MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 21/1980

Combinatorics

4. 5. bis 10. 5. 1980

Die Tagung fand unter der Leitung von D. Foata (Strasbourg) statt. Das Schwergewicht in der Thematik der Vorträge und Diskussionen lag im Bereich der kombinatorischen Enumeration und der erzeugenden Funktionen. Das enge und fruchtbare Verhältnis zu Gebieten der additiven Zahlentheorie (Partitionen), der Darstellungstheorie (symmetrische Gruppe, symmetrische Funktionen, Charaktere, etc.) und der Speziellen Funktionen (hypergeometrische Reihen, orthogonale Polynome, etc.) wurde besonders betont, sei es einerseits in der Anwendung spezifischer Methoden und Resultate dieser Disziplinen auf kombinatorische Fragestellungen, wie auch andererseits in der kombinatorischen Interpretation analytischer/algebraischer Resultate. Das Institut mit seinen vielfältigen Möglichkeiten und seiner entspannten Atmosphäre bot den Teilnehmern einen idealen Rahmen für intensive Kontakte und erfolgreiche Arbeit. Besonderer Dank gebührt dem Leiter, Herrn Prof. Dr. M. Barner, und seinen Mitarbeitern für ihre Unterstützung bei der Vorbereitung und der Durchführung der Tagung, eingeschlossen ihre Geduld bei der Verminderung organisatorischer Reibungen.

Vortragsauszüge

G. E. ANDREWS:

Ramanujan's "Lost" Notebook

In Ramanujan's "Lost" Notebook (cf. Amer. Math. Monthly 86(1979), 89-108) there are numerous q - identities with implications for additive number theory and combinatorics. The object of this talk is to describe the mathematical setting of these results and to describe some of the most surprising. For example, suppose that $\Sigma_{n\geq o} C_n q^n$ is the power series expansion of the Rogers - Ramanujan continued fraction:

$$\Sigma_{n\geq o} C_n q^n = 1 + \cfrac{q}{1 + \cfrac{q^2}{1 + \cfrac{q^3}{1 + \ddots}}}$$

and let $B_{k,a}(n)$ denote the number of partitions of n of the form $n = \lambda_1 + \lambda_2 + \ldots + \lambda_s$ where $\lambda_i - \lambda_{i+1} \geq o$, $\lambda_i - \lambda_{i+k-1} \geq 2$ and at most a-1 of the λ's equal 1.

Ramanujan asserts a number of analytic results which directly imply that $C_{5m} = B_{37,37}(m) + B_{37,13}(m-4)$, $C_{5m+1} = B_{37,32}(m) + B_{37,7}(m-6)$, and similar results for C_{5m+2}, C_{5m+3} and C_{5m+4}.

D. M. BRESSOUD:

A combinatorial technique for partition theory

A general technique for obtaining both direct correspondence proofs of partition identities and interpretations of generating functions is to consider re-orderings of underlying partitions.

This technique has yielded a correspondence proof of Schur's partition theorem and a proof that Andrew's multisum expression for the Alder polynomials generates the partition function of Gordon's identity. The theory behind this technique will be explained and several examples will be given.

D. I. A. COHEN:

Ramanujan's unwritten notebook

The ghost of the mathematician has communicated a wide range of results using the technique of PIE-sums (Partition Inclusion-Exclusion).

Theorem 1: The number of partitions of n into an even number of distinct parts $\equiv 0, \pm x \pmod{y}$ minus the number of partitions of n into an odd number of distinct parts $\equiv 0, \pm x \pmod{y}$ is zero unless $n = \frac{1}{2}k(yk \pm (y - 2x))$ in which case the difference is $(-1)^k$.

Corollaries of this include Euler's theorem (even-odd) and other amazing things (including the Jacobi triple product).

Let A be the collection of the multisets a_i. Let $\bar{a}_i$ be the sum of the numbers in a_i. Let L_A be the lattice formed by the union of the a_i. Let μ be the Möbius function of this lattice.

Theorem 2: The number of partitions of n which do not contain any set a_i is

$$\sum_{\pi \in L_A} \mu(\pi) p(n - \bar{\pi}) = \sum_{j=0}^{\infty} c_j p(n-j) \quad , \text{ where } c_j = \sum_{\bar{\pi}=j} \mu(\pi) .$$

Theorem 3: Let $A = \{a_i\}$, $B = \{b_i\}$ where the a_i are disjoint and the b_i are disjoint and $\bar{a}_i = \bar{b}_i$ for all i. Then the number of partitions of n which include no a_i is equal to the number of partitions of n which include no b_i.

Thousands of corollaries now appear. As a sample:

The number of partitions of n into parts no one of which is an odd multiple of 3

= the number of partitions of n where there are no two consecutive non-multiples of 3

= the number of partitions of n which do not contain an odd number m and its double 2m.

R. CORI:

Planar maps and well labeled trees

We present a bijection between planar maps and well labeled (or Motzkin) trees: these trees are such that positive integers are assigned to the vertices in such a way that labels differ by at most one on adjacent vertices. As a consequence of this result a combinatorial proof of the formula (of W.T. Tutte) enumerating rooted planar maps with m edges is established.

D. DUMONT:

Les fonctions elliptiques de Jacobi: une approche combinatiore

Deux extensions des fonctions elliptiques sn, cn, dn, sont étudiées: les polynomes de Schett à trois variables (X_n), et quatre fonctions Sn(u), Cn(u), Dn(u) et En(u) qui sont définies par le système différentiel suivant:

$$\frac{d}{du}\,Sn(u) = Cn(u)Dn(u)En(u) \qquad Sn(o) = o$$

$$\frac{d}{du}\,Cn(u) = a^2 Sn(u)Dn(u)En(u) \qquad Cn(o) = 1$$

$$\frac{d}{du}\,Dn(u) = b^2 Sn(u)Cn(u)En(u) \qquad Dn(o) = 1$$

$$\frac{d}{du}\,En(u) = c^2 Sn(u)Cn(u)Dn(u) \qquad En(u) = 1\ .$$

On montre deux théorèmes combinatoires sur ces deux extensions, en termes de <u>pics de cycles</u> dans les permutations. Soit σ une permutation de $\{1,2,...,n\}$. Un entier p est dit pic de cycle si $\sigma^{-1}(p) < p > \sigma(p)$. En dénombrant les permutations suivant leurs nombres de pics de cycle pairs et impairs, on obtient les coefficients des polynomes $(X_n(x,y,z))$ définies par

$$X_o = x$$

$$X_n = yz\frac{\partial}{\partial x}X_{n-1} + zx\frac{\partial}{\partial y}X_{n-1} + xy\frac{\partial}{\partial z}X_{n-1}\ .$$

En se restreignant aux permutations en cycles des longeurs paires, on obtient les coefficients de Cn(u).

Le polynome X_n est aussi le polynome énumérateur des arbres binaires étiquetés de taille n suivant leurs nombres de branches de longeur paires, de branches gauches de longeurs impairs, et de branches droites de longeurs impaires.

Ch. F. DUNKL:

Orthogonal polynomials in several discrete variables

Consider the subgroup of S_N, the symmetric group on a set with N elements, which leaves two given disjoint subsets invariant. This subgroup is isomorphic to $S_{\ell_1} \times S_{\ell_2} \times S_{\ell_3}$ where $\Sigma_i \ell_i = N$, a Young subgroup of S_N, and will be donoted by H_ℓ . The space of functions, expressed as polynomials, on S_N which are invariant under H_ℓ is decomposed with respect to the irreducible representations of S_N and of $S_{\ell_1} \times S_{\ell_2+\ell_3}$. It is useful to consider the intermediate group because there is generally more than one invariant in each irreducible representation. The decomposition is performed by means of differential operators. Further, the functions on S_N can be restricted to the coset--space $H_r \backslash S_N$ (where $H_r = S_{r_1} \times S_{r_2} \times S_{r_3}$, $\Sigma_i r_i = N$) and thus $H_r \backslash S_N / H_\ell$ - invariants are obtained. Next, orthogonal (as functions on $H_r \backslash S_N$) isomorphic copies of the same representation of S_N are constructed. Finding the H_ℓ - invariants amounts to solving certain difference equations. The resulting expressions involve Hahn-polynomials in one and two variables.

D. FOATA:

Divisibility and congruence properties of the q - Euler numbers

For $n = m\,2^{\ell}$ with m odd and $\ell \geq o$ let $Ev_n(q) = \Pi_{o \leq j \leq \ell}(1 + q^{m2^j})$ and define $D_n(q) = \Pi_{1 \leq i \leq n} Ev_i(q)$ for n odd and

$$= (1 + q^2)\,\Pi_{1 \leq i \leq n} Ev_i(q) \quad \text{for n even.}$$

It is shown that for every $n \geq 1$ the polynomial $D_n(q)$ divides the q - tangent number $T_{2n+1}(q)$.

As for the q - Euler number $E_{2n}(q)$ the congruence

$$E_{2n}(q) \equiv q^{2n(n-1)} \bmod (q+1)^2$$

is derived by means of the classical combinatorial interpretations of the tangent and secant numbers in terms of alternating permutations.

I. GESSEL:

Symmetric functions and permutation enumeration

The descent set of a permutation π is the set $\{i:\pi(i) > \pi(i+1)\}$. To certain sets of permutations we can associate symmetric functions which record the descent sets of these permutations. Thus $\Pi_{i,j} \frac{1}{1-x_i y_j}$ counts all permutations by their descent sets and the descent sets of their inverses, and $\Pi_i \frac{1}{1-x_i} \Pi_{i<j} \frac{1}{1-x_i x_j}$ counts involutions by their descent sets. From these symmetric functions generating functions for counting permutations by number of descents and sum of descent (greater index) can be obtained.

J. GILLIS:

Applications of Hermite polynomials

Given k finite sets of distinguishable elements, with cardinals $n_1, n_2, \ldots, n_k$ respectively we define

$P_{n_1,\ldots,n_k}$ = the number of k-partite graphs of degree 1 which can be constructed of them,

$E_{n_1,\ldots,n_k}$ ($\Omega_{n_1,\ldots,n_k}$) = the number of graphs of degree 1 which can be formed from the Σn_i elements by joining them in pairs such that the number of edges joining pairs of elements from the same set is even (odd) ,

$$D_{n_1,\ldots,n_k} = E_{n_1,\ldots,n_k} - \Omega_{n_1,\ldots,n_k} .$$

It is shown that

$$P_{n_1,\ldots,n_k} = \{2^{\Sigma n_i}\pi\}^{-1/2} \int_{-\infty}^{+\infty} e^{-x^2} \Pi_{1 \le i \le k} H_{n_i}(x)\, dx,$$

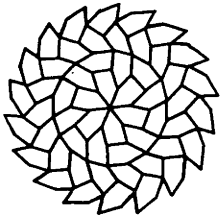
where the $H_{n_i}(x)$ are Hermite polynomials, and

$$D_{n_1,\ldots,n_k} = \sqrt{\frac{2}{\pi}} \int_{-\infty}^{+\infty} e^{-2x^2} \Pi_{1 \le i \le k} H_{n_i}(x)\, dx .$$

Asymptotic estimates are obtained for the special case $n_1 = \ldots = n_k = r$. Moreover it is shown that

$$D_{r,r,\ldots,r} > o.$$

for any even k.



M. D. HIRSCHHORN

Finite continued fractions of the Rogers-Ramanujan type, with applications to partitions

In generalizing certain results concerning continued fractions due to Ramanujan and Gordon, I was led to consider

$$1 + a + b + \frac{cq-a|}{|1+a+bq} + \cdots \frac{cq^n-a|}{|1+a+bq^n} = \frac{P_{n+1}(a,b,c,q)}{P_n(a,bq,cq,q)} .$$

Using the ordinary generating function, one can find various explicit expressions for the P_n, the simplest of which is

$$P_n = \sum_{l,m\geq 0} (-1)^m q^{\binom{m+1}{2}} a^m (c/a)_m a^l (-bq^m/a)_l \begin{bmatrix} n-m \\ m \end{bmatrix} \begin{bmatrix} n-2m \\ l \end{bmatrix} .$$

A study of the polynomials P_n has already led to

(i) a new polynomial identity implying the Rogers-Ramanujan identities,

(ii) a combinatorial explanation of two hitherto mysterious identitied of Slater,

(iii) a new proof of Sylvester's partition theorem.

M. HOARE:

Combinatorial models, collective games and orthogonal polynomials

Three types of model games are discussed which lead to solutions in orthogonal polynomials of a discrete variable:

A. Two person markovian games.

B. Single player against an (infinite) bank.

C. Collective binary games between players paired at random.

Evolution equations for each can be written giving the distribution of wealth as a function of time. (Linear for A,B; non-linear for C).

An interesting case of A is compound Bernoulli trials. A player has i B.T's for k 'successes' at probability α, followed by $N-k$ second chances for $j-k$ 'successes' with probabilities β. The transition matrix $K_{ji} = \Pr\{i \to j\}$ is a convolution of two binomial distributions and defines a Markov chain. The spectrum of $\underline{K}$ is $\lambda_n = \alpha^n(1-\beta)^n$ and its left eigenvectors are Krawtchouk polynomials $K_n(i,N,\gamma)$ with $\gamma = \beta/[1-\alpha(1-\beta)]$. The stationary distribution ($\lambda_o = 1$) is a binomial with parameter γ.

Under class C we consider binary games with negative hypergeometric distributed outcomes, the 'degree of freedom' parameters p,q now measuring the 'skill' of the players. The resulting 'Boltzmann'-type evolution equation can be solved under certain conditions (e.g. $p = q$) and leads to a negative binomial stationary distribution of wealth. The result, though over-simplified, has certain implications for actual collective games such as the ELO system for rating chess masters.

M. E. H. ISMAIL:

Orthogonal polynomials and combinatorial identities

A sequence of polynomials $P_n(x)$ is orthogonal if and only if there exists a positive measure $d\mu$ such that

$$\int_{\infty}^{\infty} P_n(x)\, P_m(x)\, d\mu(x) = \lambda_n \delta_{m,n} \quad ,$$

where $\lambda_n > o$, $n = o, 1, 2, \ldots$.

A necessary and sufficient condition for orthogonality is that $P_n(x)$ satisfies a three term recurrence relation

$$P_{n+1}(x) = (A_n x + B_n) P_n(x) - C_n P_{n-1}(x)$$

with $A_n A_{n-1} C_n > 0$, $n = 1, 2, \ldots$.

Using Markov's theorem and some recent results on orthogonal polynomials I present a method to recover the measure $d\mu$ from the three term recurrence relation. The orthogonality relations can be thought of as combinatorial identities. One of the examples treated leads to a q-analogue of Euler's formula

$$e^{\alpha z} = \sum_{n \geq o} \frac{\alpha(\alpha+n)^{n-1}}{n!} (z e^{-z})^n \quad .$$

This work is joint with Richard Askey, University of Wisconsin - Madison.

K. W. J. KADELL

A combinatorial approach to Selberg's integral

In 1944 A. Selberg evaluated the important multivariate extension

$$(*)\qquad \int_0^1\cdots\int_0^1 \prod_{j=1}^{n} t_j^{x-1}(1-t_j)^{y-1}\,\Big|\prod_{i<j}(t_i-t_j)\Big|^{2k}dt_1\ldots dt_n$$

$$= \prod_{j=1}^{n}\frac{\Gamma(x+(j-1)k)\,\Gamma(y+(j-1)k)\,\Gamma(1+jk)}{\Gamma(x+y+(n+j-2)k)\,\Gamma(1+k)}$$

of the beta integral. It includes some later results of Mehta and Dyson which occur in physics. R. Askey has recently observed that (*) is equivalent to I. MacDonald's conjecture for BC_n. We give an equivalent combinatorial problem. Such a solution may extend to several (of many) q - analogues of (*) conjectured by R. Askey. These include

$$\int_0^1\cdots\int_0^1 \prod_{j=1}^{n} t_j^{x-1}\frac{(qt_j)_\infty}{(q^y t_j)_\infty}\prod_{i<j} t_i^{2k}\left(\frac{q^{1-k}t_j}{t_i}\right)_{2k} d_qt_1\ldots d_qt_n$$

$$= q^{k\binom{n}{2}+2k^2\binom{n}{3}}\prod_{j=1}^{n}\frac{\Gamma_q(x+(j-1)k)\,\Gamma_q(y+(j-1)k)\,\Gamma(1+jk)}{\Gamma_q(x+y+(n+j-2)k)\qquad\Gamma(1+k)}$$

which yield q - analogues of some conjectures of Dyson and I. MacDonald.

T.H. KOORNWINDER:

Special functions on the symmetric group and on SU(2), a unification of two different group theoretic interpretations

Krawtchouk polynomials $K_n(x;p,N)$ and Hahn polynomials $Q_n(x;\alpha,\beta,N)$ are orthogonal polynomials on a finite set $\{0,1,\ldots,N\}$, which can be expressed as hypergeometric functions. Krawtchouk polynomials have a group theoretic interpretation as spherical or intertwining functions on wreath products of the symmetric group, most generally on $S_N \circ (S_k)^N$ with respect to the subgroups $S_N \circ (S_p \times S_{k-p})^N$ and $S_N \circ (S_{k-1})^N$. Hahn polynomials have a group theoretic interpretation as intertwining functions on S_N with respect to the subgroups $S_p \times S_{N-p}$ and $S_q \times S_{N-q}$. On the other hand, the Clebsch-Gordan coefficients for SU(2) can be expressed in terms of Hahn polynomials such that the orthogonality relations for the C-G coefficients become the orthogonality relations for the Hahn polynomials. First I show a similar group theoretic interpretation for Krawtchouk polynomials, which seems to be new: the matrix elements of the irreducible unitary representations of SU(2) can be expressed in terms of Krawtchouk polynomials, such that, for each fixed element of SU(2), the orthogonality relations for the rows or columns of the representation matrix become the orthogonality relations for Krawtchouk polynomials. Next, I give an intrinsic connection between the two group theoretic interpretations for Krawtchouk polynomials described above, by only using the group theoretic characterizations of these polynomials and not their expression as special functions. I also succeeded in giving a similar intrinsic connection for Hahn polynomials.

A. LASCOUX:

The Kazhdan-Lusztig plactic representation of the symmetric group

(joint work with M.P. Schützenberger)

Kazhdan and Lusztig have shown that for each partition I, there exists a graph Γ_I which describes the representation of the symmetric group S_n associated to this partition. More precisely, the set of vertices is the set of standard Young tableaux of shape I (read as words) and it is labeled by subsets of $\{1,2,\ldots,n-1\}$ (the label S(t) of a tableau t is the set $\{ x \in \{1,2,\ldots,n-1\} ; \cdot\cdot x+1 \cdot\cdot x \cdot\cdot$ is a subword of t $\}$. Now there is a very simple rule to write the Coxeter generators according to the graph and the labeling. It was Kazhdan's and Lusztig's conjecture that the matrices for these generators contained only o,+1,-1; this we solved. The main problem is to build the edges of the graph, and this is done by using properties of the inverse of the plactic relations which are described in "Monoide Plaxique, Combinatorics - Napoli 1978", and used in connection with the finite linear groups.

P. LEROUX:

(Two talks are given, presenting work being done by a team which includes A. JOYAL, G. LABELLE, J. LABELLE at Université du Québec à Montréal)

1 - Homogeneous Gaussian coefficients

Examples of triangular categories, which are special cases of Möbius categories, are presented. They correspond to the familiar triangular arrays of numbers which occur in enumerative combinatorics: binomial, $\binom{n}{k}$; Stirling numbers of 2^{nd} kind, $S(n,k)$, and 1^{st}; Gaussian, $\left[{n \atop k}\right]_q$, $\left[{n \atop k}\right]_{x,y}$, etc.

2 - A structural calculus for generating functions

Set theoretic operations are defined on "structure types" corresponding to addition, multiplication substitution and derivation of exponential generating functions. A "one slide proof" in color is given of Cayley's formula for the number n^{n-2} of trees on an n-element set.

S.C. MILNE:

Hypergeometrics series well poised in SU(n) and a generalization of Biedenharn's G-functions

We first proove that Holman's multivariable generalization of classical well poised hypergeometric series satisfies a general contiguous relation. We then make use of this general relation and Biedenharn's "path sum formula" to give not only an analytical but also a group theoretical formulation of a class of multivariable special fuctions ${}_{(n-2)}G_q^{(n)}(X)$ which provide a U(n) generalization of Biedenharn and Louck's $G(\Delta;X)$ functions for U(3). A detailed study of their symmetries and zeros implies that these generalized G-functions are polynomials. This fact is equivalent to the existence of an SU(n) generalization of Holman's SU(3) extension of Whipple's transformation of classical hypergeometric series. A further application of our general contiguous relations yields

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an elementary proof of Holman's U(n) generalization of the $_5F_4(1)$ summation theorem. Suitable "q-analogs" of $_{(n-2)}G_q^{(n)}(X)$ may well be related to a "U(n) analog" of the Rogers-Ramanujan identities.

W. OBERSCHELP:

Rook polynomials and matching polynomials

We give an operator theoretical formula for developing the matching polynomial $M(x) = \sum_{\nu=1}^{n} m_\nu x^\nu$ for each graph with n nodes. This formula keeps track of the different edges which occur in those matchings, and can be viewed as an algebraic description of an algorithm which generates all matchings of a given graph. For the special case of bipartite graphs the formula develops into the rook theory, developed by the author in 1973 and by Goldman, Joichi and White (JCT 1976). The connection to a theory of matching polynomials by Farrell (JCT-B 1979) is described as well as the connection to the calculation of the permanent. Since calculation of the permanent is a "hard" problem (Valiant, Theor. Comp. Sci, 1979), there is no hope of giving simple calculations for matching and rook polynomials. The theory is also possible for directed graphs and those with edge valuations.

O. PRETZEL:

Combinatorial dimension of partially ordered sets

Definitions:

1. Order R on poset S : $a \leq b \ (S) \longrightarrow a \leq b \ (R)$;

   base on S : set $\{R_1, \ldots R_n\}$ of total orders on S s.th.

   $$a \leq b \ (S) \longleftrightarrow \forall_i \quad a \leq b \ (R_i) \qquad ;$$

   dim S := minimum cardinality of a base on S .

2. Location of S : $x = (D,U)$, D down set of S,

   U = up set of S , $D < U$;

   L(S) := set of locations ;

   order on $S \cup L(S)$: order on S preserved,

   $$s \in S < (>) \ x = (D,U) \longleftrightarrow s \in D (s \in U);$$

   U(S) : $(D,U) \leq (D',U') \longleftrightarrow D \subseteq D', U \supseteq U'$ [universal extension of S]

   I(S) : $(D,U) < (D',U') \longleftrightarrow U \cap D' \neq \emptyset$ [fuzzy version of s].

3. Double antichain: Hasse diagram is a complete bipartite graph.

Theorems:

1. dim U(S) = max cardinality of double antichain subset of S

2. $S \subseteq X \longrightarrow$ dim X $\leq$ dim $X \setminus S$ + dim U(S)

3. (Bogart, Rabinovich, Trotter) S chain of height n

   $$\longrightarrow \quad \dim I(S) \sim \log_a(n) \quad (1<a<2, \text{ fixed})$$

4. S double antichain with parts of size $o \leq m \leq n$:

   $$\dim I(S) = \max_{\substack{r \leq n \\ s \leq m}} \left\{ \binom{n}{r} , \left\lceil \frac{2}{3} \left(\binom{m}{s} + \binom{n}{r} \right) \right\rceil \right\} .$$

D. RAWLINGS:

A generalization of the Worpitzky identity with applications to the permutation enumeration

A generalization of the Worpitzky identity for the Eulerian numbers provides a set-up for converting certain generating functions on finite sequences into (t,q)-generating series on permutations. The two additional indeterminates in the permutation case arise in connection with the descent number and the major index of the inverse of the permutation. As an example, the following (t,q)-generalization of the classic result of André on down-up permutations

$$\sum_{n\geq 0} \frac{T_n(t,q)u^n}{(t;q)_{n+1}} = \sum_{r\geq 0} t^r\left[\frac{2 - i[\pi(u,r) - \pi(-u,r)]}{\pi(u,r) + \pi(-u,r)}\right]$$

where $\pi(u,r) = \prod_{k=0}^{r}(1 + i\,u\,q^k)$, $(t;q)_{n+1} = (1-t)(1-tq)\ldots(1-tq^n)$,

and $i = \sqrt{-1}$,

my be derived from a result due to Carlitz on down-up sequences. Furthermore, the polynomial $T_n(t,q)$ does generalize the q-tangent and q-secant numbers obtained when counting down-up permutations by inversions.

A. RONVEAUX:

Survey on Polynomial Harmonic Functions invariant under the Cubic Group

Polynomial invariant functions under the cubic group (O_{24}) can be build systematically using the so called integrity basis of the cubic group, but these functions are not harmonic. On the other side, solutions of Laplace equation in cubic geometry ask to know all invariant harmonic functions. Such a set can be obtained by summation of Spherical Harmonics but this process gives an overcomplete basis.
This survey indicates an analytical approach in cartesian coordinates.

R. STANLEY:

The Number of Faces of a Simplicial Convex Polotype

In 1971 McMullen (Israel J. Math. 9 (1971), 559-570) conjectured that a certain condition on a vector $(f_o, f_1, ..., f_{d-1})$ of integers was necessary and sufficient for the existence of a simplicial convex d-polytope with f_i faces of dimension i. In 1979 Billera and Lee proved the sufficiency of McMullen's condition. We will outline a proof of necessity, thereby completely verifying McMullen's conjecture. The proof uses the theory of toric varieties as developed by Demazure, Mumford, and others, together with a version of the hard Lefschetz theorem due to Steenbrink.

D. STANTON:

Partial orders and q-Krawtchouk polynomials

There are two different families of q-Krawtchouk polynomials which are eigenvalues of q-Hamming schemes. Partial orders related to both possibilities are given. A lowering operator on the partial order is used to derive explicit formulas for the polynomials. Some irreducible representations of classical groups over a finite field are obtained as a corollary.

D. STOCKHOFE:

Certain bijections on the set of partitions of a natural number

Let $\alpha = (\alpha_1,\dots,\alpha_n)$ be a partition of n, $n \in \mathbb{N}$. Then for each $1 \leq i \leq n-1$, $q \in \mathbb{N}$ uniquely determined integers $t_i \geq 0$, $0 \leq \kappa_i \leq q-1$ are given by the equation $\alpha_i - \alpha_{i+1} = t_i q + \kappa_i$. For each q a bijection L_q on the set of partitions P(n) can be constructed such that for each $k,\ell \in \mathbb{N} \cup \{0\}$ the set of partitions with $\Sigma_i t_i = k$ and ℓ parts divisible by q is mapped onto the set of partitions with $\Sigma_i t_i = \ell$ and k parts divisible by q. It turns out that L_q is something in between conjugation of partitions (q=1) and the identity mapping ($q \geq n$). Some partition identities with difference conditions to the parts were derived by this construction, as special cases (q=2) one obtains results of Euler, Sylvester, Fine. The group generated by $L_1,\dots,L_{n-1}$ is the symmetric group on P(n).

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G. VIENNOT:

Determinants in colors and pictures

We give an interpretation of a general determinant with the weight of some configuration of non-crossing paths. For some particular weights and paths, this general theorem gives a "bijective" proof for many determinants involved in combinatorics: determinants with some binomial coefficients, Stirling numbers, up-down sequence of permutations, number of Young tableaux, "Kreweras dominance theorem", plane partitions, Schur functions, Hankel determinants of moments of orthogonal polynomials.

D. WHITE:

Monotonicity and Unimodality of the Pattern Inventory

It is shown that the Kostka numbers respect a natural partial order (reverse domination) on the integer partitions. It is then proved that the number of Pôlya patterns with a given weight also respects this order. Immediate consequences include the unimodality of the number of graphs with n vertices and k edges and the unimodality of the coefficients of the expansion of the Gaussian coefficient $\left[{m+n \atop n}\right]$.

Berichterstatter: V. Strehl

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