MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 14/1981

Gewöhnliche Differentialgleichungen

22. bis 28. 3. 1981

Die achte Tagung "Gewöhnliche Differentialgleichungen" fand wieder unter der Leitung der Herren H.W. Knobloch (Würzburg) und R. Reißig (Bochum) statt. Die Teilnehmerzahl betrug 41 (Bundesrepublik Deutschland: 20 , Ausland: 21 , darunter USA: 8). Bei der Aufstellung der Teilnehmerliste fand eine Koordinierung mit den Herren Mawhin und Peitgen statt, die zuvor die Tagung "Topologische Methoden in der Nichtlinearen Funktionalanalysis ..." organisierten. Verschiedenen Gästen aus USA war der Besuch beider Veranstaltungen möglich. Es wurden 34 Vorträge gehalten, 15 von deutschen und 19 von ausländischen Teilnehmern.

Eine grobe Aufteilung der Vorträge auf Spezialgebiete ergibt folgendes Bild: Verschiedene Aspekte der Verzweigungstheorie (4), Stabilitätsprobleme allgemeiner und spezieller Art (2), Systeme mit Verzögerung, bestimmte Lösungstypen und funktionalanalytische Hilfsmittel zur Untersuchung der Lösungen (4), Dynamische Systeme und Qualitative Theorie (4), Grenzzyklen autonomer Differentialgleichungssysteme (2), Resonanzprobleme bei nichtautonomen Differentialgleichungen (5), lineare und nichtlineare Randwertaufgaben (4), Differentialgleichungsprobleme in der Kontrolltheorie (1), allgemeine und spezielle numerische Methoden im Zusammenhang mit Differentialgleichungen (3), spezielle Typen von Differentialgleichungen (2), abstrakte Differentialgleichungen (2), Anwendung auf Schwingungen (1).

Außerhalb des Vortragsprogrammes führte Herr Reeb (Straßburg) eine Informations- und Diskussionsveranstaltung mit dem Thema "Introduction to Nonstandard Analysis" durch.

Die teilweise dicht gedrängte Vortragsfolge ließ die allgemeine Tendenz erkennen, die Vortragsdauer auf Kosten der Diskussionszeit in unangemessener Weise auszuweiten. Von den Tagungsleitern wurde für die nächste Tagung eine Tendenzwende durch geeignete organisatorische Maßnahmen in Aussicht gestellt.

Vortragsauszüge

B.Aulbach

Approaching invariant manifolds with asymptotic amplitude and phase

When a solution x(t) of an autonomous differential system approaches an invariant manifold M the following question comes up: does x(t) approach M with asymptotic amplitude and phase, meaning, that $x(t)-x_o(t)$ tends to 0 as t tends to infinity for a particular solution $x_o(t)$ on M or does x(t) ignore the flow on M? Which of these cases occurs depends on the relation of the flow near M to the flow on M. The lecture discusses the following result: If the invariant manifold M is compact and normally hyperbolic with a certain stability on M then any solution that approaches M does it with asymptotic amplitude and phase.

J.Baumgarte

Stabilization of mechanical equations of motion

When a given system of differential equations is integrated by numerical and automatic integration it may occur that the solution at hand satisfies an analytical relation which is a corollary of the differential equations but which is unknown to the automatic computer. Examples of such relations are first integrals (inner constraints) or the analytical relations generated by outer holonomic or non-holonomic constraints provided that the Lagrange equations of the first kind are used. It is shown that, in general, the computed numerical values of the solution satisfy such analytic relations with poor accuracy. It is the aim to show how the analytical relations can be satisfied in a stabilized manner in order to improve the numerical accuracy of the solution of the differential equations. The basic idea is applied to mechanical problems.

Ch.Conley

The Fuller index for periodic orbits

In 1968, B.Fuller defined an index for isolated sets of periodic orbits of a differential equation on a manifold. This leads to an index theory closely analogous to that for fixed points. In Fuller's approach the index is first defined for isolated non-degenerate solutions and then genericity arguments, a "homotopy invariance" theorem and a sum formula are used to extend the definition. In this talk an alternate approach is described wherein the index is defined as (a sequence of) "intersection numbers". The definition is somewhat more direct than Fuller's and leads to a slightly sharper invariant in that the group $H_1(M)$ plays a more explicit role. If $H_1(M) = 0$, it is equivalent to Fuller's index.

W.Eberhard

An equiconvergence principle for singular multipoint boundary value problems

Sei $l\,y$ der lineare Differentialausdruck $l\,y = y^{(2n)} + \sum_{k=2}^{2n} (\alpha_{2n-k} + f_{2n-k}(x))\,y^{(2n-k)}$ mit komplexen Konstanten α_j und $f_j \in L^1(0,\infty)$ für $j=0,1,\dots,2n-2$. Seien in $0=a_1 \leq a_2 \leq \dots \leq a_n$ lineare Randausdrücke $U_\nu y = y^{(k_\nu)}(a_\nu) + \sum_{j=0}^{k_\nu-1} \beta_{\nu j}\, y^{(j)}(a_\nu)$; $\nu = 1,2,\dots,n$ $(0 \leq k_\nu \leq 2n-1; \beta_{\nu j} \in \mathbb{C})$ vorgegeben, und sei D die Funktionenmenge $D = L^2(0,\infty) \cap \{ y \mid y \in C^{2n-1}[0,\infty),\ y^{(2n-1)}$ absolut stetig, $U_\nu y = 0$ für $\nu=1,2,\dots,n$ und $l\,y \in L^2(0,\infty) \}$. Für den i.allg. nicht-selbstadjungierten Differentialoperator $L: D \to L^2(0,\infty)$ mit $L\,y = l\,y$ gilt dann der Äquikonvergenzsatz: Sei $f \in L^1(0,\infty)$ mit supp $f \subset (a_n,\infty)$. Dann gilt gleichmäßig für $x \in [a_n,\infty)$:

$$\lim_{R\to\infty} \int_0^\infty f(t) \cdot \left[\frac{\sin R(x-t)}{\pi\,(x-t)} - \frac{1}{2\pi i} \int_{C_R} 2n\varrho^{2n-1}\, \Gamma(x,t,\varrho^{2n})\, d\varrho \right] dt = 0,$$

wobei C_R einen Kreisbogen mit Öffnungswinkel $\frac{\pi}{n}$ um $\varrho=0$ bedeutet. Ein analoger Äquisummierbarkeitssatz läßt sich für Rieszsche Mittel beweisen.

D.Flockerzi

Bifurcation of higher dimensional tori - Remarks on a paper by G.Sell

In his paper (ARMA 69, 1979) G.Sell is considering an equation $X'=F(X,\alpha)$ in $\mathbb{R}^{n+k}$ possessing a k-dimensional invariant torus $\tau(\alpha)$. He states several hypotheses that imply the bifurcation of a (k+1)-dimensional invariant torus $\tilde{\tau}(\alpha)$ from $\tau(\alpha)$, but the proof and the form of $\tilde{\tau}(\alpha)$ he presents are not correct. Starting from a differential system (1) $\dot{\psi} = \bar{\omega} + \alpha\Psi(\psi,x,z,\alpha)$, $\dot{x} = A\,x + X(\psi,x,z,\alpha)$, $\dot{z} = B(\psi)\,z + Z(\psi,x,z,\alpha)$ with $(\psi,x,z)\in T^k\times\mathbb{R}^2\times\mathbb{R}^{n-2}$ and using Sell's hypotheses we transform (1) into (2) $\dot{\varphi} = \omega+\varepsilon O(|z|) + O(\varepsilon^2)$, $\dot{\varrho} = \varepsilon^2[R(\varrho)+O(|z|)] + \varepsilon O(|z|^2) + O(\varepsilon^3)$, $\dot{z} = B(\varphi)\,z + \varepsilon O(1)$ with $(\varphi,\varrho,z)\in T^{k+1}\times\mathbb{R}\times\mathbb{R}^{n-2}$. To show that $z = O(\varepsilon)$ we prove the existence of an integral manifold M_ε for (2) for small $|\varrho|$. Afterwards we establish an integral manifold S_ε for the flow on M_ε. This way we obtain an a-priori bound for $|\varrho|$ and thus the torus $\tilde{\tau}(\alpha)$ given by $\{(\varphi,\varepsilon(\varrho_0+f(\varphi,\varepsilon)),\varepsilon g(\varphi,f(\varphi,\varepsilon),\varepsilon)) : \varphi\in T^{k+1}\}$ where $\varrho_0 > 0$, $\varepsilon^2 = \alpha$ (or $-\alpha$) and f,g are of order $O(\varepsilon)$.

G.Freiling

Singular perturbations of multi-point eigenvalue problems

We consider eigenvalue problems of the type $\varepsilon^p \sum_{j=n+1}^{m} y^{(j-n-1)p} \cdot b_j(x,\lambda,\varepsilon) + \sum_{j=0}^{n} b_j(x,\lambda,\varepsilon) = 0$, $0\leq x\leq 1$ (1) with multi-point boundary conditions $\sum_{j=0}^{m-1}[A_{\nu j}(\lambda,\varepsilon)y^{(j)}(0) + \sum_{l=1}^{K-1} C_{\nu l j}(\lambda,\varepsilon)y^{(j)}(x_l) + B_{\nu j}(\lambda,\varepsilon)y^{(j)}(1)]$, $1\leq\nu\leq m$ (2). Here ε is a small positive parameter, $p\in\mathbb{N}$, and all coefficients are assumed to be infinitely differentiable in x, to depend analytically on λ and to have asymptotic expansions with respect to ε as $\varepsilon\to 0$. Using asymptotic expansions for a fundamental system of solutions of the differential equation (1), we can show under certain hypotheses on the differential equation and on the boundary conditions that the eigenvalues and eigenfunctions of (1) , (2) tend to eigenvalues and eigenfunctions of the "reduced problem". In the proof we must distinguish between the "non-exceptional case" and the "exceptional case".

R.E.Gaines

Solutions to systems of nonlinear ordinary differential equations lying in cones

We consider systems of nonlinear ordinary differential equations of the general form (1) $\dot{x} = f(t,x)$ where $f: [0,T] \times \mathbb{R}^n \to \mathbb{R}^n$ is continuous. We give various conditions under which (1) has solutions lying in a prescribed cone K. The cone can, for example, be taken to be $\{x:[0,T] \to \mathbb{R}^n \mid x(t) \geqq 0$ for $t \in [0,T]$ and $x(0)=x(T)\}$. Existence is established by converting (1) to an operator equation and applying degree theoretic methods.

R.Grimmer

On a differential equation related to integral equations

Let X be a Banach space and F be a Banach space of functions defined on $\mathbb{R}^+$ into X. Let $\delta: F \to X$ be given by $\delta F = f(0)$ and D be the differentiation operator d/ds on F. The differential equation (DE) $\dot{x} = A x + \delta y$, $\dot{y} = B x + Dy$, $\dot{} = d/dt$, on $X \times F$ is considered under various hypotheses on A and B. The reason for studying this equation is its intimate relation to Volterra integral equations in a Banach space. The equation (DE) is considered in the autonomous and nonautonomous linear cases and in the autonomous nonlinear case. The main object is to determine when a semigroup or evolution operator is generated. In the linear autonomous case it is assumed that A generates a semigroup on X. Similar assumptions are made in the other cases.

P.Habets

The Picard boundary value problem for nonlinear second order vector differential equations

In this paper, existence conditions are presented for the Picard boundary value problem $x'' + f(t,x,x') = 0$, $x(a) = x(b) = 0$, where f is a continuous vector function. The main result is basically the following. Let φ, F be scalar functions (φ being strictly positive) such that: $\varphi'' + F(t,\varphi,\varphi') \leqq 0$ on $[a,b]$; $\langle x,f(t,x,y)\rangle \leqq \varphi(t)\, F(t,\varphi(t),\varphi'(t)) + |y|^2 - [\varphi'(t)]^2$ for $|x| = \varphi(t)$, $\langle x,y\rangle = |x|\varphi'(t)$. Then, provided that $|x'|$ is bounded when $|x|$ is bounded, the Picard boundary value problem has at least one solution. The main tool in the proof of that theorem is Leray-Schauder degree theory, the main differences with respect to earlier results lying in the choice of the homotopy used and of the set with respect to which the degree is computed. As particular cases, growth at most linear in x,y for $|f(t,x,y)|$ is considered on one hand. On the other hand, the scalar case is revisited and classical results about upper and lower solutions are easily deduced from a slightly modified version of the above mentioned basic theorem.

J.K.Hale

Dynamic behavior from the bifurcation function

Three problems are discussed. I. Stability in critical cases and bifurcation from a simple eigenvalue. II. The ω-limit set of a gradient like system. III. Stability of traveling waves. For problem I., it is shown that the bifurcation function obtained by the method of Liapunov-Schmidt considered as a one dimensional vector field generates the same flow as the one on the center manifold. Also, if there is a family of equilibrium points, then there is a first integral of the equation. The latter remark gives stability of traveling waves. It is also basic to a proof of the following result of Matano: For $u_t = u_{xx} + f(u)$, $0 < x < 1$, with separated boundary condition at $x = 0$, $x = 1$, any bounded solution has an ω-limit set that consists only of a point.

S.Invernizzi

Periodic solutions of systems of Rayleigh or Liénard type

Let f(t,x) ba a nonlinear term in a vector ordinary differential equation; in the study of the periodic BVP, the "sign condition" $\varepsilon_k f_k(t,x)x_k \geqq 0$, $|x_k| \geqq M$, $|\varepsilon_k| = 1$, is a quite common assumption, but it is not invariant, for instance, with respect to the orthogonal group $O(n,\mathbb{R})$, and this is not satisfactory from a physical point of view. Thus we propose, for an ordinary differential system of Rayleigh type, an "angle condition" ($O(n,\mathbb{R})$-invariant) which is more general than the "sign condition" and (with other assumptions) implies the existence for the periodic BVP. Moreover, with the same point of view, we propose an extension to systems of the classical result by Lazer in JMAA 21 (1968) concerning nonlinear 1-dimensional forced oscillations (an example: the periodic BVP for the system $x'' + Cx' + b(x)Ax = e(t)$ is solvable if e has zero mean, C is symmetric, b is real-valued, positive for large $\|x\|$ and vanishing at ∞, and A is any $n \times n$ nonsingular matrix. Remark: If $n = 2$, and $a_{11} = a_{22} = 0$, $a_{12} = -a_{21} = -1$, the k-th component of $b(x)Ax$ changes sign in each set defined by $|x_k| \geq M$).

F.Kappel

An axiomatic state space theory for infinite delay equations

Since the state of an infinite delay equation does not get smoother as time evolves, the question for an appropriate state space is much more crucial as compared with the bounded delay case. By an appropriate state space we mean a state space which enables one to prove standard theorems on existence, uniqueness and continuous dependence of solutions or to prove fundamental results in stability theory (e.g. that bounded trajectories are precompact). In the talk we discuss a slightly modified version of axioms for a state space which originally where given by J.K.Hale and J.Kato (Funkc.Ekv.21,1978). It turns out that the solution semigroup S_t corresponding to the trivial equation $\dot{x}=0$ plays a central role. An example given by G.Seifert (JDE 22, 1976) demonstrates what happens if S_t is not a C_o-semigroup on the state space. For more details the reader is referred to a joint paper with W.Schappacher (JDE 37,1980).

H.Kielhöfer

Degenerate bifurcation of stationary or periodic solutions and their exchange of stability

For the evolution equations (1) $\frac{du}{dt} + G(\lambda,u) = 0$, $G(\lambda,0) = 0$, depending on a real parameter λ in some Banach space E we prove the following theorem: Let G be analytic in λ and u and let $\mu(\lambda)$ be a simple eigenvalue of $G_u(\lambda,0)$ with Re $\mu(0) = 0$. Then 1. The degeneracy Re $\mu^{(j)}(0) = 0$; $j=0,\dots,m-1$, Re $\mu^{(m)}(0) \neq 0$ is quantitatively reflected in the bifurcation equation. 2. If m is odd then $(\lambda,u) = (0,0)$ is a bifurcation point of stationary (periodic) solutions of (1) provided that Im $\mu(0) = 0$ ($\neq 0$). 3. At most m nontrivial bifurcating branches can exist. 4. All branches can be constructed with the aid of Newton's diagram. 5. There is a quantitative Principle of Exchange of Stability saying that the eigenvalue (Floquet exponent) of $G_u(\lambda,u)$ of any bifurcating stationary (periodic) branch is quantitatively determined by the bifurcation equation.

U.Kirchgraber

On a procedure to compute level lines

Let $\mathbb{G}$ be a bounded region in $\mathbb{R}^n$ and G a smooth map from $\bar{\mathbb{G}}$ into $\mathbb{R}^{n-1}$ satisfying the following assumptions: (i) rank $\frac{\partial G}{\partial x} = n-1$, (ii) $G(x) \neq 0$ for $x \in \partial\mathbb{G}$. Let $\mathbb{M} = \{x \in \mathbb{G} \mid G(x)=0\} \neq \emptyset$ and consider the maximal connected set $\Gamma(\xi)$ of ξ in $\mathbb{M}$ for $\xi \in \mathbb{M}$. The purpose of the paper is to describe a procedure to determine $\Gamma(\xi)$ approximately. To this end consider the following system of o.d.e's (1) $\dot{x} = g(x) - K\, G_x^T \Lambda^{-1}(x)\, G(x) = f(x)$ where $g^*(x)$ is the exterior product of grad G_1, grad G_2, ..., grad G_{n-1}, $g(x) = |g^*|^{-1} g^*$, $\Lambda(x) = G_x \cdot G_x^T$, $K > 0$. Let $F(t,x)$ denote the flow of (1). <u>Theorem 1</u>. $\Gamma(\xi) = C(\xi) = \{F(t,\xi) \mid t \in \mathbb{R}\}$ where $F(t,\xi) = p(t)$ is an orbitally exponentially stable periodic solution for (1) (with period $T > 0$, say). In order to determine $\Gamma(\xi)$ approximately we use a one-step numerical integration procedure of order p to equ.(1) (like Runge-Kutta's method) i.e. a map $x \in \mathbb{G} \mapsto \tilde{F}(h,x) \in \mathbb{R}^n$ with $\tilde{F}(h,x) - F(h,x) = O(h^{p+1})$, uniformly in x and for all $h > 0$, sufficiently small. <u>Theorem 2</u>. There are positive constants H, L and a function $\hat{p}(h,t)$: $(0,H) \times \mathbb{R} \to \mathbb{R}^n$, T-periodic, bounded by $L h^p$, lipschitzian with the Lipschitz constant $L h^p$ such that $\tilde{\Gamma}(h) = \{p(t) + \hat{p}(h,t) \mid t \in \mathbb{R}\}$ is an invariant, stable, attractive manifold for the map $x \mapsto \tilde{F}(h,x)$. This leads to the following conclusion: While it is not possible in general to determine $\Gamma(\xi)$ by the above method, it is possible to determine a curve $\tilde{\Gamma}(h)$ with the same topological properties as $\Gamma(\xi)$ and which is $O(h^p)$-close to $\Gamma(\xi)$.

A.C.Lazer

Periodic solutions of differential equations which model semi-discrete diffusion processes

We study the differential system $u' = A(t)u + f(t,u) + h(t)$ where A, f, and h are periodic with the same period, u is an n vector, the elements of A satisfy certain sign restrictions and each component of f depends only on the corresponding component of u. Such systems arise when one discretizes a reaction diffusion equation in the space variables. Under certain conditions on f we can characterize the h for which there is a periodic solution.

R.Lemmert

Nicht-differenzierbare Ober- und Unterfunktionen bei Differentialgleichungen 2.Ordnung

Es wird die Möglichkeit diskutiert, Glattheitsvoraussetzungen für Ober- und Unterfunktionen bei der 1.Randwertaufgabe bei gewöhnlichen Differentialgleichungen abzuschwächen.

N.G.Lloyd

The number of limit cycles of certain polynomial differential systems

Consider the set E_n of equations (1) $\dot{x} = P(x,y)$, $\dot{y} = Q(x,y)$, where P and Q are polynomials of degree n. Let r(P,Q) be the number of limit cycles of (1). It is a well-known and long-standing problem to estimate $r(n) = \sup\{r(P,Q)\ ;\ (P,Q) \in E_n\}$. This talk will describe some recent, and continuing,work on this question for small values of n. For instance, examples of quadratic systems with four limit cycles are known; they are obtained by perturbing an equation which has a center to produce three small-amplitude limit cycles and four in total. It can be shown that four limit cycles are the most that can be produced by this technique. In cubic systems, five (or fewer) limit cycles with small amplitude can be thus produced. The techniques used involve formal computation on a computer.

J.Mawhin

Non-uniform non-resonance and resonance conditions for some Liénard differential equations

We consider the periodic boundary value problem (L) $x'' + f(x)x' + g(t,x) = e(t)$, $x(0)-x(2\pi) = x'(0)-x'(2\pi)=0$, where $e \in L^1(0,2\pi)$, $f: \mathbb{R} \to \mathbb{R}$ is continuous, $g: [0,2\pi]\times \mathbb{R} \to \mathbb{R}$ satisfies the Caratheodory conditions and is such that, for some γ and Γ in $L^1(0,2\pi)$, one has $\gamma(t) \leq \liminf_{|x|\to\infty} x^{-1}g(t,x) \leq \limsup_{|x|\to\infty} x^{-1}g(t,x) \leq \Gamma(t) \leq 1$ uniformly in $t \in [0,2\pi]$, with the last inequality strict on a subset of $[0,2\pi]$ of positive measure. We then show the existence of at least one solution in the following cases: 1) $\int_0^{2\pi}\gamma(t)\,dt > 0$; 2) $\int_0^{2\pi}\gamma(t)\,dt = 0$, $g(t,x)\,x \geq 0$ for large $|x|$ and $\int_0^{2\pi} e(t)\,dt = 0$; 3) $\int_0^{2\pi}\gamma(t)\,dt = 0$, $\gamma(t) \not\equiv 0$ and f constant. This generalizes earlier results of Lazer, Reissig, Chang, Martelli, Gupta, Amaral and Pera, the author and others. It is a joint work with J.R.Ward jr.

M.E.Muldoon

Nonlinear equations satisfied by zeros of special functions

This talk contains results of some joint work with S.Ahmed on sums of the form $\sum_{j=1, j\neq k}^{\infty} (x_j - x_k)^{-m}$ and $\sum_{j=1}^{\infty} (x_j - \alpha)^{-1}$ where $\{x_j\}$ is the sequence of zeros of an appropriate solution of a second order linear differential equation having a singularity at α. The results generalize some of those obtained by F.Calogero and others for various special functions; for references, see S.Ahmed, et al., Nuovo Cimento 49 B, 1979, pp.173-199. Some attention is paid to the problem of characterizing the solutions in question by means of formulas for sums of the above kind.

K.Nixdorff

Stand der Berechnung von Waffenrohrschwingungen mit dem Einkörperbalkenmodell

Der Vortrag gliedert sich in: 1. Physikalische Situation, Meßverfahren, Modelle zur Berechnung. 2. Mathematische Formulierung des Einkörperbalkenmodells. 3. Gegebene Daten. 4. Ermittlung der Einspannungsparameter. 5. Ermittlung der statischen Durchbiegung. 6. Für die dynamische Durchbiegung berücksichtigte Kräfte. 7. Ermittlung der dynamischen Durchbiegung mit einem Differenzenverfahren. 8. Instabilität. 9. Optimierung der Schrittweiten. Leider liefert die Meßtechnik nicht alle Daten, die das Modell braucht. Fehlende Daten müssen aus gemessenen errechnet werden. Das verwendete Differenzenverfahren bringt verschiedene Schwierigkeiten, die unter Berücksichtigung der Meßergebnisse angegangen werden.

H.O.Peitgen

Special periodic solutions of delay equations and their numerical computation

This paper is joint work with R.D.Nussbaum. The problem is to study "Special Periodic Solutions" (SPS) for the equation (1) $\dot{u}(t) = -\lambda f(u(t-1))$ where $\lambda \in \mathbb{R}$, $f: \mathbb{R} \to \mathbb{R}$ is C^1 (for simplicity), $f'(0) = m_0$, $m_\infty^1 s + o(s) \geq f(s) \geq m_\infty^2 s + o(s)$ as $s \to \infty$, and f has a finite number of zeros. A function $u: \mathbb{R} \to \mathbb{R}$ is called SPS iff u is a solution of (1) and $u(t+2) = -u(t)$ and $u(-t) = -u(t)$. We show:

Existence. (i) There exists a continuum Σ_0 of SPS for (1) bifurcating from the trivial solution at $(\lambda_0, 0)$ and the projection of Σ_0 onto the λ-space fills the interval (λ_0, ∞), $\lambda_0 = \pi/2m_0$. (ii) There exists a continuum Σ_∞ of SPS for (1) bifurcating from ∞ in $[\lambda^1_\infty, \lambda^2_\infty]$ and the projection of Σ_∞ onto the λ-space fills the interval $(\lambda^2_\infty, \infty)$, $\lambda^i_\infty = \pi/2m^i_\infty$. (iii) $\Sigma_0 \cap \Sigma_\infty = \emptyset$.
Numerical approximation of SPS. As a typical numerical approximation we investigate the trapezoidal rule to compute SPS of (1). After a sequence of transformations we obtain the following system of equations: (2) $A\,x - \mu\, B\, F(x) = 0$. Here A and B are regular matrices, F incorporates f, $x \in \mathbb{R}^n$, $\mu = \mu(\lambda)$. A careful analysis of (2) for $\mu \sim \infty$ provides solutions which approximate solutions of (1) as well as spurious solutions, i.e. solutions of (2) which do not approximate any solution of (1). We can count the number of spurious solutions and classify two different structural types of them.

R.M.Redheffer

Asymptotic stability for prey - predator equations of generalized Volterra type

The class of equations addressed is that for which the fundamental matrix $P = (p_{ij})$ satisfies $p_{ii} \leqq 0$, $p_{ij}\, p_{ji} < 0$ whenever $(i-j)\, p_{ij} \neq 0$. Also $(a_i p_{ij}) \leqq 0$, in the ordering given by quadratic forms, for some positive vector a. The main result asserts then that the long-time behavior is governed by a set of disjoint trees in the graph of P which is unique and easily constructed in any specific case. This decomposition into trees indicates a decoupling of the variables for $t \to \infty$ which is far from obvious at first glance. As an extremely special case, the methods give a substantially complete solution to the problem of stability for systems in $n \leqq 6$ unknowns. Both the characterization of matrices P satisfying $(a_i p_{ij}) \leqq 0$ and the analysis of stability involve a novel blend of topological and combinatorial consideration. The work was done jointly with Zhou Zhimihy, Zhongsan University, China.

R.Reißig

Bemerkungen über Stabilität und Übergangsverhalten linearer Systeme

Für ein lineares System $x' = A(t)x + f(t)$; $t \geq 0$, $x \in \mathbb{R}^n$, $A(t)$ und $f(t)$ meßbar und über endlichen Intervallen Lebesgue-integrierbar, werden Sätze von Halanay und Reghis verallgemeinert: Perron-Stabilität (d.h. $\sup_{t \geq 0} |\int_0^t X(t,s)\, f(s)\, ds| < \infty$, falls $f(t)$ stetig und beschränkt) ist äquivalent zur gleichförmigen asymptotischen Stabilität nach Ljapunov, d.h. $|X(t,s)| \leq M\, e^{-b(t-s)}$ für $t \geq s \geq 0$, $b > 0$. Die Beschränktheit der Koeffizienten-Matrix, die Halanay voraussetzt, wird nicht zu dem vorgeschlagenen Beweis benötigt. Entsprechendes gilt für den Satz: Wenn $\int_0^\infty [|f(t)|\, e^{at}]^p\, dt < \infty \implies \sup_{t \geq 0} e^{bt} |\int_0^t X(t,s) f(s) ds| < \infty$

(a und b positiv , $p \geq 1$), dann folgt die Abschätzung der Fundamentalmatrix $|X(t,s)| \leq M\, e^{-bt}\, e^{as}$, falls man für $p > 1$ fordert: $\sup_{t \geq 0} \int_t^{t+1} |A(u)|\, du < \infty$. Die Abschätzung schließt gleichförmige bzw. ungleichförmige asymptotische bzw. schwache Stabilität ein.

E.O.Roxin

Die Komplement-Stetigkeit von Mengen-wertigen Funktionen und deren Anwendung in der Kontrolltheorie

Ist $F: R^n \longrightarrow (R^n)$ (= kompakte Untermengen des R^n), so wird die Stetigkeit von $F(x)$ gewöhnlich im Sinne der Hausdorffschen Metrik des (R^n) verstanden. Fordert man auch die Stetigkeit von der Komplementärmenge $F^c(x)$, so ergibt sich ein strengerer Stetigkeitsbegriff. Dieser ist genau die Annahme, unter welcher gewisse Sätze der Kontrolltheorie bewiesen werden können.

H.Rüßmann

On the continuation of invariant tori of dynamical systems

One difficulty in the perturbation theory of quasiperiodic solutions of dynamical systems (Kolmogorov-Arnold-Moser-Theory) lies in the appearance of the "Small Divisors", the smallness of which is measured by certain approximation functions. The question is treated what approximation functions are admissible such that the K-A-M-technique is applicable. As an example, the one-dimensional Schrödinger equation with a quasiperiodic potential is considered.

W. Schappacher

Linear delay equations in Banach spaces

We study the linear delay equation $\frac{d}{dt}x(t) = f(x(t),x_t)$, $t\geq 0$, $x_o = \varphi$ (1) in a reflexive Banach space X. As usual, x_t is defined by $x_t(s) = x(t+s)$. Assuming that (1) is uniformly well posed, we investigate properties of the associated solution semigroup and its infinitesimal generator.

B.V. Schmitt

Sur la structure de l'équation de Duffing sans dissipation

One takes advantage of the internal symmetries of the Duffing equation without dissipation, to locate, numerically, some of its periodic solutions, harmonic as well as subharmonic, of a precise type: solutions which are either even functions, or are out-of-phase of odd functions. Maps are given (similar to the stability charts of the Mathieu equation), which show the location of the initial conditions, in the space of parameters, of the different types of solutions; the stability of the harmonic periodic solutions appears on these maps.

K. Schmitt

Positive solutions of nonlinear elliptic eigenvalue problems

We consider a nonlinear elliptic eigenvalue problem $L u + \lambda f(u) = 0$, $x \in \Omega \subset \mathbb{R}^n$, $u|_{\partial\Omega} = 0$, where Ω is a bounded domain with smooth boundary, L is a uniformly elliptic second order operator with Hölder continuous coefficients and $f: \mathbb{R} \to \mathbb{R}$ is Lipschitz continuous with $f(0)=0$. Under certain assumptions on f we prove the existence of unbounded solution continua which bifurcate from the trivial solution, respectively from infinity. These results are applied to study some similar problems recently considered by Ambrosetti and Hess and by Hess.

U. Staude

Die Eindeutigkeit des Grenzzykels bei dem Fitzhugh'schen Differentialgleichungssystem

1955 hat R. Fitzhugh ein zweidimensionales autonomes Dgl-System als Approximation für die Hodgkin-Huxleysche Nervengleichung aufgestellt und untersucht. Zwischen 1976 und 1979 haben

I.D.Hsü, W.C.Troy und P.Negrini-L.Salvadori die Hopfsche Verzweigungstheorie auf dieses System angewendet und nachgewiesen, daß für $\mu = -\mu_0$ und $\mu = \mu_0$ $(\mu_0 > 0)$ von der stationären Lösung periodische Lösungen abzweigen. In Abhängigkeit von festen Parametern des Systems liegt entweder Verzweigung nach außen (Fall 1) vom Intervall $(-\mu_0, \mu_0)$ vor oder nach innen (Fall 2). Das Fitzhugh-System kann auf ein verallgemeinertes Liénard-System transformiert werden. Dann kann gezeigt werden, daß im Fall 1 mindestens ein stabiler Grenzzykel für $\mu \in I$, $(-\mu_0, \mu_0) \subset I$, vorliegt und im Fall 2 mindestens ein stabiler Grenzzykel nur für $\mu \in (-\mu_0, \mu_0)$. Die Anwendung der Eindeutigkeitssätze für Grenzzyklen von Sansone-Conti und dem Vortragenden sichern jedoch Eindeutigkeit des Grenzzykels für $\mu \in [-\mu_1, \mu_0)$, $-\mu_0 < -\mu_1 < -\frac{1}{2}\mu_0$. Mit einer zusätzlichen Voraussetzung kann im Fall 1 auch noch erreicht werden, daß $\mu_1 = \mu_0$. (Die Untersuchungen wurden zusammen mit Herrn E.Kaumann/Mainz durchgeführt.)

A.Vanderbauwhede

Bifurcation problems in the presence of first integrals

It is well known that for bifurcation problems for ordinary differential equations, the presence of first integrals may result in a reduction of the bifurcation equations (see e.g. J.K.Hale, Ordinary Differential Equations, p.271-272, for the case of periodic solutions). We give an abstract approach to this type of problems; our results clarify the kind of condition that should be satisfied in order to get a reduction in the bifurcation equations. We also obtain conditions for the existence of one- or multi-parameter families of solutions. We apply the method to general non-linear boundary value problems for ordinary differential equations, including the problem of periodic solutions. For this last case an alternative formulation can be given.

P.Volkmann

Existenzbeweis für ein Anfangswertproblem im Banach-Raum unter Verwendung von Näherungslösungen

Sei E ein Banach-Raum; für x, y $\in$ E werde $[x,y]_- = \lim_{h\to 0-} \frac{1}{h}(\|x+hy\| - \|x\|)$ gesetzt. Betrachtet wird das Anfangswertproblem (*) $u(0)=a$, $u'(t)=f(t,u(t))$ $(0 \leq t \leq T)$ mit $a \in E$, $f=g+k$, wobei $g,k: [0,T] \times E \to E$ stetig sind, k kompakt, g beschränkt und $[x-y, g(t,x)-g(t,y)]_- \leq$

$L\|x-y\|$ ist.- <u>Satz</u>. Sind $u_n:[0,T] \to E$ $(n=1,2,3,\ldots)$ stückweise stetig differenzierbar mit $u_n(0)=a$, $\|(u_n)'_{\pm}(t) - f(t,u_n(t))\| \leq \frac{1}{n}$, so konvergiert eine Teilfolge der u_n auf $[0,T]$ gleichmäßig gegen eine Lösung von (•).- <u>Folgerung</u>. Ist M eine abgeschlossene Teilmenge von E, gilt $a \in M$ und $\liminf_{h \to 0+} \frac{1}{h} \operatorname{dist}(x+hf(t,x),M) = 0$ $(x \in M, 0 \leq t \leq T)$, so besitzt (•) eine Lösung $u: [0,T] \to M$.

H.O.Walther

<u>Chaos in differential delay equations</u>

The differential delay equation (1) $\dot{x}(t) = -\alpha x(t) + f(x(t-1))$, with $\alpha > 0$ and f a hump function  , displays a variety of phenomena, depending on α and on the shape of the hump: Stable equilibria, stable-looking periodic solutions, and also quite irregular behavior of solutions. The latter has been observed numerically, and is of interest in mathematical biology. In order to understand such "chaotic" behavior, U.an der Heiden and myself studied equation (1) with a hump which is close to a step function but nevertheless smooth. We obtained an unstable periodic orbit, and a particular solution which is homoclinic to it. Moreover the homoclinic merges into the periodic orbit in finite time: The Poincaré map corresponding to the closed orbit is not a diffeomorphism but admits a snap-back repeller in the sense of Marotto which implies chaos of the Li-Yorke-type.

H.Werner

<u>The calculation of singularities to solutions of algebraic differential equations</u>

Standard methods for the numerical calculation of solutions to ordinary differential equations are inadequate in the neighborhood of singularities because they are usually based upon polynomial or linear concepts. Using the known properties of algebraic differential equations in the complex domain it is possible to form trial approximations that are better suited and even allow for appropriate singularities. Patching up such functions furnishes non-linear splines. Using these splines gives excellent empirical estimates for the location of the sought singularities. Under the assumption that the numerical calculations are sufficiently precisely performed and the error brought forward is small we asymptotically estimate the difference between true and approximate singularity. Introducing a parameter we find monotonicity with respect to this parameter which allows either for asymptotic inclusions or for extrapolation techniques. This is also demonstrated by examples.

M.Willem

Forced oscillations of the pendulum equation

Let $g: \mathbb{R} \to \mathbb{R}$ be a continuous T-periodic function and let $f: \mathbb{R} \to \mathbb{R}$ be a 2π-periodic measurable function such that $\int_0^{2\pi} f^2(t)\,dt < \infty$. Under the assumption $\int_0^T g(u)\,du = \int_0^{2\pi} f(t)\,dt = 0$ it is proved that there exists a 2π-periodic weak solution for the equation $\ddot{u} + g(u) = f(t)$. When $g(u) = \sin u$, it is shown that, for every f such that $\int_0^{2\pi} f(t)\,dt = 0$, there exists an $\varepsilon > 0$ such that, if $h: \mathbb{R} \to \mathbb{R}$ is continuous and if $\max |h(u)| < \varepsilon$, then there exists a 2π-periodic solution of $\ddot{u} + \sin u = f(t) + h(u)$.

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