MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 23/1980

Kombinatorik geordneter Mengen

18.5. bis 24.5.1980

In den letzten Jahren haben sich bei vielen Untersuchungen im Bereich der (halb-)geordneten Mengen besonders kombinatorische Methoden und Fragestellungen als erfolgreich und richtungsweisend erzeigt. Deshalb war es wünschenswert, im Rahmen einer Tagung nicht nur Interessierte aus dem Gebiet der geordneten Mengen selbst sondern auch aus den Gebieten Kombinatorik, Informatik u.ä. zusammenzubringen. Wir sind dem Mathematischen Forschungsinstitut Oberwolfach dankbar, eine solche Tagung zu ermöglichen.

Bei der Tagung unter Leitung der Herren M. Aigner (Berlin) und R. Wille (Darmstadt) erwies sich der Umstand eines Teilnehmerkreises mit Vertretern aus verschiedenen Forschungsgebieten als sehr anregend und nützlich. Fragen der Theorie geordneter Mengen wurden unter vielfältigen Aspekten vorgestellt und diskutiert. Im Vordergrund standen dabei folgende Problemkreise: allgemeine Darstellungstheorie geordneter Mengen, Strukturfragen spezieller Ordnungen, kombinatorische Eigenschaften von Ordnungen, kombinatorische Probleme auf Ordnungen und Verallgemeinerungen klassischer kombinatorischer Fragestellungen vom Spezialfall einer zugrunde liegenden trivialen Ordnung weg auf den Fall einer nicht-trivialen Ordnung.

Die nachfolgenden Vortragsauszüge mögen einen detaillierteren Einblick in die angesprochenen Probleme und deren Zusammenspiel geben.

Leider erlaubt dieser Tagungsbericht aus technischen Gründen keinen Vortragsauszug aus dem vierhändigen Zusammenspiel des Ehepaars Rado am Klavier. Es sei deshalb ausdrücklich darauf hingewiesen, dass auch ordentliches Kombinieren von Tönen bei der Tagung nicht unberücksichtigt geblieben ist.

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Vortragsauszüge

M. AIGNER: Producing posets

Suppose we are given a linear order on n elements and let P be a poset, $|P| = n$. We say that a sequence of comparisons produces P if at the end of the sequence we arrive at a poset Q which contains P as an embedded subposet. Various results concerning the complexity of P (with respect to P-producing algorithms) are presented. Sample result: Let $C_{par,t}$ be the parallel complexity of producing the first t elements in order. Then $C_{par,t} \leq \log n + (t-1)$.

R. BONNET: Rigid chains and some applications

Let C be a class of structures, κ and μ be cardinals. C satisfies the property $D(\kappa,\mu)$ whenever there is a family $(M_i)_{i\in I}$ of structures in C such that: 1) $|I| = \mu$, 2) each M_i is of cardinality κ, 3) each M_i is rigid (i.e., has only one automorphism), 4) if $M \in C$ is embeddable into M_i and M_j, $i \neq j$, then M is of cardinality $< \kappa$.
THEOREM. We assume G.C.H.. Let κ be a successor cardinal. Let C be the class of chains (resp. all posets with chains and antichains of cardinality $< \kappa$; Boolean algebras; symmetric graphs with cliques and induced discrete subgraphs of cardinality $< \kappa$). Then C satisfies $D(\kappa,2^\kappa)$.
To prove this, it is sufficient to show that the class of chains satisfies $D(\kappa,2^\kappa)$.
CONJECTURE. (ZFC, without GCH) For what κ, $\kappa>\omega$, does the class of chains C_o satisfy $D(\kappa,2^\kappa)$?

G. BRUNS: Covering a Boolean algebra by subalgebras

CONJECTURE. If a Boolean algebra B is the (set-)union of subalgebras $B_1,\dots,B_k$ and if $|B_1\cap\dots\cap B_k| = 2^m$ $(<\infty)$ then $|B| \leq 2^{k+m-1}$.
If every B_i has the property that all atoms of B_i except one

are also atoms of B and if $m = 1$ the conjecture follows from a theorem by Lovász (Acta Sc. Hung. 21 (1970), 443-446). I have proved the conjecture under the additional assumption that $B_i \cap B_j = \{0,1\}$ (and hence $m = 1$) whenever $i \neq j$. I have proved the conjecture if $k \leq 4$ and m arbitrary and if $k = 5$ and $m = 1$.

T. BRYLAWSKI: Intersection numbers for matroid embeddings into uniform combinatorial geometries

A uniform combinatorial geometry G is a finite geometric lattice which has the same number $W(i,j)$ of flats of corank j in every upper interval of rank i. Examples include projective and affine geometries (Gaussian coefficients), partition lattices (Stirling numbers), and perfect matroid designs. A matrix equation relates the matroid structure M of a set S embedded into G to the intersection numbers of the embedding: $P \cdot W = T \cdot I$ where $T(i,j) = \binom{i}{j}$, $P(i,j)$ is the number of i-element subsets of S of corank j, and $I(i,j)$ counts the flats of G of corank j which contain i points of S. Applications include the Bruck theorem for subprojective planes (using the fact that I must be nonnegative). All $3k + n^2 - 1$ affine identities which hold among the coefficients of I for all embeddings of k-point spanning subgeometries into the projective space $PG(n,q)$ are computed.

G. DORN: Preferences, cube-lattices and partial orders

Any complete relation R on $N = \{1,..,n\}$ can be represented as an $\binom{n}{2}$-tupel $(x_{ij})_{i<j}$ where $x_{ij} = 1,-1,0$ resp. iff (i,j), (j,i) belong to R or not. R is transitive iff $x_{ij}x_{jk} - x_{jk}x_{ik} - x_{ik}x_{ij} = -1$. The transitive complete relations are the quasi-orders (or "preferences"). The lattice of all complete relations on N is isomorphic to the lattice of all subcubes of the $\binom{n}{2}$-dimensional cube $C\binom{n}{2}$. The vertices (or points) of $C\binom{n}{2}$ are the strict complete relations on N (i.e., $x_{ij} \neq 0$). The

transitive points (i.e. the linear orders on N) can be characterized as some regular subgraph of $C\binom{n}{2}$. The properties of the simple majority rule on profiles of preferences can be described in lattice-theoretical language and sets K of preferences in which the simple majority rule always gives transitive results can be characterized. Such K's occur as the complements of some representing sets or as complete subgraphs of some special graph. Their determination seems to be an NP-complete problem.

D. DUFFUS: <u>Matchings in modular lattices</u>

A <u>matching</u> f of a finite lattice L is a bijection of J(L) (the set of join-irreducibles of L) to M(L) (the set of meet-irreducibles of L) such that $x \leq f(x)$ for all $x \in J(L)$. I. Rival has conjectured that every finite modular lattice has a matching; this is verified in some special cases.
For an element x of a finite lattice L let $\bar{x} = \vee(y\in L: y$ covers x in L), let $\underline{x} = \wedge(y\in L$: x covers y in L), and let S(L) $= \{x\in L: (\underline{x}) = x\}$. Then $L = \cup\{[x,\bar{x}] : x\in S(L)\}$. If L is modular then $[x,\bar{x}]$ is a complemented modular lattice and , therefore, is the direct product of projective geometries. By exhibiting complete (graph) matchings in bipartite graphs associated with projective geometries, we show that <u>if L is a finite modular lattice and the width of S(L) is at most two then L has a matching</u>.

M. ERNE: <u>On the cardinality of topologies and the number of cliques in a graph</u>

By a generalized Fibonacci sequence we mean a sequence (g_n) with $g_o = 1$ and $g_{n+1} = g_n + g_{r_n}$ where (r_n) is any integer sequence with $r_n \leq n$ $(n \in N_o)$. For a fixed $n \in N_o$, let F_n denote the set of all integers k occuring as the n-th member of some generalized Fibonacci sequence. Furthermore, let G_n

denote the set of all numbers k for which there exists a graph with n vertices and exactly k cliques (i.e., complete subgraphs). Similarly, let Q_n^o denote the set of all k for which there is a partially ordered set with n points and exactly k antichains. Finally, the set of all cardinalities of T_o-topologies on an n-point set is denoted by T_n^o. Then $F_n \subset G_n$ and $T_n^o = Q_n^o \subset G_n$ for all $n \in N_o$. $F_n = G_n = Q_n^o$ for $n \leq 7$; $F_n \neq G_n = Q_n^o$ for $n = 8,9$ (and probably for $n = 10$). For $n \leq 9$, the difference of any two consecutive numbers in F_n, G_n or Q_n^o is always a power of 2.

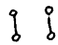
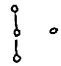
U. FAIGLE: Projective geometry on partially ordered sets

A set of axioms is presented for a projective geometry as an incidence structure on partially ordered sets of points and lines. This set includes, in particular, the triangle axiom and is equivalent to the set of axioms for classical projective incidence geometry in the case where the points and lines are trivially ordered. It is shown that the lattice of linear subsets of a projective geometry is modular and that every modular lattice of finite length is isomorphic to the lattice of linear subsets of some projective geometry.

C. GREENE: Partially ordered sets and Young tableaux

We consider partial solutions to the following questions, all related to the order-theoretic properties of permutations and Young tableaux. (1) Find an algorithm for generating random linear extensions (order-preserving labellings) of a finite poset P. (2) Given k permutations chosen at random from S_N, what is the expected length of the largest common increasing subsequence? (3) Define a canonical mapping from posets P of size N to Young tableaux on N symbols which (a) coincides with that given by the Robinson-Schensted correspondence when P is the order determined by a single permutation, and (b) has shape $\Delta = \{\Delta_1 \geq \ldots \geq \Delta_l\}$, which is the partition of $|P|$ determined by the k-chains and k-antichains of P.

P.L. HAMMER: Threshold orders

A finite poset is called a toset ("threshold ordered set") if there exist real numbers $v_1,\dots,v_n$ associated to its elements and a positive number v_o such that $i \succ j$ iff $v_i - v_j \geq v_o$. Tosets are first characterized by the absence in their Hasse diagrams of induced subgraphs isomorphic to  or to  . Tosets are then characterized as being those finite posets which have the following property: The elements can be indexed (say, for simplicity, as $e_1,\dots,e_n$) so that $i > j$ implies $e_j \not\succ e_i$, and for every $i \in \{1,\dots,n\}$, there is an index $k(i)$, s.t. $e_i \succ e_j$ iff $j \leq k(i)$. A polynomial algorithm can be derived for recognizing graphs which can be oriented so as to become tosets.

E. HARZHEIM: Generalizations of some theorems of combinatorial topology to products of linearly ordered continua

Let $C_1,\dots,C_n$ be linearly ordered continua (i.e., without steps and gaps) each having no first and no last element, C their cartesian product with the product topology of their order topologies. A parallelotope P of C is a set $I_1 \times \dots \times I_n$, where each I_k is a closed interval of C_k . Let ∂P be the boundary of P . Then the following generalizations of the Jordan-Brouwer theorem are valid: I) If $f\colon \partial P \to C$ is continuous and injective then $C - f(\partial P)$ has at least two connectivity components. II) If $f\colon P \to C$ is injective and continuous, then $f(P)$ does not separate C .
From I) and II) one can further derive the following generalization of Brouwer's theorem on the invariance of domain (open set): III) Let U be an open subset of C , $f\colon U \to C$ continuous and injective, then $f(U)$ is open. The cases $n=2$ of I) resp. III) were proved by Löttgen/Wagner resp. de Groot.

A.P. HUHN: On contraction lattices of graphs

Contraction lattices of series-parallel networks generate the variety of all lattices. It follows in particular that contraction lattices of planar graphs do not satisfy any non-trivial lattice identity. Two different proofs can be given. One of the proofs goes along the line of Freese's and Nation's analogous theorem for congruence lattices of semilattices, the other makes an effective use of Pudlak's proof of the Grätzer-Schmidt-theorem. Modularity, n-distributivity and dual n-distributivity is also characterized in contraction and bond lattices. A theorem of Jakulik is reformulated to characterize distributivity of semimodular lattices in terms of unions of the lattice.

G.O.H. KATONA: Extremal problems with forbidden subgraphs in the graph of subsets

Let X be a finite set with n elements. Define the directed graph $G = (2^X, E)$, where $(A,B) \in E$ iff $A,B \in 2^X$ and $A \supsetneq B$. The following (too) general question is raised. What is $\max|\mathcal{A}|$ where $\mathcal{A} \subset 2^X$ is such that the induced subgraph $G_{\mathcal{A}}$ does not contain any of the given directed graphs $H_1,\dots,H_k$ as subgraphs. The case $k = 1$, when H_1 consists of one edge only is known as Sperner's theorem. A theorem of Erdös' answers the case when H_1 is a directed path of length $l+1$. T. Tarján and the speaker solved the case $H_1 =$ , $H_2 =$  and obtained partial results and conjectures for other cases.

D. KELLY: Dimension of partially ordered sets

A finite poset is n-irreducible if it has dimension n, and removal of any element lowers the dimension. The embedding property for n-irreducible posets is: (E_n) "For any n-irreducible poset P and $m > n$, there is an m-irreducible poset that contains P as a subposet."
A construction technique for irreducible posets is given that, together with a knowledge of all 3-irreducible posets, allows us to prove (E_3).

E. KÖHLER: Zur Klassifikation von Klängen

Sei $v \in N$ gegeben. Dann heisst jedes Transitivitätsgebiet von $\mathcal{P}(Z_v)$ bezgl. Z_v ein Klang über Z_v. Ein Transitivitätsgebiet von $\binom{Z_v}{k}$ bezgl. Z_v heisst k-Klang über Z_v für $0 \leq k \leq v$. $K_{k,v} :=$ Menge der k-Klänge über Z_v, $K_v :=$ Menge der Klänge über Z_v. Falls $y \subset x \in \kappa \in K_{k,v}$ mit $y \in \tau \in K_{t,v}$ ist, heisst τ ein t-Teilklang von κ. Eine Klassifikation der Klänge ist dann dadurch gegeben, dass man für $\kappa \in K_{k,v}$ und jedes $0 \leq t \leq k$ die Menge $\kappa^t := \{\tau$ Teilklang von $\kappa : \tau \in K_{t,v}\}$ bestimmt. Für $v \,/\, 0 \bmod(2,3,5,7)$ und $k \leq 4$ wurde so eine Klassifikation von $K_{k,v}$ gegeben.

E.C. MILNER: The cofinality of partially ordered sets

A subset A of a partially ordered set $(P,<)$ is cofinal iff $(\forall x \in P)(\exists a \in A)(x \leq a)$. Recently M. Pouzet proved the following structural theorem for the cofinal subsets of a partially wellordered set (pwo). THEOREM. If $(P,<)$ is pwo, then P contains a cofinal subset which is order isomorphic to a finite union of a finite product of well ordered chains.
An immediate corollary of this is that the cofinality of a pwo set $(P,<)$, $cf(P) = \min\{|A| : A \text{ cofinal in } P\}$ is a regular cardinal number (or 1). A direct proof of this corollary will be sketched in the case $cf(P) = \kappa$ and $cf(\kappa) > \omega$. The proof breaks down when $cf(\kappa) = \omega$ and for this it seems to be necessary to prove the full strength of Pouzet's theorem. In attempting to give a direct proof (with N. Sauer) we were led to the following. CONJECTURE. If $(P,<)$ is any poset and $cf(P) = \kappa > cf(\kappa)$, then $(P,<)$ contains an antichain of size $cf(\kappa)$. (Pouzet's theorem implies this for the case $cf(\kappa) = \omega$).

B. MONJARDET: Metrics on posets

In lattice theory, this subject goes back to Glivenko (1936) and is related to the study of "valuations" or "norms" (Menger 1928, Birkhoff 1933). Now, it seems somewhat obsolete (the recent books do not mention it). One must find developments elsewhere, e.g., in "distance geometry" (Blumenthal 1953, Blumenthal-Menger 1969). In poset theory it seems to back to Haskins-Gudder (1972). But uses of metrics on posets appear in several domains: social choice theory (Kemeny 1969, Bogart 1973-75), psychometrics (Restle 1959), social networks (Flament 1963), data analysis (Boorman 1970), information theory (Kampe de Feriet 1973) etc., and lead to interesting results or problems (Barbut-Monjardet 1970). We give a review of recently obtained results (Comyn-Vandorpe, Bordes, Monjardet 1976, Barthelemy 1978 -79) for finite posets along three lines: 1. Quasi-valuations, shortest paths and metrics on posets. 2. Metric characterizations of posets. 3. Applications of previous results and related or open problems.

E. NELSON: Order-theoretic characterization of grammatical similarity

A context-free grammar is a 4-tuple $G = (V, \Sigma, P, S)$ where V is a finite set, $\Sigma \subset V$, $S \in V - \Sigma$ and P is a finite subset of $(V-\Sigma) \times V^*$. A (rooted) tree is a finite subset $t \subset \omega^*$ such that for all $u \in \omega^*$, all $n \in \omega$, if $un \in t$ then $u \in t$ and $uk \in t$ for all $k < n$. A G-tree is a function $l: t \to V$ (t a finite tree) s.t. if $u \in t$ is not a leaf and if k is the largest number with $uk \in t$ then $(l(u), l(u0) l(u1) \ldots l(uk)) \in P$. A G-phrase is a G-tree with all leaves labelled from Σ, and a G-sentence is a G-phrase with root labelled S. The language of G, LG, consists of all words in Σ^* which are obtained as the product of the labels of the leaves of a sentence. Kuroda (Inf. and Control 30(1976),307-378) proposes a classification of context-free grammars by certain topologies associated with them. We concentrate on the order-theoretic description. We prove: every context-free grammar is isomorphic to one with language = $\{\emptyset\}$;and discuss examples. The talk is a report on joint work with D. Wood, also of McMaster U..

A. PASZTOR: On epis in the category of ω-complete posets

POS(ω) is the category of ω-complete posets, i.e., of posets containig the l.u.b.'s of their ω-chains, together with ω-continuous maps, i.e., with maps which preserve l.u.b.'s of ω-chains. An ω-continuous map $f:P \to Q$ is dense if the closure of $f(P)$ (i.e., the least ω-complete subset of Q containing $f(P)$) is Q. J. Meseguer conjectured that in $POS(\omega)$ the epis are exactly the dense maps. Of course, the dense maps are epis. But a counterexample presents an epi which is not dense.

P. PUDLAK: On congruence lattices of lattices

This contribution concerns the old problem: Is every algebraic distributive lattice isomorphic to the congruence lattices of a lattice? Let $\mathcal{D}$ ($\mathcal{D}^{\vee}$) be the category of finite distributive lattices with embeddings preserving 0 (with 1-1 mappings preserving join and 0 respectively). Let $\mathcal{L}$ be the category of finite lattices with embeddings. Denote by $\mathfrak{I}$ the identical embedding of $\mathcal{D}$ into $\mathcal{D}^{\vee}$, and by Con the functor from $\mathcal{L}$ to $\mathcal{D}^{\vee}$ such that (i) for a lattice L, $Con(L)$ is the congruence lattice of L, (ii) for an embedding $\varphi: L_1 \to L_2$, $Con(\varphi)$ is the mapping from $Con(L_1)$ to $Con(L_2)$ which maps each $\theta \in Con(L_1)$ to the smallest congruence of L_2 that contains the image of θ.
THEOREM. There exists a functor $\mathcal{F}: \mathcal{D} \to \mathcal{L}$, and a natural equivalence τ from $\mathfrak{I}$ to $Con \circ \mathcal{F}$. As a corollary, we get a theorem of E.T. Schmidt which answers positively the question above for lattices in which the set of compact elements is closed under meets.

W. OBERSCHELP: Properties of almost all parametric relations

A result of Blass-Harary (J.Graph Th. 3,1979) and Fagin (J. Symb. logic 41, 1976) for graphs, relations, tournaments and plexes is generalized to parametric relations introduced by the author earlier (Lecture Notes 579). We prove that all parametric relations have the Blass-Fagin (BF) property, i.e., given any first order condition C in the language of the parametric condition P the ratio

$$r(n) := \frac{c_{C\wedge P}(n)}{c_P(n)} \longrightarrow 0 \text{ or } 1 \text{ if } n \longrightarrow \infty .$$

Here $c_{C \wedge P}(n)$ is the number of models with n vertices for $C \wedge P$, $c_P(n)$ is the total number of models for P. The proof uses the idea of Blass: We adopt the technique of elimination of qualifiers and use additional richness-assumptions in P which are almost true; only finitely many such assumptions are needed. The result extends to isomorphism types since we have proved (loc. cit.) that all parametric relations are almost rigid. PROBLEM. Is the condition of transitivity (which is not parametric) BF ?

R.W. QUACKENBUSH: <u>Order-primal algebras</u>

A finite non-trivial algebra, $\mathfrak{A} = (A;F)$ is <u>order-primal</u> if there is a partial order, $\leq$, on A such that every operation on A isotone with respect to $\leq$ is a polynomial of $\mathfrak{A}$. We restrict ourselves to partial orders with 0 and 1; let be order-primal. THEOREM(Deneke, Knoebel): $HSP(\mathfrak{A})$ is equationally complete; in fact, every non-trivial $\mathfrak{B} \in HSP(\mathfrak{A})$ has a subalgebra isomorphic to $\mathfrak{A}$. THEOREM(Davey, Duffus, Quackenbush, Rival): If $\leq$ is a lattice ordering, then $HSP(\mathfrak{A})$ is isomorphic (as a category) to Bounded Distributive Lattices. Let A_n be the linear sum of $\bullet$, n copies of $\bullet\,\bullet$, and $\bullet$; let $\mathfrak{A}_n$ be the corresponding order-primal algebra. THEOREM(Knoebel): $HSP(\mathfrak{A}_2)$ is congruence (4- but not 3-)distributive; $HSP(\mathfrak{A}_3)$ is not congruence distributive. CONJECTURE: $HSP(\mathfrak{A})$ is congruence distributive iff for all $a,b \in A$, either $\inf(a,b)$ or $\sup(a,b)$ exists. Spec(K) is the set of cardinalities of the finite members of the class of structures K. TRIVIALITIES: Spec(HSP($\mathbf{2}$)) is all positive integers, Spec(HSP($\mathbf{2}\times\mathbf{2}$)) consists of all squares, Spec(HSP($\mathbf{2}\times\mathbf{3}$)) contains no primes. THEOREM(McKenzie): Spec(HSP($\mathbf{3}$)) is cofinite. CONJECTURE: Spec(HSP($\mathfrak{A}$)) is cofinite if $\mathfrak{A}$ is directly irreducible and otherwise has zero density.

R. RADO: <u>Theorems for intervals of ordered sets</u>

An <u>interval</u> of the ordered set $(S,<)$ is a set $A \subset S$ such that, whenever $x,z \in A$ and $x < y < z$, then $y \in A$. One of the results to be discussed is the following. THEOREM. Let I be a set and, for every $n \in I$, A_n be an interval of the ordered set $(S,<)$. Let $k \in \{1,2,\dots\}$. Then there is a partition $I = I_1 \cup I_2 \cup \dots \cup I_k$ such that for $1 \leq i \leq k$ and $m,n \subset I_i$, $m \neq n$, always $A_m \cap A_n = \emptyset$, iff (1) $A_{n_1} \cap A_{n_2} \cap \dots \cap A_{n_{k+1}} = \emptyset$ whenever $\{n_1,n_2,\dots,n_{k+1}\}_{\neq} \subset I$.

REMARK: (1) is equivalent to saying that whenever $n_1,\dots,n_{k+1} \in I$, there is a partition $\{n_1,\dots,n_{k+1}\} = I_1 \cup \dots \cup I_k$ such that for $1 \leq i \leq k$ and $m \neq n \in I_i$, always $A_m \cap A_n = \emptyset$.

I. RIVAL: A structure theory for ordered sets

An effort is made to fashion a classification scheme for ordered sets (finite and infinite) aimed at distinguishing combinatorial properties of ordered sets. The principal novelty in this effort stems from the central role assigned to "retractions".

B. SANDS: Counting antichains in finite partially ordered sets

We show that for every $l > 1$ there is a number $r < r(l)$ such that every poset P of length $l-1$ contains an element which is in at least $1/r$ of the antichains of P. For $l=2$, r can be taken to be 8.807; in general r satisfies $(r-1)^l \leq (2-2l/r)^r$. In the other direction, when $l=2$ we can show that r must be at least 4.3865.

W.T. TROTTER: Dimension and rank for finite posets

A realizer R of a poset (X,P) is a collection $R = \{L_1,\dots,L_t\}$ of linear extensions of P so that $P = \cap R$. A realizer R is irredundant if $\cap S \neq \cap R$ for every nonempty proper subfamily $S \subset R$. The dimension and rank of a poset (X,P) are respectively the minimum and maximum cardinality of an irredundant realizer of P. Recent research has shown that the study of these invariants is directly related to many well known topics in graph theory and combinatorial mathematics including interval graphs, planar graphs, matchings and 1-factors, partitions and coverings, extremal problems, and systems of distinct representatives. In this paper, we discuss aspects of dimension and rank which involve algorithmic and structural problems for acyclic directed graphs. These problems are motivated by the alternate definitions given by Maurer, Rabinovitch, and Trotter of dimension and rank in terms of the digraph of non-forcing pairs.

B. VOIGT: Some recent results in Partition-theory for lattices

(All theorems mentioned were obtained jointly with H.-J. Prömel). A has the partition-property w.r.t. the class $\mathfrak{C}$ iff for every $B \in \mathfrak{C}$ there exists $C \in \mathfrak{C}$ s.t. for every δ-coloring of the A-subobjects of C there exists a monochromatic B-subobject. THEOREM. The class of finite linear lattices has the part. prop. w.r.t. singleton vertices and only w.r.t. these. THEOREM. The class of finite geometric arguesian lattices has the part. prop. w.r.t. singleton vertices and only w.r.t. these. THEOREM. The class of finite modular lattices has the part. prop. w.r.t. singleton vertices and not w.r.t. any sublattice of a finite geometric arguesian lattice which contains more than one element. THEOREM. Given $\varepsilon > 0$ and $k \in \mathbb{N}$ there exists $n \in \mathbb{N}$ s.t. for every $S \subset \mathcal{P}(n)$ with $|S| > \varepsilon 2^n$ there exists a lattice embedding $\phi: \mathcal{P}C(2) \to \mathcal{P}(n)$ s.t. $\phi(\mathcal{P}C(2)) \in S$.

R. WILLE: The selection property for bound subsets of ordered sets

The selection property for bound subsets plays an important role in the study of order varieties. An order variety is a class of (partially) ordered sets closed under direct products and retracts. For characterising order varieties the following theorem is helpful. THEOREM. Let $\mathfrak{K}$ be a class of ordered sets having the selection property for bound subsets. Then an ordered set P belcngs to the smallest order variety containing $\mathfrak{K}$ if and only if for

every pair (A,B) of a down set A and an up set B with $A^* \cap B_* = \emptyset$ and $A \subset B_*$ there exists an order-preserving map $f:P \to K$ for some $K \in \mathcal{R}$ such that $f(A)^* \cap f(B)_* = \emptyset$.
In general, a subset S of an ordered set P is a bound subset if $S = (S^*)_* \cap (S_*)^*$ and $S \neq \emptyset$; $\mathfrak{B}(P)$ is the set of all bound subsets of P and $S \leq T$ in $\mathfrak{B}(P)$ if $S_* \subset T_*$ and $S^* \supset T^*$. Now, P has the selection property for bound subsets if there is an order-preserving map $f: \mathfrak{B}(P) \to P$ with $f(S) \in S$ for all $S \in \mathfrak{B}(P)$. Examples and counterexamples are given.

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