# MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

## T a g u n g s b e r i c h t 25/1980

## Topologische Dynamik

1.6. bis 7.6.1980

Die Tagung fand unter der Leitung von J.Auslander (College Park) und M.Denker (Göttingen) statt. Sie war die erste ihrer Art in Oberwolfach und erfreute sich großer internationaler Beteiligung.

Die Schwerpunkte dieser Tagung bildeten Transformationen auf dem Einheitsintervall und Strukturfragen minimaler Flüsse. Daneben wurden die Theorie der Markoff-Ketten, Rekurrenzeigenschaften (inclusiv ihr Zusammenhang zur kombinatorischen Zahlentheorie) und einige maßtheoretische Fragestellungen in Verbindung mit dem Thema der Tagung behandelt. Die 20 Vorträge wurden an einem Nachmittag durch eine "problem session" ergänzt.

Die idealen Möglichkeiten, die das Forschungsinstitut in jeder Hinsicht bietet, ließen die Tagung zu einem vollen Erfolg werden. Die Teilnehmer der Tagung danken dem Direktor des Institutes, Herrn Prof.Dr. M.Barner, und seinen Mitarbeitern für die organisatorische Unterstützung bei der Vorbereitung und Durchführung der Tagung.

Vortragsauszüge:

A.BECK:

Special Flows in the Plane I: Flows Without Stagnation Points

In my book, "Continuous Flows in the Plane" (Springer 1974), I classified continuous flows in the plane using certain criteria and depending on the set of stagnation points (q.v.). Now, I am investigating the classification of structures of continuous flows carrying additional assumptions. The specific conditions are infinite differentiability ($C_\infty$), piecewise linearity, and semi-analyticity.
Among the flows without stagnation points, the $C_\infty$ flows have the same structure as the continuous ones. The piecewise linear flows are more restricted, and the semi-analytic are more restricted still.

L.BLOCK:

Periods of periodic points of maps of the circle which have a fixed point

For a continuous map $f$ of the circle to itself, let $P(f)$ denote the set of positive integers $n$ such that $f$ has a periodic point of (least) period $n$. Results are obtained which specify those sets, which occur as $P(f)$, for some continuous map $f$ of the circle to itself having a fixed point. These results extend a theorem of Šarkovskii, on maps of the interval, to maps of the circle which have a fixed point.

S.CHEN:

Entropy of Geodesic Flow and Exponent of Convergence of some Dirichlet Series

We investigate the relation between the geodesic flow $f = (f_t)$ and its topological entropy $h(f)$ on the unit tangent bundle $T_1M$ of a compact Riemannian manifold $M$ and the exponent of convergences of the following two functions: (1) the Dirichlet series

$$\sum_{g \in G} (\cosh\ d(x_o, gx_o) + 1)^{-t},\ t \in \mathbb{R},$$

where $G$ is the covering group of the universal covering $\tilde{M} \to M$, $d$ is the distance and $x_o$ is a fixed point in $\tilde{M}$, and (2) the Selberg zeta function

$$Z(s) = \prod_{g\in P} \prod_{k=0}^{\infty} (1 - e^{-(s+k)T(g)}),\ s \in C ,$$

where $T(g)$ is the translation length of $g$ and $P$ is the set of primitive elements of $C$. If $M$ has nonpositive sectional curvature, then the exponent of convergence is the topological entropy $h(f)$ of the geodesic flow. Our result is extended to Anosov flows of compact manifolds with associated zeta function of Smale.

E.M.COVEN:

Periodic, Recurrent and Non-wandering Points for Maps of the Interval

Let $f$ be a continuous map of a compact interval to itself. Let $P,R$ and $\Omega$ denote the periodic, recurrent and non-wandering points, respectively, of $f$. Theorem (Coven-Hedlung) $\bar{P} = \bar{R}$ and hence $\Omega - \bar{P}$ is nowhere dense. Z.Nitecki [Proc.Amer.Math.Soc., to appear] has shown that if $f$ is piecewise monotone, then $\Omega - \bar{P}$ consists entirely of isolated points. Theorem (Coven-Madden-Nitecki) $\Omega = \bar{P}$ holds $C^0$-generically. L.-S.Young [Invent.Math.54 (1979), 179-187] has constructed an example with $\Omega - \bar{P} \neq \emptyset$. However, in this case $\Omega(f) = \Omega(f^n)$ for all $n \geq 1$. We present examples of the more delicate phenomenon $\Omega(f) \neq \Omega(f^n)$ for some $n \geq 2$. These examples illustrate the main ideas of the proof of the following result. Theorem (Coven-Nitecki) $\Omega(f) = \Omega(f^n)$ for all odd $n \geq 1$.

J.CUNTZ:

K-theoretic Invariants for Topological Markov Chains

Let $\Sigma$ be a finite set, $A = (A(i,j))_{i,j\in\Sigma}$ a matrix with entries in $\{0,1\}$ and $\sigma_A$ the corresponding shift transformation. A and B are called flow-equivalent if the suspensions $F_A = X_A \times \mathbb{R}/\sigma_A \times T_1$ and $F_B = X_B \times \mathbb{R} / \sigma_A \times T_1$ ($T_1$ = unit translation on $\mathbb{R}$) equipped with this natural orientation are isomorphic.
With $A$ one can associate a certain non-commutative $C^*$-algebra $\mathcal{O}_A$ (W.Krieger and the author, Inv.Math. 1980). This algebra is an invariant of flow equivalence for $A$. For any $C^*$-algebra $\mathcal{A}$ one can define Abelian groups $K_i(\mathcal{A})$ (i=0,1) and $Ext^i(\mathcal{A})$ (i=0,1). In the case $\mathcal{A} = C(X)$, $X$ a compact space, these are just the ordinary K-groups for $X$, i.e. generalized cohomology groups, and the Brown-Douglas-Fillmore Ext-groups, i.e. the corresponding homology groups.
Thanks to recent results on algebraic topology for non-commutative $C^*$-algebras (partially due to the author) one can compute

$K_0(\bar{O}_A) \simeq Z^{\Sigma}/(1-A)Z^{\Sigma}$ $\quad Ext^0(\bar{O}_A) \simeq Ker(1-A)$ on $Z^{\Sigma}$

$K_1(\bar{O}_A) \simeq Ker(1-A)$ on $Z^{\Sigma}$ $\quad Ext^1(\bar{O}_A) \simeq Z^{\Sigma}/(1-A)Z^{\Sigma}$

Thus, one recovers the invariants of flow equivalence discovered before by Bowen and Franks. One also obtains new invariants for reducible Markov chains. Assume for instance that $A = \begin{pmatrix} A_1 & X \\ O & A_2 \end{pmatrix}$ where $A_1$ and $A_2$ are irreducible. Then there is an exact sequence $\rho_A : O \to \bar{O}_{A_2} \to \bar{O}_A \to \bar{O}_{A_1} \to O$. Equivalence classes (for an appropriate notion of equivalence) of such exact sequences form a group denoted by $Ext(\bar{O}_{A_1}, \bar{O}_{A_2})$. The couple $[\rho_A]$, $Ext(\bar{O}_{A_1}, \bar{O}_{A_2})$ is an invariant of flow equivalence for $A$ which can easily be computed from the matrix $A$.

F.M.DEKKING:

Substitution Minimal Flows

We slightly generalize Gottschalk's definition of a substitution maximal flow. The extended class of flows comprises a minimal flow studied by Chacon, and by del Junco.

In this class one can give simple examples of flows (not maximal that are 1-mixing but not 2-mixing (similar to those given by S.Goodman, B.Marcus. An example of a minimal flow with this property is given by M.Keane and myself.

Finally, we discuss the computation of sequence entropy for substitution minimal flows.

H.FURSTENBERG:

Recurrence in Topological Dynamics and Combinatorial Number Theory

A number of theorems in topological dynamics when applied to symbolic systems (subshifts) have implications for combinatorial number theory. Multiple Recurrence Theorem: Let $T_1, T_2, \ldots, T_l$ be commuting maps of a compact metric space $X$ to itself, $\exists\ x \in X$ and a sequence $n_k \to \infty$ with $T_i^{n_k} x \to x$ for $i=1,2,\ldots,l$.

When applied to a symbolic system: $X \subset \Omega = \Lambda^Z$, $\Lambda$ finite, $T\omega(n) = \omega(n+1)$, $T_i = T^i$, one obtains van der Waerden's theorem:

Theorem: If $Z = C_1 \cup \ldots \cup C_n$ is a finite partition, some $C_j$ contains arbitrarily long arithmetic progressions.

Let $(X,T)$ be a compact metric dynamical system. A point $x \in X$ is uniformly recurrent if for any neighbourhood $V$ of $x$ there is a sequence $(n_k)$ with $n_k \to \infty$ and $(n_{k+1}-n_k)$ bounded for which $T^{n_k} x \in V$. Two points $x,y \in X$ are proximal if $\lim\inf d(T^n x, T^n y) = 0$.

Theorem (Auslander-Ellis): If $x$ is any point in a compact metric dynamical system, $\exists$ a point $y$ proximal to $x$ such that $y$ is uniformly recurrent.

We say that a set $H \subset \mathbb{N}$ is a <u>central set</u> if for some system $(X,T)$ and some point $x \in X$ and a uniformly recurrent $y$ proximal to $x$, there is a neighbourhood $V$ of $y$ with $\{n : T^n x \in V\} \subset H$.
Applying the Auslander-Ellis theorem to symbolic systems one obtains:
<u>Theorem</u>: If $\mathbb{N} = C_1 \cup \ldots \cup C_n$ is a finite partition, some $C_j$ is a central set.
The significance of central sets for combinatorial number theory comes from the following results.

<u>Theorem</u>: If $H$ is a central set, there is a sequence $h_i \in H$ such that for all $i_1 < i_2 < \ldots < i_k$, $k$ arbitrary, $h_{i_1} + h_{i_2} + \ldots + h_{i_k} \in H$.
<u>Theorem</u>: If $H$ is a central set and $\sum a_{ij} x_j = 0$ is a regular system in the sense of Rado, there exists a solution $x_1, \ldots, x_m$ coming from $H$.
These now imply Hindman's theorem and Rado's theorem for finite partitions of $\mathbb{N}$.

S.GLASNER:

## Structure of Minimal Flows

The structure theorem for minimal flows which represents a minimal flow as a weakly mixing extension of a PI-flow (see [V]), can be viewed as a completion of an important step in the theory of minimal flows. However, a much finer structure theory for minimal flows is clearly desirable, especially so in the case of weakly mixing flows and weakly mixing extensions.
Some recent results in this direction ([E-G],[G], [G-W]) concerning the notions of Pure Weak Mixing and $D^{\perp\perp}$ will be described.

[V] - W.A.Veech, Topological Dynamics, Bull.A.M.Soc.<u>83</u> (1977),775-830.
[E-G] - R.Ellis & S.Glasner,Pure Weak Mixing, T.A.M.S. <u>243</u> (1978), 135-146.
[G] - Minimal Skew Products, to appear in the T.A.M.S.
[G-W] - S.Glasner & B.Weiss, A Weakly Mixing upside-down tower of Isometric Extensions, to appear.

F.HOFBAUER:

## The Transformation $T : x \to ax(1-x)$ on $[0,1]$

The goal is to determine the topological transitive T-invariant closed subsets of $[0,1]$. To this end certain shift spaces $(X,\sigma)$ are used which arise from f-expansions and whose topological structure can be determined with the aid of infinite transition matrices. Then these results are brought back to the original transformation $T$ on $[0,1]$.

One gets the following: $[0,1]$ is the disjoint union of a wandering set and $\bigcup_{1\le i\le K} L_i$ for some $K$ with $1 \le K \le \infty$ (for $K = \infty$ we have also an $L_\infty$).
The $L_i$'s are topologically transitive, closed, T-invariant subsets of $[0,1]$ and $L_i \cap L_j$ is at most finite for $i \neq j$. For $i < K$, $L_i$ is a periodic orbit or a Cantor set, which has Lebesgue-measure zero and is isomorphic (as a topological dynamical system) to a finite type subshift. $L_K$ is a periodic orbit, a Cantor set or a finite union of intervals. It contains the limit sets of almost all $x \in [0,1]$. In the case $K = \infty$, $L_\infty$ is a Cantor-set which contains no periodic points.
During the conference, I learned about papers of Jonker and Rand (preprint), and Guckenheimer (Comm.Math.Phys.) which contain similar things

A.KATOK:

Every transformation with finite entropy has a realisation on every manifold

Theorem: Let $M$ be a topological manifold, dim $M > 1$, $\lambda$ - a continuous Borel probability measure on M positive on open sets; $T : (X,\mu) \to (X,\mu)$ an automorphism of a Lebesgue space $X$ with finite entropy.
Then there exists a $\lambda$-preserving homeomorphism $f$ of $M$ which is metrically isomorphic to T.
For $M = \mathbb{T}^2$ (two-dimensional torus), this result was proved by D.Lind and J.-P.Thouvenot in 1975. A simplified version of the trick described in [1], § 2, allows to extend the method of Lind-Thouvenot to the disc $D^2$ with extra condition $f|_{\partial D^2} = \text{id}$. The transition from $D^2$ to an arbitrary manifold is based on the construction from [3], § 1 (also simplified) and on the following general statement.
Proposition: Let $X$ be a compact metric space, G a locally compact group, $U \subset X$ a non-empty open set, $f : X \to X$ a topologically transitive homeomorphism preserving a measure $\mu$ which is positive on open sets. Then there exists a continuous map $g : X \to G$ and a $\mu$-measurable (Borel) map $\psi : X \to G$ such that (i) $g(x) = \psi(f(x)) \cdot \psi^{-1}(x)$ (ii) g is equal to identity outside $U$ (iii) for arbitrary open sets $A \subset X$ and $H \subset G$ the set $A_H = \{x \in A : \psi(x) \in H\}$ has positive measure.
(References: (1) A.Katok, Ann.of Math.110 (1979), 529-547.(2) M.Brin, J.Feldman, A.Katok, Bernoulli diffeomorphisms and group extensions of

dynamical systems with non-zero characteristic exponents, to appear in Ann. of Math.)

G.KELLER:

A Short Proof for a Central Limit Theorem for Certain Piecewise Expanding Maps of the Interval

Let $T : [0,1] \to [0,1]$ be a transformation for which there exists a partition $0 = a_0 < \dots < a_N = 1$ of $[0,1]$ such that $T_{|(a_{i-1},a_i)}$ is of class $C^1$ $(i=1,\dots,N)$, $|T'| \geq \alpha > 1$, and such that $|T'|^{-1}$ is a function of bounded variation. By means of an ergodic theorem of Ionescu-Tulcea and Marinescu (Ann.Math.,1950) we obtain a spectral representation for the Perron-Frobenius-Operator $P_T$ associated with $T$, which allows us to apply a CLT for stationary processes due to Gordin, resulting in the following theorem:
If $T$ is weakly mixing for its unique absolutely continuous invariant probability-measure $m$ on $[0,1]$, and if $f : [0,1] \to \mathbb{R}$ is of bounded variation or Hölder-continuous, $\int f\,dm = 0$, then $\sigma_j := \lim_{n\to\infty} 1/\sqrt{n} \cdot (\int (S_n f)^2 dm)^{1/2}$ exists - where $S_n f = \sum_{K=0}^{n-1} f \circ T^K$ - and if $\sigma_j > 0$, then
$m(x \mid (1/\sigma_j\sqrt{n}) S_n f < z) \xrightarrow{n\to\infty} 1/\sqrt{2\pi} \int \exp(-u^2/2)du \quad (\forall\ z \in \mathbb{R})$

W.KRIEGER:

Full Shifts and Their Homomorphic Images of Maximal Entropy

The following theorem is given: Let $T$ be a topological Markov chain such that for all $n \in \mathbb{N}$ the number of periodic points of period $n$ of $T$ equals $N^n$. Then $T$ is a homomorphic image of the full N-shift. This theorem is the converse of a theorem of B.Marcus (Factors and extensions of full shifts, Monatshefte). The proof uses the notion of a (future) dimension of a topological Markov chain as introduced in W.Krieger, On dimension functions and topological Markov chains, Inventiones 56, 239-250 (1980), and it consists of three parts that can be described as follows: Constructing a synchronization mechanism, providing a suitable lemma of Rohlin-Kakutani type, and the actual coding procedure.

D.McMAHON:

Structure Theorems in Topological Dynamics

There are two keys to proving general structure theorems for minimal flows. The first is a shadow diagram - given $\varphi : X \to Y$ there exists $\varphi' : X' \to Y'$ such that $X'$, $Y'$ are (strongly) proximal extensions of $X, Y$ and $\varphi$ is special, in this case $\varphi'$ is open and has a relative invariant measure, RIM, (see the paper of Glasner on RIM's). Then for any minimal flow $X$ there is a (strongly) proximal extension $X^*$ of $X$ ; a flow $Y^*$ that is obtained from the singleton flow via almost periodic (equicontinuous) extensions, proximal extensions and inverse limits; and a homomorphism $\varphi^* : X^* \to Y^*$ such that no nontrivial almost periodic extension of $Y^*$ is a factor of $X^*$. The second key to the structure theorem is that $\varphi^*$ is weakly mixing when $X$ is metric; that is, $oc(x,x') \supseteq R(\varphi^*)$ for some $(x,x')$ in $R(\varphi^*)$ where $R(\varphi^*) = \{(x,x') : \varphi^*(x) = \varphi^*(x')\}$. We give the outline of an easy proof of this theorem in the case that $Y^*$ is the singleton flow and $X = X^*$. This proof generalizes easily to the general case.

J.MOULIN-OLLAGNIER:

Riemann-integrable Functions in Topological Dynamics

Given a dynamical system $(X,G)$ where $X$ is compact and $G$ is a group of homeomorphisms of $X$, we say that a bounded real-valued function f on X is Riemann-integrable if the set $D(f)$ of all its discontinuity points satisfies $\mu(D(f)) = 0$ for every $\mu \in \mathcal{M}(X,G)$, the convex compact set of all Borel regular probability measures on X, invariant under the action of G.

Riemann-integrable functions still enjoy many interesting properties that were previously known to hold for continuous functions. Here are some examples:

- the ergodic minimax theorem: $\sup_{\mu\in\mathcal{M}(X,G)} \mu(f) = \lim_{\mathcal{M}} \sup_{u\in X} 1/|A|. (\sum_{g\in A} f\circ g(n))$ when $G$ is amenable and $\mathcal{M}$ is its ameaning filter. (For details see: Une nouvelle démonstration du théorème de Følner. J.M.O. and D.Pinchon CRAS (1978))
- a well-known corollary of this theorem for continuous functions: if $\mathcal{M}(X,G) = \{d\mu\}$ $1/|A| \sum_{g\in A} f \circ g \underset{\mathcal{M}}{\to} \mu(f)$ uniformly.
- the variational principle $\sup_{\mu\in\mathcal{M}(X,G)} (h(\mu)+\mu(f)) = P(f)$

(For a proof of the variational principle for amenable groups, see: The variational principle, J.M.O. and D-Pinchon: Studia Math. 1980). As a final remark, let us say that the algebra $\bar{R}(X)$ together with its group $G$ of automorphisms is an invariant of the equivalence of dynamical systems, in the sense given to this work by R.Adler and B.Marcus in their paper, Topological Entropy and Equivalence of dynamical systems.

K.PETERSEN:

Speed of Mixing

Let $(X,\mathcal{B},\mu)$ be a nonatomic probability space and $T : X \to X$ a strongly mixing measure-preserving transformation.

(1) Given any speed function $\phi(n) > 0$ with $\lim_{n\to\infty} \phi(n) = 0$, there is a measurable set $A \subset X$ such that

$$\limsup_{n\to\infty} \frac{\mu(T^n A\cap A) - \mu(A)^2}{\phi(n)} = \infty.$$

(2) If $T$ has positive entropy, then in fact there is a measurable set $A \subset X$ with

$$\lim_{n\to\infty} \frac{\mu(T^n A\cap A) - \mu(A)^2}{\phi(n)} = \infty.$$

(3) Suppose that $T$ is ergodic. Call a function $f \in L^2(X)$ with $\int f d\mu = 0$ consistent if $\langle f, fT^k\rangle \geq 0$ for all $k$. Then the consistent functions are contained in the orthocomplement of the eigenfunctions.

K.SCHMIDT:

Almost Invariant Sets

Let $G$ be a countable group, $(X,\mathfrak{F},\mu)$ a nonatomic probability space, and $(\varphi, x) \to \varphi x$ a measure-preserving ergodic action of $G$ on $(X,\mathfrak{F},\mu)$. A sequence $(B_n) \subset$ is called asymptotically invariant if $\overline{\lim}\, \mu(B_n)(1-\mu(B_n)) > 0$ and $\lim_n \mu(B_n \,\Delta\, \varphi B_n) = 0$ for every $\varphi \in G$.

Proposition 1

$G$ is amenable iff every finite measure preserving ergodic action of $G$ on $(X,\mathfrak{F},\mu)$ has asymptotically invariant sequences.

Proposition 2 (Connes-Weiss)

$G$ has Katznelson property $T$ iff no finite m.p. ergodic action of $G$ on $(X,\mathfrak{F},\mu)$ has asymptotically invariant sequences.

Proposition 3

G has property T iff, for every f.m.p. ergodic action of G on $(X,\mathcal{J},\mu)$, and for every Abelian (amenable) group A, every cocycle for G with values in A is cohomologous to one with values in a compact subgroup of A.

The talk containes several examples, including two actions of $SL(2,\mathbb{Z})$: one on the 2-torus without asympt.inv.sequences, and one on the 3-torus with asympt.inv. sequences.

W.SZLENK:

Absolutely Continuous Invariant Measures for Almost Expanding Mappings of An Interval

Let $f : \langle 0,1\rangle \circlearrowleft$ be a differentiable mapping. Set $C_n^o = \{x : f^{n'}(x) = 0\}$ $C_n = C_n^o \cup \{0,1\}$, $f^n = \underbrace{f\circ\ldots\circ f}_{n\text{-times}}$, We assume that

A.1. $f \in C^2$

A.2. Card $C_1^o < +\infty$

A.3. $f''|_{C_1^o} \neq 0$

A.4. There exists a number $\lambda_o > 1$ such that

$$\bigcup_{n=1}^{\infty} f^n(C_1) \subset \{x : |f'(x)| \geq \lambda_o\}.$$

A.5. There exist: $\lambda_1 > 1$, $\lambda_2 > 1$, $n_o$ such that if $D = \{x : |f'(x)| < \lambda_1\}$, $f^n(x) \in D$ for $n \geq n_o$, then $|f^{n'}(x)| \geq \lambda_2$.

Theorem 1:

If the assumptions A.1.-A.5. are fullfilled, then there exists an invariant measure absolutely continuous with respect to the Lebesgue measure.

Theorem 2:

The condition A.5. ist equivalent to the following one:

A.6. There exists a number $\lambda_4 > 1$ such that for every periodic point $x : f^p(x) = x$ the following inequality holds: $|f^{p'}(x)| \geq \lambda_4^p$.

J.P.TROALLIC:

Théorèmes de DELEEUW et GLICKSBERG, ELLIS, NAMIOKA, VEECH: Une Approche Commune

Nous montrons que divers résultats importants de DELEEUW et GLICKSBERG,

ELLIS, NAMIOKA, BERGLUND et HOFMANN, VEECH découlent de la propriété topologique suivante:
Soit $(X,G)$ un système dynamique minimal localement compact, $Y$ un espace topologique, $Z$ un espace uniforme et $\varphi : X \to C_{s}(Y,Z)$ une application continue. Soit $K$ un ensemble de parties compactes de $Y$ et $V$ la structure uniforme sur $X$ la moins fine rendant l'application $\varphi : X \to C_{K}(Y,Z)$ uniformément continue. Si le groupe $G$ est $V$-uniformément équicontinu, $\varphi : X \to C_{K}(Y,Z)$ est une application continue.
Nos démonstrations ne font appel à aucun théorème de point fixe (contrairement à certaines des démonstrations initiales).

J. de VRIES:

Compactifications of Topological Transformation Groups

It is well-known that every completely regular Hausdorff space $X$ can be embedded in a compact Hausdorff space; e.g. $X$ can be embedded in $\beta X$, its Stone-Cech compactification. However, usually a continuous action of a topological group $G$ on such a space $X$ cannot be extended to a continuous action of $G$ on $\beta X$. For a large class of topological transformation groups (ttg's) $\langle G,X,\pi\rangle$, including all ttg's with $G$ locally compact and $X$ completely regular Hausdorff, the action $\pi$ can be extended to some compactification of $X$:
Theorem: Let $\langle G,X,\pi\rangle$ be a ttg such that there exists a separated uniformity on $X$, generating the topology of $X$, and a neighbourhood $U_o$ of $e$ in $G$ such that $\{\pi^t : t \in U_o\}$ is -equicontinuous. Then there exists an equivariant embedding of $\langle G,X,\pi\rangle$ in a ttg $\langle G,Y,\sigma\rangle$ with $Y$ a compact Hausdorff space. Moreover, $Y$ can be chosen such that

$$w(Y) \leq \max(w(G/G_o), w(X)),$$

where $G_o := \{t \in G : \pi^t = 1_X\}$ and $w(\cdot)$ denotes the weight of a topological space.
Remark: If $G$ is locally compact, then the conditions of the theorem are fulfilled for every completely regular space $X$, and the estimation for $w(Y)$ can be improved to $w(Y) \leq \max\{L(G/G_o), w(X)\}$, where $L(\cdot)$ denotes the Lindelöf degree of a space
Corollary: Let $\langle G,X,\pi\rangle$ satisfy the conditions of the theorem (e.g. $G$ locally compact). Then for $x \in X$ the following are equivalent:
(i) $x$ is an almost periodic point;
(ii) $x$ is a discretely almost periodic point;
(iii) the orbit of $x$ can equivariantly be embedded in a compact minimal set.

J.v.d.WOUDE:

## Highly Proximal Extensions and Relative Disjointness

A homomorphism $\varphi : X \to Y$ of minimal transformation groups (m.t.g's) is called highly proximal (h.p) if $\varphi$ is irreducible (i.e. $\varphi[\bar{A}] = Y \Rightarrow \bar{A} = X$). If X is metric then $\varphi$ is h.p. iff $\varphi$ is almost 1-1. Two m.t.g's X and Y are h.p. equivalent if they have a common h.p. extension. The unique maximal element in the equivalence class of X is called $X^*$. Then $X = X^*$ iff X is an image of the universal m.t.g. under an open map. If the acting group T is discrete then $X = X^*$ iff X is extremely disconnected. We prove that if T is discrete then $X = X^*$ is distal iff X is finite.

Let $\varphi : X \to Z$ and $\psi : Y \to Z$ be homomorphisms of mtg's then $\varphi$ and $\psi$ are disjoint ($\varphi \perp \psi$) if $R_{\varphi\psi} = \{(x,y) | \varphi(x) = \psi(y)\}$ is minimal (and so $X \perp Y$ iff $X \times Y$ is minimal). It is known that $X \perp Y$ iff $X^* \perp Y^*$. Our objective is a relativized version of this fact. (i.e. $\varphi \perp \psi$ iff $\varphi^* \perp \psi^*$ where $\varphi^* : X^* \to Z^*$ is the induced map). We prove that $\varphi \perp \psi$ implies $\varphi^* \perp \psi^*$ and if $Z = Z^*$ or $R_{\varphi\psi}$ has a dense set of almost periodic points then $\varphi \perp \psi$ iff $\varphi^* \perp \psi^*$ and $\varphi \stackrel{\cdot}{\perp} \psi$ iff $\varphi^* \stackrel{\cdot}{\perp} \psi^*$ (i.e. $R_{\varphi\psi}$ is ergodic iff $R_{\varphi^*\psi^*}$ is). Moreover, if $R_{\varphi\psi}$ has a dense set of almost periodic points then $R_{\varphi^*\psi^*}$ does.

T.S.WU:

## A Note On The Measurable Subsets In Compact Topological Groups

Let G be a compact connected topological group and $\mu$ be its (normalized) Haar measure. Given $\delta > 0$, let $A_1, A_2, \ldots$ be a sequence of measurable subsets of G such that $\mu(A_i) \geq \delta$ for $i=1,2,3,\ldots$
We show that there exists an integer $n$ such that $A_1, A_2, \ldots, A_n = G$.

## Problem Session

Let $T$ be a piecewise $C^2$-map of the interval with $|T'| > 1$. Wong (Ann.Prob.7(1979)) has shown that for a class of Hölder continuous functions $f$ a central limit theorem holds: Set $S_n(f) = f + f\circ T + \ldots + f\circ T^{k-1}$. Then $1/\sqrt{n}\,(S_n f - \int S_n f d\mu)$ converges weakly to a normal distribution. In many cases such results follow from an invariance principle. It does not seem to be too difficult to redefine the process $S_n f$ on a richer probability space together with a Brownian motion $B_t$ so that $P(|S_{[t]}-B_t| \geq \sqrt{t}\,\varphi(t)) \to 0$ for some $\varphi(t) \to 0$, or to obtain $S_{[t]}-B_t = O(\sqrt{t}\,\psi(t))$ a.s. where $\psi(t) \to 0$. Then find the best possible functions $\varphi$ and $\psi$. (DENKER)

(2) A well-known conjecture about the transformation $f_t : x \to \frac{1}{2}(t-x^2)$ on $[-1-\sqrt{1+t}, 1+\sqrt{1+t}]$ says that the map $t \to h_{top}(f_t)$ is monotone, $t \in [4,8]$. It is shown in "The topological entropy of $x \to ax(1-x)$" by F.Hofbauer (to appear in Monatshefte f.Math.) that this is equivalent to the following problem about the polynomials $P_k(t)$ given by $P_o(t)\equiv 0$ and $P_{k+1}(t) = \frac{1}{2}(A-P_k(t)^2)$. One can easily deduce from this recursion formula that (*) $P_{k+m}(t) = P_m(t)$ and $P'_{k+m}(t) = P'_m(t)$ for $m \geq 1$ if $P_k(t) = 0$. Let $4 = z_o < z_1 < \ldots < z_{n(k)-1} < z_{n(k)} = 8$ be the zeros of $P_1P_2\ldots P_k$, together with the number 8. Using (*) one can determine the sign of $P_{k+1}(z_i)$ and $P_{k+1}(z_{i+1})$. If one of these numbers is zero, we take the sign of $P_{k+1}(z_i+\varepsilon)$ or $P_{k+1}(z_{i+1}-\varepsilon)$ respectively for arbitrary small $\varepsilon > 0$. If $z_{i+1} = z_{n(k)} = 8$ one easily shows from the recursion formula that $P_{k+1}(8) = -4$. The problem is now to show for $0 \leq i \leq n(k)-1$ and $k \geq 1$ by induction on $k$ that $P_{k+1}$ has no zero in $(z_i, z_{i+1})$ if $P_{k+1}(z_i)$ and $P_{k+1}(z_{i+1})$ have the same sign, and that $P_{k+1}$ has only one zero in $(z_i, z_{i+1})$, if $P_{k+1}(z_i)$ and $P_{k+1}(z_{i+1})$ have different signs. (HOFBAUER)

(3) Automorphisms of the 2-shift. Let $X = \{0,1\}^{\mathbb{Z}}$ be the space of two-sided sequences of 0's and 1's, and let $\sigma : X \to X$ be the shift. It is well-known that every automorphism of $(X,\sigma)$, i.e., every shift-commuting homeomorphism of X, can be expressed as $\sigma^k \circ f$ for some $k \in \mathbb{Z}$ and some uniformly finite-length block map (i.e.,code). Thus the group G of automorphisms is at most countable. The theorem of Curtis, Hedlund and Lyndon (Hedlund, Math.Systems Theory,1969) states that G contains a copy of evry finite group.
The subgroup G' generated by $\sigma$ and the elements of finite order

contains all known automorphisms. <u>Problem</u>: Is $G' = G$.

(4) Positively expansive maps on manifolds.

A continuous map $f : M \to M$ of a compact manifold is called positive expansive if there is a constant $e > 0$ such that if $x \neq y$, then $d(f^n(x), f^n(y)) \geq e$ for some $n \geq 0$. A $C^1$-endomorphism $g : M \to M$ is called expanding if there exists $c > 0$ and $\lambda > 1$ such that $\|Dg^n(v)\| \geq c\lambda^n \|v\|$ for all $v \in TM$ and all $n \geq 1$.

<u>Problem</u>: Is every positively expansive map of a compact manifold topc logically conjugate to an expanding endomorphism?

(COVEN)

(5) Let $\beta > 1$ be real and let $T_\beta : [0,1) \to [0,1)$ be the transformation $T_\beta x = \beta x \pmod 1$. A point $x \in [0,1)$ is called <u>periodic</u> if the set $\{T_\beta^n x : n \geq 0\}$ is finite, and $Per(\beta)$ will denote the set of periodic points for $T_\beta$. The following results are known.

1. Let $\beta$ be a Pisot number (i.e. an algebraic integer $> 1$ with all conjugates of modulus $< 1$). Then $Per(\beta) = \mathbb{Q}(\beta) \cap [0,1)$, when $\mathbb{Q}(\beta)$ is the smallest subfield of $\mathbb{R}$ containing $\beta$.
2. If $Per(\beta) \supset [0,1) \cap Q$, then $\beta$ is either a Pisot- or a Sale number (= an algebraic integer $> 1$ with all conjugates of modulus $\leq 1$)

<u>Problem</u>: If $\beta$ is a Salem number, is it true that $\beta \pmod 1 \in Per(\beta)$? Is it true that $Per(\beta) = \mathbb{Q}(\beta) \wedge [0,1)$? (SCHMIDT)

(6) Consider aperiodic and irreducible topological Markov chains T and T'. If T is a factor of T' and T' a factor of T are then T and T' conjugate?

(KRIEGER)

(7) Let $f$ be a $C^r$ $(r\geq 1)$ diffeomorphism of a compact manifold. Can $f$ at the same time have positive topological entropy and be strictly ergodic? I think that the answer is negative. Moreover the following conjecture may be true. For any $h$; $0 \leq h < h(f)$ there exists a probability f-invariant Borel ergodic measure such that $h_\mu(f) = h$. (A.Katok: Hyperbolicity, entropy and minimality for smooth dynamical systems, preprint, Univ. of Maryland, 1979

(8) Let $f$ be a map of an interval $I$ into itself of class $C^2$ (or $C^{1+\varepsilon}$) except for finite number of points with one-sided derivativ (finite or infinite) at all points; $\mu$ - an absolutely continuous f-

invariant ergodic measure such that $h_\mu(f) = 0$.

Conjecture: f as an automorphism of $(I,\mu)$ is metrically conjugate to an interval exchange transformation.

Stronger version: There exists an f-invariant set $A \subset I$ $\mu(A) = 1$ such that $f|_A$ is topologically conjugate to an interval exchange transformation.

Question: Find a counterexample to this conjecture for $C^1$ maps.

(9) Let $\Gamma$ be a finite graph $f : \Gamma \to \Gamma$ a continuous map $P_f = \{n \in \mathbb{N} : \exists\, x \in \Gamma \quad f^n x = x, \quad f^k x \neq x,\ k = 1,\dots,n-1\}$. Describe possible sets $P_f$ for a given graph.

For the interval the answer is given by Šharkovski, for the circle with the extra assumption

$P_f \ni 1$ by Block.

Let $f : I \to I$ be an interval exchange transformation. (i) If f is weakly mixing with respect to the Lebesgue measure then it is topologically mixing. (ii) If f is strictly ergodic and topologically weakly mixing then it is metrically weakly mixing. (iii) If f has an eigenfunction then this function is continuous in the "symbolic topology" (the weakest topology which is stronger than the standard one and which makes f a homeomorphism. (iv) Discrete part of the spectrum of f may contain one irrational frequency and finite number roots of unity. Similar question can be asked for substitution minimal sets.

(KATOK)

(10) Non-wandering sets for powers of a diffeomorphism: Recall the definition of the non-wandering set of a map $f : M \to M$ :
$\Omega(f) = \{x \in M \mid \exists\, x_i \to x,\ n_i \in \mathbb{N}.\ni.f^{n_i}x_i \to x\}$. It is easy to see that a point x belongs to the nonwandering set of a power of f, $g = f^N$, if it belongs to $\Omega(f)$ and furthermore the "return times" $n_i$ of $x_i$ to the vicinity of x can be chosen all divisible by N. Sawada ("On the iteration of diffeomorphisms without $C^0$ $\Omega$-explosions," Proc.Amer.Math. Soc. 79 (1980), 110-122) recently constructed an example of a diffeomorphism $f : S^2 \to S^2$ such that $\Omega(f^2) \neq \Omega(f)$. Further examples of diffeomorphisms of surfaces and maps of the interval with $\Omega(f^N) \neq \Omega(f)$ for various $N > 1$ were given by Coven and Nitecki ("Non-wandering sets of the powers of maps of the interval", to appear). These are

based on examples built on the Möbius band and the annulus; in the latter case $f$ interchanges the two boundary circles. These examples embed in any closed surface of positive genus, and in any higher-dimensional manifold, but they do not embed in the two-dimensional disc. Problem: Does there exist a diffeomorphism $f : D^2 \to D^2$ for which $\Omega(f^N) \neq \Omega(f)$ for some $N > 1$?

(11) Homtervals: Suppose $M = I$ or $S^1$ is a one-dim. manifold and $f : M \to M$ is a continuous map. A homterval for f is a non-degenerate interval $J \subset M$ such that (i) the iterates of J are disjoint: $f^nJ \cap f^mJ = \emptyset$ for $n \neq m$; (ii) for each $n > 0$, $f^n|J$ is a homeomorphism; and (iii) J does not belong to the stable set of a periodic sink. (The terminology, due to Misiurewicz, is a transcription of his Polish term, "homcinek".) Homtervals were first encountered by Poincaré and Denjoy in the context of aperiodic, non-transitive homeomorphisms of the circle. Denjoy showed that there exist $C^1$ diffeomorphisms of $S^1$ with homtervals, but if the derivative of f has bounded variation (in particular, if $f$ is $C^2$), then $f$ has no homtervals. Recently, the question of existence of homtervals for certain maps of the interval has aroused some interest in the work of Misiurewicz ("Absolutely continuous measures for certain maps of the interval", IHES 1979), and Guckenheimer ("Sensitive dependence to initial conditions for one-dimensional maps", Comm.Math.Physics 70 (1979),133-160). The question arises because its intimate connection with the question of whether a symbolic dynamics for for such a map separates points. Misiurewicz and Guckenheimer (op.cit.) have shown that certain interval maps, characterized especially by a negative Schwartzian derivative:

$$Sf = f'''/f' - 3/2(f''/f')^2 < 0$$

have no homtervals (Misiurewicz assumes also that the orbit of every critical point is bounded away from the critical set, while Guckenheimer assumes a single critical point), and it can be shown that for any $C^2$ map $f : I \to I$, no homterval can have an orbit bounded away from all critical points. On the other hand, the Denjoy example on $S^1$ can be adapted to give an example of a $C^1$ map $f : I \to I$ which possesses homtervals (see Coven-Nitecki, op.cit., for an example with two critical points; this can be modified to give an example with a unique critical point.

Problem 1: Does there exist a $C^2$ Denjoy homeomorphism - i.e., is there a homeomorphism $f : S^1 \to S^1$ with irrational rotation number that possesses homtervals but is $C^2$ ? Note that such an example must have critical points. An answer to the question above might also help solve Problem 2: Does there exist a $C^2$ map $f : I \to I$ with homtervals?

(NITECKI)

(12) Strange series. It can be proved that a series like $\sum_{k=1}^{\infty} \frac{\langle x+k\alpha\rangle - 1/2}{k}$ where $\alpha$ is irrational and $\langle y\rangle$ is the fractional part of $y$, converges for almost all $x \in [0,1]$. Does the equicontinuity of the maps $y \to \langle y+k\alpha\rangle$ then force it to converge for all x?

(13) Weak mixing and density zero. If $T : X \to X$ is a weakly mixing measure-preserving transformation and $A,B \subset X$ are measurable, then there is a set $N \subset \mathbb{N}$ of density zero such that

$$\lim_{\substack{n\to\infty \\ n \notin N}} \mu(T^n A \cap B) = \mu(A)\mu(B).$$

S.Kakutani has observed that it is possible in fact to choose a single set N of density zero which works simultaneously for all A,B. Of course it is not N itself but rather some equivalence class, [N], that is important - for example, only the tails of N matter. Which equivalence classes [N] can arise? How are transformations with the same [N] related?

(14) Expected multiple recurrence time. Furstenberg's ergodic Szemerédi theorem implies that if $T : X \to X$ is a (finite) measure-preserveing transformation and $A \subset X$ is a set of positive measure, then almost every point of A is multiply recurrent with respect to A: for almost all $x \in A$, given $l$ there is an n such that

$$x, T^n x, T^{2n} x, \dots, T^{ln} x \in A.$$

Let $r_l(x)$ be the smallest such n. If T is ergodic, is there any kind of formula for $\int_A r_l(x)d\mu(x)$ ? (Recall Kac' Theorem: $\int_A r_1(x)d\mu(x) = 1.$)

(PETERSEN)

(15) Suppose $\varphi : (X,t) \to (Y,T)$ is a homomorphism of minimal flows having a RIM, $\lambda$. Is there a RIM $\mu$ of $\varphi$ such that (support $\mu_y)r = \varphi^{-1}(y)r$ for some $r \in J$. What if $X = M$ the universal minimal set.

What if $X$ is metric and there is a RIM, $\alpha$, on $\varphi \circ \psi$, $M \xrightarrow{\psi} X \rightarrow Y$ that induces $\lambda$, $\hat{\psi}(\alpha) = \lambda$.

(16) Suppose $\varphi : (X?T) \rightarrow (Y,T)$ is an open homomorphism of metric minimal flows having a RIM. Given $y$ does $\exists$ $x,x' \in \varphi^{-1}(y)$ such that the smallest closed invariant equivalence relation containing $(x,x')$ equals $R(\varphi) = \{(x,x') : \varphi(x) = \varphi(x')\}$. What if $Y$ is a singleton and $x' \notin xT$.

(McMAHON)

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