MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

TAGUNGSBERICHT 4/81

"MODELLTHEORIE"

(Mengentheoretische Topologie)

11.1. bis 17.1.1981

Die Tagung, die unter Leitung der Herren M.M. Richter (Aachen) und E.-J. Thiele (Berlin) stattfand, behandelte mengentheoretische Fragen der Mengentheoretischen Topologie, darunter insbesondere das "Normal-Moore-space-problem" und damit zusammenhängende Fragen. Parallel zu dieser Tagung fand im Institut eine Tagung über Abelsche Gruppen statt. Beide Tagungen waren bereits in engem Kontakt zwischen den Tagungsleitern vorbereitet und die Programme aufeinander abgestimmt worden; infolgedessen wurden die beiden Übersichtsvorträge der Modelltheorietagung (Devlin, Tall) sowie der Überblicksvortrag von Herrn Eklof aus der Gruppentagung gemeinsam vor beiden Auditorien gehalten; auch darüber hinaus ergaben sich noch Programmabstimmungen.

Diese Kombination von zwei Tagungen mit gemeinsamen Interessengebieten wurde von den Teilnehmern der Modelltheorietagung als sehr nützlich und angenehm empfunden. Die Zusammenarbeit zwischen den Tagungsteilnehmern war sehr erfreulich und verlief völlig reibungslos. Wir halten derartige Paralleltagungen nicht nur für durchführbar, sondern im gewissen Umfang sogar für erstrebenswert.

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Vortragsauszüge

H. BRANDENBURG, Characterizations of developable spaces

Following R.H. Bing a topological space X is called 'developable' if it has a sequence $(A_n \mid n \in \mathbb{N})$ of open covers such that for each $x \in X$ the collection $\{St(x,A_n) \mid n \in \mathbb{N}\}$ is a neighbourhood base, where $St(x,A_n) := \bigcup \{a \mid a \in A_n, x \in a\}$. The significance of this class of spaces is best explained by the fact that a T_1-space is metrizable iff it is a collectionwise normal developable space. We presented new characterizations of developable spaces which can be viewed as analogues, for developable spaces, of the Urysohn metrization theorem, the Nagata-Smirnov metrization theorem, and Nagata's "double sequence Theorem", respectively.

K.J. DEVLIN: 1. Combinatorial principles in set theory (G.V.)

I gave my standard introductory talk on $V = L$ and some of its combinatorial principles.

K.J. DEVLIN, Proper forcing

I discussed generalizations of Martin's axiom due mainly to S. Shelah.

E. HARZHEIM, Generalizations of the Jordan-Brouwer theorem.

Let $X_1, \dots, X_n$ be nonempty linearly ordered sets without steps and gaps (hence continuously ordered) each equipped with the order topology having the set of open intervals as a basis. Let $X := X_1 \times \cdots \times X_n$ be their product space. A parallelotope of X is a set $I_1 \times \cdots \times I_n$, where I_ν is a closed interval of X_ν. Then the following generalizations of the Jordan-Brouwer theorem and the theorem of the invariance of domain are valid:

Theorem 1. If P is a parallelotope of X and $f: P \to X$ is an injective and continuous mapping, then $X \setminus f[P]$ is connected.

Theorem 2. If B is the boundary of a parallelotope P of X and $f: B \to X$ is injective and continuous, then $X \setminus f[B]$ has at least two connectivity components.

Theorem 3. Let $M \subset X$ be open, $f: M \to X$ injective and continuous, then $f[M]$ is an open subset of X.

I. JUHASZ, A survey of S- and L-spaces

The lecture gave an overview of the most recent developments in S- and L-spaces.

P. KOEPKE, An application of the core model

B. KOPPELBERG, Some consequences of Fisher's axiom

Let π be any ordinal. Then Fisher's axiom gives the following two results:

(1) There is a sequence of cardinals of order type π and an inner model such that in this model each cardinal of the sequence has an ω_1-complete uniform ultrafilter.

(2) There is a sequence of cardinals of order type π and an inner model such that in the model each cardinal of the sequence

is measurable and no other measurable cardinals occur in the model.

A. KRAWCZYK, Certain Boolean algebras $P(\omega)/_I$

The talk based on a joint paper by W. Just and A. Krawczyk "On certain Boolean algebras $P(\omega)/_I$". Results:

Theorem 1. (CH) If I and J are F_σ-subsets of Cantor-space, then $P(\omega)/_I$ and $P(\omega)/_J$ are isomorphic Boolean algebras.

Consider a function $f:\omega \to \mathbb{R}^+$ such that (i) $\sum_{\nu<\omega} f(\nu) = +\infty$, (ii) $\lim_{\mu\to\infty} \left(f(\mu) / \sum_{\nu\leq\mu} f(\nu) \right) = 0$. Denote $d_f^*(a) := \limsup_{\mu\to\infty} \left(\sum_{\nu\in\mu\cap a} f(\nu) / \sum_{\nu<\mu} f(\nu) \right)$, $I_f := \{a \mid d_f^*(a) = 0\}$ and $\rho_f(a,b) := d_f^*(a \dot{-} b)$, where $a,b \subset \omega$.

Theorem 2. (CH) Suppose $f,g:\omega \to \mathbb{R}^+$ satisfy (i) and (ii). Then there exists $\varphi: P(\omega)/_{I_f} \xrightarrow{\cong} P(\omega)/_{I_g}$ being simultaneously an isomorphism of Boolean algebras and an isometry (with respect to ρ_f and ρ_g).

Theorem 1 extends a result of Erdös and Monk, Theorem 2 gives a solution of a question of Erdös (see question 65 in van Rouwen, Monk and Rubin).

Theorem 3. If I is analytic, then $P(\omega)/_I$ is not complete.

W. MAREK, Partition properties of $P_\kappa \lambda$

In the talk I discussed the problem of partitioning I-stationary sets for I an ideal in $P_\kappa \lambda$. I also showed how various proofs for κ can be "raised" to $P_\kappa \lambda$.

P. NYIKOS, Fisher's axiom and the normal Moore space problem

A survey of the highlights in the history of the normal Moore space problem, followed with my proof that Fisher's axiom (every countably additive measure on a set X can be extended to a countably additive measure defined for all subsets of X) implies that every first countable normal space is collectionwise normal and hence that every normal Moore (developable) space is metrizable. Together with a recent result of W. Fleissner this essentially solves the normal Moore space problem:

Fleissner's result shows that if it is consistent that every normal Moore space is metrizable, it is consistent that there is a measurable cardinal;

on the other hand, Kunen has shown that the consistency of Fisher's axiom follows from that of there being a strongly compact cardinal.

P. NYIKOS, Problems concerning countably compact noncompact spaces

The best consistency results concerning the following still unsolved problems about the spaces in the title are presented ('space' means 'Hausdorff space'):

1. Is there a hereditarily separable example?
2. Is there one which is separable and first countable?
   - (a) and normal? (a´) and not normal?
   - (b) and locally compact?
   - (c) and locally countable?
   - (d) and a manifold?
3. Is there one which is first countable and does not contain ω_1?
4. Is there one which is a manifold of weight $>\omega_1$?

S. NEGREPONTIS, The fine structure of the countable chain condition

In this talk I discussed examples of c.c.c. spaces (e.g. Gaifman's example of a c.c.c. space without strictly positive measure, Galvin's example, under CH, of a c.c.c. space, whose square is not c.c.c., Argyros' example of a space without a strictly positive measure, having Knaster's property (K) and, under CH, not having (K_3), etc.). Most of the material is given in the book by W.W. Comfort and the author: Chain conditions in topology, Cambridge University Press, to appear in 1981.

Also I discussed briefly joint work with S. Argyros on chain conditions for spaces of measures.

Theorem. If X is compact, Hausdorff with a strictly positive

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measure, and α a cardinal with $cf(\alpha) > \omega_1$, then the space of probability measures on X, with the w*-topology, has caliber α.

The conclusion need not hold, under CH, for $\alpha = \omega_1$ (Herydon's example for Pelczynski's conjecture works).-

Some open problems were also given.

A. OSTASZEWSKI, Inductive Construction of spaces with G_δ-diagonal

We show how to construct by transfinite induction a topology on ω_1 that is right separated and has a G_δ-diagonal. Simultaneously with the construction of sets $U_n(\alpha) \subset \alpha \cup \{\alpha\}$ one constructs a function $N: [\omega_1]^2 \to \omega$ such that $n \geq N(\alpha,\beta) \Rightarrow \forall \delta \; \{\alpha,\beta\} \not\subset U_n(\delta)$ and so that if $N(\alpha,\beta) = n$ then $N(\alpha',\beta') = n$, for all $\alpha' \in U_n(\alpha)$ and $\beta' \in U_n(\beta)$. The induction proceeds in steps of length ω and at each stage α we require that any finite system of equations $N(x,\beta_i) = c_i$ with $\beta_i \in \alpha$, $c_i \in \omega$ has a solution for x in α provided the system is compatible. Compatibility is defined so that for each pair $i \neq j$ if $c_i \neq c_j$ we have $S(\beta_i,c_i) \cap S(\beta_j,c_j) = \emptyset$, where $S(\beta,c) := U_c(\beta) \cup \bigcup(U_c(\gamma) \mid \gamma \in U_c(\beta)) \cup \bigcup(U_c(\delta) \mid \delta \in U_c(\gamma), \gamma \in U_c(\beta)) \cup \cdots$.

T. PRZYMUSINSKI, Continuous images of $\beta\mathbb{N}$

Theorem. Every perfectly normal compact space is a continuous image of $\beta\mathbb{N} \setminus \mathbb{N}$.

Example. There exists a consistent example of a compact space of weight $\omega_2 \leq 2^\omega$ and π-weight ω_1 which is not a continuous image of $\beta\mathbb{N} \setminus \mathbb{N}$.

T. PRZYMUSINSKI, Fleissner's example

Fleissner's example of a normal non-metrizable Moore space from CH was described and the proof of its properties sketched. Also remarks about the newest version of Fleissner's example leading to a partial solution of the normal Moore space problem were made.

J. REINERMANN, On the characterization of some topological properties of sets via fixed points of continuous mappings

It can easily be shown that Banach's fixed point principle characterizes complete metric spaces. Analogously we discuss the question whether the well-known fixed point theorem of Brouwer, Schauder and Tychonoff (for compact mappings in a locally convex topological linear space) and of Brouder, Göhde and Kirk (for non-expansive mappings in uniformly-convex Banach spaces) characterize the function's domain of definition. The answer to the first question is 'no' in general (Spaghetti-set in a separable Banach space of dimension ≥ 2, and other star-shaped compact sets). In the non-expansive case the recent results of Buley, Schöneberg, Ray and the author are discussed. Furthermore the counterexample due to Alspach (1980) and another outstanding problem for non-expansive mappings in strictly-convex reflexive Banach spaces (for which a partial answer is given by the author several years ago) are presented.

F. TALL, Martin's axiom (G.V.)

A general introduction to the subject, aimed at the set-theoretically naive.

F. TALL, The normal Moore space problem in 1980

A survey of the results of the last year.

W. WEISS, π-weight variations

We denote by $\pi_d(X)$ the least cardinal of a collection S of open sets for the (regular topological) space X such that for each dense open $U \subset X$ there is a non-empty G in S with $G \subset U$, and by $\pi_o(X)$ the least cardinal of a π-base for some open subspace of X. These notions are due to M. van de Val and E. K. van Douwen.

Theorem 1. (van Douwen) For any regular space X:

1. $\pi_d(X) \leq \pi_o(X)$.
2. If $\pi_d(X) = \omega$ then $\pi_o(X) = \omega$.

Theorem 2. (I. Juhász and W. Weiss) For any regular space X:

1. $\pi_o(X) \leq 2^{\pi_d(X)}$.
2. $\pi_o(X) \leq \pi_d(X) \cdot \pi_\chi(X)$, where $\pi_\chi(X)$ is the π-character of X.

3. If X is compact,
4. If X is locally connected,
5. If X is a LOTS,
6. (MA) If X is separable,
7. (MA) If $\pi(X) < 2^{\omega}$ and $c(X) = \omega$,
8. (MA) If X is a dense subspace of $2^{(2^{\omega_1})}$,

then $\pi_o(X) = \pi_d(X)$.

9. CON(ZF) $\Rightarrow$ CON(ZF + $\exists X$ dense subspace of $2^{(2^{\omega_1})}$ such that $\omega_1 = \pi_d(X) < \pi_o(X)$).

P. KOEPKE, An application of the core model

The core model K was defined by T. Dodd and R.B. Jensen:
K is a transitive class containing all ordinals such that:

1. $K \models$ ZFC + GCH + V=K
2. The Covering Theorem: If there is no inner model with a measurable cardinal then K covers V, i.e. $\forall X \subset On \ \exists Y \in K \ (X \subset Y$ and $\mathrm{card}(Y) = \mathrm{card}(X) + \omega_1)$.
3. If $\pi : K \to K$ is a nontrivial elementary embedding then there is an inner model with a measurable cardinal.

We give a definition of K which does not involve finestructure theory. $M = J_\alpha^U$ is a premouse at $\varkappa$ iff $M \models$ (U is a normal measure on $\varkappa$). The low part of M is $lp(M) := V_\kappa \cap M$. $M = J_\alpha^U$ is called iterable if the iterated ultrapowers of M modulo U are all well-founded. - Then

$$K = L \cup \bigcup\{ lp(M) \mid M \text{ is an iterable premouse} \}.$$

This characterization of K along with 1.-3. allows us to prove

Theorem. The generalized Chang-property $(\omega_3,\omega_2) \Rightarrow (\omega_2,\omega_1)$ (cf. Chang-Keisler, Model Theory Ch. 7.3) implies the existence of an inner model with a measurable cardinal.

A little more work yields the existence of 0^+ from $(\omega_3,\omega_2) \Rightarrow (\omega_2,\omega_1)$. With similar methods 0^+ can be deduced from various "accessible" Jónsson cardinals (cf. Abstracts A.M.S., June 1980, p.390).

Von der Paralleltagung (Abelsche Gruppen) wurde als gemeinsame Veranstaltung abgehalten:

P. EKLOF, On the rank of Ext (G.V.)

This was a survey talk dealing with the results obtained under various set-theoretic hypotheses to:

1. Whitehead Problem (Does $Ext(A,\mathbb{Z}) = 0$ imply that A is free?)
2. Kan-Whitehead Problem (Is there an X such that $H^n(X) \simeq \mathbb{Q}$? or equivalently, is there an A such that $Ext(A,\mathbb{Z}) \simeq \mathbb{Q}$?
3. Fuchs' Problem 39 (Characterize the structure of $Ext(A,\mathbb{Z})$!)

Among the models of set theory considered with respect to these problems are models of $ZFC + V = L$, of $ZFC + MA + \sim CH$, and models of $ZFC + GCH$ obtained with proper forcing methods.

Edgar Rothacker (Bochum)

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