MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 18/1980

Mathematische Logik

20.4. bis 26.4.1980

Unter der Leitung von E.Specker (Zürich) und W.Felscher (Tübingen) fand in der Woche vom 20.4. bis 26.4.1980 im Forschungsinstitut in Oberwolfach die diesjährige Tagung über Mathematische Logik statt. Es wurden 28 Vorträge über verschiedene Gebiete der Mathematischen Logik gehalten.

Vortragsauszüge

E.BÖRGER: *On conservativity of reduction procedures*
(joint work with S.O.Aanderaa and H.R.Lewis)
A *Krom* formula of pure quantification theory is a formula in conjunctive normal form such that each conjunct is a disjunction of at most *two* atomic formulas or negations of atomic formulas. Every class of Krom formulas that is determined by the form of their quantifier prefixes and which is known to have an unsolvable decision problem for satisfiability is here shown to be a *conservative* reduction class. Therefore both the general satisfiability problem, and the problem of satisfiability in finite models, can be effectively reduced from arbitrary formulas to Krom formulas of these several prefix types.
This solves in particular the question whether the Krom class $\forall\exists\forall\forall$ is a conservative reduction class. Other Krom classes to which this class has been reduced by a conservative reduction procedure are therefore shown to be conservative reduction classes too.
(See S.O.Aanderaa, E.Börger, Yu.Gurevich: Prefix classes of Krom formulae with identity, AMLG $\geq$ 1980; H.R.Lewis, W.D.Goldfarb: The decision problem for formulas with a small number of atomic subformulas, JSL 1973.)

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H.G.CARSTENS: *Rekursive Kombinatorik*

Es wurde ein Überblick über die bislang behandelten kombinatorischen Konstruktionen gegeben. Als neue Ergebnisse wurden dargestellt

1. Kriterien für die Existenz rekursiver Pfade in stark rekursiven Bäumen
2. Für die chromatische (χ) und die rekursiv chromatische (χ_r) Zahl von stark rekursiven Graphen endlichen Geschlechts gilt
   $\chi_r \leq \max\{8, \chi\}$
3. Für die Heawoodzahl ($H(j)$) und die rekursive Heawoodzahl ($H_r(j)$) stark rekursiver Graphen vom endlichen Geschlecht j gilt
   $H_r(0) = 5 \neq H(0)$
   $j > 0$: $H_r(j) = H(j)$

D.van DALEN: $\mathbb{R}^s$, *varations on a theme of Troelstra*

One can construct an extension of the intuitionistic continuum $\mathbb{R}$, by means of "singletons", i.e. sets S with the properties (i) $\forall xy\,(Sx \wedge Sy \rightarrow x=y)$, (ii) $\neg\neg\exists x\, Sx$. The resulting set of singletons, $\mathbb{R}^s$, contains $\mathbb{R}$. One can define the operations and relations in a natural way. $\mathbb{R}^s \subseteq \mathbb{R}^{ee}$ (Troelstra's *extended reals*), and $\mathbb{R}^s = \mathbb{R}^{ee}$ iff the double negation shift holds for $\forall rr'(r < r' \rightarrow r \in T \vee r' \notin T)$ (for "Dedekind cuts" T). Facts concerning the extension of functions $\mathbb{R} \rightarrow \mathbb{R}$ are established. $\mathbb{R}^s$ has a stable equality, but carries no apartness.The relation between $\mathbb{R}$, $\mathbb{R}^s$ and $\mathbb{R}^{ee}$ is such that all $f: \mathbb{R}^{ee} \rightarrow \mathbb{R}^s$ and $g: \mathbb{R}^s \rightarrow \mathbb{R}$ are constant. Finally, $\mathbb{R}^s$ is (contrary to $\mathbb{R}^{ee}$) not order-complete.

U.FELGNER: *Gruppen, die Quantoren - Elimination zulassen*

Eine Gruppe G nennen wir kurz eine QE - Gruppe, falls die Theorie 1.Stufe von G (in der Sprache mit $\cdot, ^{-1}, 1$) Quantoren - Elimination zuläßt. Zusammen mit G.Cherlin habe ich die auflösbaren und die endlichen einfachen QE-Gruppen bestimmt. Unter den p-Gruppen gibt es nur im Falle $p = 2$ nicht-abelsche QE-Gruppen, und diese sind die Quaternionen-Gruppe Q der Ordnung 8, die Sylow 2-Gruppe der $PSU_3(4^2) = U_3(4)$ der Ordnung 64 und noch einige unendliche 2-Gruppen vom Exponent 4. Unter den endlichen einfachen Gruppen sind $PSL_2(5)$ und $PSL_2(7)$ die einzigen QE-Gruppen. Die Klasse der auflösbaren QE-Gruppen ist groß und kann mit den Begriffen $O(G)$, $O_{2',2}(G)$ und $O_{2',2,3}(G)$ beschrieben werden.

W.FELSCHER: *On Substitutions and their Applications*

Consider the sequential calculus for (classical) first order logic. If the quantifier rules are formulated for substitutions which are *free* (for the respective formula) then it is well known that the calculus is incomplete. If, instead, substitutions involving a built-in renaming of bound variables are used, then the calculus is semantically complete but the usual cut elimination procedure, making use of the purification of pure-sequent proofs, breaks down. In this communication a new (primitive) recursively defined operation for renaming bound variables, together with a corresponding substitution operation, is defined for which it can be shown that it gives a complete calculus with cut elimination. The basic proof theoretical lemma is, of course, that if $M \Rightarrow N, v(y)$ is the premiss of a quantifier rule with y as an Eigenvariable, then a proof of this sequent can be transformed into a proof of the sequent $M \Rightarrow N, sub(y,t|v)$ where t is an arbitrary term.

J.Y.GIRARD: *The proof - theory of small admissibles*

This is an exposition of the methods and of the results of inductive logic, when applied to small admissibles. On one side, a new kind of cut-elimination theorems, on the other side, an explicit construction of these admissibles together with explicit bounds for the α - recursive functions.

G.HASENJAEGER: *Varianten zur vollständigen Induktion*

Für $\bar{n} := \{x \mid x < n\}$, wobei "$\bar{n} = n$" nicht vorausgesetzt wird, schließt die $\bar{n}$-Relativierung $\alpha^{\bar{n}}$ jeder genau im Endlichen gültigen PL1-Formel α ebenso wie die übliche Induktion $Ind(A^1)$ in Standard-Deutung alle Nicht-Standard-Modelle aus.

Beispiele (bei denen zur Vereinfachung auch mengentheoretische Notation gebraucht wird)

$$\alpha_1 : A \neq \emptyset \wedge B \neq \emptyset \rightarrow \neg 2\times A\times A \xleftrightarrow[R^5]{bij} B\times B.$$

$$\alpha_2 : A \neq \emptyset \wedge B \neq \emptyset \rightarrow \neg\{0\}\times A\times A\times A \cup \{1\}\times B\times B\times B \xleftrightarrow[R^7]{bij} C\times C\times C.$$

Die Einsetzungen in $\alpha_1^{\bar{n}}$ bzw. $\alpha_2^{\bar{n}}$: $A/\bar{a}$, $B/\bar{b}$, $(C/\bar{c},)$ $n/a+b(+c)$, in Verbindung mit Einsetungen für R^5 bzw. R^7, welche aus

$$Add(a,b,c) := \lambda xyz((x=0 \wedge y<a \wedge z=y \vee x=1 \wedge y<b \wedge z=a+y) \wedge z<c) \quad \text{und}$$

$\text{Mlt}(a,b,c) := \lambda xyz(x<a \wedge y<b \wedge z<c \wedge z=x+ay)$ komponiert sind, liefern bekannte Sätze (mit elementaren Eigenschaften von +,•). *Frage:* Wie "stark" sind solche Formeln $\alpha_i^{\overline{n}}$ sonst?

G.F.van der HOEVEN: *Projections of lawless sequences*

In the universe of sequences of natural numbers, we can distinguish subtypes, like e.g. the lawlike and the lawless sequences. Lawless sequences are relatively simple, and have a convincing axiomatization, (*LS*), due to Kreisel. For e.g. the definition of the reals they are not suitable. For that purpose we want to have a notion of sequence which is closed under pairing and continuous functional application, and for which continuity principles (like $\forall\alpha\exists x$- and $\forall\alpha\exists\beta$-continuity) are plausible.

Universes constructed from lawless sequences and lawlike objects (universes of projections) can be used to imitate a primitive notion of sequence, thus "reducing" this notion to lawlessness. Such a reduction gives a better insight in the primitive concept, and makes it possible to prove continuity properties formally, in *LS*. E.g. the reduction of Troelstra's notion of GC-sequence, yields a projected universe which models (provably in *LS*) a close variant of the system *CS*.

G.JÄGER: *Constructible Hierarchy and Proof Theory*

In a first step we reduced subsystems of analysis and set theory to theories of iterated admissible sets. In a second step we introduced a system RS of ramified set theory which is the proof theoretical analogue of (a segment of) the constructible hierarchy. We showed that you can embed the theories of iterated admissible sets into RS and proved a cut elimination theorem for RS. Finally we used these methods for the ordinal analysis of various subsystems of analysis and set theory; we indicated how to calculate the proof theoretical ordinal of the theory $(\Delta^1_2\text{-CA}) + (\text{BI})$.

A.W.JANKOWSKI: *Embedding of Logical Structures*

We shall say that $L = \langle M, \simeq, \mathcal{L}, \models, \mathcal{D}\rangle$ is a *pseudological stucture* provided that i) M is a class of *models*, ii) $\simeq$ is a binary relation between models, iii) $\mathcal{L}$ is a class of *formulas*, iv) $\models$ is a binary relation between models and formulas, v) $\mathcal{D}$ is a class of *deduction rules*.

A pseudological stucture is *consistent* provided that there is no $\alpha \in \mathcal{L}$ such that α is a tautology (i.e. $\alpha \in Cn(\emptyset)$, where Cn is the *semantical consequence* in L) and α is an antitautology (i.e. $Cn(\{\alpha\}) = \mathcal{L}$). L *satisfies compactness theorem* provided that:

$$\alpha \in Cn(A) \Rightarrow (\exists A_0)(A_0 \subseteq A \ \& \ \overline{\overline{A_0}} < \omega \ \& \ \alpha \in Cn(A_0)).$$

L is said to be a *logical structure* provided that: i) $\simeq$ is equivalence relation, ii) if $\mathfrak{A} \simeq \mathfrak{B}$ then $\mathfrak{A} \equiv \mathfrak{B}$, where $\equiv$ is *elementary equivalence relation* in L, iii) for every $A \subseteq \mathcal{L}$ we have $C(A) \subseteq Cn(A)$, where C is the *syntactical consequence*, iv) if formula α is a tautology (resp. an antitautology) then for every model $\mathfrak{A}$ we have $\mathfrak{A} \models \alpha$ (resp. non $\mathfrak{A} \models \alpha$).

Example: Let μ = <Rel,Fnt,Const>, where Rel (Fnt, Const) is a non-empty set of relation (function, constant) symbols. Let $L_\mu = \langle M_\mu, \simeq_\mu, \mathcal{L}_\mu, \models_\mu, \mathcal{D}_\mu \rangle$, where: M_μ is the class of relational structures of type μ, $\simeq_\mu$-isomorphism, $\models_\mu$-classical satisfaction relation, $\mathcal{D}_\mu$-all possible rules of deduction in classical logic over $\mathcal{L}_\mu$-classical sentences.

We shall say that L_1 *is included in* L_2 provided that: $M_1 \subseteq M_2$, $\simeq_1 = \simeq_2 \cap (M_1 \times M_1)$, $\equiv_1 = \equiv_2 \cap (M_1 \times M_1)$, $\models_1 = \models_2 \cap (M_1 \cap \mathcal{L}_1)$, $Cn_1 = Cn_2 \upharpoonright \mathcal{L}_1$, if $\alpha \in \mathcal{L}_1$, then α is a tautology (antitautology) in L_1 iff α is a tautology (antitautology) in L_2, $\mathcal{D}_1 \subseteq \mathcal{D}_2 \upharpoonright \mathcal{L}_1$.

We shall say that L_1 *is embeddable in* L_2 provided that there exists L_3 such that L_1 is isomorphic to L_3 and L_3 is included in L_2.

Theorem: If L is a pseudological structure with countable set of formulas, then L is embeddable in L_μ iff L is a consistent logical structure which satisfies compactness theorem.

H.KOTLARSKI: *On Cofinal Extensions of Models of Arithmetic*

Let $M,N \models PA$, $M \subseteq N$. N is a *cofinal extension* of M iff $\forall a \in N \ \exists b \in M \ N \models a<b$. By Gaifman's theorem, in this situation $M \prec N$.

Theorem 1. (Smorynski & Stavi). Let N be a cofinal extension of M. Then

(i) if M is recursively saturated then N is recursively saturated.

(ii) if M is ω-saturated then N is ω-saturated.

I stress that this fails for end-elementary extensions. κ-saturation for uncountable κ is not preserved under cofinal extension,more exactly

Theorem 2. Every non-standard M has a cofinal extension which is not ω_1-saturated.

The construction used to prove thm.2. does not give a simple extension of M ($N \succ M$ is a *simple extension* iff N is the Skolem closure of $M \cup \{d\}$ for a single element d).

Theorem 3. If M is ω_1-saturated and N is a simple cofinal extension of M then N is ω_1-saturated.

Theorem 4. If M is saturated then there exists a simple cofinal extension which is still saturated.

I do not know if the assumption in theorem may be weakened to "M is κ-saturated" and get a κ-saturated simple cofinal extension.

Resplendency is not preserved under cofinal extensions, nevertheless

Theorem 5. If $X \subseteq M$ is such that $(M,X) \models$ induction in $L_{PA} \cup \{X\}$ and N is a cofinal extension of M then N has a subset X^N such that $(M,X) \prec (N,X^N)$.

This result together with some auxiliary tricks may be used to derive

Theorem 6. If M is the number-theoretic part of some model of Kelley-Morse set theory then every cofinal extension of M is the number-theoretic part of some model of ZF set theory. Similarly for third-order and second-order arithmetic.

In the countable case the assumption of theorem 6 may be weakened to "M is ZF-expandable" (M is A_2-expandable) by theorem 1.

P.H.KRAUSS: *Model Companions of Filtral Varieties*

V a filtral variety, M the simple members, M_+ the simple or trivial members, $IP_{bs}M_+$ ($IP_{bs_0}M_+$) isomorphs of Boolean subdirect products of members of M_+ (with no isolated non-trivial factors), V^a (V^e) the V-algebraically (existentially) closed members of V.

(1) $IP_{bs_0}M_+$ ($IP_{bs}M_+$) is an $\forall\exists$(positive) class

(2) $IP_{bs}(M_+)^a \subseteq V^a$

(3) $V^a \subseteq IP_{bs}M_+$

(4) $IP_{bs_0}(M_+)^a \subseteq V^e$

(5) $V^e \subseteq IP_{bs_0}M_+$

(6) If N is a model companion of M_+, then $V^a = IP_{bs}N_+$ is an $\forall\exists$ positive class and $V^e = IP_{bs_0}N_+$ is a model companion of V

(7) If M_+ has a model companion, then V has a model completion iff M has the amalgamation property.

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H.LUCKHARDT: *Intuitionistic propositional operators*

Four groups of results are discussed.

I. Uniformity: $\bigwedge p \bigvee n A(p,n) \to \bigvee n \bigwedge p A(p,n)$. Let $\Omega^{(\sim)}$ denote {(lawlike) propositions}.- S has a countable non-trivial partition P, $f: \Omega \overset{ext}{\to} S \Rightarrow f: \Omega \to$ one P-cell of S. $f: \Omega \overset{ext}{\to} \Omega \Rightarrow [f \text{ inj} \Leftrightarrow f = id]$. $\forall n > 0 \neg\exists$ ext inj $f: \Omega^n \to \Omega$. For separable metric spaces T with cardinality ≥ 3: card Ω and card T are incomparable.

II. Intuitionistic propositional logic with propositional quantifiers over Ω *resp.* $\Omega^{\sim}$*:* Do these logics coincide? Axiomatizability implies decidability. $\bigvee q(p \leftrightarrow \neg\neg q \vee \neg q) = \neg\neg p$. Extensional choice among propositions is not generally valid.

III. Bases: For operators $\bigvee \underline{q}(G\underline{q} \overset{\to}{\leftrightarrow} p)$ the basis property can be characterized by definability. Most of these operators have no $(\top, \bot)$-invariant basis.

IV. New extensional monadic propositional operators: The $\bigvee q(p \leftrightarrow P_i q)$ (P_i from the Rieger-Nishimura lattice) are new and distinct for $i = 7$ and ≥ 9. Further new operators use quantification over $\Omega^{\sim}$.

K.POTTHOFF: *Consistency Properties und Forcing*

Das Konzept der Consistency Property wird in verallgemeinerter Form als Forcingstruktur und damit als Bestandteil einer allgemeinen Forcingmethode benutzt. Damit kann dann der Modell-Existenz-Satz sowohl für die modelltheoretischen klassischen Sätze (Vollständigkeitssätze für $L_{\omega,\omega}$, $L_{\omega_1,\omega}$, Interpolationssätze für $L_{\omega,\omega}$ und $L_{\omega_1,\omega}$, Löwenheim-Skolem etc.) als auch für die bekannten Forcing-Strukturen wie endliches oder unendliches Forcing genutzt werden. Diese beiden Forcing-Methoden stellen sich so als Spezialfall einer sehr allgemeinen modelltheoretischen Methode heraus und sind daher besser motivierbar.

C.RAUSZER: *Logical Semantics for Data Base Protection*

In system being considered the access to the data base is limited for some type of users. In the paper only territorial aspect of protection is discussed. "Territorial" could be cleared as follows: If a user has an access to an area, then he can obtain a full information within this area. The area to which a user has an access is identified with his position in a certain hierarchy. Such an area

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will be called, informally, the priority of a user. The differences between their priorities are reflected by the different answers which users obtain to the same query.

By a P-system we mean an information storage and retrieval system introduced by Marek and Pawlak with distinguished family $\mathcal{D}$ of sets of objects. Elements of $\mathcal{D}$ are interpreted as values of priorities, i.e. an accessible area for a priority p is $D_p \in \mathcal{D}$.

The language (for queries) consists of two lewels: terms and formulas. Terms are interpreted as sets of objects of a P-system, whereas formulas are interpreted as truth values. It should be emphasized that the priorities are elements of a formal language and they are treated as formulas.

It turns out that the role analogous to that of Boolean algebra and classical logic for Marek - Pawlak systems is played in the case of systems considered here by pseudo-Boolean algebra with distinguished set of elements and an intermediate logic. That means that conditions on objects can be stated in a common form of "Boolean formulas" but the "inner logic" of considered systems is stronger than intuitionistic and not classical one. So terms are ruled by axioms of Boolean algebra enriched by certain specific axioms. For formulas we have the axioms of intuitionistic logic and certain axioms for priorities.

As a model for our language we consider P-system. Some algebraic properties of P-system are the main tools of proving the completeness theorem. We can prove that the formalized system is decidable and we show some connections between the formalization given by Marek and Pawlak and introduced above.

W.RAUTENBERG: *On the expressive power of implicationless formulas*

1. There is a countable partition of the lattice H of intermediate logics as to whether logics in H have the same $\neg,\wedge,\vee$ - reduct. Maximal elements of the partition classes will be presented.

2. The $\neg,\wedge,\vee$ - fragmentary intuitionistic logic $L_{\neg,\wedge,\vee}$ determinates its consequence $\vdash^{i}_{\neg,\wedge,\vee}$ uniquely (E.Capińska). Nevertheless, there is no (hidden) deduction theorem as follows from the

Theorem: Let $\vdash$ be a standard consequence, $\gamma_1, \gamma_2, \ldots$ a sequence of formulas such that (i) $\nvdash \gamma_n$, (ii) $\gamma_i \vdash \gamma_{i+1}$, (iii) for each finite matrix A there is some n such that $\models^{A} \gamma_n$ (Gödel property), and suppose $\vdash$ has the strong finite model property (shown for $\vdash^{i}_{\neg,\wedge,\vee}$ by A.Wroński), then $\vdash$ admits no deduction theorem, also not in a very general sense.

M.v.RIMSCHA: *Transitivitätsbedingungen*

Wozu können die Transitivitätsbedingungen nützlich sein?

Für das System ZF^{0} + *Sext* (ZF ohne Fundierung + starke Extensionalität) ist bislang keine Mächtigkeitsdefinition bekannt (in ZF^{0} + *Sext* ex. i.A. keine Rangfunktion!). Eine Möglichkeit, zu einer Mächtigkeitsdefinition zu kommen, ist, zusätzlich die Bedingung *Tr* zu fordern:

Tr $\forall x \exists u$ (u ist transitiv $\wedge$ x ist bijektiv abbildbar auf u).

Die Kardinalität von x läßt sich dann wie folgt definieren:

KT(x) := {u | u ist transitiv $\wedge$ x ist bijektiv abbildbar auf u}

Außer *Tr* soll noch eine Verschärfung *Tr'* betrachtet werden:

Tr' $\forall x \exists u$ (u ist transitiv $\wedge$ u ist Teilmenge der transitiven Hülle von x $\wedge$ x ist bijektiv abbildbar auf u)

Thema: Welche Beziehungen bestehen zwischen *Tr* bzw. *Tr'* und anderen Axiomen der ZFC - Mengenlehre?

Resultate: $ZFC^{0} \vdash$ *Tr*

ZF^{0} + *U1* $\vdash$ *Tr* (*U1*: "Jede extensionale Relation ist als transitive Menge darstellbar")

ZFC $\vdash$ *Tr'*

ZFC^{0} + *U1* $\vdash \neg$*Tr'*

Tr ist unabhängig von ZF

Tr ist unabhängig von ZF^{0} + *OP* + P17

(*OP*: "Ordnungsprinzip"; P17: "Die Potenzmenge einer wohlordnbaren Menge läßt sich wohlordnen")

Tr ist unabhängig von ZF^{0} + *Sext*

Tr' ist unabhängig von ZFC^{0} + *Sext*

ZF + *Tr* $\vdash$ "Dedekind - endlich $\equiv$ endlich"

U.R.SCHMERL: *Aspekte beweistheoretischer Feinstruktur in Systemen der prädikativen Analysis*

Mit Hilfe geeigneter Formalisierungen werden Hierarchien formaler Systeme RA^{α}_{β} der verzweigten Analysis konstruiert, die alle untereinander bezüglich der Herleitbarkeit von Formeln einer beliebigen Schicht γ vergleichbar sind. Dies wird durch sog. Feinstrukturrelationen ausgedrückt, deren wichtigste die Gestalt

$$\forall \gamma < \omega^{\alpha_1}: \quad RA^{\alpha+\omega^{\alpha_1}}_{\beta} \equiv^{\alpha+\gamma} RA^{\alpha+\gamma}_{\varphi_{1+\alpha_1}(\beta)}$$

hat. Dabei gibt der obere Index der Theorien RA^{α}_{β} die Schicht des Systems, der untere seine Lage in der Hierarchie $(RA^{\alpha}_{\beta})_{\beta}$ an; $\equiv^{\alpha+\gamma}$ be-

deutet, daß beide Systeme die gleichen Theoreme der Schichten $\leq \alpha + \gamma$ haben. Diese Relationen charakterisieren somit die beweistheoretische Stärke der Systeme RA^{α}_{β} für beliebige Schichten γ, $\gamma \leq \alpha$.
Da verschiedene Systeme der prädikativen Analysis in die R^{α}_{β} eingebettet werden können und sich diese andererseits in ersteren interpretieren lassen, ergeben die Feistrukturrelationen ein Mittel zur Analyse auch von ungeschichteten Systemen. Einige Beispiele dafür werden angegeben.

D.SCHMIDT: *An algebraic characterisation of P*

In view of the following theorem of R.V.Book (J.ACM 25, 23 - 31) we try to obtain a similar characterisation of P.
Book's theorem: NP is the smallest class of languages which contains all the regular languages and is closed under intersection and homomorphic replication.
We are able to characterise P as follows: P is the smallest class of languages which contains all the regular languages and is closed under intersection, concatenation, Kleene * and piecewise invertble homomorphhic replication.
The notion of closure under piecewise invertible homomorphic replication is too complicated to define here, but in any case it is rather tailored to the proof of this theorem and we hope a simpler characterisation of P can be found.

W.SCHÖNFELD: *Upper bounds for proof - length*

We specify a proof system for relation equations and a procedure SEARCH which searchs systematically for a proof to a given equation. Let $\mathcal{T}$ be the class of relation equations α corresponding to a slight extension of the prefix class $\exists\ldots\exists\forall\exists\ldots\exists$. By $lg(\alpha)$ we denote the length of α.
Theorem: For every $\alpha \in \mathcal{T}$ the following are equivalent:
(i) $\models \alpha$
(ii) There is a proof found by SEARCH with not more than

$$2^{lg(\alpha)(2^{2^{lg(\alpha)}}+2)} \text{ many sequents.}$$

The proof consists of a description of possible loops in the search process.

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J.SCHULTE MÖNTING: *Algebraic Consequence Operators*

(joint work with W.Felscher)

We consider subclasses of the class J of implicational algebras, i.e. of all p.o.sets with a greatest element e and one binary operation $\supset$ such that $a \supset b = e$ iff $a \leq b$ holds. Such a class A defines a consequence operator cn_A on the term algebra T.

1. cn_A is *equational* if it can be computed "from below" syntactically from the appropriate set of equations. Equational consequence operators can be characterized by a representation property like the Stone theorem. Equational definability depends on the size of the term algebra.

2. For a calculus cn_A, the class of *test algebras* is the largest subclass S of J such that $cn_A = cn_S$. S can be characterised by closure properties.

Theorem (using PI): Let A be the class of test algebras of cn_A. Then the following are equivalent:

(i) A is quasi-equational (i.e. definable by equations and implications of equations)

(ii) cn_A is definable by axioms and finitary rules

(iii) cn_A is finitary

(iv) A is elementary.

3. We say that an algebra is *small* if it is a homomorphic image of the term algebra. A class A *admits principal filters* if, for every $A \in A$, $a \in A$, the segment $[a, e_A]$ is the kernel of a homomorphism to an algebra $B \in A$.

Theorem: Let A be the class of test algebras of cn_A. Then cn_A is *deductive* iff the class of small algebras of A admits principal filters.

The above investigations remain valid if we consider, for a signature Δ (including the implication), the clss J_Δ of those Δ-algebras whose reduct is in J. The class of *Quasi-Post-algebras* is the subclass of Heyting algebras which satisfies $a \supset (b \cup c) = (a \supset b) \cup (a \supset c)$.

Corollary: Let C be the class of (i) Diego algbras

or (ii) relatively pseudocomplemented $\cap$-semilattices

or (iii) Quasi-Post-algebras

Then every subclass of C which is (a) elementary and (b) the class of test algebras of its calculus is even equational.

W.SCHWABHÄUSER: *Eine Klasse unentscheidbarer Modelle der allgemeinen affinen Geometrie*

Die (ebene) allgemeine affine Geometrie GA_2 (erster Stufe) wurde 1965 von Szczerba und Tarski eingeführt (s.Fund.Math. 104 (1979)). Die Modelle sind (bis auf Isomorphie) die Einschränkungen $\mathfrak{A}_2(\mathfrak{F})|S$ von affinen Ebenen über reell-abgeschlossenen Körpern $\mathfrak{F}$ auf gewisse nicht-leere, konvexe, offene Punktmengen S. Eine Charakterisierung aller Modelle ist bisher nicht gelungen. Die Unentscheidbarkeit von GA_2 und andere Resultate wurden gezeigt mit Hilfe spezieller Modelle $\mathfrak{T}_N(\mathfrak{F})$, die durch abzählbar viele Ecken festgelegt werden (s.o.,S.178). *Satz:* $\mathfrak{T}_N(\mathfrak{F})$ ist ein Modell von GA_2 genau dann, wenn $\mathfrak{F}$ ein Modell der Theorie R_2 (angeordnete Körper mit Stetigkeitsaxiomen schwacher zweiter Stufe, s. Schwabhäuser, Fund.Math. 103 (1979)) ist. Gleichwertige Bedingungen für $\mathfrak{F}$ (s.dort) sind: (i) $\mathfrak{F}$ ist ein Modell der Theorie R_N (formal erster Stufe, aber mit dem Prädikat N für die Eigenschaft, natürliche Zahl zu sein). (ii) Die "natürliche Übersetzung" $s(\mathfrak{F}) \subseteq P\omega$ von $\mathfrak{F}$ ist ein Modell der Arithmetik A_ω zweiter Stufe.

H.SCHWICHTENBERG: *Homogene Bäume und subrekursive Hierarchien*

Die schwach wachsende (oder punktweise) und die stark wachsende Hierarchie g_α bzw. f_α sind - für einen mit Fundamentalfolgen versehenen Abschnitt der Ordinalzahlen - wie folgt definiert:

$g_0 n = 0$, $g_{\alpha+1} n = (g_\alpha n) + 1$, $g_\alpha n = g_{\alpha_n} n$ für α Limes, und

$f_0 n = 2^n$, $f_{\alpha+1} n = f_\alpha^n n$, $f_\alpha n = f_{\alpha_n} n$ für α Limes. J.-Y.Girard hat gezeigt, daß f_{ε_0} und g_α mit $\alpha = \varphi_{\varepsilon_{\Omega+1}} 0$ denselben Kalmár-elementaren Grad haben. Dieses Resultat wird mit Hilfe des von H.Jervell eingeführten Begriffes eines homogenen Baumes bewiesen.

D.SIEFKES: *Computing with Recursion*

When Gödel in 1936 defined the "computable" functions through sets of recursive equations, he changed 'computing' from the naive way of Dedekind and Skolem to handle primitive recursive definitions, into a formal business. The unique total solution of a set of arbitrary recursive equations need not be a computable function, as was shown later by Büchi and Kalmár. Gödel restricted "computing" to applying two formal steps: replacement and substitution. Since then two ways are common to impose "meaning" on a set of recursive equations: the syntactic way, to specify computation steps through a formal system (Gödel, Church - Kleene, Post, Turing), and the semantic way, to prove that "fixed points" exist (Kleene, Scott - Strachey). We introduce a universal programming language which unites these two ways. Terms are built from function constants by the operators composition, product, and test. Programs are sets of recursive definitions: equations with function variables on the left side. The evaluation of programs follows the term structure, and is thus (up to parallel processing) deterministic, and natural. The language accomodates arbitrary data structures, and models arbitrary cost definitions.

M.STEIN: *Interpretations of Heyting's Arithmetic*

In this lecture we introduced a set of interpretations of Heyting's arithmetic by which one can show that Heyting's arithmetic is conservative over the fragment in which formulae $\exists x^{\sigma} A$ are allowed only if $d(\sigma) < n$ and formulae $\forall y^{\tau} B$ are allowed only if $d(\tau) < m$, with $n \leq m$. As examples for the definition we give the following cases:

Let $A^{(n,m)} \equiv \exists v^{\sigma} \forall w^{\tau} \overline{A}$, $B^{(n,m)} \equiv \exists y^{\nu} \forall z^{\mu} \overline{B}$. Then

$$(A \wedge B)^{(n,m)} \equiv \exists V^{(l)\sigma} Y^{(l)\nu} \forall w^{\tau} z^{\mu} \overline{(A \wedge B)}$$

$$\overline{(A \wedge B)} \equiv r^{l} (\overline{A}[Vr] \wedge \overline{B}[Vr])$$

Here l is the type $n-1$; $r^{l} \equiv \Lambda$, if $n = 0$.

$$\left.\begin{array}{l} (\exists x^{\delta} A)^{(n,m)} \equiv \exists X^{(l)\delta} V^{(l)\nu} \forall W^{(l)(k)\tau} \overline{(\exists x^{\delta} A)} \\ \overline{(\exists x^{\delta} A)} \equiv \exists r^{l} \forall u^{k} \overline{A}[Xr, Vr, Wur] \end{array}\right\} \text{ if } d(\delta) \geq n$$

$$\left.\begin{array}{l} (\exists x^{\delta} A)^{(n,m)} \equiv \exists V^{(l)\sigma} \forall W^{(k)(l)\tau} \overline{(\exists x^{\delta} A)} \\ \overline{(\exists x^{\delta} A)} \equiv \exists r^{l} \exists x \forall u^{k} \overline{A}[Vr, Wur, x] \end{array}\right\} \text{ if } d(\delta) < n$$

W. THOMAS: *Fragments of arithmetic with relations of finite valency*

It is shown how a method due to Läuchli and further developed by Shelah, concerning the (weak) monadic theory of orderings $(A,<,R)$ with unary R, can be extended to binary R when considering only the *first*-order theory. Applications are:

Theorem 1: If $R_1,\dots,R_n$ are binary relations over $\mathbb{N}$ of finite valency, then in $(\mathbb{N},<,R_1,\dots,R_n)$ addition and multiplication are not definable.

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Theorem 2: If $f : \mathbb{N} \to \mathbb{N}$ is recursive and $f(n+1) - f(n)$ is nondereasing, the the first-order theory of $(\mathbb{N}, <, f)$ is decidable.
Finally, a combinatorial result is presented which yields a normal form theorem for sentences of theories as in Theorem 1 as well as the following "complexity bound": If the R_i are recursive (and of finite valency), then the first-order theory of $(\mathbb{N}, <, R_1, \ldots, R_n)$ is in the Boolean algebra over Σ_3.

A.S.TROELSTRA: *On a new propositional operator in intuitionistic propositional logic*

The operator $*(P) \equiv \exists Q\,(P \leftrightarrow \neg Q \vee \neg\neg Q)$, discussed by G.Kreisel in Rome in 1977 is an interesting example of a new propositional operator in intuitionistic propositional logic, not definable from $\to, \wedge, \vee, \perp$. For full topological models over Cantor-space and the reals, $*(P) \leftrightarrow \neg\neg P$ classically, while conflicting with Church's thesis. Over $[0,1]$, $*(P)$ is topologically undefinable; this argument may be translated into non-validity, with respect to the notion of intuitionistic validity uniformly in a parameter (i.e. a parameter ranging over $[0,1]$).

H.VOLGER: *Algebraische Hüllenoperatoren und Durchschnittseigenschaften von Theorien* (gemeinsam mit G.L.Cherlin)

M.O.Rabin hat 1962 die konvexen Theorien syntaktisch charakterisiert. Eine Theorie T heißt konvex, falls die Klasse Mod(T) der T-Modelle unter beliebigen Durchschnitten abgeschlossen ist. T heißt *↓-konvex*, falls Mod(T) abgeschlossen ist unter Durchschnitten von absteigenden Ketten von Substrukturen. D.M.R.Park hat 1964 gezeigt, daß eine Theorie genau dann ↓-konvex ist, wenn T eine $\forall\exists$-Theorie ist und Mod(T) abgeschlossen ist unter denjenigen Substrukturen, die abgeschlossen sind bzgl. eines algebraischen Hüllenoperators Ȧlg.
Die syntaktische Charakterisierung der ↓-konvexen Theorien war ein offenes Problem.
Mit Hilfe einer Reihe neuer algebraischer Hüllenoperatoren gelingt es diese Resultate zu erweitern. Wir zeigen, daß T genau dann ↓-konvex ist, wenn Mod(T) abgeschlossen ist unter denjenigen Substrukturen, die abgeschlossen sind bzgl. eines algebraischen Hüllenoperators $Alg^{\exists}$. Dies liefert ein nützliches Kriterium für ↓-Konvexität

und führt auf eine syntaktische Charakterisierung der $\downarrow$-konvexen Theorien. Führt man nun für eine Klasse von teilweise geordneten Mengen eine zugehörige *Konvexitätseigenschaft* ein, so kann man zeigen, daß die beiden obigen Beispiele die einzigen nichttrivialen Konvexitätseigenschaften sind. Dabei verwenden wir eine Invariante für teilweise geordnete Mengen, die mißt bis zu welchem Grad die Menge nach unten gerichtet ist.
Verwendet man allgemeinere Einbettungen zwischen Strukturen, so erhält man auch Park's Resultate über elementare Konvexität. Auch Rabin's Resultat läßt sich mit unseren Methoden beweisen. Überdies kann man zeigen, daß im allgemeinen höchstens $\omega+2$ verschiedene Konvexitätseigenschaften existieren.- Die syntaktische Charakterisierung der elementar-konvexen Theorien bleibt ein offenes Problem.

Anhang Die Autorin des folgenden Beitrages mußte ihre Tagungs-Teilnahme leider absagen.

Z.ADAMOWICZ: *Axiomatizations of forcing with an application to Peana arithmetic*

We introduce the notion of a quasi-forcing relation and of a consistency relation and define it by means of algebraical conditions. Forcing in set theory provides an example of a quasi-forcing relation. We develop a general theory of quasi-forcing and consistency relations and show their mutual correspondence. Then it is shown that in the Kirby-Paris theory of indicators in Peano Arithmetic there occurs a consistency relation. Thus we are able to define an appropriate quasi-forcing relation. However, the consistency relation is definable arithmetically by means of the Paris game and the quasi-forcing relation is not definable in PA. We show several analogies between the quasi-forcing relation that we have defined for Peano Arithmetic and forcing in set Theory. We also derive from our methods a few observations about models of Arithmetic.

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