetric n.r. and let each proper ideal of  $N$  be prime. Then either the set of the ideals of  $N$  is totally ordered with respect to inclusion or  $N$  admits exactly two minimal ideals  $I, J$ ; the set of the other ideals of  $N$  is totally ordered with respect to inclusion.

G. FERRERO:

Steiner neo-fields.

A Steiner neo-field (SNF) is a structure  $(N; +, \cdot)$  such that  
1)  $(N; +)$  is a loop with zero 0 and the identities  $a+a = 0$ ,  
 $a+b = b+a$ ,  $a+(a+b) = b$ ; 2)  $(N \setminus \{0\}; \cdot)$  is a group, and  
 $0a = a0 = 0$ ; 3) the product is (one-sided) distributive with  
respect to the sum.

The SNF are useful in studying Steiner triple systems.

The following problems are of interest in geometry: To obtain  
the SNF with given additive loops, or given multiplicative  
groups, or given automorphisms. The second problem is diffi-  
cult because there are a lot of SNF having the same multi-  
plicative group: for example, a certain construction yields  
 $2^{k^2+(6k+1)^2-1}$  SNF with the same multiplicative group  $C_{6k+1}^2$ .

Any finite nilpotent group  $G \neq C_9$  is the multiplicative  
group of a SNF iff it has order  $6k+1$  or  $6k+3$ .

An abelian divisible group is the multiplicative group of a  
SNF iff it is without torsion two. Another typical result  
from the works of Ferrero-Gallina-Scapellato is the following  
Theorem. Let  $\mathcal{P}$  be the set of the prime numbers  $p = 6k+1$ ;  
let  $G$  be a  $\mathcal{P}$ -group, and let the order of each element of  $G$   
be a prime power. Let  $\mathcal{F} \subseteq \text{Aut } G$  be a group of odd order.  
Then there exists a SNF  $N$  whose multiplicative group is  
isomorphic to  $G$  and with a group of automorphisms which  
operate as  $\mathcal{F}$  on  $N \setminus \{0\}$ , and moreover, which admits as sub-  
neo-fields each union of a multiplicative subgroup and  $\{0\}$ .

Y. FONG:

A theorem on strictly semi-perfect near-ring modules.

Let  $N$  be a zero-symmetric near-ring. Here we define a  
(strictly) semiperfect near-ring  $N$ -module  $M$  by the following  
two properties: (1)  $M$  is projective and (2) every factor  
module of  $M$  has a (strictly) projective cover. By applying  
the result of A. Oswald (Ph. D. Thesis, Theorem 3.1,

p. 32, 1973) we obtain a similar result which states: If  $U$  is a submodule of a strictly semi-perfect near-ring module  $M$  then there exists a submodule  $P$  of  $M$  such that  $P$  is a strictly projective cover of  $M/U$  and a submodule  $V$  of  $M$  with (i)  $M = V \oplus P$  with  $V \subseteq U$  and (ii)  $P \cap U$  is a strictly small submodule of  $P$  and hence of  $M$ . The last statement in (ii) guarantees the following theorem: If  $M$  is strictly semi-perfect then  $M/J(M)$  is semi-simple. Here  $J(M)$  is the radical of  $M$ .

D. GRÖGER:

Bemerkungen zur Galoistheorie in Quaternionen-Fastkörpern.

Es sei  $Q_K(+, \cdot)$  der Quaternionen-Körper über dem kommutativen Körper  $K$  und  $\varphi$  ein gekoppelter Homomorphismus von  $Q_K(+, \cdot)$  mit kommutativer Dickson-Gruppe  $\Gamma_\varphi$ , die nur innere Automorphismen enthalte. Eine vorgenommene Charakterisierung von  $\varphi$  ermöglicht Aussagen über das Verhalten von Automorphismen und Teilfastkörpern des abgeleiteten "Quaternionen-Fastkörpers"  $Q_K^{\varphi}(+, \cdot) = Q_K(+, \circ)$ . Man erhält schließlich:  
 $K$  ist galoissch in  $Q_K(+, \circ) \iff \Gamma_\varphi$  liegt in einer Kleinschen Vierergruppe.

Die unterschiedliche Gestalt von  $\varphi$  in den Fällen  $K = \mathbb{R}$  und  $K = \mathbb{Q}$  bewirkt, daß der reelle Quaternionen-Körper hierfür keine interessanten Beispiele liefert, wohingegen mit dem rationalen Quaternionen-Körper ein solches konstruiert werden kann, in welchem eine "galoissche Beziehung" zwischen Untergruppen der Galois-Gruppe und Teilfastkörpern nicht besteht.

R. D. HOFER:

Maximal left ideals in near-rings of continuous functions.

Let  $(G, +)$  be a (not necessarily abelian) topological group and let  $\mathcal{N}_G$  be the (right) near-ring, under pointwise induced addition and composition, of all continuous selfmaps

of  $G$  which leave the identity element (denoted by  $0$ ) of  $G$  fixed. For  $f \in \mathcal{N}_0(G)$  let  $Z^*(f) = \{x \in G : x \neq 0 \text{ and } f(x) = 0\}$  and for any nonzero  $p \in G$  let  $M_p = \{f \in \mathcal{N}_0(G) : f(p) = 0\}$ . If  $G$  is discrete, Marjory Johnson [Maximal right ideals of transformation near-rings, J. Austral. Math. Soc. 19(1975), 410 - 412] showed that a left ideal  $M$  is maximal iff either  $M = M_p$  for some nonzero  $p$ , or is maximal among the ideals  $I$  for which  $Z^*(f) \cap Z^*(g)$  is infinite for every  $f, g \in I$ .

Theorem. Suppose  $G$  is a compact, first countable, 0-dimensional, Hausdorff group. (I). If  $M$  is a maximal left ideal in  $\mathcal{N}_0(G)$  then  $\{Z^*(f) : f \in M\}$  is an ultrafilter. (II). If  $\mathcal{A}$  is an ultrafilter of sets of the form  $Z^*(f)$ , then

$\{f \in \mathcal{N}_0(G) : Z^*(f) \in \mathcal{A}\}$  is a maximal left ideal in  $\mathcal{N}_0(G)$ .

H. KARZEL:

On the construction of near-fields.

The first examples of proper near-fields (which were finite) were given by L.E. Dickson at the beginning of this century. H. Zassenhaus determined all finite near-fields in 1936. In 1940 Kalscheuer constructed continuous near-fields. The concepts "Dickson near-field" and "coupling map" were defined in 1964. With the exception of seven finite near-fields all known examples of near-fields are Dickson. The Dickson near-fields were then studied by F. Pokropp who constructed many classes of Dickson near-fields, by W. Kerby in connection with ordering, by H. Wefelscheid in connection with valuation theory and topology, by J. Misfeld in connection with topological normal near-fields. A general theory of near-fields, mainly Dickson near-fields was developed by H. Wähling, who gave many new construction methods for coupling maps and hence for Dickson near-fields. An interesting task is now to find methods which enable us to construct near-fields which are non Dickson. In the simplest case where the near-field  $F$  is of dimension 2 over its kernel  $K$  we have to find two functions  $f, g: K^2 \rightarrow K$  which satisfy some

functional equations. For the finite case this was done partially with the help of a computer. Now it would be of interest to get solutions for  $K = \mathbb{Q}$ .

H. KAUSCHITSCH:

Near-rings of formal power series.

Let  $R$  be a commutative ring with identity. By  $R[[x]]$  we denote the ring of formal power series in one indeterminate over  $R$ . The set  $R_+[[x]]$  of formal power series of positive order is a subring of  $R[[x]]$ , in which the substitution  $f \circ g$  is always defined.  $(R_+[[x]], +, \circ)$  is a zero-symmetric near-ring, in general not commutative. Therefore, we consider first permutable power series ( $f \circ g = g \circ f$ ) and commutative subsemigroups of  $(R_+[[x]], \circ)$ . If  $R$  is a field of characteristic 0, a complete answer can be given. Then near-ring ideals and their connections to the ring- and composition ideals (i.e. ideals of the TO-algebra  $(R_+[[x]], +, \cdot, \circ)$ ) were considered. If  $R$  is a field  $F$ , then all near-ring ideals are composition ideals iff  $\text{char}(F) \neq 2$ . All near-ring ideals  $I$  are of the form  $I = x^k F[[x]]$ ,  $k \geq 1$ , so that all homomorphic images of the power series near-ring are polynomial near-rings. In this case ( $\text{char}(F) \neq 2$ ) the left ideals are exactly the ring ideals.  $(F_+[[x]], +, \circ)$  fulfills the ACC, but does not fulfill the DCC on ideals, and hence, by a theorem of Betsch, the near-ring of formal power series cannot be completely reducible.- Some of these results may be extended to the case, that  $R$  is noetherian. Finally, all D-ideals (i.e. ideals of the  $\Omega$ -group  $(R_+[[x]], +, \cdot, \circ, ')$ , where  $'$  denotes the formal derivation) were determined for  $R$  a field and  $R = \mathbb{Z}$ .

W. KERBY:

Embeddability of  $k$ -transitive groups in  $(k+1)$ -transitive groups.

If  $H$  is a sharply  $k$ -transitive group, the question arises



as to the number, if any, of  $(k+1)$ -transitive groups  $G$  with the property that the stabilizers  $G_a$  are isomorphic as permutation groups to  $H$ . If  $k \geq 3$ , then the only sharply  $(k+1)$ -transitive groups are  $\text{Sym}(k+1)$ ,  $\text{Alt}(k+3)$ ,  $M_{11}$  and  $M_{12}$  and we have unique embedding in the above sense. If  $k = 1$  and there is at least one group  $G$  with  $G_a \simeq H$ , then in general there are several non isomorphic such groups and we do not always have unique embedding. If  $k = 2$  no examples are known to the author where the embedding is not unique. The only method presently known to the author of constructing sharply 3-transitive groups leads to groups  $G$  such that the stabilizer  $G_{a,b}$  of two distinct points contains a commutative subgroup  $A$  with  $[G_{a,b}:A] = 2^n$  or infinite. One has the following

Theorem. If  $H$  is a sharply 2-transitive group, which is embeddable into a sharply 3-transitive group  $G$  constructable by the method mentioned above and  $[G_{a,b}:A]$  is finite, then there does not exist another such group  $\bar{G}$  not isomorphic to  $G$ , such that  $\bar{G}_a \simeq H$ .

R. LOCKHART:

The Addition of Endomorphisms and Default Properties.

The algebra of endomorphisms for certain structures is considered. In fields one has matrix algebras and one loses invertibility, although there is a complete characterisation of the invertible elements via determinants. In near-fields (or more generally, near-rings with identity hosted by abelian groups) one loses associativity in the matrix algebra. We give a determination of the associativity subgroup for this case.

We discuss Fröhlich's non-abelian homological algebra, which is based upon the notion of a distributively generated near-ring. Two criticisms of his approach are made and from them, we come to the definition of a pseudo-homomorphism as a mapping from a group to itself which is a homomorphism

modulo the commutator subgroup. The set of all pseudo-  
endomorphisms forms a near-ring under the usual operations;  
and we advance this as a structure upon which to base a non-  
abelian homological algebra which would generalise the work  
of Fröhlich. Various connections with standard homological  
algebra are indicated, and we conjecture that the distribu-  
tively generated near-ring of endomorphisms of a group is  
dense in the near-ring of pseudo-endomorphisms in a topology  
which generalises the finite topology of Jacobson. Near-rings  
which have the property of pseudo-distributivity, which is  
characteristic of pseudo-endomorphisms, are named Fröhlich  
near-rings, by us. We suggest these as a useful generali-  
sation of distributively generated near-rings.

C. G. LYONS:

Reduction theorems for endomorphism near-rings.

Let  $G$  be a group,  $E(G)$  be the endomorphism near-ring of  $G$ ,  
and  $\text{Soc}(G)$  be the socle of  $G$  which is the sum of the minimal  
fully invariant subgroups of  $G$  provided such subgroups exist.  
It is the aim of this paper to study the structure of  $E(G)$ .  
However, the results are stated and proved for the more  
general D.G. near-ring  $(R, S)$  with  $\text{Inn}(G) \subseteq S \subseteq \text{End}(G)$ .  $\text{Soc}(G)$   
is the sum of the minimal  $(R, S)$ -subgroups of  $G$  and it is  
assumed to exist. We seek to describe  $R/N$  and  $N$  where  $N$  is  
the annihilator of the  $R$ -series  $G \supset \text{Soc}(G) \supset \{0\}$ .  $\text{Soc}(G)$  is  
the direct sum of  $A$  and  $B$ , each of which is a direct sum of  
minimal summands which are abelian and perfect, respectively.  
We consider  $GN$  projected onto the summands of  $\text{Soc}(G)$  and get  
partial descriptions of  $N$ . In particular, if  $H$  is a perfect  
minimal summand of  $B$  and  $\{g_1, g_2, \dots, g_n\}$  is any finite set  
of elements from distinct cosets of the centralizer of  $H$   
in  $G$ , then the condition  $g_i N \pi_H \neq \{0\}$  for  $1 \leq i \leq n$  implies  
that  $N$  has a transitive-type property on  $H$ . The minimal  
abelian summand case is less tractable in the general case.

However, when  $R$  satisfies the DCCR the abelian case is strengthened and the summand  $B$  of  $\text{Soc}(G)$  is shown to be finite.

S. J. MAHMOOD:

Subdirect decomposition of d.g. near-rings.

A subdirect product of d.g. near-rings is defined to be a d.g. near-ring  $(R,S)$ , isomorphic to a sub-d.g. near-ring of  $(T,U)$ , where  $\{p_\lambda: (T,U) \longrightarrow (T_\lambda, U_\lambda)\}_{\lambda \in \Delta}$  is the product, such that  $p_\lambda|(R,S)$  is an epimorphism for each  $\lambda \in \Delta$ . The restriction of maps to the generating semigroups gives a subdirect decomposition of semigroups, which can be trivial. But for every non-trivial subdirect decomposition of semigroups, there exists a non-trivial d.g. subdirect decomposition. Even though a d.g. near-ring is not an algebra, Birkhoff's result about subdirect decomposition of algebras is shown to be true in this case.

If  $(R,S)$  is a d.g. subdirect product of  $\{(T_\lambda, U_\lambda): \lambda \in \Delta\}$ , then, using a result of category theory, and a result on upper faithful d.g. near-rings, it is proved that the upper faithful d.g. near-ring  $(\bar{R}, S)$  is a d.g. subdirect product of the upper faithful d.g. near-rings  $\{(\bar{T}_\lambda, U_\lambda): \lambda \in \Delta\}$ . Finally, for every d.g. subdirect decomposition of a faithful d.g. near-ring with each factor faithful, which is possible in view of the previous result, a subdirect decomposition in the category of groups is obtained.

C. J. MAXSON:

Recent Results on Centralizer Near-rings.

Let  $V$  be a group and  $S$  a semigroup of endomorphisms of  $V$  which includes the zero endomorphism. Under function addition and composition, the set

$$\mathcal{Z}(S;V) = \{f:V \longrightarrow V \mid f \circ \sigma = \sigma \circ f, \forall \sigma \in S\}$$

is a near-ring, called the centralizer near-ring determined by  $S$  and  $V$ . For the work in this investigation all algebraic

structures are finite, further, all near-rings are zero-symmetric and with identity. A semisimple near-ring  $N = N_1 \oplus \dots \oplus N_t$  is strong iff each simple summand  $N_i$  is either a non-ring or a field. A semigroup  $S$  of endomorphisms of  $V$  is fixed point free iff i)  $\bigcap_{\alpha \in S} \text{Ker } \alpha = \{0\}$ ; ii)  $\text{Ker } \alpha = \text{Ker } \alpha^2 = \dots \quad \forall \alpha \in S$ ; iii) For  $\alpha, \beta \in S$ , if there exists  $v \in V$  such that  $\alpha(v) = \beta(v) \neq 0$ , then  $\alpha = \beta$ .

Using these concepts a Wielandt-Betsch type structure theorem for strong semisimple near-rings is obtained.

Theorem.  $N$  is a strong semisimple near-ring  $\iff \exists$  a group  $V$  and a fixed point free semigroup  $S$  of endomorphisms of  $V$  such that  $N \cong \mathcal{C}(S;V)$ .

This result leads to a study of  $\mathcal{C}(S;V)$  where  $S$  is a completely regular inverse semigroup of endomorphisms. In this situation, those  $\mathcal{C}(S;V)$  that are semisimple are characterized in terms of  $S$  and  $V$ .

J. D. P. MELDRUM:

#### Free near-rings.

We extend the results of Fröhlich on free d.g. near-rings to zero-symmetric left near-rings. From now on near-ring will mean zero-symmetric left near-ring. Starting with a set  $X$  we construct a free additive group on a set of symbols which can be completely described. A multiplication is defined on the free group  $F$  thus obtained. This makes  $F$  a near-ring, the free near-ring on  $X$ , possessing all the appropriate universal properties. By a modification of this construction the free product of two near-rings can be defined. We also define similarly the free  $R$ -module on a set  $X$  and the free  $R$ -module product of two  $R$ -modules, for a given near-ring  $R$ . In these two cases we get free sums of copies of  $(R,+)$  coming in. Finally we use these results to define a group near-ring  $R(G)$  for a given near-ring  $R$  and a given multiplicative group  $G$ . This group near-ring is shown to have the property that all representations of  $G$  as a group of  $R$ -automorphisms

of an  $R$ -module  $M$  are in 1-1 correspondence with representations of  $R(G)$  on  $M$  which map  $G$  into the group of automorphisms of  $M$ .

R. MLITZ:

Radical properties of near-rings which are not defined by ideals.

Radicals of near-rings and, more generally, of  $\Omega$ -groups may arise in a natural way from properties which do not yield ideals. Thus, consider a variety  $V$  of  $\Omega$ -groups and a binary relation  $\ll$  on  $V$  satisfying some supplementary conditions (chosen such that in the case of zero-symmetric near-rings each of the relations "L is a subnear-ring, a (normal)  $N$ -subgroup, a (normal) invariant  $N$ -subgroup, a left ideal, a right ideal, an ideal in  $N$ " is a special case of " $L \ll N$ "). Define a  $\ll$ -radical on  $V$  to be a general radical (in the sense of universal algebra) satisfying the supplementary condition that an element of  $V$  is radical-free iff it does not contain radical  $\ll$ -subgroups  $\neq \{0\}$ . Characterizations of the radical and the radical-free classes are indicated. It turns out that every  $G \in V$  contains a (unique) maximal  $\ll$ -subgroup  $RG$  which is radical.  $RG$  is a lower bound for the radical; it is equal to the radical iff it is an ideal.

W. B. MÜLLER:

Formal differentiation and formal integration in composition rings.

A composition ring  $(R, +, \cdot, *)$  is an algebra, where  $(R, +, \cdot)$  is a ring,  $(R, +, *)$  is a right near-ring, and  $(x \cdot y) * z = (x * z) \cdot (y * z)$  for all  $x, y, z \in R$ .

A formal differentiation of  $R$  is defined as a mapping  $D: R \rightarrow R$  which satisfies

- (1)  $D(f+g) = D(f) + D(g)$
- (2)  $D(f \cdot g) = D(f) \cdot g + f \cdot D(g) \quad (\forall f, g \in R)$
- (3)  $D(f * g) = (D(f) * g) \cdot D(g)$ .

All formal differentiations are determined in  $A[x]$ , the polynomial ring in  $x$  over a commutative ring  $A$  with 1,  $A(x)$ , the ring of quotients of  $A[x]$ ,  $A[[x]]$ , the ring of formal power series over  $A$ , and  $A^A$ , the full function ring over  $A$ .

For near-rings  $(N, +, *)$  all mappings  $D: N \rightarrow N$  which fulfill (3) are determined.

A formal integration of  $R$  is defined as a mapping  $I: R \rightarrow R$  which satisfies

- (i)  $I(f+g) = I(f) + I(g)$
- (ii)  $I(a.f) = a.I(f)$ , where  $a$  is a constant ( $a*0 = a$ ),
- (iii)  $I(I(f).g) = I(f).I(g) - I(f.I(g))$ ,
- (iv)  $I((f*I(g)).g) = I(f)*I(g)$ , for all  $f, g \in R$ .

All formal integrations in  $A[x]$ ,  $A(x)$ ,  $A[[x]]$ , and  $A^A$  are determined.

A. OSWALD:

Right ideals in near-rings of group mappings.

Let  $G$  be a group and  $S^*$  be a group of fixed point free automorphisms of  $G$ . Let  $S = S^* \cup \{o\}$  where  $o$  is the zero endomorphism of  $G$ . Then with pointwise addition and mapping composition as addition and multiplication, respectively, the set  $M_S(G) = \{f: G \rightarrow G \mid (y\alpha)f = (yf)\alpha, y \in G, \alpha \in S\}$  is a left near-ring. Let  $\mathcal{R}$  be the set of right ideals and  $R$  the set of normal right subgroups of  $M_S(G)$ .

We can partition  $G$  into subsets of the form  $yS^*$  and the number of such disjoint subsets is denoted by  $|G:S^*|$ . A set  $\{x_\lambda\}$  consisting of one  $x_\lambda$  from each different  $yS^*$  excluding  $oS^*$  is called a basis for  $G$  over  $S^*$ .

A group  $G$  is non-normal relative to  $S^*$  if, for some  $y \in G$ ,  $G \neq N(yS^*) = \{x_i \in G \mid x_i + yS^* - x_i \in yS^*\}$ .

Theorem. If  $|G:S^*|$  is finite and  $G$  is non-normal relative to  $S^*$  then  $\mathcal{R} = R$ .

This generalised part of a result of M. J. Johnson.

G. PILZ:

Polynomial near-rings.

Given any algebra  $G$  out of some variety  $\mathcal{U}$  of  $\Omega$ -groups, one can form  $G^{\mathcal{U}}[x]$ , the algebra of all polynomials in  $x$  over  $G$  in  $\mathcal{U}$ . Roughly spoken,  $G^{\mathcal{U}}[x]$  consists of "everything which can be built up by  $G \cup \{x\}$  using the operations and laws of  $\mathcal{U}$ ". If the group part of  $G$  is written additively,  $G^{\mathcal{U}}[x]$  is a near-ring w.r.t. addition and substitution. Concerning the structure of this near-ring, one can see that all near-ring radicals are contained in  $\{p \in G^{\mathcal{U}}[x] \mid \forall g \in G: p \circ g \in \mathfrak{R}(G)\}$ , where  $\mathfrak{R}(G)$  is the radical of  $G$  (i.e. the intersection of all maximal ideals of  $G$ ). If  $\mathcal{G}$  is the variety of all groups,  $G^{\mathcal{G}}[x] =: G[x]$  is a "generalized distributively generated near-ring" (g.d.g.n.r.), which means that  $G[x]$  is additively generated by constant and distributive elements. D.g. near-rings and abstract affine near-rings are other examples. For finite g.d.g.n.r.'s  $N$  with identity ( $N_0$  not a ring),  $N$  is primitive iff  $N$  is simple iff  $N \cong M(G)$  or  $N \cong M_0(G)$ , where  $G$  is finite, non-abelian and invariantly simple and  $M(G) = (G^G, +, \circ)$ . Every near-ring can be embedded into some  $G^{\mathcal{U}}[x]$ , but not always into some  $G[x]$ . Every abstract affine near-ring is isomorphic to a polynomial near-ring in the variety of (ring-)modules. Also,  $G^{\mathcal{U}}[x]$  plays a very useful role in determining generated ideals in  $\Omega$ -groups.

S. D. SCOTT:

Tame near-rings.

Tame near-rings are those that have a unitary faithful  $N$ -group in which every  $N$ -subgroup is a submodule. If  $V$  is such an  $N$ -group then for  $v$  and  $w$  in  $V$  and  $\alpha$  in  $N$  there exists  $\beta$  in  $N$  such that  $(v + w)\alpha - v\alpha = w\beta$ . Examples may be found by taking semigroups of endomorphisms of a group that contain all inner automorphisms. The near-ring generated by such endomorphisms provides an example. Also zero-symmetric polynomial maps over an  $\Omega$ -group are

a further example. In both these cases the  $\beta$  in the above expression can be chosen independently of  $w$ . Such  $N$ -groups are called compatible and a refinement of this notion gives 2-tame near-rings (and  $N$ -groups). A primitive 2-tame near-ring on  $V$  is either a ring or dense in  $M_0(V)$ . Also minimal condition for right ideals gives all possible chain conditions. In the case of  $I(V)$ , minimal condition yields finiteness. Questions concerning maximal condition for  $I(V)$  arise. In the case where  $V$  is a simple group it seems likely that the introduction of a topology on  $V$  gives  $V$  finite. Indeed this topology carries over to type 2  $N$ -groups.

Y.-S. SO:

Near-rings of polynomials over groups.

If  $G$  is a group then the polynomials in  $x$  over  $G$  are of the form  $g_0 + z_1x + g_1 + z_2x + \dots + z_nx + g_n$  ( $g_i \in G, z_i \in Z$ ). The set  $G[x]$  of all polynomials over  $G$  forms a near-ring w.r.t. addition and substitution. The zero-symmetric part  $G_0[x]$  turns out to be a distributively generated near-ring. In studying homomorphisms, one sees that a group homomorphism from  $G$  to  $H$  can be extended to a near-ring homomorphism from  $G[x]$  to  $H[x]$  in a natural way. Conversely, under a certain condition (which is weaker than the one used in the case of universal algebra), all near-ring homomorphisms from  $G[x]$  to  $H[x]$  arise in this way. In order to get results on the radical  $J_2$  of  $G[x]$ , we have to know enough strictly maximal left ideals. We present some classes of these. In this way we obtain  $\{p = g_0 + z_1x + \dots + z_nx + g_n \in G[x] \mid \forall g \in G: p \circ g \in \beta(G) \wedge \sum z_i = 0\}$  as an upper bound for the near-ring radical of  $G[x]$  (where  $\beta(G)$  is Baer's group radical of  $G$ ). A lower bound is given by the ideal generated by  $\beta(G)$ . In the variety of abelian groups  $A$ , the corresponding polynomial near-ring is abstract affine, having  $\beta(A)$  as radicals ( $J_0 = J_1 = J_2$ ).



H. WÄHLING:

Dickson near-fields with prime rank.

1) Let  $(F, +, \cdot)$  be a skew-field,  $\varphi$  a coupling map on  $(F, +, \cdot)$ ,  $(F, +, \circ)$  the  $\varphi$ -derivation of  $(F, +, \cdot)$ ,  $K_F$  the kernel,  $Z_F$  the centre of  $(F, +, \circ)$ ,  $U_\varphi = \{x \in F^* \mid \varphi_x = 1\}$ ,  $P_\varphi$  the fixed field of  $\Gamma_\varphi = \{\varphi_a \mid a \in F^*\}$  and  $C$  the centre of  $(F, +, \cdot)$ .

If  $[F:K_F] = p$  is a prime and  $K_F = Z_F$ , then the following is valid:

(A) If  $\cdot$  is commutative, then  $K_F = P_\varphi$  and  $-$  with  $\nu: F^* \rightarrow P_\varphi^*$  the norm of  $F/P_\varphi$  - there exists an epimorphism  $\tau$  from  $(\nu(F^*), \cdot)$  onto  $\Gamma_\varphi$  such that  $\varphi = \tau \circ \nu$  and  $\tau(k^p) = 1$  for all  $k \in K_F^*$ .

(B) If  $\cdot$  is not commutative and if one turns  $-$  if necessary - from  $\cdot$  to the dual of  $\cdot$ , then  $K_F = P_\varphi$  is a maximal commutative subfield of  $(F, +, \cdot)$  with

$$F = U_\varphi \cdot K_F, \quad C^* = U_\varphi \cap K_F, \quad [K_F:C] = p \quad \text{and}$$

$$\varphi_{uk} = i_k : \begin{cases} F \rightarrow F \\ x \rightarrow k^{-1}xk \end{cases} \quad (u \in U_\varphi, k \in K_F^*).$$

2) (A) Let  $(F, +, \cdot)$  be a field,  $\Gamma < \text{Aut}(F, +, \cdot)$  finite with fixed subfield  $P$ ,  $\nu$  the norm of  $F/P$  and  $\tau: \nu(F^*) \rightarrow \Gamma$  an epimorphism. Then  $\varphi = \tau \circ \nu$  is a coupling map on  $(F, +, \cdot)$  with  $K_F = P = P_\varphi$  and  $\Gamma_\varphi = \Gamma$ ; and  $K_F = Z_F$  if and only if

$$\tau(k [F:P]) = 1 \quad \text{for all } k \in P^*.$$

(B) Let  $(F, +, \cdot)$  be a skew-field with centre  $C \neq F$  and  $U, K' < (F^*, \cdot)$  with  $F^* = UK'$  and  $U \cap K' < C$ . Then

$$\varphi : \begin{cases} F^* \rightarrow \text{Aut}(F, +, \cdot) \\ uk \rightarrow i_k \end{cases} \quad (u \in U, k \in K')$$

is a coupling map on  $(F, +, \cdot)$ . If  $K = K' \cup \{0\}$  is a maximal commutative subfield of  $(F, +, \cdot)$ , then  $P_\varphi = K = K_F = Z_F$ .

H. WEFELSCHIED:

Sharply 3-transitive groups which possess a  $\text{PSL}(2, K)$  as a Zassenhaus-transitive subgroup.

A permutation group  $(H, M)$  is called a ZT-group (Zassenhaus-transitive) iff the group  $H$  operates 2-transitively on  $M$  and  $M_{a,b} \neq \{\text{id}\}$ ,  $M_{a,b,c} = \{\text{id}\}$  for different  $a, b, c \in M$ . A subgroup  $H \leq G$  of a permutation group  $(G, M)$  is called a ZT-subgroup of  $G$  iff  $(H, M)$  is Zassenhaus-transitive. One can show that the set of all ZT-subgroups of a sharply 3-transitive group forms a lattice. In all known examples this lattice has a smallest element which is a  $\text{PSL}(2, K)$ . In a certain sense a sharply 3-transitive group  $(G, M)$  which possesses a  $\text{PSL}(2, K)$  as a ZT-subgroup is determined by the following fact:

Theorem: Let  $(G, M)$  be a sharply 3-transitive group which satisfies the following conditions:

- (i)  $G_{a,b}$  contains an involution.
- (ii) There exists a ZT-subgroup  $H < G$  which is isomorphic as a permutation group to some  $\text{PSL}(2, K)$  (i.e.  $(H, M) \simeq (\text{PSL}(2, K), \bar{K})$ ).

Then the group  $(G, M)$  is isomorphic as a permutation group to a group consisting of elements of the form  $\alpha$  and  $\beta$ , which are constructed in the following way:

On the field  $K$  there exists a mapping

$$\varphi: \begin{cases} K \longrightarrow \text{Aut}(K, +, \cdot) \\ a \longrightarrow a_\varphi \end{cases}$$

such that  $F := (K, +, \circ)$  with  $a \circ b := a \cdot a_\varphi(b)$  is a near-field.

$$\alpha: \begin{cases} \bar{K} \longrightarrow \bar{K} := K \cup \{\infty\} \\ x \longrightarrow a + m \circ x & (a, m \in K, m \neq 0) \\ \infty \longrightarrow \infty \end{cases}$$

$$\beta: \begin{cases} \bar{K} \longrightarrow \bar{K} \\ x \longrightarrow a + \mathcal{C}(b + m \circ x) & (a, b, m \in K, m \neq 0) \\ -m^{-1} \circ b \longrightarrow \infty \\ \infty \longrightarrow a \end{cases}$$

where  $m^{-1_0}$  is the inverse of  $m$  with respect to  $(\circ)$  and  $\phi(z) := z^{-1_0}$  (inverse with respect to  $(.)$ ).

Remark: The condition (i) says:  $\text{char } F \neq 2$ .

H. J. WEINERT:

Structures of right quotients.

In this paper, we sketch a very convenient way to deal with rings, near-rings, semirings, ... of quotients, reducing all tedious considerations to results on semigroups. For this purpose, we call any algebra  $(S, +, \cdot)$  with two binary operations such that  $(S, \cdot)$  is a semigroup a "structure".

Definition.  $(T, +, \cdot)$  is called a structure of right quotients of a structure  $(S, +, \cdot)$  with respect to a subsemigroup  $(\Sigma, \cdot)$  of  $(S, \cdot)$  iff  $(T, \cdot)$  is a semigroup of right quotients of  $(S, \cdot)$  w.r.t.  $(\Sigma, \cdot)$ .

Facts on semigroups of right quotients (briefly s.r.q.)

A s.r.q.  $T$  of  $S$  w.r.t.  $\Sigma$  is defined to be a semigroup containing  $S$  and an identity  $1$  such that each  $\alpha \in \Sigma$  has an inverse  $\alpha^{-1} \in T$  and  $T = \{a\alpha^{-1} \mid a \in S, \alpha \in \Sigma\}$ . Such a s.r.q.  $T$  exists iff (i) each  $\alpha \in \Sigma$  is cancellable in  $S$  and (ii)  $a \sum \alpha S \neq \emptyset$  holds for all  $a \in S, \alpha \in \Sigma$ .

(Ore Asano-Condition). In this case,  $T$  is uniquely determined by  $S$  and  $\Sigma$ , up to  $S$ -isomorphisms, and we write  $T = Q_R(S, \Sigma)$ . Using Greek and Roman letters as above, calculations in  $T$  are given by

$$(1) \quad a\alpha^{-1} = b\beta^{-1} \iff \exists x, \xi : \alpha x = \beta \xi \quad \text{and} \quad ax = b\xi$$

$$(1') \quad \iff \forall u, v : \alpha u = \beta v \implies au = bv$$

$$(2) \quad a\alpha^{-1} \cdot b\beta^{-1} = (a\alpha^{-1})(\beta\tau)^{-1} \quad \text{for any } \alpha\tau = b\tau \text{ by (ii).}$$

For a short proof of all this (based on the equivalence of the right sides of (1) and (1')) and more results on s.r.q. see H.J.Weinert, On the extension of partial orders on semigroups of right quotients, Trans. Amer. Math. Soc. 142 (1969), 345 - 353. Here we only need that a finite number of elements  $a\alpha^{-1}, b\beta^{-1}, \dots$  can be written with the same denominator, according to

$$(3) \quad a\alpha^{-1} = (ax)(\beta\xi)^{-1} = c\gamma^{-1} \quad \text{for any } \alpha x = \beta\xi \text{ by (ii)}$$

$$b\beta^{-1} = (b\xi)(\beta\xi)^{-1} = d\gamma^{-1}.$$

The basic statement on structures of right quotients.

Let the structure  $(S, +, \cdot)$  be right distributive (briefly r.d.) and consider any s.r.q.  $(T, \cdot) = Q_r(S, \Sigma)$ . Then there exists exactly one extension  $+$  on  $T$  of  $+$  on  $S$  such that  $(T, +, \cdot)$  is also r.d., given by

$$(4) \quad a\alpha^{-1} + b\beta^{-1} = (ax + b\xi)(\beta\xi)^{-1} \quad \text{for any } \alpha x = \beta\xi.$$

By (3), it is enough to prove this statement using equivalently

$$(4') \quad c\gamma^{-1} + d\gamma^{-1} = (c + d)\gamma^{-1}.$$

Moreover, the proof of the following statements becomes trivial by (4'):

Corollary 1. For  $(S, +, \cdot)$  and  $(T, +, \cdot)$  as above, if  $(S, +)$  is commutative, or associative, or (left) cancellative, or a group - the same property holds for  $(T, +)$ . If  $(S, +, \cdot)$  is left distributive, so is  $(T, +, \cdot)$ .

Only if the existence of some special elements is involved, one has to go back to (4); for instance, we have

Corollary 2. If  $(S, +)$  has a neutral element  $o$ , and if  $o\Sigma = o$ , then  $(T, +)$  has a neutral element  $0$  (which is equal to  $o\beta^{-1}$ , for each  $\beta$ ), and conversely.

H. J. WEINERT:

Cancellativity in generalized ring structures.

As a far-reaching generalization of (associative) rings we consider any algebra  $(S, +, \cdot)$  with two binary operations such that  $(S, \cdot)$  is a semigroup. Denote by  $\mathcal{L}$  [ $\mathcal{R}$ ] the set of all left [right] cancellable elements of  $(S, \cdot)$ , by  $o$  the neutral element of  $(S, +)$ , and write  $S^* = S \setminus \{o\}$  meaning  $S^* = S$  if  $(S, +)$  has no neutral element.

To avoid trivial rubs, assume  $|S| \geq 2$ . Then the following ring-like

Definition  $(S, +, \cdot)$  is multiplicatively left cancellative iff  $S^* \subseteq \mathcal{L}$

is justified by the

Theorem  $S^* \subseteq \mathcal{L}$  is equivalent to one of the following cases:

- $\alpha$ )  $S$  has no neutral element, and  $(S, \cdot)$  is left cancellative.
- $\beta$ )  $S$  has a neutral element  $o$ , and  $(S, \cdot)$  is left cancellative.
- $\gamma$ )  $S$  has a neutral element  $o$ ,  $So = oS = o$ , and  $(S^*, \cdot)$  is a semigroup which is left cancellative.

In this context, "left" and  $\mathcal{L}$  may be replaced by "right" and  $\mathcal{R}$ , by "two-sided" and  $\mathcal{L} \cap \mathcal{R}$ , and by "mixed" and  $\mathcal{L} \cup \mathcal{R}$ . Moreover,  $S^* \subseteq \mathcal{L} \cup \mathcal{R}$  implies  $S^* \subseteq \mathcal{L}$  or  $S^* \subseteq \mathcal{R}$ .

As a matter of fact, all these statements are valid dealing only with a semigroup  $(S, \cdot)$  and one special element  $o \in S$ , being exceptional for whatever reason. (Cf. H.J. Weinert, On left, right, and two-sided cancellable elements in semigroups, Semigroup Forum 16 (1978), 97 - 103).

For semirings  $(S, +, \cdot)$ , the 9 classes  $(\alpha, \mathcal{L})$ ,  $(\alpha, \mathcal{R})$ ,  $(\alpha, \mathcal{L} \cap \mathcal{R})$ ,  $(\beta, \mathcal{L})$ ,  $\dots$ ,  $(\gamma, \mathcal{L} \cap \mathcal{R})$  are mutually distinct and not empty. For right distributive near-rings one only has the 3 cases  $(\beta, \mathcal{R})$ ,  $(\gamma, \mathcal{R})$ , and  $(\gamma, \mathcal{L}) = (\gamma, \mathcal{L} \cap \mathcal{R})$ , hence a left cancellative r.d. near-ring is also right-cancellative and zero symmetric.

Similarly, we justify the following ring-like

Definition A left or right distributive semi-near-ring  $(S, +, \cdot)$  is called a semi-near-field iff  $(S^*, \cdot)$  is a group.

Theorem. For any semi-near-ring  $(S, +, \cdot)$ , the following statements are equivalent:

- a)  $(S^*, \cdot)$  is a group.
- b) There exists an element  $e_L \in S$  such that for all  $a \in S^*$  one has  $e_L a = a$  and  $a^* a = e_L$  for some  $a^* \in S$ .
- c) For all  $a, b \in S^*$  there are  $x, y \in S$  such that  $ax = b$  and  $ya = b$ .

In the non-trivial case with a zero (which is hard to prove),  $e_L$  is in  $S^*$  and  $a^*$ ,  $x$ ,  $y$  may be chosen in  $S^*$ . Moreover,  $|S| \geq 3$  implies  $S \in (\gamma, \mathcal{L} \cap \mathcal{R})$ , which is not true in general for the 10 semi-near-fields  $S$  of order 2.

H. WIELANDT:

Die Anfänge der Theorie der Fastringe.

Die Grundtatsachen der Strukturtheorie bis zum Analogon des Satzes von Wedderburn über einfache Ringe sind zweimal veröffentlicht worden: (1) durch den Vortragenden, ausgehend von Fittings "Endomorphismenbereichen" nicht-abelscher Gruppen, auf der Gruppentagung in Hamburg 1937 (ein Auszug ohne Beweise erschien in Deutsche Math. 3), (2) durch D. W. Blackett in seiner bei E. Artin entstandenen Dissertation (Princeton University 1950). Da Artin zu den Veranstaltern der Hamburger Tagung gehört hatte, stellt sich die Frage nach einem möglichen Zusammenhang.

Ein Briefwechsel im Juni 1980 mit Zassenhaus, der in Hamburg Artins Assistent gewesen war, und mit Blackett hat ergeben: Artin war 1937 über den Inhalt des Vortrags unterrichtet, hatte aber 1950 offenbar keine Erinnerung mehr daran; Blackett hat das ihm von Artin vorgeschlagene Thema "Endliche Fastkörper" selbständig abgewandelt.

R. ZEAMER:

Some Near Ring Extensions of  $\mathbb{R}$ .

Let  $\mathbb{R}(S) = \bigstar_{s \in S} \mathbb{R}_s$ , where  $S$  is an arbitrary set and  $\mathbb{R}_s := \{rs \mid r \in \mathbb{R}\}$ . Define a norm  $\|\cdot\|: \mathbb{R}(S) \rightarrow \mathbb{R}^+$  by  $\|w\| := \inf \left\{ \sum |r_i| \mid w = \sum (r_i s_i)^{g_i}, g_i \in \mathbb{R}(S) \right\}$ .

Using Van Kampen diagrams it is shown that  $d(u,v) := \|u - v\|$  defines a distance function on  $\mathbb{R}(S)$  with respect to which  $\mathbb{R}(S)$  becomes a topological group. Moreover, any monoid on  $S^0$  extends to a multiplication on  $\mathbb{R}(S)$  which makes  $\mathbb{R}(S)$  a near-ring d.g. by  $\bigcup \mathbb{R}_s$  and topological with respect to  $d$ . A completion  $\overline{\mathbb{R}(S)} \supseteq \mathbb{R}(S)$  is then constructed in which all Cauchy sequences whose elements are of bounded length, converge. The before mentioned near rings on  $\mathbb{R}(S)$  all extend to topologically d.g. near rings on  $\overline{\mathbb{R}(S)}$ .

Using this fact it is then shown that the endomorphisms of  $\overline{\mathbb{R}(S)}$  as an  $\mathbb{R}$ -module are continuous and all extensions of maps  $\varphi: S \longrightarrow \overline{\mathbb{R}(S)}$  such that the lengths of the  $\varphi(s)$  are bounded.

$\mathcal{A} \subseteq K(S), \mathbb{R}(S),$  or  $\overline{\mathbb{R}(S)}$  is called a full ideal ( $K \subseteq \mathbb{R}$  subring) iff  $\mathcal{A}$  is an ideal with respect to every monoid on  $S^0$ . For  $S$  infinite, full ideals and subgroups invariant with respect to  $\mathbb{R}$ , on  $K$ -endos of  $\mathbb{R}(S), \overline{\mathbb{R}(S)},$  or  $K(S)$  respectively, are the same.

A variety  $V \subseteq Z(S)$  ( $Z(S)$  the free group on  $S$ ) is called regular if  $v \in V, (\frac{1}{n})v \in Z(S) \implies (\frac{1}{n})v \in V$ .

The closed full ideals of  $\overline{\mathbb{R}(S)}$  are partitioned into a disjoint union of sublattices,  $\{I_V\}_V$  a regular variety, where  $I_V := \{\mathcal{A} \mid \mathcal{A} \text{ closed full ideal of } \overline{\mathbb{R}(S)}, \mathcal{A} \cap Z(S) = V\}$ .

Each  $I_V$  has a minimal element,  $\overline{VQ(S)}$ . The elements of the lower central series are shown to be regular yielding a descending sequence of full closed ideals of  $\overline{\mathbb{R}(S)}$ ,  $\{\overline{D_n(Q(S))}\}_{n \geq 1}, \{D_n\}_{n \geq 1}$  the lower central series.

This shows there are plenty of complete topologically d.g. near rings extending  $\mathbb{R}$ .

J. L. ZEMMER:

Affine Transformations on a Total Near-Ring.

A (left) near-ring  $N$  is called a total near-ring if it is a subnear-ring of a (left) near-field  $K$  and has the property that  $\beta \in K$  implies either  $\beta \in N$  or  $\beta^{-1} \in N$ . This paper is concerned with the affine transformations  $\xi \rightarrow \alpha \xi + \beta$  where  $\alpha \neq 0, \beta \in N, N$  a total near-ring. These transformations form a semigroup under composition. It is clear that this semigroup can be embedded in a sharply doubly transitive group of permutations acting on an appropriate set (i. e. a near-field containing the given total near-ring). The two main results of this paper are

(1) necessary conditions that a transformation semigroup be embedable in a sharply doubly transitive group of permutations and (2) necessary and sufficient conditions that a transformation semigroup be isomorphic to the semigroup of affine transformations on a total near-ring.

Berichterstatter: G. Betsch



Liste der Tagungsteilnehmer

- Bell, Prof. H. E., Dept. of Mathematics, Brock University,  
St. Catherines, Ontario L2S 3A1, Canada
- Betsch, Dr. G., Mathematisches Institut der Universität  
Tübingen, Auf der Morgenstelle 10, D-7400 Tübingen 1
- Clay, Prof. J. R., Dept. of Mathematics, University of  
Arizona, Tucson, Arizona 85 721, USA
- Ferrero Cotti, Prof. C., Istituto di matematica, Università  
degli Studi, I 43 100 Parma
- Ferrero, Prof. G., Istituto di Matematica, Università degli  
Studi, I 43 100 Parma
- Fong, Prof. Yuen, Dept. of Mathematics, National Cheng-Kung  
University, Tainan, Taiwan, Rep. of China
- Gröger, D., Institut für Math., Technische Universität  
Hannover, Welfengarten 1, D-3000 Hannover 1
- Hofer, Prof. R. D., Dept. of Mathematics, State University  
College, Plattsburgh, N.Y. 12 901, USA
- Karzel, Prof. Dr. H., Institut für Mathematik, Technische  
Universität München, Arcisstraße 21, D-8000 München 2
- Kautschitsch, Doz. Dr. H., Institut für Mathematik,  
Universität Klagenfurt, Universitätsstraße 65,  
A-9010 Klagenfurt
- Kerby, Prof. Dr. W. E., Mathematisches Seminar, Universität  
Hamburg, Bundesstraße 55, D-2000 Hamburg 13
- Lockhart, Dr. R., 139, Old Dover Road, Blackheath, S.E.3.8.SY,  
London, England
- Lyons, Prof. C. G., Dept. of Mathematics and Comp. Sc.,  
James Madison University, Harrisonburg, Virginia 22 801,  
USA
- Mahmood, Dr. S. J., Dept. of Mathematics, Quaid-i-Azam  
University, Islamabad, Pakistan  
(und Dept. of Math., The University, Edinburgh EH9 3JZ,  
Scotland)

- Maxson, Prof. C. J., Dept. of Mathematics, Texas A & M University, College Station, Texas 77 843, USA
- Meldrum, Dr. J. D. P., Dept. of Mathematics, The University, Edinburgh EH9 3JZ, Scotland
- Mlitz, Doz. Dr. R., Institut für Angewandte Mathematik, Technische Universität Wien, Gusshausstraße 27 - 29, A-1040 Wien
- Müller, Prof. Dr. W. B., Institut für Mathematik, Universität Klagenfurt, Universitätsstraße 65, A-9010 Klagenfurt
- Oswald, Dr. A., Dept. of Mathematics, Teesside Polytechnic, Middlesbrough TS1 3BA, Cleveland, U.K.
- Pilz, Prof. Dr. G., Institut für Mathematik, Johannes-Kepler-Universität Linz, A-4045 Linz-Auhof
- Scott, Dr. S. D., 100 Beacon Ave., Campbells Bay, Auckland, New Zealand
- So, Dr. Yong-Sian, Institut für Mathematik, Johannes-Kepler-Universität Linz, A-4045 Linz-Auhof
- Wähling, Prof. Dr. H., Institut für Mathematik, Technische Universität München, Arcisstraße 21, Postfach 20 24 20, D-8000 München 2
- Wefelscheid, Prof. Dr. H., Fachbereich Mathematik der Universität Duisburg, Lotharstraße 65, D-4100 Duisburg
- Weinert, Prof. Dr. H. J., Institut für Mathematik, Technische Universität Clausthal, D-3392 Clausthal-Zellerfeld
- Wielandt, Prof. Dr. H., Mathematisches Institut der Universität Tübingen, Auf der Morgenstelle 10, D-7400 Tübingen 1
- Wiesenbauer, Dr. J., Institut für Algebra und Mathematische Strukturtheorie, Technische Universität Wien, Argentinierstraße 8, A-1040 Wien
- Zeamer, Dr. R. W., Dept. of Mathematics, Queen Mary College, Mile End Road, London E1 4NS, England
- Zemmer, Prof. J. L., Dept. of Mathematics, University of Missouri, Columbia, Missouri 65 211, USA