

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 22/1980

Gruppen und Geometrien
11.5. bis 17.5.1980

Die Tagung fand unter Leitung von Herrn B. Fischer (Bielefeld) und Herrn D.G. Higman (Ann Arbor) statt. Es haben 33 Mathematiker teilgenommen, und es wurden 25 Vorträge gehalten.

Wie den folgenden Auszügen zu entnehmen ist, wurden Themen aus der Theorie der endlichen Gruppen, der Graphentheorie und der Geometrie behandelt. Insbesondere wurden dabei Zusammenhänge zwischen Eigenschaften kombinatorischer Strukturen und denen ihrer Automorphismen-Gruppe dargestellt. Außerdem wurde über die neuentdeckten Beziehungen zwischen endlichen einfachen Gruppen und Modulfunktionen berichtet.

An einem der Abende fand ein Problemkreis statt, in dem von den Teilnehmern aktuelle Probleme und Fragen gestellt und erörtet wurden.

Neben diesen offiziellen Veranstaltungen hatten die Teilnehmer Gelegenheit zu gemeinsamer Arbeit und Diskussionen in kleinerem Kreis.

Liste der Tagungsteilnehmer

Babai, L. (Budapest) Bannai, E. (Columbus) Batten, L.M. (Brüssel) Baumann, B. (Bielefeld) Bender, H. (Kiel) Cohen. A.M. (Amsterdam) Damerell, R.M. (Egham) Fischer, B. (Bielefeld) Hähl, H. (Tübingen) (Tübingen Hering, Ch. Higman, D.G. (Ann Arbor) Hähl, H. Tübingen Hering, Ch. (Tübingen) Janko, Z. (Heidelberg) Lefevre-Percsy, Chr. (Brüssel) Liebler, R. (z.Z. Tübingen) Livingstone, D. (Birmingham) Mathon, R.A. (Totonto) McKay, J. (Montreal) Neumaier, A. (Berlin) Norton, S. (Cambridge) Percsy (Mons) Ronan, M.A. Chicago) Rowlinson, P. (Stirling) Smith, St.D. (Chicago) Stellmacher, B. (Bielefeld) Strambach, K. (Erlangen) Suzuki, M. (Urbana) Timmesfeld, F. (Gießen) Virotte, M.M. (Paris) Wagner, A. (Birmingham) Walter, J.H. (Urbana). Weiss, R. (Berlin) Wilbrink, H.A. (Amsterdam) Zara, F. (Amiens)

Vortragsauszüge:

L. BABAI : Coherent configurations and the order of primitive permutation groups

Let G be a primitive but not doubly transitive permutation group of degree n. H. Wielandt proved that $|G| < 4^n$ and asked if the exponent n could be replaced by $c\sqrt{n} \log n$. We solve this problem apart from a $\log n$ factor.

Theorem. For G as above, $|G| < \exp(4\sqrt{n} \log^2 n)$.

The proof is purely combinatorial. We translate the problem to coherent configurations (the digraph version of association schemes) and prove a stronger result about them. The arguments use hypergraph cover and considerations relating degree and diameter of regular graphs. No groups occur in either the statement or the proof of the main result.

Recently P.J. Cameron has greatly improved on the above bound, assuming the classification of finite simple groups. His results motivate conjectures on coherent configurations, in particular on strongly regular graphs.

E. BANNAI: On (P U Q)-polynomial schemes with large diameters

A list of known P-polynomial (i.e., distance-transitive graphs) or Q-polynomial schemes with large diameters was presented. They are divided into 3 categories:
(I) analogues of J(v,k), (II) analogues of H(n,q), and (III) variations of (I) and (II). They are closely related to classical groups, and they should play the same role in combinatorics as the classical groups do in finite group theory.

Conjecture I If d is sufficiently large, then any P-polynomial scheme has structure of Q-polynomial scheme, and vice versa.

(This was checked for all known examples with large diameters. In particular the odd graph O_k has a Q-polynomial structure which seems to have been unnoticed before. So the above list is in fact the list of known (P U Q)-polynomial schemes with large diameter d).

Conjecture 2 If d is sufficiently large, then any (P U Q)-polynomial scheme is in the above list.

Finally a recent result of D. Leonard which says that all the parameters of any (P U Q)-polynomial scheme are expressed by using only 6 parameters k, a_1 , a_2 , c_2 , 0 and μ_1 was mentioned.





L.M. BATTEN: Quadrics and the Higman-Sims geometry

It may be the case that some of the so-called 'sporadic' finite simple groups can be shown to belong to families of groups. In this paper we derive a connection between the Higman-Sims group and those groups associated with quadrics, via corresponding geometries. An axiom system is introduced in which when lines all have two points we obtain precisely the Higman-Sims geometry, and when lines have more than two points, we obtain certain generalized quadrangles which include quadrics of the type $\P_4(q)$.

B. BAUMANN: Some properties of the Thompson subgroup

Let P be a p-group (p a prime). Then $J(P) = \langle A \leq P \mid A$ is elementary abelian of maximal order > is the Thompson subgroup of P. The following theorem holds:

Theorem. Let G be a finite group and P a Sylow 2-subgroup of G. Assume $G = \langle J(P)^G \rangle$, $C_G(O_2(G)) \subseteq O_2(G)$, $V = \langle \Omega_1(Z(P))^G \rangle$ and $\overline{G} = G/C_G(V)$. Then there is a subgroup $H = H_1 \times \ldots \times H_n \times E(\overline{G})$ of \overline{G} . Let H_{n+1}, \ldots, H_m be the components of \overline{G} (i.e. $E(\overline{G})$ is a central product of H_{n+1}, \ldots, H_m). Then

(i) H, ₫ G (i=1,...,m), (ii) G/H is a 2-group,

(iii)
$$H_1 \times ... \times H_n \times Z(E(\overline{G})) = Sol(\overline{G})$$
, (iv) $H_1 \simeq E_3$ (i=1,...,n),

(v) $[[v,H_i],H_i] = 1$ (i + j).

A.M. COHEN: A near octagon associated with the Hall-Janko group

Let P be the set of involutions of the Hall-Janko group that are central in a 2-Sylow subgroup and let L be the set of triples in P consisting of pairwise commuting involutions.

Then (P,h) is a near octagon on 315 points with 3 points per line and 5 lines per point without 4-cycles.

Study of the generalized subhexagons leads to Buekenhout's diagram and in a way that does not look very promising for the existence of analogs of 'Yanushka's lemma' on generalized subquadrangles.

R.M. DAMERELL: Factorisation of Polynomials

The study of algebraic combinatorics leads to many number-theoretical problems of the following type: given an infinite family F of polynomials, to determine which members of F have factors of small degree. No satisfactory method is yet known for the solution of these problems, but the method of local analysis sometimes succeeds. This talk will describe the methods for factorising polynomials over p-adic fields, with illustrations from various combinatorial problems. Most of the work described here was done jointly with E. Bannai and M. Georgiacodis.

H. HÄHL: A new class of 8-dimensional locally compact translation planes
We discuss toplogical 8-dimensional translation planes over R admitting
SL₂(C) as collineation group. (All such planes can be explicitly determined;
they are in fact the first known examples of non-classical 8-dimensional
translation planes over R in which the connected component of the collineation
group has no fixed point.) We give a description of these planes and of the
action of their collineation groups, and we explain the role played by these
planes in the classification of all 8-dimensional translation planes over R
having large collineation groups.



Z. JANKO: Problem der Existenz der projektiven Ebene der Ordnung 12

Es wird bewiesen, daß die Automorphismengruppe einer Ebene der Ordnung 12 eine sehr kleine {2,3}-Gruppe sein muß. Die Hauptschwierigkeit im Beweis ist die Behandlung der Automorphismen der Ordnung 5, 11 und 13.

C. LEFEVRE-PERCSY: Foldings of thin geometries

Tits defines "foldings" of thin chamber complexes and uses them in the study of Coxeter complexes and their relation with buildings.

We consider foldings of a particular class of chamber complexes:

Geometries, in the sense of Buekenhout and Tits.

We prove, with S. Guillitte, that every folding of a thin and "strongly connected" geometry is associated to a reflection. By constructing a class

of counter-examples, we show that this is not true if the connexity of

R. LIEBLER: Autotopism Group Representations

the geometry is somewhat weaker.

If G is an autotopism group of a finite plane II, then G fixes a triangle P,Q,R and normalizes the groups of (P,PQ)-perspectivities, (Q,PQ)-perspectivities and (R,RP)-perspectivities. Since each of these is elementary abelian, they may be viewed as G-modules. The main theorem discussed here asserts that these modules are all free just in case G fixes a point not on the triangle. This and related results will appear in the Journal of the London Mathematical Society.

D. LIVINGSTONE: Kirdar's Identity

If $[\lambda]$ and $\{\alpha\}$ denote the same partition of n with $[\lambda]$ an abbreviation for the set of parts $\lambda_1 \geq \lambda_2 \geq \lambda_3 \ldots$ and $\{\alpha\}$ one for the set of repetition numbers a_1, a_2, \ldots, a_1 being the number of λ_r equal to i, then

This simple identity is an immediate consequence of a theorem in



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representation theory. But, taken out of this context, it seems that it could be difficult to prove.

R. A. MATHON: On Geometric Association Schemes with 3 classes

A 3-dimensional partial geometry Pa G_3 is a linear incidence structure consisting of points P, lines L, and planes M satisfying

- (i) Every line L E L contains s+1 points.
- (ii) If d(x,y) = i then exactly t_i+1 lines through y contain points at distance i-1 from x, i = 2,3 (x,y $\in P$).
- (iii) If d(x,L) = j then $|\{y \in L | d(x,y) = j\}| = \alpha_j$, j = 1,2 $(x \in P, L \in L)$.
- (iv) Every pair of intersecting lines from L is contained in a unique partial geometry with parameters (s,t₂,α₁) (= plane).

Then the following statements hold:

- 1. The point graph of a $Pa G_3$ is the graph of a distance regular association scheme with 3 classes.
- 2. The planes of a Pa G_3 are either nets $(\alpha_1 = t_2)$, or duals of Steiner 2-designs $(\alpha_1 = t_2+1)$, or generalized quadrangles $(\alpha_1 = 1)$. In the last case $\alpha_2 = 1$ and $t_3 \le s^3 + t_2(s^2 - s + 1)$.

Examples are given of geometries with planes of any of these types.

J. MCKAY: Coxeter graphs, character tables, platonic solids, and singularities

I shall describe the representation graph of a group and classify a generalization of the Coxeter graph related to affine Dynkin diagrams which has connections with the character tables of certain finite groups themselves related to the structure of rational singularities as studied by Klein. One can obtain the finite rank Dynkin diagrams in a uniform way from a single universal diagram together with 'folding' - an incidence - and weight-preserving map from a graph to its quotient under an automorphism. There appear to be connections with the Monster group.



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S. NORTON: Modular Functions associated with groups

It was shown how the j-function invariant under the modular group I could yield many other modular functions by replacing its coefficients by Monstercharacters of the same degree, and that the process could be replaced on these functions by replacing their coefficients by characters of smaller groups. (However further iteration gives no more functions, as far as is known!). It is apparently true that all such functions are invariant under a genus zero discrete subgroup of $PGL_2(2)$ containing some $\Gamma_2(n)$ (but not z+z+k for non-integral k). It is also true that these functions obey certain formulae, known as the replication formulae, associated with character identities on the above groups. Some computation work was illustrated identifying 72 functions of odd level satisfying the replication formulae with Hauptmoduls of modular groups of the type above. A conjecture was made that the two classes of functions are identical, although this has not been proved in either direction. The existence of functions of order 58 and 82 suggests, as a wild possibility, the existence of a (presumably) new sporadic group associated with $\sqrt{j(2z)-1728}$ (of order 2) containing elements of orders 29 and/or 41.





N. PERCSY: Locally embeddable geometries

Several recent results prove that certain locally embeddable geometries are actually embeddable (in some generalized projective space). So are locally projective geometries (Wille), locally affine ones (Mäurer), locally strongly embeddable or locally "categorically" embeddable ones (Kantor).

All geometries considered here are geometric lattices; embeddings are isometric in Kantor's sense (i.e. order- and dimension-preserving injective mappings).

We prove a necessary and sufficient condition for a geometry G of rank > 4 to be embeddable: the condition demands that G be locally embeddable and that these embeddings satisfy certain compatibility conditions. This theorem provides a unique general proof of the above ones and has new applications, e.o. to locally affine-projective geometries.

M.A. RONAN: Representations of Chevalley Groups using Buildings

If Δ is a building whose vertices have types $i \in I$, then for any subset $J \subset I$ we let Δ^J denote the subcomplex spanned by the vertices of types $j \in J$. We will show that the reduced homology of Δ^J is trivial except in dimension equal to the dimension of Δ^J .

We consider the action of $G = \operatorname{Aut} \Delta$ on this top homology group. For example, if J = I it is well-known (Curtis-Solomon) that one obtains the Steinberg representation, although in general the representation on $\operatorname{H}_{\mathbf{m}}(\Delta^J)$ (m=|J|-1) turns out to be reducible. However, in the case of Δ_n , if $J = \{1, \ldots, k\}$ where the nodes of the diagram are numbered in increasing order $\frac{1-2}{2} \cdots \frac{n-1-n}{2}$, then the representation on $\operatorname{H}_{k-1}(\Delta^J)$ is irreducible. We will consider some other special cases, one of which is closely related to a representation of $\operatorname{M}_{2\Delta}$.

P. ROWLINSON: The uniqueness of certain automorphic graphs

It is shown that there is only one automorphic graph Γ with intersection array {12, 10, 5; 1, 1, 8}. It is possible to find $|\operatorname{Aut}(\Gamma)|$ by inspecting the subgraph of Γ induced by the α - ρ paths of length 3, where α and ρ are two fixed points of Γ at distance 3 apart.





The uniqueness of an automorphic graph with array { 1, 9, 9; 1, 2, 4} follows from a result of Yamushka on generalized hexagons.

St. D. SMITH: 2-local geometries for sporadic (and classical) groups

Mark Ronan and the author have studied natural geometries for certain sporadic simple groups, provided by low-dimensional GF(2)-modules; these exhibit many similarities with buildings for Chevalley groups, and can be axiomatized by Dynkin-like diagrams. In fact, these geometries correspond to chamber systems in the sense of Tits, provided by 2-constrained 2-locals of the groups.

For example, one can show that the relevant geometries characterize $M_{2\Delta}$ and Co.1, providing new constructions of these groups. The minimal modules for He and Ru display certain interesting features.

Finally, this approach seems to provide some useful methods for exhibiting the action of the parabolics in a Chevalley group, defined over GF(2), on its various irreducible GF(2)-modules.

B. STELLMACHER: Automorphism groups of trees

G be a group and H₁ and H₂ be finite subgroups of G such that

- $G = < H_1, H_2 >$. (*)
- (2) H_1 and H_2 have no non-trivial normal subgroups in common. (3) $|H_1/H_1 \cap H_2| = 2^{u_1}+1$, and there exists $L_1 \subseteq H_1$ such that $O_2(H_1) \subseteq L_1$ and $L_1/O_2(H_1) \cong L_2(2^{u_1})$ (i=1,2).

In his paper "Automorphisms of trivalent graphs" D. Goldschmidt considered this situation for u, = u, and described the structure of H1, H2. For this purpose he transformed this problem into a problem about edge-transitive groups of automorphisms of a trivalent tree, whose vertex stabilizers are finite.

This method yields similar results in the more general case of hypothesis (*).



M. SUZUKI: Transfer

A general method of obtaining the transfer homomorphism is discussed. A typical result is the following. Let G be a finite group, S an S_p -group for some prime p, and let A be a normal subgroup of S. Suppose that H is a subgroup of G which contains $N_G(A)$ and that A is of Sylow type in H (i.e. $A^X \subset H$ for some $x \in G$ implies that $A^X = A^h$ for some $h \in H$). Then, one of the following three cases occur: (1) $G/O^p(G) \simeq H/O^p(H)$, (2) for some conjugate H^X , $A \cap H^X$ is a proper subgroup of A and $A \cap H^X = N_A(B)$ for some subgroup B of $A \cap H^X$ with index $|A \cap H^X : B| = p$, or (3) p=2 and for some conjugate H^X , $|A:A \cap H^X| = 2$ and there is a normal subgroup B of A such that $|A \supset A \cap H^X| \supset B$ with |A:B| = 4. This generalizes a recent lemma of Solomon-Wong.

A. WAGNER: Invariants of reflection und transvection groups

Theorem: Let $G \subseteq GL(V,k)$ where V is a finite dimensional vector space over the field k of characteristic O. If G is generated by reflections or transvections then the ring of invariants of G is generated by algebraically independent elements (i.e. is a polynomial ring).

The proof is short and elementary.

- Note: 1. If G is finite the theorem is well-known: Shephard and Todd (1954); Chevalley (1955)
 - 2. If G is finite then the converse of this theorem is true: Shephard and Todd (1954)
 - 3. If G is infinite then the converse of this theorem is false.
 - 4. If the characteristic of k is not zero then this theorem is false: Nakajima (1979).

J.H. WALTER: Characterizations of Chevalley Groups

The purpose of the talk is to discuss the characterization of Chevalley groups defined over fields of odd characteristic. Let Chev(q) be the set of Chevalley group defined over a field P_q^m for some odd prime q. Define

Chev*(q) = Chev(q) - {
$$PSL(2,q)$$
}, q > 3,
Chev*(3) = Chev(3) - { $PSL(2,3)$, $PSL(3,3)$, $PSU(3,3)$, $PSp(4,3)$, $PSL(4,3)$,
 $PSU(4,3)$, $G_2(3)$ }.



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Theorem. Let G be a finite group such that $F^*(G)$ is simple. Suppose some involution centralizer has a 2-component of type $Chev^*(q)$, q odd. Then $F^*(G)$ has type Chev(q) or type $L_q(4)$.

The proof rests on Aschbacher's classification of Chevalley groups over fields of odd characteristic which requires that an involution centralizer have an intrinsic SL(2,q)-component. The concept of a strongly intrinsic component is introduced and its role in characterizing the groups of

R. WEISS: A geometric characterization of certain groups of Lie type

Let $\Gamma = (V,E)$ be an undirected graph. For each $x \in V$ let $\Gamma_i(x)$ be the set of vertices at a distance of at most i from x, let $G_i(x)$ be the set of elements in G (G some subgroup of $\operatorname{aut}(\Gamma)$) fixing each $u \in \Gamma_i(x)$, let $\Gamma(x) = \Gamma_i(x)$ and $G(x) = G_0(x)$. An s-path is an (s+1)-tuple (x_0,\ldots,x_g) of vertices x_i such that $x_i \in \Gamma(x_{i-1})$ for $1 \le i \le s$ and $x_i + x_{i-2}$ for $2 \le i \le s$. For each s-path (x_0,\ldots,x_g) let $G(x_0,\ldots,x_g) = G(x_0) \cap \ldots \cap G_i(x_g)$ and $G_i(x_0,\ldots,x_g) = G_i(x_0) \cap \ldots \cap G_i(x_g)$.

(i) $G_1(x_1,...,x_{n-1})$ acts transitively on $\Gamma(x_n)-\{x_{n-1}\}$ and

Theorem 1: Suppose for some $n \ge 3$

type Chev(3) is discussed.

(ii) $G_1(x_0, x_1) \cap G(x_0, ..., x_n) = 1$ for every n-path $(x_0, ..., x_n)$. Then n = 3, 4, 6 or 8, assuming $|\Gamma(x)| \ge 3$ for each $x \in V$.

Theorem 2: Let Γ and G be as in Theorem 1. Suppose, too, that Γ is finite and that

(iii) $G_1(x_1,...,x_{n-1}) \cap G(u) = 1$ for $u \notin \Gamma(x_1) \cup \Gamma(x_2)$ when n = 3, for $u \notin \Gamma_{n/2}(x_{n/2})$ when $n \ge 4$, for every n-path $(x_0,...,x_n)$ and (iv) $|\Gamma(x)|$ is odd and not too small for every $x \in V$. Then Γ is a generalized n-gon.

Theorem 3: Suppose that Γ is connected and finite, that G acts transitively on V and that $G(x)^{\Gamma(x)} \overset{\sim}{\succeq} L_3(p)$ (acting on $1+p+p^2$ points) for every $x \in V$. Then $G_5(x,y) = 1$ for every $\{x,y\} \in E$.

H.A. WILBRINK: 2-transitive Minkowski planes

We show that all Minkowski planes admitting an automorphism group with the property that any pair of non-parallel points can be mapped onto any other pair of non-parallel points, are known.

Adressen der Tagungsteilnehmer

Professor Eiichi Bannai Professor László Babai Department of Mathematics Dept. Algebra, The Ohio State University Eötvös University Columbus, Ohio 43210 Budapest 8, Pf. 323 IISA H-1445 Ungarn Belgien

Professor Dr. H. Bender Mathematisches Seminar Universität Olshausenstr. 40-60

2300 Kiel 1

· Dr. R.M. Damerell Maths. Dept., Royal Holloway College Egham Surrey

England

Prof. Dr. D.G. Higman University of Michigan Ann Arbor, Mich. 48104

Prof. Dr. Robert Liebler

USA . Prof. John McKay Concordia University.RM961 1455 Maisoneuve West,

Colorado State University

Fort Collins Co. 80524

Montreal, Quebec, H3G IM8 Canada ' Professor M.A. Ronan University of Illinois at Chicago Circle Math. Dept.

Professor Andries B. Brouwer Math. Centre Tweede Boerhaavestr. 49 1091 AL Amsterdam. Niederlande

Dr. H. Hähl Mathem. Institut Universität Tübingen Auf der Morgenstelle 10 7400 Tübingen 1

Prof. Dr. Zvonimir Janko Mathematisches Institut Universität Heidelberg Im Neuenheimer Feld 288 6900 Heidelberg

Prof. Dr. Donald Livingstone

The University of Birmingham

Dr. Arnold Neumaier Pachbereich Mathematik Technische Universität 1000 Berlin 12

P.O. Box 363

Great Britain

Birmingham BI5 2TT

Professor P. Rowlinson Dept. of Mathematics University of Stirling Scotland

Professor Lynn M. Batten U. Libre de Bruxelles Serive de Géométrie C.P. 216 Campus Plaine, Blvd. de Triomphe, 1050 Bruxelles

Dr. Arjeh M. Cohen Mathematisch Centrum 2^e Boerhaavestraat 49 1091 AL Amsterdam Niederlande

Prof. Dr. Ctristoph Hering Universität Tübingen Mathematisches Institut Auf der Morgenstelle 10 7400 Tübingen 1 Dr. Lefevre-Perscy

U.L.B.-Campus Plaine Departement de Mathematiques C.P.216 B.1050 Bruxelles Belgien Prof. Dr. Rudolf A. Mathon Dep. of Computer Science University of Toronto Toronto, MSS 1A7

Dr. Simon Norton DPMMS. 16 Mill Lane Cambridge England

Canada

Professor Stephen D. Smith Bept. of Mathematics University of Illinois at Chicago Cirle Chicago, 111. 60680 USA

DFG Eorschutt Aneinschaft Forschu**USA**neinsc

Professor Dr. K. Strambach Mathematisches Institut Universität Erlangen-Nürnberg Bismarckstr. I 1/2 8520 Erlangen

Professor Michio Suzuki University of Illinois Math. Dept. Urbana, Illinois 61801 USA

Prof. Dr. F.-G. Timmesfeld Mathematisches Institut Universität Arndtstr. 2 6300 Giessen

Frau Prof.Dr. M.M. Virotte Université Paris 7. 2 Place Tussier F. Paris 5° Frankreich

Professor A. Wagner Dep. of Pure Math. University of Birmingham Birmingham 15 G.B. Professor John H. Walter University of Illinois at Urbana-Champaign Dept. of Mathematics Urbana, Ill. 61801 USA

Professor H.A. Wilbrink Mathematisch Centrum Tweede Boerhaavestraat 49 1091 AL Amsterdam Niederlande

Professor Dr. R. Weiss II. Mathematisches Institut der FU Königin-Luise-Str. 24/26 1000 Berlin 33

Professor Dr. Zara UER de Mathématiques 33 Rue Saint Leu, 80039 Amiens Cedex France Frankreich Prof. Dr. B. Fischer Fakultät für Mathematik Universität Bielefeld Universitätsstr. 4800 Bielefeld I



