MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 11/1981

ANWENDUNGEN TOPOLOGISCHER METHODEN IN DER NICHTLINEAREN FUNKTIONALANALYSIS, NICHTLINEAREN ANALYSIS UND NUMERISCHEN MATHEMATIK

1.3. bis 7.3.1981

Die Tagung stand unter der Leitung von J. Mawhin (Louvain) und H.O. Peitgen (Bremen). Sie wurde von 47 Teilnehmern besucht, von denen 36 Vorträge hielten.

In den letzten Jahren hat sich das junge Gebiet der nichtlinearen Funktionalanalysis einer zunehmenden Aufmerksamkeit erfreut. Diese Entwicklung hat sich auch in einigen sehr interessanten Tagungen über diesen Gegenstand in Oberwolfach widergespiegelt. Innerhalb der nichtlinearen Funktionalanalysis haben sich seit den bahnbrechenden Arbeiten von Poincaré topologische Methoden als besonders fruchtbar erwiesen. Das Ziel der Tagung war, insbesondere diese Methoden in charakteristischen Anwendungen innerhalb der

- nichtlinearen Funktionalanalysis
- nichtlinearen Analysis
- numerischen Mathematik

herauszustellen, kennenzulernen und zu diskutieren. Die Auswahl der Teilnehmer hatte zum Ziel, abstrakte Theorie und konkrete Anwendungen ausgewogen und in ihrer befruchtenden Wechselwirkung darzustellen.

Besondere Aufmerksamkeit wurde den folgenden Themen geschenkt:

- <u>Verzweigungstheorie</u> (globale Verzweigung, Hopf-Verzweigung, Multiparameter-Verzweigung, Verzweigung und Symmetrien)

- Variationsmethoden (Morse-Theorie, Lusternik-Schnirelman-Theorie, Conley-Index)

- Abbildungsgradmethoden (Kontinuitätsmethoden, mehrwertige Abbildungen, Zusammenhangseigenschaften von Lösungen parametrisierter Familien)

- Fixpunkttheorie (Kegelabbildungen, Fixpunktindex, kohomologische Indextheorien)

- Operatoren von monotonem Typ (verallgemeinerte Abbildungsgradtheorien, Orlicz-Räume, Akkretive Operatoren)

- Komplexität dynamischer Systeme (Chaos bei Delay-Gleichungen , Kaskadenverzweigungen, Chaosgeneratoren)

- Numerische Analysis (Kontinuitätsmethoden, Auflösung von Verzweigung)

Als Folge der besonderen Zielsetzung der Tagung wurden Mathematiker von unterschiedlichstem Hintergrund zusammengeführt. Diese Tatsache gab der Tagung eine besondere wissenschaftliche Spannung, die sich in sehr intensiven Diskussionen und regem Austausch von Ideen äußerte.

Die Organisatoren haben die reibungslose, freundliche und ausgezeichnete innere Organisation des Instituts sehr schätzen gelernt. Sie gab ihnen Gelegenheit, sich ganz den wissenschaftlichen Aspekten der Tagung zu widmen.

Vortragsauszüge

J. ALEXANDER:

Global Continuation and Bifurcation Results depending on an infinite number of parameters

We consider continuous $f: A \times X \to X$ where A and X are Banach spaces (A = parameter space). Let $\mathcal{M}$ be a collection of finite-dimensional subspaces of A, all containing a fixed $M_o \in \mathcal{M}$ with the properties

i) $M_1, M_2 \in \mathcal{M} \Rightarrow \exists M_3 \in \mathcal{M}, M_3 \supset M_1 \cup M_2$,
ii) $\bigcup_{M \in \mathcal{M}} M = A$.

The M are called slices of A. Similarly, $M \times X$ is called a slice of $A \times X$. A map f is compact on slices if $f/M\times X$ is compact for each $M \in \mathcal{M}$. We prove continuation and bifurcation results analogous to those for A finite-dimensional and present applications to a nonlinear Sturm-Liouville problem arising from analysis of a bent rod and to a problem on elastic strings.

A. AMBROSETTI:

Minimal critical points and applications to some nonlinear problems

Let us consider a C^1- functional $f: E \to R$ (E Banach space) of the form $f(u) = - a(u,u) + b(u)$, where a is a continuous, symmetric, bilinear form and b is a nonlinear term satisfying some "convexity" conditions. Setting $M = \{u \in E: \langle f'(u),u\rangle = 0\}$, we show that M is a C^1- manifold in E and the stationary points of f on E are the critical points of f on M. Moreover if f assumes its minimum on M in a point v, then $a(v,v) = \max \{a(u,u): u \in E,\ u \neq 0,\ b(su) = b(sv)\ \forall s \geq 0, \langle b'(u),u\rangle = \langle b'(v),v\rangle\}$. This last property can be usefully applied in some nonlinear problems: 1) The existence of vortex rings in an ideal fluid with prescribed vortex-strength parameter; 2) The existence of solutions of given minimal period for a class of convex Hamiltonian systems; 3) The existence of n different modes of vibration for a Hamiltonian system with n degrees of freedom on a convex energy surface, according to a recent theorem of Ekeland and Lasry.

M.S. BERGER

Nonlinear Desingularization

Singular Points of Nonlinear Fredholm Operators of Positive Index

Nonlinear Desingularization: We discuss a new class of bifurcation phenomena for which Thom's theory of singularities does not apply. Examples occur in the theory of vortex rings and certain Yang-Mills theories.

Singular Points of Nonlinear Fredholm Operators: We show that global topological results are needed to study local questions about nonlinear Fredholm operators of positive index at a singular point.

F.E. BROWDER

The generalized degree of mapping and strongly nonlinear elliptic problems

A general theory of degree of mappings is described with values in an abelian group Γ and a procedure for creating degree theories of this type involving general approximation schemes. Applications include degree theories for A-proper mappings including maps from a reflexive Banach space X to its conjugate space X^* which are in the class $(S)_+$ and pseudo-monotone. New applications are given including operators T from X to

2^{X^*} of the form $T = T_o + f$ with T maximal monotone, f bounded and pseudo-monotone. The central application considered is a map T of the form $A + g(x,u)$ from $W_o^{m,p}(\Omega)$ to its dualspace $W^{-m,p'}(\Omega)$ with A regular elliptic, i.e.

$$A(u) = \sum_{|\alpha|\le m} (-1)^{|\alpha|} D^{\alpha} A_{\alpha}(x,u,\ldots,D^m u)$$

defining a pseudo-monotone bounded map while the only densely defined strongly nonlinear term $g(x,r)$ merely satisfies the sign condition $g(x,r)\cdot r \geq 0$.

S.N. CHOW und J. MALLET-PARET

Interval mappings and singularly perturbed delay differential equations

We investigate the relationship between mappings $f: I \to I$, where I is an interval, and the delay differential equation

(1) $$\varepsilon\dot{x}(t) = -x(t) + f(x(t-1)).$$

Periodic points of the mapping f are related to periodic solutions of (1) by means of a boundary layer equation. In some cases the periodic solutions tend univormly to square waves as $\varepsilon \to 0$; in other cases a non-uniform convergence resembling a "Gibbs' Phenomenon" is shown: New bifurcation phenomena, relating to period-doubling bifurcations in the interval mapping, are also studied.

PH. CLEMENT und L.A. PELETIER

On Continua of Positive Superharmonic Solutions to Semilinear eigenvalue problems

We consider the problem:

$$- \Delta u = \lambda g(\cdot, u, \Delta u) \;, \quad \Omega \subset R^N \text{ open, bounded}$$
$$u = 0 \quad \text{on } \partial\Omega$$

Under fairly general conditions on g, there exists a connected set of positive solutions $\mathcal{C}$ (in $R^+ \times C^1$ topology) which is unbounded in $R^+ \times C^1$, (in $R^+ \times C^\circ$ if g does not depend on ∇u.) We are interested in the asymptotic behaviour as $\lambda \to \infty$ of the solutions. We prove that if $\exists\, c: R^+ \to R^+$ increasing such that $|g(x,u,p)| \leqslant c(|u|)(1 + |p|^2)$ and if for $(\lambda,u) \in \mathcal{C}$ $g(x,u,\nabla u) \geqslant 0$, then for each λ (such that $(\lambda,u) \in \mathcal{C}$) there exists a minimal solution $\underline{u}(\lambda)$. If moreover there exists $M > 0$ such that $u(\lambda)(x) \leqslant M$, then $\bar{u}(x) := \sup_{(\lambda,u) \in \mathcal{C}} u(x), x \in \Omega$

satisfies the "reduced" equation

$$g(x,\bar{u}(x), \nabla u(x)) = 0 \quad \text{a.e.}$$

When g does not depend on ∇u, we give a condition which guarantees the existence of M. In this case we prove that not only the minimal solutions converge to $\bar{u}$ but any sequence $(\lambda_n, u_n) \in \mathcal{C}$, $\lambda_n \to \infty$, as $n \to \infty$.

C. CONLEY

Connection Matrices for Morse Decompositions

A Morse decomposition of a compact invariant set determines not just one, but several different filtrations of the set. Each filtration gives some indication of how the Morse sets are connected by the flow. This information can be collected in a (generally incompletely defined) matrix called the connection matrix and this information is stable under perturbation.
However, for nearby equation it may be that there are more entries in the connection matrix which are determined by the perturbed equation. The idea hence is to determine all possible ways in which the new entries can appear under perturbation. This counts the possible ways the given system can "bifurcate" at this level of description.

E. FADELL

Ljusternik-Schnirelman Category, Krasnoselsk'ii Genus and Cohomological Index Theories

Let E denote a free G-space, where G is a compact Lie group. Assume further that Σ is the class of closed invariant subsets of E. Then, for $A \in \Sigma$ we have the index theories $cat_B A^*$, where $B = E/G$, $A^* = A/G$; G-genus A; and finally $Index_\alpha A$, where α is an appropriate characteristic class.

Thm.: In general $Cat_B A^* =$ G-genus A, for all $A \in \Sigma$. On the other hand, $Index_\alpha A \leq$ G-genus A and one can contruct examples A such that $Index_\alpha A <$ G-genus A, and in fact one can make the disparity as large as you please. $Index_\alpha$ behaves well for "intersection theorems" while on the other hand examples show that the corresponding "intersection theorems" are false for G-genus. Other phenomena also indicate that G-genus picks up delicate "homotopy information" of the G-space, while the more crude index theory $Index_\alpha$ is more "computable".

C. C. FENSKE

A generalization of the fixed point index

Let (X,d) be a metric space and $f: X \to X$ a continuous mapping. Assume that there exists a compact invariant set $A \supset Fix\ f$ and a decreasing sequence of compact ANR $Z_n \subset X$ with $A = \bigcap_{n=1}^{\infty} Z_n$ and a sequence of retractions $\rho^n_{n+1} : Z_n \to Z_{n+1}$ with the following property: For $m > n$ call $\rho^n_m := \rho^{m-1}_m \cdots \rho^n_{n+1}$, then for each $\varepsilon > 0$ there is an n_o such that $d(x,\rho^n_m x) < \varepsilon$ whenever $x \in Z_n$ and $m > n \geq n_o$. For each n choose a neighbourhood Ω_n of Z_n in X and a retraction $r_n : \Omega_n \to Z_n$. We may choose Ω_n so small that for each $\varepsilon > 0$ there is an n_o such that for all $m > n \geq n_o$ and all $x \in \Omega_n$ we have $d(x,\rho^n_m r_n x) < \varepsilon$. If $U \subset X$ is open and $fx \neq x$ for $x \in \partial U$ then for n sufficiently large there is an

m_n such that $\mathrm{ind}(Z_m, \rho^n_m r_n f, Z_m \cap U)$ is defined whenever $m \geq m_n$. We show that this index does not depend on m, so we denote the common value by $\mathrm{ind}_A(X,f,U)$. Moreover, $\mathrm{ind}_A(X,f,U)$ does not depend on the choice of $(\Omega_n, Z_n, \rho^n_{n+1}, r_n)$, and it satisfies the normalization property: $\mathrm{ind}_A(X,f,X) = \Lambda(f|A)$ if we use Čech homology.

G. FOURNIER

Fixed Point Principles for Cones

We give some fixed point theorems of the Krasnosel'skii type for cones in linear normed spaces which generalize theorems obtained by R.D. Nussbaum and H.O. Peitgen & G. Fournier. We also give attempts of generalization of these theorems to eventually k-set-contraction Fréchet differentiable maps.

R.E. GAINES

Degree theoretic methods in problems of optimal control

We consider the problem

$$(1) \qquad \min_{x \in A_n[0,1]} \left\{ l(x(0),x(1)) + \int_0^1 L(x(t),\dot{x}(t))\,dt \right\}$$

where $A_n[0,1] = \{x\colon [0,1] \to R^n \mid x \text{ is absolutely continuous}\}$ and $l, L\colon R^n \times R^n \to R \cup \{+\infty\}$ are proper, lower semicontinuous and convex. Associated with (1) is

$$(2) \qquad \begin{cases} \dot{x} \in \partial_p H(x,p) \\ -\dot{p} \in \partial_x H(x,p) \\ (p(0), -p(1)) \in \partial l(x(0), x(1)) \end{cases}$$

where $H(x,p) = \sup_v \{v \cdot p - L(x,v)\}$ and ∂_p, ∂_x, and ∂l are appropriate subdifferentials. If (x,p) is an absolutely continuous solution to (2) and some weak regularity conditions are satisfied, then x is a solution to (1) and the minimum is finite. In this lecture we discuss degree theoretic methods for establishing existence of solutions to (2).

K. GEBA

Homotopy Theory and the Hopf Bifurcation Theorem

The aim of the lecture is to present a proof of the Hopf Bifurcation Theorem which uses stable homotopy theory. The method admits various generalizations.

M. GOLUBITSKY

Bifurcation with O(3) symmetry including applications to the Bénard Problem

David Schaeffer and I have adapted the Singularity Theory of Thom and Mather to study problems in steady-state bifurcation theory in the presence of a compact symmetry group. In this lecture I will describe an application of these techniques to the Bénard problem in spherical geometry. The Bénard problem concerns convection in a viscous fluid when it is heated from below. The fluid is assumed to be confined in a spherical shell with outer radius R_O and inner radius ηR_O. We use the Boussinesq approximation where there is a trivial solution representing pure heat conduction radially outward. As the temperature on the inner sphere (i.e., the Rayleigh number R) is increased this trivial solution looses stability, say at $R = R^*$. The Bénard problem is the study of the resulting bifurcation. When $\eta = 0.3$ (a choice motivated by considering convection in the molten layer of the earth) we show that under certain conditions multiple stable steady-states exist for $R > R^*$ and that these states can be non-axisymmetric. Our analysis relies on the O(3) symmetry inherent in this problem.

L. GORNIEWICZ

A remark on the Krasnoselskii's translation operator along trajectories of ordinary differential equations

We will present a generalization of the Krasnoselskii's translation operator along trajectories of ordinary differential equations. We will define it as a multi-valued, admissible map. Therefore the topological degree for such an operator is possible. If we assume that the right side of an ordinary differential equation is an linearly bounded, continuous and ω-periodic, with respect to first variable, map, then the sufficient condition for such an equation to have an ω-periodic solution is $\mathrm{Deg}(\Phi_{a+\omega,a}) \neq \{0\}$, where $\Phi_{a+\omega,a}$ denotes the multi-valued translation operator along the equation $y'(\cdot) = f(\cdot, y(\cdot))$, $f: [a,b] x R^n \to R^n$. Some applications of the above result are given.

J.P. GOSSEZ

Weak and strong derivatives in Orlicz spaces

The study of elliptic boundary value problems involving nonlinearities in the upper order terms which are not of polynomial type is usually carried out in the setting of Orlicz-Sobolev spaces. In this talk we discuss some questions which arise in this approach and which are related to the classical L^p result of Meyers and Serrin on the equality of weak and strong derivatives. Among other things we show that $W^m L_M(\Omega) \cap C^\infty(\Omega)$ is dense in $W^m L_M(\Omega)$ with respect to the so-called modular convergence.

J.K. HALE

Effect of domain on stability in a parabolic equation

For a scalar parabolic equation $u_t = \Delta u + f(u)$ in Ω, a bounded domain, with $\partial u/\partial n = 0$ on $\partial\Omega$, the only stable equilibrium solutions are constants if Ω is convex. This is true regardless of the function f. If Ω is not convex, it is known that stable solutions can exist and be nonconstant if f is an appropriate nonlinear function. We discuss this as a bifurcation problem for domains which depend on a parameter.

G. HETZER

On a semilinear operator equation at resonance in a Hilbert space

We consider the solvability of the semilinear operator equation

$$Lu = Bu$$

in a Hilbert space H, if $L: H \supseteq \mathrm{dom}(L) \to H$ is selfadjoint and linear and $B: H \to H$ is a demicontinuous operator, which satisfies

$$\|Bu - \tfrac{1}{2}(\alpha+\beta)u\| \leq \tfrac{1}{2}(\beta-\alpha)\|u\| + \gamma(\|u\|), \ \forall u \in H,$$

where $\alpha, \beta \in \mathbb{R}, \alpha < \beta$, $\sigma(L) \cap [\alpha, \beta] = \{\alpha\}$, $\alpha \notin \sigma_e(L)$ and $\gamma: \mathbb{R}^+ \to \mathbb{R}^+$ fulfils $\gamma(r)/r \to 0$ for $r \to \infty$. Moreover it is supposed that either

(I) ran(L) is closed, $\sigma(L)$ is an isolated pure point spectrum with

$$\sigma_e(L) = \{0\} \quad \text{and} \quad \mathrm{sgn}(\alpha)\cdot B \ \text{is monotone}$$

or

(II) L is bounded from below, $\sigma_e(L) \neq \emptyset$ and B fulfils

$$\langle Bu - Bv, u - v\rangle \geq -\inf(\sigma_e(L))\|u - v\|^2, \ \forall u,v \in H.$$

Finally an abstract Landesmann/Lazer type condition and some further hypothesis concerning B are required.

H.G. JEGGLE

Approximation of bifurcation branches

Motivated by numerical studies done by Peitgen, Saupe and Schmitt this communication is concerned with nonlinear eigenvalue problems

(P) $T(u,\lambda) \equiv Lu - \lambda N(u) = 0$, $u \in E$ (R-Banach space), $\lambda \in R$,

and similar finite dimensional approximations

(P_h) $T_h(u_h,\lambda) \equiv L_h u_h - \lambda N_h(u_h) = 0$, $u_h \in E_h$ (R-Banach space), $\lambda \in R$

where h is a discretization parameter. The spaces E, E_h and operators T, T_h are supposed to be related in a generalized sense of A-properness.

Using to Krasnoselskii – Rabinowitz theorem and a stabilization lemma for the degree one can see that (P) and (P_h) have the same global bifurcation structure in the compact case and in bounded sets of $E \times R$ in the unbounded case. In both alternatives the bifurcation branches of (P_h) converge to the original ones.

Based on the Lyapunov-Schmidt bifurcation equations and the convergence theory of linear eigenvalue problems we construct convergent approximations for simple bifurcation points and locally for the associated solution branches. Assuming that the bifurcation branches of (P) behave "nicely", i.e., are nonsingular, it is possible to use an extended implicit function theorem and a continuation argument to construct isolated bifurcation branches for (P_h) which converge to those of (P). With regard to the numerically irrelevant solutions we demonstrate that they disappear in the sense that their norm becomes arbitrarily large, if a simple compactness condition is satisfied. Examples are semilinear second order ordinary differential and Laplace operator boundary value problems and their difference approximations. The paper is based on joint work with Klaus Schmitt.

W. MARZANTOWICZ

On the Nonlinear Elliptic Equations with Symmetry

Let $S(V)$ be a sphere in a real orthogonal representation V of a compact Lie group G. If G is finite we denote by $\mu(V)$ the highest common divisor of the orders of G-orbits on $S(V)$. We consider a nonlinear elliptic problem $P(u) = \phi(u)$ with a compact perturbation ϕ. We pose this problem in classical G-Banach spaces constructed from the G-space of V-valued smooth functions in a bounded region $\Omega \subset R^n$. Our main theorem asserts that if

a) the problem $P(u) = \phi(u)$, $B(u) = 0$ on $\partial\Omega$ is well-posed and G-equivariant,

b) $\mu(V) > 1$ if G is finite or $V^T = \{0\}$ for some torus $T \subset G$ if G is infinite,

c) coker $P \subsetneq$ ker P,

then the problem $P(u) = \phi(u)$ has a solution with arbitrary norm in the appropriate function space.

R. NUSSBAUM

A nonlinear integral equation

I shall discuss the integral equation

(i) $u(x) = f(x) + \lambda \int_x^1 u(y)u(y-x)dy, \quad 0 \leq x \leq 1$

where $f(x)$ is a given continuous function and λ a real parameter. Earlier work has been done on (1) by B.J. Levin, M.G. Krein, G. Pimbley and R. Ramalho; the results I shall give generalize, sharpen and (in some cases) correct previous theorems. As an example I mention the following theorem.

Theorem 1. Suppose that $f(x) \geq 0$ for $0 \leq x \leq 1$, $f(1) \neq 0$, and $f(x)$ is continuous on $[0,1]$. Then for $0 < \lambda < \lambda_+ := (2\int_0^1 f(x)dx)^{-1}$, (1) has precisely two nonnegative, continuous solutions u_λ and v_λ. One has that $u_\lambda(x) \leq v_\lambda(x)$ for all x, and the maps $\lambda \mapsto u_\lambda \in C[0,1]$ and $\lambda \mapsto v_\lambda \in C[0,1]$ extend continuously to the intervals $[0, \lambda_+]$ and $(0, \lambda_+]$ respectively. Equation (1) has no real-valued solutions for $\lambda > \lambda_+$ and $u_{\lambda_+} = v_{\lambda_+}$.

D. PASCALI

Problems of the Stefan type

Some Stefan problems with certain boundary value conditions are written as evolution equations involving nonlinear operators of subdifferential type. This form guarantees the existence of some smooth solutions of corresponding problems.

J. PEJSACHOWICZ

Connectivity properties of the solution set of parametrized families of compact vector fields

We shall prove the following connectivity result:
Let X be a Banach space, U be an open subset of $X \times R^n$, locally bounded over R^n. Let $F: \bar{U} \to X$ be a compact map and let $\mathcal{S}$ be the solution set of the equation $x-F(x,\lambda) = 0$. Assume that the degree of the compact vectorfield $I-F_\lambda$ is different from zero for some $\lambda \in R^n$.
Then there exists a maximal connected subset $\mathcal{C}$ of $\mathcal{S}$ such that the following property holds:
If $p: \mathcal{C} \to R^n$ is the restriction to $\mathcal{C}$ of the projection over R^n and $\eta \in \check{H}^n_c(R^n)$ is a generator of the n-th Čech-cohomology group with compact supports then $p^*(\eta) \neq 0$ in $\check{H}^n_c(\mathcal{C})$. In particular i) $p: \mathcal{C} \to R^n$ is essential (as proper mapping) and hence onto, ii) topological dimension of $\mathcal{C}$ is at least n.

H. PETERS

Complex behaviour of solutions of retarded model equations

We discuss equations of the type (✱) $\dot{x}(t) = -f_\alpha(x(t-1))$ where f_α is a suitably chosen step function which models the nonlinearities $f_\alpha(x) = \alpha x.(1+x^8)^{-1}$ resp. $f_\alpha(x) = \alpha.\sin(x)$.

The model functions

$$\text{(N1)} \quad f_\alpha(x) = \begin{cases} \alpha\,, & 0 < x < 1 \\ 1\,, & 1 \leqslant x \end{cases} \quad ; \quad \text{(N2)} \quad f_\alpha(x) = \alpha.\mathrm{sign}(\sin \pi x))$$

$$f_\alpha(-x) = - f_\alpha(x)$$

reduce the infinite dimensional periodicity problem for equation (✱) to a one-dimensional difference equation $x_{n+1} = S_\alpha x_n$, $S_\alpha : [0,1] \to [0,1]$.

For certain parameters α a theorem of Li and Yorke ("Period three implies chaos") can be applied, thus proving chaotic behaviour for the model equations (caused by a "hump" in the nonlinearity (N1) and by interferences between "slowly" and "rapidly" oscillating solutions in the case of the nonlinearity (N2)). Moreover we show a parameter-depending transition from periodic solutions to "periodic solutions of the second kind" $(x(t+p) = x(t)+C)$ in the case of (N2).

K. SCHMITT

Finite dimensional approximations of nonlinear elliptic boundary value problems

We consider the nonlinear elliptic boundary value problem

$$Lu + \lambda f(u) = 0, \quad x \in \Omega \;, \quad u/_{\partial\Omega} = 0,$$

where the nonlinearity f has several zeros and is asymptotically linear at ∞ . Using some global topological perturbations we prove (in case $f'(0) > 0$) that the problem posseses solution branches, one bifurcation from zero and one from infinity and we show how the latter may be obtained from the former. If, on the other hand, $f'(0) < 0$, we show how a solution branch, bifurcating from infinity, may be obtained by perturbation techniques from the solution set of the problem considered first.

If one considers finite dimensional approximations of such problems one discovers a much richer solution structure, in fact there will exist many solutions which do not approximate solutions of the associated infinite dimensional problem .

H.W. SIEGBERG

Chaotic mappings on S^1

The "Period three implies chaos" result of Li and Yorke and Šarkovskii is generalized for continuous mappings on the circle using tools of algebraic topology. Moreover, we give estimates for the topological entropy of chaotic mappings on S^1.
The following theorem summarizes the results:

Let $f: S^1 \to S^1$ be a continous mapping with a fixed point and points of period two and three.
Then f is "chaotic", and the topological entropy h(f) satisfies $h(f) \geq \log 2/3$.

G. SKORDEV

Fixed point index and chain approximations

Report on a joint work with H.W. Siegberg. In order to define the fixed point index for a map of a simplicial complex into itself using local Lefschetz number one only needs some chain map which approximates the original map in a natural way. Using this simple remark and acyclic model arguments we obtain a fixed point index for compositions of u.s.c. acyclic maps. This index has all important properties - normalisation, homotopy invariance, additivity, commutativity and mod-p. Using these properties it is possible to extend this index for u.s.c. acyclic maps of a compact ANR and to obtain in this case a fixed point index with all properties.

J.A. SMOLLER

Stability and Bifurcation of Steady-State Solutions for Predator - Prey Equations with Diffusion

We consider the predator prey equations with diffusion

$$\begin{aligned} u_t &= u_{xx} + f(u) - uv \\ v_t &= dv_{xx} + v\,[-v+m(u-\gamma)] \end{aligned} , \quad \begin{pmatrix} u \\ v \end{pmatrix} (\pm L) = 0$$

on $-L < x < L$. We consider the cases $d \geq 0$ and study the existence and bifurcation of steady state solutions ($u_t \equiv v_t \equiv 0$), as well as their stability. Some new bifurcation diagrams are obtained, as well as the existence of a continuum of steady state solutions.

M. STRUWE

Infinitely many critical points for functionals which are not even and applications

In generalization of work by Lusternik and Schnirelman, Krasnoselskii and others a perturbation theorem is proved asserting the existence of infinitely many critical points of functionals E satisfying a Palais-Smale type condition and bounded from below which are perturbed from symmetry.
As an application multiplicity results are derived for superlinear elliptic boundary value problems of the kind

$$-\Delta u = f(x,u) \quad \text{in } \Omega \; ; \quad u|_{\partial\Omega} = 0$$

where essentially f behaves like $u|u|^{p-1}$ for large values of u with $p \in (1,p^*)$ for some $p^* > 1$. Then main tools are Lusternik-Schnirelman type methods and energy estimates.

A.T. TROMBA

Degree theory and Plateau Problem

The set of all minimal surfaces spanning wires is shown to be an algebraic variety, the strata being determined by the singularities in the surface itself. A degree theory for a Fredholm map on this variety is described and the degree is one. This degree represents the "algebraic number" of minimal surfaces spanning a wire.

G. VIDOSSICH

The geometric boundary value problem of Stampacchia

Stampacchia started his mathematical career by working on ODE. He wrote about 4 or 5 papers on the subject. In one of the latest ones he introduced the following setting for BVP: Given $f: I \times R^n \to R^n$ and $V_1, \dots, V_n \subseteq I \times R^n$, find a solution x of $x' = f(t,x)$ such that the graph of x meets each V_i. By an appropriate choice of the V_i's, it is possible to show that some well known BVP are special cases of this, as well as some others like to find the solution of $v'' = f(t,v,v')$ which is tangent to two given stright lines. I present a generalization of Stampacchia's result with some open problems.

A. VIGNOLI

Unbounded connected branches of eigenfunctions for nonlinear operator equations

The class of O-epi (zero-epi) maps has been introduced and studied in [1] . In this context we have the following result which is part of recent work in collaboration with M. Furi.

Proposition 1. Let E,F be infinite dimensional Banach spaces and let $f: E \to F$ be O-epi, bounded, proper on bounded closed sets and let $h: E \to F$ be compact and such that $h^{-1}(B(0,\alpha))$ is bounded for some $\alpha > 0$. Then the set $S = \{x \in E: f(x) = \lambda h(x)$ for some $\lambda \geq 0\}$ has an unbounded connected component starting from $f^{-1}(0) \cup h^{-1}(B(0,\alpha))$.

This result is false in finite dimensions. Take $\varphi: \mathbb{C} \to \mathbb{C}$ defined by $\varphi(z) = iz$. Clearly, the set $S = \{z \in \mathbb{C}: z = \lambda\varphi(z)$ for some $\lambda \in R\} = \{0\}$.

As an illustration of the above proposition consider the integral operator $h: L^2(0,1) \to L^2(0,1)$ defined by $h(x)(t) = \int_0^1 f(t,s)x^2(s)ds$, where $f: [0,1] \times [0,1] \to R$ is continuous and $f(t,s) \geq \alpha > 0$ for all $(t,s) \in [0,1] \times [0,1]$. In view of the fact that the identity $I: E \to E$ is 0-epi (see [1]) we get that the set $S = \{x \in L^2(0,1): x = \lambda h(x)$ for some $\lambda \geq 0\}$ has an unbounded component starting from $\{0\}$.

[1] M.Furi - M.Martelli - A.Vignoli: Annali Mat.Pura Appl. 124(1980), 321-343

H.O. WALTHER

Chaos in differential delay equations

We show chaotic behavior of the Li-Yorke-type for solutions to $\dot{x}(t) = f(x(t-1))$, where $f < 0$ on $(-\infty,-1) \cup (1,\infty)$ and $0 < f$ on $(-1,0)$. Such equations model control of high frequency generators by phaselocking; chaos has been observed numerically for certain nonlinearities by UEDA.

We construct a solution which is homoclinic to an unstable periodic orbit. This allows to apply arguments, due to Li-Yorke-Marotto, to the Poincaré map which belongs to the periodic orbit. The same structure can be shown to exist in more complicated equations, like

$\dot{x}(t) = f(x(t-1)) - \alpha x(t)$ where f is a hump function

as in the models for populations of red blood cells which have been introduced by Lasota, Wazewska, Mackey and Glass.

J.R.L. WEBB

Approximation solvability and accretive operators

Recent progress on approximation solvability methods for accretive operators will be discussed. In particular, the A-properness of accretive operators and a topological degree argument are used to prove surjectivity theorems in $(\pi)_1$ Banach spaces whose duals are uniformly convex. Unlike previous results in this area, weak continuity of the duality mapping is not required. Nevertheless the techniques are fairly elementary, employing finite dimensional approximations.

F. WILLE

Fixed point theorems for distance separating maps in Banach spaces

Conjecture of H. Hopf (Portug. Math. 4, 136-137, 1945):
"Let $f: S^n \to S^n$ be a continuous mapping and let $\alpha \in (0,\pi]$ be a real number such that $f(x) \neq f(y)$ if $\sphericalangle(x,y) = \alpha$, then $\deg f \neq 0$." A proof of this statement is given. By the method of Leray and Schauder we get the following fixed point theorems.

Theorems 1. X real Banach space, B unit ball with center 0 in X, $G: B \to X$ continuous compact, $f(x) = x - G(x)$ satisfying

$$\inf_{\substack{\|x-y\| = a \\ \|x\| = \|y\| = 1}} \left\| \frac{f(x)}{\|f(x)\|} - \frac{f(y)}{\|f(y)\|} \right\| > 0 \quad \text{for some } a \in (0,2] \quad (f(x) \neq 0 \text{ if } x \text{ [illegible] } 1)$$

Then G has a fixed point $x_0 \in B$: $x_0 - G(x_0) = 0$.
A different proof of the Hopf conjecture in the case $\alpha = \pi/2$ is given. Therefore we get in this case a stronger result:

Theorem 2. X real Hilbert space, B, G, f defined as above, satisfying $\frac{f(x)}{\|f(x)\|} \neq \frac{f(y)}{\|f(y)\|}$ if $x \cdot y = 0$, $\|x\| = \|y\| = 1$ (instead of the "inf-condition" above). Then G has a fixed point. Further conclusions and examples are given.

M. WILLEM

Critical point of indefinite functionals

Let be H a real Hilbert space, let $L: D(L) \subset H \to H$ be a self-adjoint operator and let $\psi : H \to R$ be a Gâteaux-differentiable function. Under a coercivity assumption and a growth condition on ψ the study of

$$Lu + D\psi(u) = 0$$

is reduced to the study of the critical point of a coercive functional. The spectrum of L is not assumed to be bounded from above or from below and the continuous spectrum is not necessarily empty. Applications are given to existence and multiplicity results for nonlinear boundary value problems on infinite intervals.

Berichterstatter: J. Mawhin, H.O. Peitgen

Liste der Tagungsteilnehmer

James C. Alexander
Department of Mathematics
University of Maryland
College Park, Maryland 20742
USA

Eugene Allgower
Mathematics Department
Colorado State University
Fort Collins, Colorado 80523
USA

Herbert Amann
Mathematisches Institut
Universität Zürich
Freiestraße 36
CH-8032 Zürich

Antonio Ambrosetti
SISSA - International School
Advanced Studies
Strada Costiera, 11
Miramare, Trieste, Italy

Jürgen Appell
FUB, FB 19
1. Math. Institut
Hüttenweg 9
D-1000 Berlin 33
West Germany

Mel S. Berger
Department of Mathematics
University of Massachusetts
Amherst Mass 01003
USA

Felix E. Browder
Department of Mathematics
University of Chicago
5734 University Avenue
Chicago, Illinois 60637 USA

Philippe Clément
Technische Hogeschool Delft
Julianalaan 132
Delft
Nederland

Ch. Conley
Mathematics Research Center
University of Wisconsin-Madison
610 Walnut Street
Madison, Wisconsin 53706
USA

Charles Conley
zur Zeit Mathematisches
Institut der Ruhr Universität
Bochum
4630 Bochum West Germany

Klaus Deimling
FB 17 der GH Paderborn
Warburgerstraße 100
D-4790 Paderborn
West Germany

Christian Fabry
Université de Louvain
Institut Mathematique
Chemin du Cyclotron 2
B-1348 Louvain-la-Neuve
Belgien

Edward Fadell
Dept. of Mathematics
University of Wisconsin
Madison, Wisconsin 53706
USA

Jean-Pierre Gossez
Department of Mathematics,CP214
University of Brussels
1050 Brussels
Belgique

Gilles Fournier
Département de Mathématiques
Université de Sherbrooke
Sherbrooke, P.O.
Canada

R.E. Graines
Department of Mathematics
Colorado State University
Fort Collins, Colorado 80523
USA

C.C. Fenske
Mathematisches Institut
der Justus-Liebig-Univers.
Arndtstraße 2
D-6300 Giessen West Germany

K.P. Hadeler
Lehrstuhl für Biomathematik
Universität Tübingen
D-7400 Tübingen
West Germany

Kazimierz Geba
Institute of Math.
University of Gdansk
80952 Gdansk
Poland

J.K. Hale
Division of Applied Mathematics
Brown University
Providence, RI 02912
USA

Kurt Georg
Inst. für Angew. Math.
Wegelerstraße 6
5300 Bonn
West Germany

Georg Hetzer
Lehrstuhl C für Mathematik
RWTH Aachen
Templergraben 55
5100 Aachen West Germany

Martin Golubitsky
Department of Mathematics
Arizona State University
Tempe, AZ 85281
USA

Hansgeorg Jeggle
Fachbereich Mathematik
Technische Universität Berlin
1000 Berlin 12
West Germany

Lech Górniewicz
Department of Mathematics
Higher Pedagogical School
76200 Stupsk
Poland

Franz Kappel
Institut für Mathematik
Universität Graz
Elisabethstraße 11
A-8010 Graz Austria

Klaus Kirchgässner
Math. Institut A
Universität Stuttgart
Pfaffenwaldring 57
7000 Stuttgart 80 West Germ.

Edward M. Landesman
Applied Sciences
University of California
Santa Cruz, California 95064
USA

John Mallet-Paret
Division of Applied Mathematics
Brown University
Providence RI 02912
USA

Waclaw Marzantowicz
Institute of Math.
University of Gdansk
80952 Gdansk
Poland

Ivar Massabò
Dipartimento di Matematica
Università della Calabria
Arcavacata di Rende
87030 Roges (Cosenza)
Italy

J. Mawhin
Université de Louvain
Institut Mathematique
Chemin du Cyclotron 2
B-1348 Louvain-la-Neuve
Belgien

Roger Nussbaum
Dept. of Mathematics,
Rutgers University
New Brunswick, New Jersey
USA 08903

Dan Pascali
Technische Hochschule Darmstadt
Fachbereich Mathematik
Arbeitsgruppe 6
Schloßgartenstr. 7
6100 Darmstadt West Germany

Heinz-Otto Peitgen
Fachbereich 4 - Mathematik
Universität Bremen
Postfach 330 440
2800 Bremen 33
West Germany

Jacobo Pejsachowicz
Univers. della Calabria
Dipartimento di Matematica
87030 Roges (Cosenza)
Italy

Hubert Peters
Fachbereich Mathematik
Universität Bremen
2800 Bremen
West Germany

Hans-Willi Siegberg
Fachbereich Mathematik
Universität Bremen
2800 Bremen
West Germany

G.S. Skordev
Department of Mathematics
University of Sofia
Sofia 1113
ul. "G. Bonchev" 8
Bulgaria

G.S. Skordev
zur Zeit FB 4 - Mathematik
Universität Bremen
2800 Bremen
West Germany

Joel Smoller
University of Warwick
Coventry CV 4 - 7AL
England

G. Vidossich
International Centre for
theoretical Physics
P.O. Box 586
34100 Trieste, Italy

Klaus Schmitt
Department of Mathematics
University of Utah
Salt Lake City, Utah 84112
USA

Alfonso Vignoli
Istituto Matematico
"Guido Castelnuovo"
Universita di Roma
Citta Universitaria
00100 Roma, Italy

Michael Struwe
Institut f. Angew. Analysis
Beringstraße 6
D - 5300 Bonn 1
West Germany

Hans-Otto Walther
Mathematisches Institut
der Universität München
Theresienstr. 39
D-8000 München 2 West Germany

Charles Stuart
Dept. de Mathématiques
E.P.F.L.
61, Avenue de Cow
CH-1007 Lausanne

J.R.L. Webb
Department of Mathematics
University of Glasgow
Glasgow G12 8QW
Scotland, U.K.

A. Tromba
Department of Mathematics
University of California
Santa Cruz, California 95064
USA

Friedrich Wille
Fachbereich Mathematik
der Gesamthochschule Kassel
Heinrich-Plett-Str. 40
D-3500 Kassel West Germany

A. Vanderbauwhede
Institut voor Theoretische
Mechanika
Rijksuniversiteit Gent
Krijgslaan, 271
B-9000 Gent Belgien

M. Willem
Université de Louvain
Institut Mathematique
Chemin du Cyclotron 2
B-1348 Louvain-la-Neuve
Belgien