

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T A G U N G S B E R I C H T 2/1981

MATHEMATISCHE THEORIEN DER FLUIDE

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Die Tagung wurde von den Professoren Wolfgang Bürger (Karlsruhe) und Ingo Müller (Berlin) organisiert. 53 Teilnehmer waren anwesend, und so wurde die Kapazität des Instituts voll ausgenutzt. Das Generalthema der Tagung war die mathematische Beschreibung der Materialfunktionen und der Strömungseigenschaften von Flüssigkeiten, wobei der Schwerpunkt bei den nichtkonventionellen Flüssigkeiten und Strömungen lag.

Innerhalb dieses Rahmens wurden Beiträge geliefert zur Beschreibung von nicht-Newton'schen Flüssigkeiten, Polymerlösungen, Flüssigkristallen, viskoelastischen Stoffen, Elastomeren, Memory-Legierungen und plastischen Stoffen. Es gab dabei sowohl rein mechanische als auch thermodynamische Betrachtungen. Die 39 Vorträge lassen sich unter den folgenden Überschriften einordnen:

- i.) Stabilität
- ii.) Strömung viskoelastischer Flüssigkeiten
- iii.) Polymerlösungen
- iv.) Mathematische Grundlagenprobleme bei Strömungen von Flüssigkeiten
- v.) Materialien mit Formerinnerungsvermögen und viskoelastische Stoffe
- vi.) Superflüssigkeiten
- vii.) Irreversible Thermodynamik

Besondere Erwähnung verdienen ein sehr schöner Lehrfilm über Rheologie, den Professor Walters im Anschluß an seinen Vortrag zeigte, und eine abendliche Diskussion über superfluides Helium, an der sich diejenigen Teilnehmer beteiligten, die im Laufe der Sitzung über Superflüssigkeiten einige kontroverse Standpunkte vertreten hatten. In dieser Diskussion zeigte sich Übereinstimmung, daß die Probleme rotierenden Heliums noch einer systematischen Theorie bedürfen.

Professor R.S. Rivlin als Leiter der letzten Sitzung war so freundlich, einige Schlußworte zu sprechen, in denen er der Tagung interessante Vorträge und Diskussionen und ein gutes wissenschaftliches Niveau bescheinigte.

Vortragsauszüge

Section: Stability

D.D. JOSEPH

Instability of the State of Rest of Fluids of Arbitrary Grade  $n \neq 1$

It is shown that the rest state of a fluid of grade  $N$  or complexity  $N$  is unstable in the spectral sense of linearized theory if the ratio of the coefficients of  $A_n$  and  $A_{n-1}$  in the constitutive equation is negative. Negative ratios, and only negative ratios are implied by integral expansions of the stress.

B. STRAUGHAN

Catastrophic Instabilities in a Fluid of Grade three

Thermodynamical considerations have shown that the most general form for the stress constitutive relation of an incompressible fluid of grade three is  $T = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta(\text{tr}A_1^2)A_1$ , where  $A_1$  and  $A_2$  are the first two Rivlin-Ericksen tensors. The condition  $\alpha_1 < 0$ , which is observed experimentally and is compatible with the Clausius-Duhem inequality although it is not compatible with the free energy being a minimum in equilibrium, will be shown to lead to behaviour thought not to be physically acceptable.

S. KROZER

The Use of Nearly Viscometric Flow Concept to the Discussions of Some Hydrodynamic Stability Problems

A linear stability analysis of viscometric flow of shear thinning viscoelastic fluids with short memory time was discussed, taking into account the concept of the nearly viscometric flow. The discussion was based on a derived, for such conditions modified, Orr-Sommerfeld equation. An approximate simple stability criterion was obtained for Couette as well as for channel flow. The flow in a pipe becomes more unstable for lower shear velocities on the wall than the flow in a two dimensional channel. The results of the evaluation agree with known experimental results.

F.M. LESLIE

An Instability in Oscillating Shear Flow of Nematic Liquid Crystal

The continuum theory proposed by Ericksen and Leslie for flow phenomena in nematic liquid crystals describes a variety of dynamic effects in these anisotropic liquids rather well. In the context of viscometry nematics divide into two classes, those which align uniformly in shear flow and those which do not, and theory distinguishes the two according to the sign of a particular viscous coefficient. This paper studies the likely behaviour in oscillatory simple shear flow making some reasonable approximations and predicts that certain non-flow aligning materials are unstable in this flow, the critical amplitude for the instability being independent of frequency, but directly proportional to the gap width. Agreement with existing experimental results is reasonably good.

P.J. BARRATT

Thermal Instabilities in Nematic Liquid Crystals

Continuum theory is used to investigate the onset of both stationary and oscillating convective instabilities in a thin sample of nematic liquid crystal confined between two horizontal flat plates when a temperature gradient is applied perpendicular to the plates. We consider the cases in which the anisotropic axis is initially aligned perpendicular to the plates and allow for the presence of a uniform magnetic field applied parallel to the initial alignment. Using a Fourier series method, we first derive an expression for determining the critical temperature gradient at which stationary convection sets in and show that this yields an almost analytic solution. A remarkable result is that one must heat the upper plate! Secondly we employ Chebyshev polynomials to obtain accurate values for the gradient thresholds at which oscillatory convection is possible in MBBA when heating from below. The variation of threshold with magnetic field strength is examined and it is shown that the nature of the instability changes from oscillatory to stationary convection at a critical magnetic field strength. Results in the latter case are shown to be in good agreement with experimental observations.

Yu. A. BEREZIN, N.N. YANENKO

On Gasdynamics with non-Monotonic Equation of State

Evolution of gas sphere in a field of gravitation is considered. The dependence of pressure on density is non-monotonic (Van-der Waals' type). The regions with  $dp/d\rho < 0$  are unstable. Qualitative analysis and the numerical solutions of the governing equations of variable type show that we have either a separation of the sphere in two parts with very different densities and a sharp boundary between them (regime 1) or a continuous radial density distribution (regime 2). In the regime 1 the gas densities out of the sharp boundary are in the stable regions with  $dp/d\rho > 0$ . During the time both density and radius of the sphere have oscillations. These gasdynamical autovibrational processes are due to non-monotonic equation of state (Van-der-Waals' type).

Section: Flow of Viscoelastic Fluids

K. WALTERS

Newtonian and non-Newtonian Flow in Complex Geometries

A flow visualization technique using an expanded laser beam and trace amounts of particulate additives is employed to study the behaviour of Newtonian and non-Newtonian elastic liquids in a number of complex geometries. Particular attention is paid to the effect of fluid elasticity on the flow characteristics. Attempts are made to simulate numerically the observed flows using finite-difference techniques. The agreement between theory and experiment in this regard is very satisfactory.

J.Y. KAZAKIA

The Influence of Vibration on Poiseuille Flow on a non-Newtonian Fluid

An incompressible, isotropic, non-Newtonian fluid flows through a straight, rigid pipe of circular cross-section, under a constant pressure gradient. It is experimentally known that moderate vibrations of the boundaries produce significant increases in the mean rate of discharge of the fluid.

This paper presents a mathematical modelling of this effect by the use of a constitutive equation of the Rivlin-Ericksen type with the assumption that the fluid is slightly non-Newtonian. We consider superposed vibrations that are longitudinal and/or rotational.

In addition, a study of the unsteady Poiseuille flow of a second order Rivlin-Ericksen fluid through a straight pipe of non-circular cross-section will be presented. It is seen that in this case the vibration of the boundaries causes lateral secondary flows.

R.S. RIVLIN

The Simple Fluid Concept

The assumption underlying and leading up to the concept of a simple fluid will be examined critically. An attempt will be made to determine which elements in the development are based on laws of physics, which are based on arbitrary assumptions restricting the class of materials or flows, and which are based on faulty mathematical reasoning.

Section: Polymer Solutions

J.R.A. PEARSON

Models for Polymer Melts

A brief survey and comparison of three statistical models for polymer melts was given:

- (a) Bead-spring or bead-rod model
- (b) Temporary junctions network model
- (c) Reptating phantom chain model

In all of these, the basic chemical unit, a linear macromolecule is treated phenomenologically and leads to an orientation distribution function for purely elastic sub-elements. Each has a characteristic length scale: bead diameter (or mean spring length) for (a), mean spring length in (b) tube diameter in (c), each characteristic line scale: from the resistance coefficient for bead motions in (a); from the rate of destruction or formation of junctions in (b); from the chain diffusion rate in a tube in (c).

A systematic affine deformation process applies: To the suspending line in (a); to the junctions in (b); to the tubes in (c). A randomizing process applies to the spring vibration distribution function in all these cases: directly in (a); to recently broken junctions in (b); to chain ends diffusing out of tubes in (c). Thus the fundamental properties of three apparently very dissimilar physical models are in effect very similar, and so are the rheological equations of state derived from them.

P.K. CURRIE

Calculations on the Doi-Edwards Model for Polymer Melts

Recently, Doi and Edwards have derived from a molecular model a constitutive equation relating the stress in a polymer melt to its history of deformation. The model is valid for all possible flows of the melt, and thus predicts a theoretical correlation between the behaviour of the melt in shear and its behaviour in other flows, such as extension. The correlation between shear and extensional flows is currently of some interest, and therefore the predictions of the Doi-Edwards model have been studied in some detail. It is shown that a single potential function  $U$  characterizes the strain-dependent memory-fading predicted by the model. An exact expression is found for  $U$ , and also a good approximation, expressed as a simple function of a simple combination of the strain invariants. Comparison with experimental results suggests that the strain-dependent memory-fading predicted by the Doi-Edwards model is too strong.

R. TAKSERMANN-KROEGER

Mathematical Description of a Polymer Network in Flow

We describe a molecular theory of the flow behaviour of condensed polymer systems. The model is based on the idea of a random network in which functions decay and reform permanently (temporary network). These two processes which are assumed to be correlated cause a viscous contribution to the deformation and flow behaviour of the polymer. The dynamics of the junctions is treated in analogy to Boltzmann's collision ansatz. We apply methods of statistical physics and of the theory of graphs. Equations are derived which describe completely the many-particle problem (the junctions are treated like particles.) The many-particle problem is simplified using the Hartree self-consistent field method. The resulting one-particle problem is governed by a set of simultaneous integro-differential equations, the Hartree equations. These equations can be treated by iteration starting with the Boltzmann relaxation time approximation. The so-obtained probability density will be used to calculate the expectation values for the observable quantities like stress tensor and viscometric functions.

A. SZANIAWSKI

Model of a Suspension of Particles Dispersed Regularly in a Continuous Phase

Considering the equations of motion of a dilute suspension with small volumetric fractions of particles  $\phi \ll 1$ , the main attention will be paid to terms of order  $\phi^n$  where  $n < 1$ . For instance the mean drag force  $D$  of uniformly moving spheres, randomly distributed in space is found to be /S.Childress/  
 $D = D_0 + A \phi^{1/2} + \dots$ . The averaged stress tensor  $\tau$  may be divided into two components:  $\tau = \tau_s + \tau_m$ . The tensor of averaged local stresses  $\tau_s = \langle \tau \rangle$  contains the Einstein term  $O(\phi)$  and eventually higher order terms of  $\phi$  and will not be an object of our interest. The problem consists in evaluating the mean momentum exchange  $\tau_m = - \langle \rho(\tilde{u}-u)(\tilde{u}-u) \rangle$ . Since the local microscale distribution of velocity  $u$  is very difficult to determine for a real distribution of real particles, artificial models should be considered. It will be shown that for a model with regular distribution of spherical particles in a cubic lattice the tensor  $\tau_m$  contains terms  $O(\phi^{2/3})$  which grow faster than the "classical" Einstein term  $O(\phi)$ . Disregarding terms containing  $O(\phi^{2/3})$  does not seem to be justified in other models.

O. HASSAGER

Step Shear Experiments, Constitutive Relations and Variational Principles

Non-Newtonian liquids are often tested in a "step shear experiment", where a material initially at rest is subjected to an instantaneous simple shear deformation and the shear and normal stress components are subsequently measured. It is shown here that the step shear experiment gives important information about the form that the constitutive relation for the material can have. For example a memory integral expansion in the Cauchy strain tensor with bounded memory functions always yields the "Lodge-Meissner relation" in a step shear experiment, independent of the number of terms included in the expansion. On the other hand many simple differential models give a different step shear relation and they can therefore not be represented by such a memory integral expansion.

A variational principle for creeping motion of incompressible liquids described by a class of KBKZ models is described. Volume forces and surface forces derivable from a potential are included. It is shown that the extremum of an integral over the deformation history of the sum of a volume integral and a surface integral is equivalent to the equations of motion and continuity being satisfied for all particles in the volume, and the boundary conditions being satisfied for all particles on the surface.

Section: Basic Problems in Fluid Flow

R.J. KNOPS

Uniqueness and Continuous Dependence of Solution

The lecture describes a method for establishing uniqueness and continuous dependence of the solution upon initial data in the exterior problem of non-linear elastodynamics under mild asymptotic decay rates at large spatial distances. While the proof does not require the strain energy to be positive in bounded parts of the exterior region, this condition must be imposed in the neighbourhood of infinity. A counter example is used to demonstrate the necessity of this condition.

These and similar results may be of some help to the overall study of phase transitions.



G. CAPRIZ

Free-Boundary Problems from the Theory of Lubrication

The theory of lubrication is the source of many interesting free-boundary problems and singular perturbation problems; some of them have almost text-book character. For others the statement itself is under discussion: for instance, it is generally conceded that the flow phenomena at the boundary of the region of cavitation need to be better understood. The question is discussed from various points of view; in particular the analytical conditions to be satisfied during slow steady flow of a viscous fluid are examined anew and equations of motion for a fluid with bubbles are proposed.

N.T. DUNWOODY, J. DUNWOODY

On the Existence of Retarded Plane Poiseuille Flow of Simple Fluids

The retarded histories of plane laminar-(Poiseuille Flow) of Hereditary Simple Fluids between two parallel plates at a finite distance apart are shown to exist at arbitrary time  $t = 0$  to any order of approximation in the retardation parameter according to the scheme of approximation of Coleman and Noll. The result obtained by Coleman and Mizel for second order fluids is reinterpreted in the above context.

K. HUTTER

An Extended Channel Model Describing Shallow water waves in Rotating Elongated Basins

Considering the channel-like shape of many lakes, a hydrodynamic model is developed. The 3-D equations are formulated in a curvilinear coordinate system along the "long" axis of the lake. Applying the method of weighted residuals and expanding the field variables with shape functions over the cross sections, approximate equations for the fluid motion are derived. The emerging equations form a cross-sectionally discretized set of spatially 1-D PDE's in the longitudinal lake direction. At first the channel equations are presented for arbitrary shape functions with arbitrary closure conditions.

Subsequently they are specialised for Cauchy-series. The free oscillation of the simplest channel model is shown to reduce to the classical Chrystal equation. A first order linear model exhibits the essential features of gravitational oscillations in rotating basins in that it allows approximate reproduction of Kelvin, Poincaré and inertial waves. Positively and negatively ret. amphidromics are shown to develop and this is borne out by numerical rotations from a real lake.

S. ZAHORSKI

On Harmonic Waves in Simple Viscoelastic Fluids

Certain cases of plane harmonic shear waves as well as one-dimensional longitudinal waves, propagating in simple viscoelastic fluids, are considered under the assumption of either linear shear response or small amounts of deformation. To this end the generating flows are treated as particular cases of the motions with proportional stretch histories. Many fundamental properties of such waves are discussed in greater detail for very low and very high /ultrasonic/ frequencies. These are the damping effects, the phase shifts, the maximum amounts of deformation, the speeds of propagation etc. The high frequency behaviour of the fluids considered essentially depends on the generalized dynamic viscosities or moduli. Moreover, the phenomena of wave reflection and refraction are analysed for the case in which the direction of propagation forms some angle with a plane interface between two non-mixing viscoelastic fluids.

Section: Shape Memory/Viscoelasticity

F. BAUMGART

New Results for Memory Alloys

The lecture shows at first shortly the shape-memory-effect in Nickel-Titanium-Alloys. The practical applications and fields of investigation considered by the Krupp-Research-Institute are:

- space application (release and step mechanism)
- medical application (orthopedic implants)
- energy conversion

Some example for continuous memory-engines are demonstrated and a very simple type is investigated. With a simple theory for memory sheets in bending it is proved that such an engine has an output of mechanical work.

F. FALK

Landau Theory for Shape Memory Alloys

The shape memory effect in alloys is due to a martensitic first order phase transition. For a one dimensional model system a phenomenological Landau free energy is presented, which accounts for the observed mechanical and thermodynamical behaviour of the shape memory alloys, e.g. the stress-strain curves, the elastic constants, the latent heat of the phase transition, and the shape memory effect. Information about phase boundaries is provided by an appropriate Ginzburg-Landau free energy density.

I. MÜLLER, K. WILMANSKI

Model for Shape Memory Materials

The objective of the paper is to construct the statistical model, simulating the behaviour of the shape memory material in thermodynamic equilibrium.

On the basis of the notion of a lattice element, introduced in the paper, the mean field potential is postulated. It is shown that the required properties of the Helmholtz free energy (first order phase transition) follow if this potential has three minima, the middle one being shallower than the remaining two.

We derive the Helmholtz free energy by means of the configurational part of the partition function. It is shown that the assumption of the unconstrained equilibrium yields the description of the shape memory material in high temperature. The lower bound for this approximation is found.

The low temperature approximation is shown to be appropriate under the assumption of the constrained equilibrium.

J.F. BESSELING

Rubber Thermo-Visco-Elasticity (large deformation)

Simple rheological models like the Maxwell and the Kelvin solids, as well as their generalizations, have proved their value in the description of creep and plasticity of metals. In the case of large deformations the stresses are then to be related to small elastic strains, which may be looked upon as small

deviations from a stressfree, geometrical natural reference state. In the case of rubber visco-elasticity we may have large elastic strains of the macromolecular network and small elastic strains of a visco-elastic fraction of the material. Then the question arises of a proper formulation of the macroscopic stress-strain-time relation. A constitutive hypothesis for the thermo-visco-elastic behaviour of rubberlike materials will be discussed.

P. STREHLOW

#### On Stress Induced Motion of Alkali Ions in Vitreous SiO<sub>2</sub>

The surface properties of glasses become more and more important for e.g. date display windows, channel plates and glass fibres for optical communication. In addition, alkali ion transport processes are used for surface hardening. This shows that the knowledge of transport processes and surface properties in glasses is important. In this context the thermodynamic mixture theory is applied to calculate the stress distribution and the alkali ion concentration of a glass sphere after annealing. Application of continuum thermodynamics on curved surfaces and solution of the corresponding boundary value problem yields a simple possibility for the determination of surface-tension. The calculated distribution of alkali concentration is confirmed by experiments on isotop activation and photometric measurements.

L.A. TURSKI

#### Hydrodynamic Concepts in the Theory of Low-Dimensional Continuous Magnetic Systems

The dynamical properties of a continuous Heisenberg chain are analyzed by means of the curvature-torsion variables rather than conventional polar angles. In contrast to the conventional approach the present one allows for the least action principle formulation and resulting equations of motion are analogous to these known from the one-dimensional hydrodynamics of a "quantum" fluid. Various consequences of these equations like the instability of a finite amplitude spin wave are analyzed. It is also shown that compressible Heisenberg chain remains completely integrable and the relations between magnetic soliton and elastic kinks are discussed in some detail.

P. HAUPT

Thermodynamic Restrictions on Material Functions of Finite Linear Viscoelasticity

The free energy functional is assumed as a quadratic form of the relative strain history. Then, the constitutive equation for the stress consists of one functional which depends linearly on the strain history and of one quadratic term. The functions occurring in integral representations of these functionals are related to the intrinsic material properties, i.e. the relaxation and creep behaviours. As a consequence of the thermodynamic equations, the free energy has to satisfy the minimum property. Moreover, the internal dissipation has to be positive. This implies the validity of an equation which restricts the general form of the material functions. As a result, the material functions are exponentials, where the corresponding relaxation spectra are continuous.

H. ZORSKI, E. KRÖNER

Balance Laws for Bodies with Cracks

The considered body contains a moving crack. On the basis of the first law of thermodynamics (with no radiation) integral balance laws are derived and surface (path) independent integrals are deduced for both static and dynamic cases and arbitrary constitutive laws. Energy release rate and forces on the crack tip are defined.

Similar considerations are carried out for the balance of linear momentum, field linear momentum and field angular momentum.

Section: Super Fluids

R.J. ATKIN, N. FOX

Low-Temperature Effects in Liquid Helium

The derivation of the equation of motion for liquid helium II usually involves the notional separation of the liquid into two ingredients, normal fluid and superfluid. The total mass density of the liquid is taken to be the sum of the

mass densities of these two ingredients. Using the techniques of generalized continuum mechanics, we show how a general theory for the liquid may be derived in a more direct manner without making such an assumption. This theory is then used to provide explanations of a number of observed effects. We also show how a special case of our theory reduces to the usual two-fluid theory.

D. LHULLIER

Superfluidity and Liquid-Vapor Mixtures

After a presentation of the main physical aspects of superfluidity, we show why the Landau "two-fluid" model should better be thought of as a model of fluid with intrinsic momentum. It is nevertheless possible to improve the description of a chemical mixture of two fluids on the basis of a (limited) analogy with superfluids. In this spirit we present a set of equations describing a liquid-vapor mixture (with phase change) when the two velocities and two pressures are different. Lastly we comment upon difficulties connected with the possibility of two different temperatures.

W. DREYER

On the Correspondence between the Rational Mixture Theory and Landau's Theory of He II

Landau was the first who laid down field equations which explain many of the extraordinary properties of superfluid Helium.

Many others followed him and used his equations as the basis for a generalized theory in order to include properties which appear as consequences of the occurrence of vortex lines.

However, the assumptions made by Landau which guide him to his "Two Fluid Equations", are not written down very explicitly, and these equations seem to be very strange in the light of a mixture theory. The Rational Mixture Theory gives us the possibility to rederive the Landau Equations from four additional assumptions.

Section: Irreversible Thermodynamics

K.-H. ANTHONY

Eine phänomenologische Feldtheorie der irreversiblen Prozesse

Unsere Bemühungen zur feldtheoretischen Beschreibung statischer Defektstrukturen sowie reversibler dynamischer Prozesse von geordneten Systemen bewegen sich durchweg im Rahmen des Lagrange-Formalismus der Felder, des Hamiltonschen Prinzips und der sehr allgemeinen Methoden der nichteuklidischen Geometrien zur Beschreibung der großen Deformationen und der Defektstrukturen. Ich sehe keine Möglichkeit, diese Theorie - und an dieser Theorie soll aufgrund ihrer klaren Methodik festgehalten werden - durch Hinzufügen der phänomenologischen Thermodynamik der irreversiblen Prozesse Onsagerscher Prägung zu einer Theorie der dissipativen Phänomene auszubauen. Ein besonders gravierendes Hindernis ist die Tatsache, daß die Onsagersche Theorie eine Extrapolation der Gleichgewichtsthermodynamik ist und damit von vornherein nur in hinreichender Nähe des Gleichgewichts anwendbar ist. Der Lagrange-Formalismus und das Hamiltonsche Prinzip der Felder, die sich für nichtdissipative Phänomene längst bewährt haben, gehen umgekehrt von dynamischen Prozessen ohne Beschränkung auf die Gleichgewichtsnähe aus und schließen die Gleichgewichtszustände in natürlicher Weise ein. Die gesamte physikalische Information des Prozesses ist in einem Langrangefunktional enthalten. Daraus folgen mittels des Hamiltonschen Variationsprinzips die Feldgleichung (z.B. Bewegungsgleichungen) und mittels des Noether-Theorems alle Bilanzgleichungen (z.B. Energie, Impuls, Drehimpuls, Ladung, Masse) sowie alle statischen und dynamischen Zustandsgleichungen. Es wird gezeigt, daß diese Methodik auch auf irreversible Prozesse übertragbar ist. Für beliebige Prozesse ergeben sich insbesondere Definitionen für die Entropiedichte, den Entropiestrom und die Entropieerzeugung. Die positive Definitheit der Entropieerzeugung wird mit der Stabilität der Prozesse im Ljapunowschen Sinne verknüpft. Zwanglos ergibt sich auch ein verallgemeinertes Prinzip der minimalen Entropieproduktion, das im Gegensatz zum entsprechenden Prinzip der Onsagerschen Theorie nicht auf die lineare Theorie beschränkt ist. Es wurde eine Integrationsmethode entwickelt, die zu einer großen Klasse von Feldgleichungen und möglicherweise sogar zu jeder Feldgleichung eine Lagrangefunktion zu berechnen gestattet.

W. MUSCHIK

Is the Principle of Objectivity Valid or Not?

An OBSERVER is

A set of frames of reference

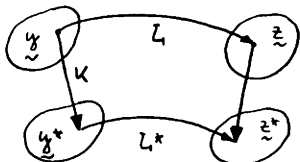
$$\mathcal{L} = (0, 0^*, 0^{**}, \dots)$$

will be called an observer, if each two frames are connected by an observer-invariant change of frame;

which are defined by:  $0 \rightarrow 0^*$

$$\begin{aligned} x^{\alpha} &= f^{\alpha}(x^{\beta}), \quad \exists f^{\alpha^{-1}}, \quad \frac{\partial x^{\alpha}}{\partial t} = 0, \quad \alpha = 1, 2, 3, 4, \quad \alpha = 1, 2, 3 \\ e_m^* &= R e_m, \quad \tilde{w}^m = K^m_i w^i, \quad R = K^{-1} e_m^* e_i \end{aligned}$$

Therefore each observer has its own (private) tensor formulation of physical quantities:  $\underline{y}, \underline{z}, \dots$  For two observers and one material the Axiom of Identification is valid for constitutive equations L and L in the following sense



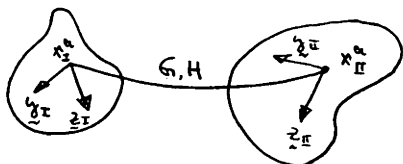
$$L^*(\cdot) = M \cdot L(K^{-1} \cdot)$$

COVARIANCE is defined by

$$M = 1 \wedge K = 1, \text{ i.e.}$$

there exists a common tensor-formulation for all observers:  $L^*(\cdot) = L(\cdot)$ .

If we consider two identical bodies I and II in different states of motion



$x_I^{\alpha}$  rests in the inertial observer  $\mathcal{L}$

$$x_{II}^{\alpha} = Q^{\alpha}(\mathcal{L}) x_I^{\alpha} + c^{\alpha}(\mathcal{L})$$

$$y_{II} = G(y_I), \quad z_{II} = H(z_I) \quad \exists G, H$$

Principle of OBJECTIVITY  $\underline{z}_I = L(\underline{y}_I) \rightarrow \underline{z}_{II} = L(\underline{y}_{II})$ .

Covariance and objectivity give  $L = HLG^{-1} = L^*$ , where L belongs to I and  $L^*$

to II. For special material  $L = \alpha 1$  we get:  $\underline{z}_I = \alpha \underline{y}_I \rightarrow \underline{z}_{II}^* = \alpha \underline{y}_{II}^*$

which contradicts I. Müller's counterexample of the rotating cylinder with

heated axis for which  $q_I^* \neq (\nabla \frac{1}{T})_{II}^*$ .



J. STICKFORTH

Eine neue Theorie viscoelastischer Fluide auf der Grundlage des Begriffs der Zwischenkonfiguration

Die kinematische Grundlage der modernen Plastizitätstheorie, nämlich das Gesetz der multiplikativen Zusammensetzung des elastischen und des inelastischen Deformationsgradienten,  $\underline{F}_{(e)}$  und  $\underline{F}_{(i)}$ , wird aus seiner nicht notwendigen Verbindung mit den inneren Zustandsgrößen der Plastizität gelöst und in den Rahmen der pseudo-linearen irreversiblen Thermodynamik eingefügt (wodurch die Begriffe elastisch und nichtelastisch tatsächlich erst physikalisch definiert werden). Es ergibt sich eine allgemeine thermomechanische Theorie großer Verformungen ohne Objektivitätsproblem und ohne funktionelle Materialgesetze; trotzdem gibt es das Phänomen Gedächtnis, weil  $\underline{F}_{(i)}$  den Charakter eines inneren Parameters wie die Reaktionslaufzahl  $\xi$  bei chemischen Reaktionen besitzt. - Die Anwendung auf eine ebene Couetteströmung liefert weitgehende Übereinstimmung der sich als geometrieabhängig erweisenden visko-metrischen Funktionen mit den wirklichen Verhältnissen.

J. KRATOCHVIL, M. TOKUDA

Fading Memory of Induced Anisotropy in Plastic Materials

Plastic response of materials is strongly influenced by the deformation history. This phenomenon manifests itself as a work-hardening and an induced anisotropy of plastic properties. Experimental results demonstrate that the memory of induced plastic anisotropy is of a fading type: an anisotropy induced by a deformation history is erased and replaced by the new one in the course of further deformation. It is shown that the mechanism of the memory of the induced anisotropy is controlled by internal stresses arising due to interaction among grains of a deformed polycrystalline aggregate. An idealized mathematical model of the polycrystal is used to evaluate the internal stresses and to model induced anisotropy behaviour. A successful

comparison of computational and experimental results is demonstrated. A possibility to build the simplified version of the model into the finite element method of solution of elastic-plastic boundary-value problems is studied.

G.A. MAUGIN

The Use of Internal Variables in the Modelling of Macromolecular Solutions

In the modelling of the anomalous behavior of the flow of deformable molecules one recognizes the fact that the peculiarities of this flow are typically due to the deformability of the molecules. The latter can be conveniently characterized, in a continuum description, by a quantity which may be a scalar, a vector or a tensor, and is considered as a so-called internal variable that satisfies a time-evolution equation while the global thermo-mechanical behavior of the fluid is still governed by the usual balance laws of continuum thermodynamics. Two cases are presented: (i) one for dilute polymer solutions where the internal variable is a second-order symmetric tensor. Then the model reveals a crucial interdependence between the way in which the fluid strain rate induces the molecular deformation and the way in which the latter contributes to the stress tensor; (ii) another one (still in the process of construction) in which the internal variable is a vector (which represents the mean extensional molecular deformation) and which aims at reproducing the phenomena of stress-induced diffusion and of the existence of concentration gradients in solutions of macromolecules.

G. LEBON, J. CASAS-VAZQUEZ, D. JOU

A Thermodynamic Model for Viscous Fluids

We present a model for non-isothermal viscous fluids in non-equilibrium. The dissipative fluxes, i.e. the heat flux and the viscous stress tensor,

are introduced as independent variables in the constitutive equations. Their rate of change is governed by first-order differential equations. Restrictions on the constitutive equations are, as usually placed by the general principles of continuum mechanics and the second law of thermodynamics. The form of the second law is not Clausius-Duhem inequality, but that proposed by Müller, wherein the entropy flux is given by a constitutive equation. Once the deterministic equations have been considered, we turn our attention to the fluctuations of the dissipative fluxes near their steady state values. Starting from the fundamental Boltzmann relation, we obtain expressions for the second moments of the fluctuation dissipation theorem and lead to a non-Markovian choice in fluctuating hydrodynamics.

A. MORRO

#### Hidden Variable Approach to Dissipative Effects in Magnetofluidynamics

The hidden variable formalism is applied so as to obtain a sound description of relaxation effects associated with viscosity, heat-conduction, and electric conduction in magnetofluidynamics. On assuming that the evolution of the hidden variables is governed by linear equations, compatibility with the Clausius-Duhem inequality leads to a satisfactory improvement of Navier-Stokes-Fourier-Ohm model. Besides allowing wave front propagation, the new scheme appears to be a suitable starting point for further generalizations such as, for example, models accounting for thermogalvano-magnetic effects and cross-effect coupling terms. It is of interest the fact that the evolution for the electric current is strongly suggested when looking at the behaviour of charges in plasmas described as mixtures.

Symmetric Form and Shock Waves for Fluidodynamics Systems

The systems of (both classical and relativistic) Fluidodynamics and Magnetohydrodynamics have a negative entropy density which is a convex function of the field variables. As a consequence one can prove the following results:

- 1) It is possible to choose a field  $\underline{U}$  (main field) such that these systems become symmetric, hyperbolic and conservative; hence the Cauchy problem is well posed in  $H^s$  ( $s > 4$ ). Furthermore the thermodynamic variables appearing in  $\underline{U}$  are the "coldness" and the "thermal potential".
- 2) The entropy grows across a shock surface.
- 3) The jump of the entropy determines the jump of all field variables.
- 4) The velocity of shocks  $s$  lies between the smallest and the largest characteristic velocity. So, in a relativistic case,  $|s| \leq c$ , where  $c$  is the light velocity in vacuum.

T. ALTS

A New Approach to a Thermodynamic Theory of Simple Fluids with Thermokinematic Constraints

A new method for the indirect introduction of thermo-kinematic constraints into a thermodynamic continuum theory of simple fluids is presented. It is applicable to all fluid materials for which the representations of the constitutive functions are known.

Detailed results are given for the simplest non-trivial example, a Navier-Stokes fluid with Fourier heat conduction subject to the constraint of incompressibility with purely thermal volume expansion. It is shown, that a unique decomposition of stress, internal energy, heat flux, entropy and entropy flux into pressure dependent and pressure independent constitutive parts - the so-called principle of determinism of constrained materials - is not valid in viscous fluids.

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