

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 12/1981

" Probability Theory and Mathematical Statistics "

8.3. bis 14.3.1981

Die Tagung stand unter der Leitung von Hans G. Kellerer (München) und Frank L. Spitzer (Ithaca/USA) und führte insgesamt fünfzig Wissenschaftler aus dem In- und Ausland zusammen.

Neben Berichten über neuere Einzelergebnisse aus Wahrscheinlichkeitstheorie und Mathematischer Statistik ergaben sich in den Vorträgen folgende Schwerpunkte: (1) Interaktionsprozesse (Zeitentwicklung und Gleichgewichtstheorie), (2) Grenzwertsätze vom Typ "large deviations", (3) Charakterisierung extremerer Wahrscheinlichkeitsmaße und Integraldarstellung. Der durch stochastische Modelle in Physik, Biologie und anderen naturwissenschaftlichen Disziplinen angelegte Themenkreis (1) hat sich im letzten Jahrzehnt zu einem der aktivsten Forschungsgebiete der Wahrscheinlichkeitstheorie entwickelt. Die zunächst mehr theoretischen Themenkreise (2) und (3) gewinnen zunehmend Bedeutung in den Anwendungen, etwa in der Mathematischen Statistik. In einer Reihe von Überblicksvorträgen wurde auch den Teilnehmern mit anderen Arbeitsrichtungen die Möglichkeit geboten, sich über den aktuellen Stand in diesen Gebieten zu informieren.

Die Tagung hat gezeigt, wie wichtig in Anbetracht der raschen wissenschaftlichen Entwicklung und der großen Entfernung der einzelnen Forschungsgruppen die Möglichkeit zu einem intensiven Gedankenaustausch ist. Dafür hat sich das Mathematische Forschungsinstitut Oberwolfach erneut als ideales Forum erwiesen.

Vortragsauszüge

S. ALBEVERIO:

Stochastic fields for hydrodynamics

We discuss stochastic solutions for the Euler equation for an incompressible fluid in a domain Λ of \mathbb{R}^2 . The equation can be written $\frac{\partial}{\partial t} \eta = B(\eta)$, with $\eta = \text{rot } u$, u being the velocity field and $B(\eta)$ a certain non linear operator. The energy $H(u) = \frac{1}{2} \int_{\Lambda} u^2 dx$ and the enstrophy $S(\text{rot } u) = \frac{1}{2} \int_{\Lambda} (\text{rot } u)^2 dx$ are invariant in time, for u a classical solution of the Euler equation. We use these invariant quantities to construct invariant measures. Let Λ be bounded, $\gamma > 0$ and let μ_{γ} be the Gaussian measure given formally by $\exp(-\gamma S(\eta)) d\eta$ i.e. white noise distribution for η i.e. $\int_{\mathcal{D}'(\Lambda)} e^{i\langle f, \xi \rangle} d\mu_{\gamma}(\Lambda) = e^{-\frac{1}{2} \gamma \langle f, f \rangle}$, $\langle \cdot, \cdot \rangle$ meaning pairing (in the sense of generalized functions). Then $B(\cdot) \in L^2(d\mu_{\gamma})$ whereas $H(\xi) = +\infty$ for a. e. $\xi \in \text{supp } \mu_{\gamma}$. However a renormalized energy $:H:_{\gamma}(\xi)$ obtained as the limit in $L^2(d\mu_{\gamma})$ as $N \rightarrow \infty$ of $\frac{1}{2} \sum_{n=1}^N \frac{1}{\lambda_n} (\langle \varphi_n, \xi \rangle^2 - \frac{1}{\gamma})$ ($-\lambda_n$: n -th eigenvalue of the Laplacian in Λ , with eigenfunction φ_n) exists, and yields the probability measures $d\mu_{\beta, \gamma} = e^{-\beta :H:_{\gamma}} d\mu_{\gamma} / \int e^{-\beta :H:_{\gamma}} d\mu_{\gamma}$ for all $\beta \geq 0$, formally equal to $\exp(-\beta H(\eta) - \gamma S(\eta)) d\eta$. There exists a flow α_t on $\text{supp } \mu_{\gamma}$ which is a suitable limit as $N \rightarrow \infty$ of the classical Euler flow, obtained by writing the Euler equations for the components of η along φ_n and retaining only the components corresponding to $|\lambda_n| < N$. The fields $\xi \circ \alpha_t$ with stochastic initial condition $\xi \in \text{supp } \mu_{\gamma}$ have $\mu_{\beta, \gamma}$ as stationary distribution and are thus stochastic (non classical) solutions of the Euler equation. The limit $\Lambda \uparrow \mathbb{R}^2$, the relation with the Navier-Stokes equation as well as other invariant measures (of Poisson type: "vortex model") is also mentioned. The results are

joint work with R. Høegh-Krohn and M. De Faria. Recent related work has been done by C. Boldrighini and S. Frigio.

R. AZENCOTT:

Asymptotic expansions for slightly perturbed dynamic systems

For dynamic systems of the type $\kappa'_t = b(\kappa_t)$, consider the slightly perturbed system $d\kappa_t^\varepsilon = \varepsilon \sigma(\kappa_t^\varepsilon) d\beta_t + b(\kappa_t^\varepsilon) dt$ where $\beta_t =$ Brownian motion, $\varepsilon \rightarrow 0$. We give asymptotic expansions of $\mathbb{P}(\kappa_{[0,1]}^\varepsilon \in A)$ for certain smooth sets A in $C_{[0,1]}$ (space of paths), as well as expansions of $\mathbb{E}(\exp \frac{\Theta(\kappa^\varepsilon)}{\varepsilon^2})$ where $\Theta : C_{[0,1]} \rightarrow \mathbb{R}$ is a continuous functional. This is done using Laplace methods in path space, stochastic Taylor expansions, and action functionals. The results apply to diffusions in small time interval, and give promising handles to grasp the problem of asymptotic expansions of densities for the two above considered situations.

M. BRAMSON:

Application of the Feynman-Kac formula to the Kolmogorov nonlinear diffusion equation

The Kolmogorov equation is a semilinear diffusion equation which first appeared in the 1930's in the context of genetics, and has more recently surfaced in connection with the maximal displacement of branching Brownian motion. As is well-known, the solution to the equation under Heaviside initial data approaches a travelling wave. It has recently been shown, that similar convergence holds for certain other initial data, although convergence may be to other waves. Here, the general solution to the problem is discussed, where necessary and sufficient conditions on the initial data are given for convergence to a travelling wave. An exact formula is also given for the position of the wave. The basic tool employed is the Feynman-Kac integral formula, which is applied in conjunction

with estimates for crossing probabilities of certain curves by Brownian motion. One obtains estimates for path integrals of Brownian motion which lead to the desired results.

L. DAVIES:

Some characterizations of the exponential and stable distributions

It is shown that many wellknown characterizations of the exponential and stable distributions reduce to the convolution equation $H(x) = \int H(x+y)\mu(dy)$ where μ is a radon measure and H a non-negative continuous function. The general solution of this equation for a locally compact separable abelian group was given by Choquet/Deny in 1959/60. A martingale proof for a locally compact separable abelian semigroup is given. For the case \mathbb{R}_+ the convolution equation with error term is studied which leads to some new characterizations of the Weibull and stable distributions.

M. D. DONSKER:

Function space integrals and large deviations

Asymptotic evaluation of Markov process expectations as the time gets large has gone through successive stages of refinement. What lies behind these results is an exponential estimate on the probabilities of large deviations from the typical behavior implied by the ergodic theorem.

These results have been applied to various problems such as the Wiener sausage and more recently the asymptotics of the Polaron problem in statistical mechanics.

R. DURRETT:

Some new results on contact processes

In this talk we survey some new results on contact processes concentrating on those obtained since the monograph of Griffeath was published.

E. EBERLEIN:

Strong approximation using the Wasserstein-distance

We consider sequences of random variables $(X_k)_{k \geq 1}$ with values in complete separable metric spaces $(S_k)_{k \geq 1}$ and having a dependence structure which is given in terms of the Wasserstein-distance of certain distributions.

Furthermore let $(G_k)_{k \geq 1}$ be a sequence of distributions on $(S_k)_{k \geq 1}$ such that G_k is not too far from the distribution of X_k in the Prohorov-distance. Then $(X_k)_{k \geq 1}$ can be approximated by a sequence of independent random variables $(Y_k)_{k \geq 1}$ such that Y_k has distribution G_k for every $k \geq 1$. This extends results of Berkes/Philipp and Philipp for sequences satisfying mixing conditions.

As an application of this approximation theorem we get a strong, i.e. almost sure invariance principle for very weak Bernoulli processes.

J. FRITZ:

Interacting diffusion processes

We consider diffusion processes in an infinite product space

$R^S = \{x = (x_k : k \in S)\}$ of type (*) $dx_k = b_k(x)dt + \sigma_k(x)dW_k$,

where W_k is a family of independent Wiener processes. SCR^d is such that

$|j-k| \geq 1$ if $j \neq k \in S$, b_k and σ_k depend only on such x_j that $|j-k| \leq r$. The process is associated to an interaction $U = \{U_v\}$ of radius r of interaction by

$b_k(x) = -c_k(x) \frac{\partial}{\partial x_k} \sum_{v \neq k} U_v(x)$. Our basic condition is that $\delta \sigma_k^2 \leq c_k \leq L \sigma_k^2$ with

some $0 < \delta < L$. The allowed singularity of U_v depends on the dimension and a

local Lipschitz condition is assumed, too. The Markov process associated to (*)

is constructed in the set $\Omega_0 CR^S$ of configurations with bounded logarithmic

energy fluctuations. These results extend to point fields in dimensions four or

less with smooth positive interactions and in dimension two with certain

singular interactions.

Stationary measures in the time-reversible case can be characterized as follows. Suppose that $\sigma_k = 1$ and $\int G\phi d\mu = 0$ for a reasonable class of smooth functions, where G is the formal generator associated to $(*)$. Then some moment conditions imply that μ is a Gibbs random field with interaction U . If $d \leq 2$ and U is dynamically superstable then moment conditions reduce to the following support condition. Let $\Omega = \{x \in \mathbb{R}^S : \lim_{|k| \rightarrow \infty} |x_k| e^{-\epsilon |k|} = 0 \text{ for } \forall \epsilon > 0\}$, then $\int G\phi d\mu = 0$ and $\mu(\Omega) = 1$ imply that μ is a Gibbs state. Results by Holley and Stroock are extended to unbounded spin systems. We can prove a similar statement for certain small random perturbations of Hamiltonian systems.

P. GAENSSLER:

Recent developments in the theory of empirical processes

Starting with Donsker's theorem (Functional CLT) for the uniform empirical process in the setting of weak convergence in nonseparable metric spaces (cf. Dudley (1967), Wichura (1968), Pollard (1979)), the natural extensions to empirical C - and F -processes (for classes C of sets resp. classes F of functions due to Dudley (1978-81) are illustrated: First, in obtaining a Functional CLT for empirical C -processes in case that C allows in a certain sense a finite-dimensional parametrization, and secondly (concerning empirical F -processes) in obtaining Funktional CLT's for weighted empirical processes (O'Reilly (1974)). Finally, some connections between empirical processes and simple point processes (via the martingale approach) are mentioned.

H.O. GEORGII:

Markov random fields and their typical configurations

We consider Markov random fields on the square lattice \mathbb{Z}^2 with compact state space which are Gibbsian for $\beta\Phi$, Φ being a continuous potential. For given $\epsilon > 0$ and each configuration σ we construct a subgraph $G_\epsilon(\sigma)$ of \mathbb{Z}^2 by drawing

a an edge between all pairs $\{i, j\}$ of adjacent sites for which $\Phi(\sigma_i, \sigma_j) \leq \min \Phi + \epsilon$.

Theorem: If β is sufficiently large then there is some Markov random field μ for $\beta\Phi$ showing all symmetries of Φ and such that μ - a.s. there is an infinite cluster of $G_\epsilon(\cdot)$ which surrounds each finite subset of \mathbb{Z}^2 . Therefore:

If Φ and ϵ are such that each cluster of $G_\epsilon(\cdot)$ shows one of $N \geq 2$ symmetry - related but mutually incompatible patterns - then for large β there are at least N mutually singular Markov random fields for $\beta\Phi$.

N.C. JAIN:

Some large deviation results for sums of independent random variables

Let X_1, X_2, \dots be i.i.d. random variables in the domain of attraction of a stable law G with a strictly positive density and satisfying the scaling property. Let $S_n = S_n/n + G$. We obtain Donsker-Varadhan type large deviation results for the normalized sums S_n . These results are then used to obtain analogues of Strassen's results for the behavior of "small" values of $\{S_n\}$.

Some examples of these applications are the following: let

$b(n) = \lfloor n / \log \log n \rfloor$, and $c(n) = a(b(n))$. Theorem 1. For $c > 0$, there exist constants $k_{c,G}$ and c_G such that $\limsup_n n^{-1} \sum_{j=1}^n \chi_{[0,c]}(|S_j| / c(n)) = k_{c,G}$, a.s., and $k_{c,G} = 1$ for $c \geq c_G$. Theorem 2. With c_G as above, $\liminf_n c(n)^{-1} \max_{1 \leq j \leq n} |S_j| = c_G$, a.s. (If G is $N(0,1)$, then $c_G = \pi 8^{-1/2}$).

For $a > 0$ there exists a constant $A_{a,G}$ such that $\liminf_n n^{-1} \sum_{j=1}^n (|S_j| / c(n))^a = A_{a,G}$, a.s. (If G is $N(0,1)$, then $A_{2,G} = 1/4$).

O. KALLENBERG:

Some surprises related to previsible sampling

From current work on exchangeability some surprising facts related to previsible sampling are presented. Here are a few simple examples:

a) Let $X = (X_1, \dots, X_n)$ be a finite population enumerated

in random order, and let $\tau_1 < \dots < \tau_\nu$ be previsible stopping times. Then $(X_{\tau_1}, \dots, X_{\tau_\nu})$ can be imbedded into a copy of X . b) Let X be a continuous process on $[0,1]$ with $X_0 = X_1 = 0$, and suppose that the process obtained by joining together the paths on $[0,s]$ and $[t,1]$ can be imbedded into a copy of X for all s and t . Then X is a mixture of Brownian bridges. c) Let X be a random sequence such that $\Theta_\tau \stackrel{d}{=} X$ for all stopping times τ . Then X is exchangeable. d) Let X be a continuous recurrent process such that $\Theta_\sigma \stackrel{d}{=} \Theta_\tau$ for every pair of stopping times σ and τ with constant $X_\sigma = X_\tau$. Then X is a mixture of diffusions.

J.F.C. KINGMAN:

Coalescence and genealogy

Models for the genetic diversity, or the geographical spread, of a reproducing population demand implicit or explicit analysis of the family relationships, the genealogy of the population. For a wide class of haploid models this can be expressed in terms of a particular continuous-time Markov process, the n -coalescent ($n = 1, 2, \dots$), having as states the equivalence relations on $\{1, 2, \dots, n\}$. The n -coalescents for all values of n can be coupled using a more complicated Markov process, which can be described in terms of random colouring from a dynamic paintbox.

U. KRENGEL:

On multiparameter subadditive ergodic theory

A subadditive multiparameter process is a stationary process $\{F_I\}$ indexed by the set of "rectangles" such that $F_I \leq \sum_{i=1}^n F_{I_i}$ holds if I is the disjoint union of I_1, \dots, I_n . Numerous interesting examples have been given by Hammersley and Smythe. The pointwise ergodic theorems given by Smythe and Nguyen require additional "strong subadditivity". A new maximal lemma for such processes yields a proof without these supplementary conditions and at

the same time provides a simple new proof of the 1-parameter special case due to Kingman. (Joint work with Akcoglu). It can be shown by example that convergence in L_2 holds in the subadditive ergodic theorem if F_1 is non-negative, but fails in general. Without non-negativity one can prove weak convergence. Some parts of the mean ergodic theory for linear operators in Banach spaces admit generalizations to subadditive processes in Banach lattices (joint work with Derriennic).

H. KÜNSCH:

Thermodynamics and maximum likelihood for Gaussian random fields

Gaussian fields are considered as Gibbsian fields. Thermodynamic functions are calculated for them and the variational principle is proved. As an application we get an approximation of log-likelihood and an information theoretic interpretation of the asymptotic behaviour of the maximum likelihood estimator for Gaussian Markov fields.

St. LAURITZEN:

Extreme point models in statistics

Many statistical models are given as the extreme points of the convex set of probability measures, satisfying various symmetry conditions. We give examples of this and discuss also some situations where the extreme points of the corresponding set only are partially known.

Th.M. LIGGETT:

Generalized potlatch and smoothing processes

The potlatch and smoothing processes were introduced by Spitzer in his 1979 Wald Lectures and were later studied by Liggett and Spitzer. In this joint work with Richard Holley, we introduce some generalizations of these processes which exhibit a form of phase transition. Our results show that phase transition

does not generally occur in one or two dimensions, but usually does occur in higher dimensions. Upper and lower bounds for the relevant critical values are obtained. As one application of our results, we obtain the limiting behavior of the critical values for the linear contact process in d dimensions as $d \rightarrow \infty$. This is done by comparing the contact process with an appropriate generalized smoothing process.

P. MAJOR:

Limit theorems for dependent random variables

We are interested in the limit behaviour of partial sums of strongly dependent random variables. When one tries to solve such problems the first step of the investigation is to find the possible limits. This leads to the problem of determining the fixed points of a rather complicated transformation. This question seems to be very hard, but in the special case when non-linear functionals of Gaussian fields are considered we can find some non-trivial fixed points. We have presented some limit theorems where the limit is non-Gaussian. We have indicated how to find examples where the norming factor is N^α , $\alpha < \frac{1}{2}$.

M. METIVIER:

Weak convergence and diffusion approximation of jump processes

The purpose of the talk is to call attention on sufficient conditions of uniform tightness for sequences of laws of stochastic processes, the prototype of which is due to D. Aldous and R. Rebolledo and on recent developments in different directions.

These theorems are very well fitted to sequences of semimartingales for the following reason: if (X^n) is a sequence of processes which can be written $X^n = M^n + A^n$ where M^n is a martingale and A^n a process with bounded

variation, the uniform tightness is in many cases immediately reduced to an easy applying of a condition of Aldous type to the sequence $(\langle M^n \rangle)$ and (A^n) of processes.

1°) Very simple examples are given to show these uniform tightness conditions combined with a characterization of the law of a diffusion as the solution of a submartingale problem gives a natural way to get convergence theorems to diffusions (Example in queueing theory, stochastic approximation,...).

2°) Extensions in two directions are mentioned:

- a) obtaining of sufficient conditions which do not imply the quasi continuity of the limit (Jacod-Memin-Métivier-Rebolledo);
- b) Extension to infinite dimensional valued processes.

J. NEVEU:

Stationary point processes and queues

For a stationary point process of arrivals marked by required services, it is almost obvious that there exists a stationary waiting time provided the expectation of service is strictly less than the expectation of the interarrival time (a two line proof is provided). This immediately implies Birkhoff's ergodic theorem with a finite or infinite invariant measure (playing the role of a Palm measure) and also the Chacon-Ornstein theorem. Different disciplines of service lead to different but equivalent proofs of this theorem.

Queues with k servers, or with impatience (eventually rejection) are also studied: there exist for them stationary characteristics (waiting time, queue length, etc.) under natural conditions only if one extends the underlying space in general. A minimal extension is provided.

A general "Little formula" is given that contains many known and recently discovered formulas.

H. ROST:

Hydrodynamical limit of a stochastic many particle system

One considers a particle system in R^d which evolves in a Markovian way, with a translation invariant interaction between the particles. Define the rescaled (in space and time) random measure

$$M^\epsilon(t) := \epsilon^d \sum_i \delta_{\epsilon X_i(t/c(\epsilon))}$$

ϵ : a scaling parameter, $c(\epsilon)$: typically a power of ϵ .

We give examples which exhibit the following macroscopic and microscopic behaviour (as ϵ tends to zero):

1) If $M^\epsilon(0) \rightarrow f(x)dx$ (weakly in probability) then for all $t > 0$

$M^\epsilon(t) \rightarrow f(x,t)dx$, where $f(.,.)$ is the solution of a kinetic (or Euler) equation

$$\frac{df}{dt} = F(f, Df, \text{higher derivatives in } x)$$

to the initial value $f(x,0) = f_0$.

2) For every $x \in R^d$ and $t > 0$ the random measure $\sum_i \delta_{X_i(t/c(\epsilon))} - x/\epsilon$ converges

to the (unique) ergodic equilibrium measure for the original dynamics with density $f(x,t)$.

D. SIEGMUND:

Large deviations for boundary crossing probabilities

For random walks s_n , $n = 1, 2, \dots$, whose distribution can be imbedded in an exponential family, a method is described for determining the asymptotic behavior as $m \rightarrow \infty$ of $P\{s_n > mc(\frac{n}{m}) \text{ for some } n < m; s_m = m\mu_0\}$ ($\mu_0 < c(1)$).

The method involves considering this conditional probability as one member of a family indexed by μ_0 and expressing the probability as the integral of the likelihood ratio of this measure with respect to another appropriately chosen member of the class. Applications are given to the distribution of the Smirnov statistic and to modified repeated significance tests.

F. SPITZER:

Products of i.i.d. positive matrices

(joint work with H. Kesten)

Let A_n be a sequence of i.i.d. positive matrices, with the property that their expectation matrix $M = E(A_k)$ has largest eigenvalue $\rho(M) = 1$. What can be said about the distribution of the products $P_n = A_1 A_2 \dots A_n$ as $n \rightarrow \infty$. In the one-dimensional case $P_n \rightarrow 0$ except in the trivial case $A_k \equiv 1$. In higher dimension there are other possibilities - e.g. if all the A_k are stochastic with probability one, then P_n is stochastic and hence cannot tend to zero. This phenomenon is related to two others

- (i) The Furstenberg-Kesten theorem which implies that $[(P_n)_{ij}]^{\frac{1}{n}} \xrightarrow{\text{a.e.}} L \leq 1$.
- (ii) The Kronecker product $R : R_{ij,kl} = E[A_{ik} A_{jl}]$, which determines whether the covariances of $(P_n)_{ij}$ tend to finite limits. They do iff $\rho(R) = 1$.

Under suitable positivity hypotheses we prove that the following five conditions are equivalent

- (1) $(P_n)_{ij} \rightarrow 0$ in measure for some pair i, j
- (2) $L = 1$ in the Furstenberg-Kesten theorem
- (3) $\rho(P_n) = 1$ w.p.1 for all n
- (4) $\rho(R) = 1$ for the Kronecker product R
- (5) P_n converges weakly to a probability measure which is not concentrated at 0.

H. STRASSER:

An invariance property of statistical experiments

It has been shown by LeCam, 1973, that weak limits of localized product experiments as a rule satisfy an invariance condition which is called translation invariance.

It is shown that such limits also satisfy another invariance condition called stability. A complete description is given of all Gaussian experiments which are stable and translation invariant. It is known that such experiments can be con-

sidered as subexperiments of Gaussian shifts where the underlying Hilbert space may be of infinite dimension. It cannot be finite dimensional if the stability exponent is $p < 2$. Moreover, any translation invariant experiment which is stable with exponent $p = 2$ must be a Gaussian shift. If $p < 2$ then it needs not be even Gaussian. As an application it is shown that for sequences of experiments with stable limits the notion of Pitman efficiency for tests can be introduced in a natural way.

D. SZASZ:

Convergence to equilibrium of the Lorentz gas

For systems with a finite number of degrees of freedom good ergodic properties involve convergence to equilibrium of non-equilibrium evolutions. For systems with an infinite number of degrees of freedom a deeper analysis of the ergodic behaviour of the evolution may give the same type of result. We show that if the initial distribution of the periodical Lorentz gas satisfies an independence condition then the convergence of the system to its equilibrium follows from the K-property of the Sinai billiard. If we use the global central limit theorem, proved by Bemimovich and Sinai for the Sinai billiard, then we can substitute this independence condition by a weaker mixing condition. To the extension of the results to the non-periodic Lorentz gas the local version of the mentioned CLT would be needed, which we could not prove because of principal difficulties. The results are joint with A. Krámli.

S.R.S. VARADHAN:

Large deviations

We consider the problem of evaluating the function space integrál

$$E\{\exp[\alpha \int_0^t \int_0^t e^{-|\sigma-s|} \frac{d\sigma ds}{|x(\sigma)-x(s)|}]\} \text{ asymptotically for large } t.$$

Here $x(t)$ is a tied down 3-dimensional Brownian motion tied down by $x(0) = x(t) = 0$. One aims to show that for large t the expectation is of the form $\exp[tg(\alpha) + O(t)]$ where $g(\alpha)$ is a function behaving like $c\alpha^2$ for large α . We study the problem using the method of large deviations. We prove that a conjecture of Pekar regarding the constant c is indeed correct.

A. WAKOLBINGER:

Simplices of probability measures

The concept of a simplex of probability measures possessing an H -sufficient statistic was introduced by Dynkin (1978).

In case of a standard Borel space (which is the case essentially treated by Dynkin) different characterizations of such simplices are given in the present talk:

- a) as invariant measures w.r. to an arbitrary countable family of Markov kernels
- b) as measurable, weakly closed sets of probability measures possessing an (a priori not unique) integral representation and a sufficient and complete statistic.

In addition, the corresponding results in case of perfect spaces are stated. Finally, the following question is dealt with: when does an element of some simplex of probability measures belong to a "sub-simplex" of Dynkin's kind (i.e. when are its representing extremal measures strongly orthogonal) and an answer in terms of its Bayes estimator is given (if the latter can be suitably defined, which is the case, e.g., in the situation of the simplex of Cox processes).

P. WEIß:

Time reversible states of spatial birth - death processes

(joint work with E. Glötzl)

For a lattice phase space Z^d it is well known that Gibbsian measures on $\{0,1\}^{Z^d}$ with respect to a local energy $e(x,\mu)$ are just the probability measures which are time reversible for the spin-flip process with the speed function $c(x,\mu) = \exp(e(x,\mu))$.

We generalize this result to general phase spaces, including in particular models with phase space R^d and Z^d . The corresponding generalization of spin-flip processes are spatial birth and death processes.

H. v. WEIZSÄCKER:

Extreme probabilities: theory and examples

The talk first discusses Douglas' extreme point criterion and a general Choquet-type result for non-compact sets of probability measures (v.Weizsäcker-Winkler Math. Ann. 79). As examples of extreme point problems the following are discussed: Extreme martingale distributions, extreme doubly stochastic measures (we characterize all mixtures of permutation measure on $[0,1]^2$) and an ergodic decomposition of quasiinvariant measures. At the end it is pointed out that weak algebraic conditions on a non-compact simplex imply almost the existence of a sufficient statistic for the extreme points in the sense of Dynkin.

S. ZABELL:

Upper bounds for large deviation probabilities

It is possible to give a very general lower bound for large deviation probabilities. This gives rise to the problems of

- 1) computing this bound in particular cases,
- 2) determining if this is the exact rate of exponential convergence to zero
- 3) if not, determining upper bounds.

If $\{\bar{X}_n\}$ is a sequence of sample means for random vectors in a locally convex TVS V , and $G \subset V$ is closed, a number of partial results is discussed for the case $n^{-1} \log P\{\bar{X}_n \in G\}$.

W.R. van ZWET:

A Berry-Esseen bound for symmetric statistics

Let X_1, X_2, \dots be i.i.d. and let $T_N = \tau_N(X_1, \dots, X_N)$ where τ_N is symmetric in its N arguments. Suppose that $E T_N = 0$, $E T_N^2 = 1$, $E |E(T_N | X_1)|^3 \leq A N^{-3/2}$ and $1 + E\{E(T_N | X_1, \dots, X_{N-2})\}^2 - 2 E\{E(T_N | X_1, \dots, X_{N-1})\}^2 \leq B N^{-3}$.

Then there exists $C > 0$ depending on A and B such that

$$\sup_x |P(T_N \leq x) - \Phi(x)| \leq C N^{-1/2}$$

when Φ is the standard normal distribution function. This result is applied to U -statistics and to linear functions of order statistics.

Berichterstatter: P. Prinz (München)

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