

MATHEMATISCHES FORSCHUNGSIINSTITUT OBERWOLFACH

Tagungsbericht 16/1981

Mathematische Logik

5.4. bis 11.4.1981

Unter der Leitung von E.Specker (Zürich) und W.Felscher (Tübingen) fand in der Woche vom 5.4. bis 11.4.1981 im Forschungsinstitut in Oberwolfach die diesjährige Tagung über Mathematische Logik statt. Es wurden 35 Vorträge über verschiedene Gebiete der Mathematischen Logik gehalten.

Mit Ende dieser Tagung hat Herr Professor Specker sein Amt als regelmäßiger Tagungsleiter abgegeben. Ihm sei im Namen aller Tagungsteilnehmer der letzten Jahre noch einmal herzlich gedankt.

Vortragsauszüge

Z.ADAMOWICZ: *Euclid's Theorem and Matiasiewicz' Theorem in Weak Arithmetic*

We construct a model M_0 for the theory PA^- , Peano arithmetic without induction in which

- 1 the set of primes is bounded,
- 2 every number > 1 has a prime divisor,
- 3 the Matiasiewicz theorem fails,
- 4 there is a certain fragment of Σ_2 and Π_2 induction for formulas without parameters.

The paper is a contribution to the solution of Wilkie's question wether in Δ_0 induction one can prove the existence of infinitely many primes.

S.BASARAB: *Algebraic Function Fields over Internal Fields*

Let us consider a mathematical structure containing the set of natural numbers \mathbb{N} , a given family of fields, polynomial rings, etc. and take

an enlargement in Robinson's sense. Given an internal field K we associate in a functorial way to each finitely generated field extension F of K an internal field extension \hat{F}/K such that $F \subset \hat{F}$. We discuss some transfer properties of \hat{F}/F . For instance, \hat{F}/F is regular, the dimension of F/K equals the internal dimension of \hat{F}/K , F/K is totally real iff \hat{F}/K is totally real, F/K is formally p -adic iff \hat{F}/K is formally p -adic, etc. As applications we show the existence of some bounds in the theory of fields.

M.BEESON: *Recursive Models for Constructive Type and Set Theories*

We construct recursive models for Martin-Löf's (type or) set theories. These models are a sort of recursive realizability; in fact, we show that for formulae of HA^ω , satisfaction in the model corresponds to m-realizability. We use the models to prove several metamathematical theorems about Martin-Löf's theories, including the consistency and independence of "all functions from R to R are uniformly continuous on compact sets". In general, uniform continuity on compact spaces is consistent. But "all functions from N^N to N^N " is refutable, as is Church's thesis, in Martin-Löf's theories, although every provably well-defined function is continuous. We make use of an interpretation due to Aczel to construct recursive (extensional realizability) models of the constructive set theories of Myhill and Friedman.

E.BÖRGER: *On the Problem of Herman/Jackowski*

The problem of Herman/Jackowski (see Discrete Math. 5(1973) 131 - 144) is whether the uniform halting problem for one-state Turing machines with n -dimensional tapes for $n > 3$ and jumping reading heads is decidable. Discussing the proof given by Herman/Jackowski in op.cit. for $n = 3$ it is argued that the reason for decidability is a purely combinatorial feature, more than a geometrical one. The connection of this problem to various other decision problems for kinds of vector addition systems is discussed, and the following two theorems are presented:

Thm.1. (M.KARPINSKI) Word, confluence and halting problem for vector games with dimension 3 are recursive.

Thm.2. (E.BÖRGER) Rational games and commutative Markov algorithms with at least 3 - letter alphabet can have combinatorial decision problems

of arbitrarily high r.e. m-degree complexity.

Corollary of thm.1. Commutative Markov algorithms with 2-letter alphabets have recursive combinatorial decision problems.

References: J.H.Conway: Unpredictable Iterations, in: Proc.Boulder Conf. on Number Theory, Univ.of Colorado, 1972, pp. 49 - 52

G.T.Herman, J.A.Jackowski: A decision procedure using discrete geometry. in: Discr.Math. 5(1973) 131 - 144

(joint work with M.Karpinski)

W.BUCHHOLZ: Zur Ordinalzahlanalyse imprädikativer Theorien
Die beweistheoretischen Ordinalzahlen der Theorien ID_v werden unter Bezugnahme auf abstrakte Bäume (im Sinne von Kleene's $\hat{0}$) charakterisiert. Dieses Vorgehen hat den Vorteil, daß keine komplizierten Berechnungen innerhalb spezieller Ordinalzahlbezeichnungssysteme erforderlich sind.

Die abstrakten Bäume der Klasse $\hat{0}_v$ (v eine rekursive Ordinalzahl) werden durch folgende Regeln erzeugt:

1. $0 \in \hat{0}_v$
2. $a \in \hat{0}_v \Rightarrow a^+ \in \hat{0}_v$ (Nachfolger)
3. $\forall i \in \mathbb{N} (a_i \in \hat{0}_v) \Rightarrow (a_i)_{i \in \mathbb{N}} \in \hat{0}_v$
4. $\mu < v \wedge \forall x \in \hat{0}_\mu (a_x \in \hat{0}_v) \Rightarrow (a_x)_{x \in \hat{0}_\mu} \in \hat{0}_v$

Für $a \in \hat{0}_v$ sei $|a|$ die Länge (oder Tiefe) von a . In $\hat{0}_v$ lässt sich in natürlicher Weise die Addition und die Exponentiation (zur Basis 2) definieren. Ferner sei $\hat{\Omega}_{\mu+1} := \{x\}_{x \in \hat{0}_\mu}$ (für $\mu < v$), $\hat{\omega} := (\hat{\Omega}_{\mu+1})_{\mu \in \mathbb{N}}$ und $\Sigma_v := \{a_i\}_{i \in \mathbb{N}}$ mit $a_0 := \hat{\omega}$ und $a_{i+1} := \hat{\Omega}_{\mu(i)+1}$, wobei $\{\mu(i) \mid i \in \mathbb{N}\}$ eine Aufzählung aller Ordinalzahlen $< v$ sei. Schließlich definiert man noch zu jedem $\sigma < v$ eine sogenannte Kollabierungsfunktion D_σ welche die Baumklasse $\hat{0}_v$ in die niedrigere Klasse $\hat{0}_\sigma$ abbildet. Die Definition von $D_\sigma(a)$ für $a \in \hat{0}_v$ erfolgt durch Rekursion nach a simultan für alle $\sigma < v$:

$$D_\sigma(0) := 0$$

$$D_\sigma(a^+) := D_\sigma(a)^+$$

$$D_\sigma((a_i)_{i \in \mathbb{N}}) := (D_\sigma(a_i))_{i \in \mathbb{N}}$$

$$D_\sigma((a_x)_{x \in \hat{0}_\mu}) := \begin{cases} (D_\sigma(a_x))_{x \in \hat{0}_\mu}, & \text{für } \mu < \sigma \\ D_\sigma(a_x) \text{ mit } x := D_\mu(a_0), & \text{für } \sigma \leq \mu \end{cases}$$

Nun lässt sich die Grenzzahl von ID_V folgendermaßen darstellen:

$$\sup_{m \in \mathbb{N}} |D_0|^{2^{\sum_{v=0}^m \omega_v}}$$

A.CANTINI: Remarks on Predicative Theories of Classifications

We study three extensions of elementary arithmetic, PI, PW, PS, of increasing expressive power. Their main non arithmetical principle is an untyped comprehension schema \underline{C} , essentially due to Gilmore, Feferman, Nepeivoda. The consistency of \underline{C} is assured by introducing a kind of strong ("predicative") negation. PW is characterized by the presence of two "prewellordering connectives", while PS contains axioms defining extensional equality and hereditary classifications.

Theorem 1: $\Sigma_1^1 - DC \leq PW$; $KP(\Sigma_1) \leq PS$.

Theorem 2: $|PS| = \phi_{\epsilon_0} 0$ and $PI \equiv PW \equiv PS$.

Note. $KP(\Sigma_1)$:= the set theory containing KP, ω -existence, ω -full induction, $\Delta_0 - DC$, $\Sigma_1 - \epsilon$ -induction.

E.CASARI: Bemerkungen über Zwischensysteme der Prädikatenlogik

Den aussagenlogischen Basen P (positive Logik), $B \equiv P + (\alpha \rightarrow \beta) \vee (\beta \rightarrow \alpha)$,

$T_n \equiv P + \alpha_1 \vee (\alpha_1 \rightarrow \alpha_2) \vee \dots \vee (\alpha_n \rightarrow \alpha_{n+1})$ werden Postulate für die Quantoren hinzugefügt: Q (übliches System), D $\equiv Q + \forall x(\alpha(x) \vee \beta) \rightarrow \forall x(\alpha(x) \vee \beta)$,

$H_1 \equiv Q + \exists x \forall y(\alpha(y) \rightarrow \alpha(x))$, $H_2 \equiv Q + \exists x \forall y(\alpha(x) \rightarrow \alpha(y))$, H $\equiv H_1 + H_2$. Die Abhängigkeitsverhältnisse zwischen den entstehenden Logiken werden festgelegt und es werden einige Eigenschaften solcher Logiken gezeigt.

z.B. in PH besitzt jede Formel eine Normalform;

$PH_1 \equiv PQ + \exists x \alpha \leftrightarrow \forall y(\forall x(\alpha(x) \rightarrow \alpha(y)) \rightarrow \alpha(y))$.

Die K(ripke) - Modelle solcher Systeme werden untersucht, z.B. BDH_1 (BDH_2) gilt in allen K - Modellen mit wohlgeordneter (dual wohlgeordneter) Indexmenge und konstanten Bereichen. Eine Konstruktion von D.Klemke (1972) modifizierend, wird gezeigt (P.Minari,1981): PD, BD, $T_n D$ ($n \geq 1$) sind vollständig charakterisiert durch die Klassen der K - Modelle mit konstanten Bereichen, bzw. konstanten Bereichen und linear geordneten Indexmengen, bzw. konstanten Bereichen und höchstens n -gliedrigen linearen Indexmengen.

G.CHERLIN: Zil'ber's Conjectures

Theorem: A totally categorical complete theory is not finitely axiomatizable.

A proof has been published by Zil'ber, with some gaps. I discuss one way to fill in these gaps, using a classification theorem for strongly minimal \aleph_0 -categorical sets derived ultimately from the classification of finite simple groups.

M. DICKMANN: *Rings of Continuous Functions*

The talk presented a brief account of Part I of the paper [1]. This deals with *real closed rings* (RCR's) which are commutative, totally ordered rings satisfying the "intermediate value theorem" for polynomials in one variable. (The algebra and model theory of RCR's is developed in Part II of [1].)

The most important examples of RCR's appearing in nature are rings of the form $C(X)/P$ for some prime ideals P in the ring $C(X)$ of all real-valued continuous functions on a (completely regular) topological space X .

There is a neat division between:

(a) those spaces X for which many (most, all) prime ideals P in $C(X)$ are such that $C(X)/P$ is an RCR. Examples of these are the F-spaces and various (larger) classes of spaces. These are not "nice" spaces from a topological viewpoint, although some of them (e.g. $\beta\mathbb{N}-\mathbb{N}$) are intensively studied in point-set topology.

(b) those spaces X - such as non-discrete metrizable spaces - for which one cannot prove in ZFC that there are non-maximal prime ideals P such that $C(X)/P$ is an RCR. However, for many spaces in this class (e.g. \mathbb{N}^* , $[0,1]$, etc.), using Martin's Axiom one can construct many non-maximal prime ideals with this property. For example:

Theorem: Let P be a non-maximal prime ideal of $C(\mathbb{N}^*)$, $P \subseteq M_\omega$.

Let U be the unique ultrafilter on \mathbb{N} such that $P_U \subseteq P$, where

$$P_U = \{ f \in C(\mathbb{N}^*) \mid Z(f) - \{\omega\} \in U \}.$$

Then $C(\mathbb{N}^*)/P \models \text{RCR}$ iff U is a P -point of $\beta\mathbb{N}-\mathbb{N}$.

Theorem: Martin's Axiom implies that there are ultrafilters U of closed sets of \mathbb{R}^+ such that:

- (a) every set of U has infinite Lebesgue measure.
- (b) U determines a non-maximal prime ideal $\gamma(U)$ of $C(\mathbb{R}^{+*})$ such that $C(\mathbb{R}^{+*})/\gamma(U) \models \text{RCR}$.
- (c) \mathbb{R}^{+*} = one point compactification of \mathbb{R}^+ ($\cong [0,1]$).

[1] G.CHERLIN - M.A.DICKMANN - Real Closed Rings

- I. Residue rings of rings of continuous functions
 - II. Model theory.
- (to appear)

K.-H.DIENER: Die Klassen H_κ und HC_κ der Mengen von erblicher Kardinalzahl κ (in ZF ohne Auswahlaxiom)

Sei Wf die Klasse der fundierten Mengen, sei $R(\kappa)$ die Menge aller fundierten Mengen x vom Rang $\rho(x) < \kappa$. Für jede Kardinalzahl κ definiert man die Klassen H_κ und HC_κ der Mengen von erblicher Kardinalzahl κ (im engeren bzw. im weiteren Sinn):

$$H_\kappa := \{ x \in Wf \mid |Tc(x)| < \kappa \}$$
$$HC_\kappa := \{ x \in Wf \mid \forall y(y \in Tc(\{x\}) \rightarrow |y| < \kappa) \} .$$

Dabei bezeichnet $Tc(x)$ die transitive Hülle der Menge x .

Durch Anwendung von Methoden und Resultaten der infinitären Universalen Algebra wird gezeigt, daß die folgenden Aussagen - bisher nur mit Hilfe des Auswahlaxioms hergeleitet - auch ohne Auswahlaxiom bewiesen werden können, also Theoreme von ZF sind:

- | | |
|-----------------------------------|--|
| a) $H_\kappa \subseteq R(\kappa)$ | b) $HC_\kappa \subseteq R(\kappa^+)$ |
| c) $\rho(H_\kappa) = \kappa$ | d) $\rho(HC_\kappa) = \begin{cases} \kappa, & \text{wenn } \kappa \text{ regulär} \\ \kappa^+, & \text{wenn } \kappa \text{ singulär} \end{cases}$ |

Hieraus folgt insbesondere, daß die Klassen H_κ und HC_κ in jedem ZF - Modell (also auch in CHURCHschen Modellen ohne überabzählbare Kardinalzahlen) Mengen vom Rang κ oder κ^+ sind. Für den Spezialfall $\kappa = \omega_1$ wurde die Aussage b) von JECH (1978) bewiesen.

Durch Angabe eines ZF - Modells wurde schließlich gezeigt, daß die in ZFC herleitbare Aussage " $HC_\kappa \subseteq H_{\kappa^+}$ (für jede Kardinalzahl κ)" kein Theorem von ZF ist.

J.DILLER: Funktionalinterpretationen der klassischen Analysis

Für eine konservative Erweiterung HA^ω der Heyting - Arithmetik endlicher Typen HA^ω um einen beschränkten Allquantor $\Lambda x \in a$ wird eine allgemeine Funktionalinterpretation J diskutiert. Jedes Modell für " $\Lambda x \in a$ " liefert dann eine "konkrete" Funktionalinterpretation, darunter die Dialectica - Interpretation D (Gödel), \wedge (Diller - Nahm), n - Interpretation (Stein) und modifizierte Realisation mr (Kreisel). Zur in-

tuitionistischen Analysis gilt der Interpretationssatz:

J interpretiert $\text{HA}_E^\omega + \{A \leftrightarrow A^J\} + \text{BR}_\sigma + \text{Rule} - \text{BI}_\sigma^D$
in $T_E + \text{BR}_\sigma + \text{folgende Rule} - \epsilon - \text{BI}_\sigma^D$:

$t(\bar{c}(Lc)) = 0$, $t(\bar{c}x) = 0 \rightarrow Q(\bar{c}x)$
 $\forall u \in g(\bar{c}x) Q(\bar{c}x * u) \rightarrow Q(\bar{c}x)$ $\vdash Q(< >)$

Für $J=0$ und $J=\wedge$ ist $\text{Rule} - \epsilon - \text{BI}_\sigma^D$ in $T_E + \text{BR}_\sigma$ herleitbar, und
J interpretiert BI_σ in $T_E + \text{BR}_\sigma$.

Für $J=mr$ und $J=n > 0$ läßt sich das Axiomschema $\epsilon - \text{BI}_\sigma^D$ in T_E formulieren, und J interpretiert BI_σ in $T_E + \text{BR}_\sigma + \epsilon - \text{BI}_\sigma^D$.

Betrachtet man als klassische Analysis $\text{HA}_E^\omega + \{A \vee \neg A\} + \{A \leftrightarrow A^{-J}\} + \text{DC}_\tau$,
so gilt: J o - interpretiert die klassische Analysis in $T_E + \text{BR}_\sigma$ für
 $J=0$ und $J=\wedge$. Für $J=mr$ und $J=n$ ist dies anscheinend nicht der Fall.
Frage: Gibt es überhaupt andere Funktionalinterpretationen der klassi-
schen Analysis?

W.FRIEDRICH: Funktionalinterpretation und Spielquantoren

Wir betrachten die folgenden Erweiterungen α^+ , α^{++} der Analysis α :
 $\alpha^+ \equiv \alpha + (C)-\wedge^0$, $\alpha^{++} \equiv \alpha + (C)-\wedge^1$, wobei $(C)-\wedge^0 [(C)-\wedge^1] \equiv$
arithmetische [analytische] Komprehension über Objekte beliebigen Typs.
Übersetzt man diese Theorien nach einem Vorschlag von Luckhardt jeweils
in eine Theorie über einen fundierten Spielquantor, so läßt sich die
Gödelsche Funktionalinterpretation hiervon in $T + \text{BR}$ bzw. einer Er-
weiterung von $T + \text{BR}$ um neue Limestypen mit entsprechenden Rekursionen
durchführen. Die Grundidee der Spielquantorinterpretation besteht darin,
den Primaussagen Aussagen mit kompliziertem Quantorpräfix zuzuordnen,
so daß im Kern rekursiv operiert werden kann, während das Präfix die
beweisbaren Unstetigkeiten schluckt. Während somit α^+ gleich stark
der Analysis α ist, ist α^{++} beweisbar wesentlich stärker. Bei dieser
Interpretation werden beweisbare Aussagen $\lambda x^1 \forall y^1 (\varphi xy = 0)$ konstruktiv
realisiert.

D.GIORGETTA: Universelle lokal-endliche Gruppen in der Mächtigkeit des Kontinuums

Durch Abwandlung einer Konstruktion von S.Shelah wird die Existenz
einer universellen lokal-endlichen Gruppe \mathcal{N} in der Mächtigkeit 2^{\aleph_0}
nachgewiesen mit der Eigenschaft, daß keine überabzählbare Untergruppe
von \mathcal{N} irgendeine nichttriviale Identität erfüllt.

J.Y.GIRARD: *The Theory of Dilators*

The concept of dilator, i.e. of functor of ON into itself commuting to direct limits and to pull-backs, is of general interest in proof theory; from such an object, one deduces a way of representing ordinals: any ordinal $\langle F(x) \rangle$ can be described by a sequence: $(z; x_0, \dots, x_{n-1}; x)$ (the object y thus denoted is $F(f)(z)$, where $rg(f)$ is equal to (x_0, \dots, x_{n-1}) ; f is chosen by the condition: $rg(f)$ minimum for inclusion); the range of F is the set of pairs $(z; n)$ such that $(z; 0, \dots, n-1; n)$ is a notation as above. To each $(z; n) \in rg(F)$, it is possible to associate a permutation σ of $0, \dots, n-1$: this permutation permits to compare any two objects $(z; x_0, \dots, x_{n-1}; x)$ and $(z; y_0, \dots, y_{n-1}; x)$, when the relative order of the points x_i and y_i is known: if i is minimum such that $x_{\sigma(i)} \neq y_{\sigma(i)}$, then these objects are compared by the order between $x_{\sigma(i)}$ and $y_{\sigma(i)}$. The comparison between $(z; x_0, \dots, x_{n-1}; x)$ and $(z'; y_0, \dots, y_{m-1}; x)$ depends on the value of an object $s(z, n; z', m) = (p, t)$. The above data permit to represent the objects of $F(x)$ in a tree-like structure (*dendroid*), and it is possible to prove a theorem: there is an equivalence of categories between DIL (dilators) and SHD (strongly homogeneous dendroids).

L.GORDEEV: *Axiom of Choice in Constructive Set Theory with Extensionality*

In Beeson's paper "Goodman's theorem and beyond" the following problem has been formulated: is ("basic" Friedman's extensional constructive set theory) \mathcal{B} extended by the axiom of choice at all finite types conservative over \mathcal{B} for arithmetic statements?

We present the (positive) solution of the problem. In fact, we prove that $\mathcal{B} + AC_{FT}^*$ is conservative over HA, where

$AC_{FT}^*: \forall x \in \omega^\sigma \exists y \in b \langle x, y \rangle \in c \rightarrow \exists f \text{ Fnc}(f) \& f: \omega^\sigma \rightarrow b \& \forall x \in \omega^\sigma \langle x, f(x) \rangle \in c \text{ for}$
all finite types σ , $\omega^\emptyset := \omega$, $\omega^{(\delta \rightarrow \tau)} := (\omega^\tau)^{\omega^\delta}$, and set-variables b, c .

The crucial point of the proof is some new model-theoretic treatment of (weak) Feferman's systems of functions and classes, which provides (particularly) the conservativity over HA of Feferman's theory EM_0 extended by the axiom of choice at all extensional finite types.

R.J.GRAYSON: *Derived Rules for Intuitionistic Higher-Order Logic*
Three examples of a model-theoretic approach to proving derived rules

are considered. The general scheme is as follows: In a theory T one constructs models M and proves a soundness theorem ($M \models T$); then, if $T \vdash \varphi$, one obtains $T \vdash (M \models \varphi)$ which yields the desired $T \vdash \varphi^*$, by suitable choice of M .

Examples: (a) The usual " φ "-forms of realisability do not correspond to "models" since they are not closed under deducibility; but a slight modification corrects this, and yields Church's Rule and the $\forall\exists\beta$ -continuity rule for intuitionistic higher-order logic (IHOL), or for intuitionistic set theory (IZF).

(b) A clever choice of space T (due to Joyal) yields a rule of local continuous choice between provably complete separable metric spaces, by considering the topological model constructed over the open sets of T . The rule fails when AC-NN is added.

(c) The rule of bar induction for IHOL (or IZF) is proved by using the model over the complete Heyting algebra of "formal opens" of Baire space. This proof applies also to systems with countable choice principles; it is due independently to S.Hayashi and M.Beeson.

W.GUZICKI: Non-equivalent Definable Quantifiers

We shall consider definable generalized quantifiers in models of set theory and second-order arithmetic. Given a model M , a definable quantifier in M will be a mapping which to any variable x and a formula φ assigns another formula of the language of the model M , denoted as $Qx\varphi$, in such a way that the following properties hold in M :

$$\forall x (\varphi(x) \rightarrow \psi(x)) \rightarrow (Qx\varphi \rightarrow Qx\psi)$$

$$Qx(\varphi \vee \psi) \rightarrow (Qx\varphi \vee Qx\psi)$$

$$Qx (x = x)$$

$$\neg \exists y Qx (x = y)$$

Definable quantifiers are used to build elementary extensions of countable models with special properties. The basic theorem due to Krivine and McAloon says that there exists an elementary extension N of M such that a formula θ has no new elements in N iff for any formula φ the following implication holds in M :

$$Qx \exists y [\theta(y) \& \varphi(x,y)] \rightarrow \exists y Qx \varphi(x,y).$$

Quantifiers which generate elementary extensions which preserve exactly the same formulas are called equivalent. The existence of nonequivalent quantifiers in models of ZFC is obvious; an almost complete description

of them is due to Mrs. Dubiel. In case of second - order arithmetic the existence of nonequivalent quantifiers strongly depends on the model. The main result is that there exist models with infinitely many nonequivalent definable quantifiers.

A.HAJNAL: *Cofinality of Partially Ordered Sets*

The cofinality of a p.o.set $P = \langle P, \leq \rangle$ is

$$\text{cof}(P) = \min\{\kappa: \exists X \subset P (|X| = \kappa \wedge X \text{ is cofinal in } P)\}.$$

M.Pouzet conjectured that if $\text{cof}(P)$ is singular, then P contains a subset X , $|X| = \text{cf}(\text{cof}(P))$ such that any two elements of X are incomparable. He proved the instance $\kappa = \omega$ of his conjecture. E.C.Milner, K.Prikry; and A.Hajnal, N.Sauer proved independently that the conjecture is true provided $\text{cof}(P) = \lambda$, $\text{cf}(\lambda) = \kappa$ and $\forall \lambda' < \lambda \forall \kappa' < \kappa \lambda'^{\kappa'} < \lambda$. The proof of the second group of authors was presented.

G.JÄGER: *Choice and Autonomously Iterated Comprehension in Analysis*

For every natural number n we fix a complete Π_n^1 -predicate $P_n(X, x)$. Given a well - ordering relation \prec in the natural numbers and an arbitrary set X we define the Π_n^1 - jump - hierarchy along \prec starting with X by the following transfinite recursion:

- (i) $Y_0 := X$
- (ii) $Y_{a+1} := \{x: P_n(Y_a, x)\}$
- (iii) for limit a : $Y_a := \text{disjoint union of } Y_b, b \prec a$.

We write $H_n^\prec(X, Y)$ to express that Y satisfies (i) - (iii).

The idea of autonomously iterated Π_n^1 - jumps is made precise in the theory $\text{Aut}(\Pi_n^1)$. Besides elementary number theory Z_2 , $\text{Aut}(\Pi_n^1)$ contains the Bar Rule (BR) and the following rule

$\text{Wf}(\prec)$

$$\overline{\forall X \exists Y H_n^\prec(X, Y)}$$

where \prec is a prim.rec. well - ordering. Then we obtain the following theorem: $(\Sigma_{n+1}^1 - \text{AC}) + (\text{BR})$ is a conservative extension of $\text{Aut}(\Pi_n^1)$ for Π_1^1 -sentences, where $i = 2, 3, 4$ for $n = 0, 1, 2+m$.

This theorem holds as well if we replace $(\Sigma_{n+1}^1 - \text{AC})$ by $(\Sigma_{n+1}^1 - \text{DC})$.



M.KARPIŃSKI: *On the problem of Weak Definability in Second Order Theories*

Given the (Monadic) *Second-Order Theory of Countable Order* - or corresponding *Second-Order Theory of the Structure* $N_2 = \langle 2^*, r_0, r_1 \rangle$, $r_i(x) = x_i$, of two successor functions. We study the problem of *Weak Definability* of these theories, the problem posed in 1968 by H.Gaifman: Given any formula $H(A_0, \dots, A_{n-1})$ of the theory, does there exist a uniform procedure for checking the existence of an equivalent weak formula $G(A_0, \dots, A_{n-1})$? "Weak" in the sense of "finite set" quantifiable.

The aim of this lecture is to answer to the Gaifman Problem in positive - by the following

- (1) The *Weak Definability* of the above theories is *recursive* (in fact in Rec_{prim}).

This yields us directly (by the classical result of M.O.Rabin that the equivalence problem of two S2S-formulas is in Rec_{prim}) the following.

- (2) There *exists* a "recursive typewriter" for printing out for arbitrary formula F of S2S at least one equivalent weak formula H (if such exists at all!).

R.KOSSAK: *Short Recursively Saturated Models of PA*

The main result is that the short recursively saturated models of PA are exactly the short elementary cuts of recursively saturated models. In proof of it we use a characterisation of short recursively saturated models which is similar to that of countable recursively saturated models in terms of non-standard satisfaction classes. The approach allows to give another proof of the result of Smoryński and Starý which says that recursive (and short recursive) saturation is preserved under cofinal extensions.

H.KOTLARSKI: *Elementary Cuts in Models of Arithmetic*

Let M be a countable recursively saturated model of Peano Arithmetic. A set $X \subseteq M$ is *closed* iff for every $b \in M \setminus X$ there exists an automorphism g of M such that $g(b) \neq b$ and $\forall x \in X g(x) = x$. This notion is taken from Galois theory.

Clearly if $X \subseteq M$ is closed then $X \prec M$. We know only rather trivial information about arbitrary $X \prec M$ which are closed, but we have a

good deal of information for elementary *cuts* of M . Namely

THEOREM 1. If $N \prec M$ is not closed then there exists $b \in M$ such that $N = M[b]$. Here $M[b] = \{x \in M : \text{for every parameter-free term } t(v) \in M \models t(x) < b\}$ is the greatest elementary cut of M which does not contain b .

It follows that only countably many $N \prec M$ are not closed, though there are continuum many elementary cuts of M .

What about closedness of cuts of the form $M[b]$? This is settled by

THEOREM 2. (i) there exist $b \in M$ such that $M[b]$ is not closed.

(ii) there exist $b \in M$ such that $M[b]$ is closed.

M.MASSERON: *Dendroids: Comparison of Hierarchies According to Girard*

Quasi-dendroids are a relaxed version of dendroids (a normalization functor associates a dendroid to each quasi-dendroid). Induction on quasi-dendroids, based on the two operations $+$ and $*$, permits to define two hierarchies, indexed by quasi-dendroids: γ (the pointwise hierarchy), λ (the iterative hierarchy) and their two-places counterparts δ and θ :

$$\gamma_0(n) = 0, \gamma_1(n) = 1, \gamma_{\sum_{i \in k} D_i}(n) = \gamma_{D_0}(n) + \dots + \gamma_{D_{k-1}}(n), \gamma_{\sum_{i \in \omega} D_i}*(n) = \gamma_{\sum_{i \in n} D_i}(n).$$

$$\theta_0(m, n) = n, \theta_1(m, n) = m+n, \theta_{\sum_{i \in k} D_i}(m, n) = \theta_{D_0}(m, \theta_{D_1}(m, \dots, \theta_{D_{k-1}}(m, n)))$$

$$\theta_{\sum_{i \in \omega} D_i}*(m, n) = \theta_{\sum_{i \in n} D_i}(m, n). \quad \lambda_D = \text{Un}(\theta_D) \text{ where } \text{Un}(f)(n) = f(n, 0) +$$

$$\sum_{p < n} (f(n-p-1, p+1) - f(n-p-1, p)), \text{ and } \delta_D = \text{Sep}(\gamma_D) \text{ where Sep is the inverse of Un.}$$

Transporting the functor Δ to strongly homogeneous dendroids, we have the following comparison properties: $\theta_D = \delta_{AD}$ and $\lambda_D = \gamma_{AD}$. With a natural choice of D such that $h(D) = \epsilon_0$, one gets $h(AD) = H$ (the Howard ordinal): this gives the most interesting particular case, which, incorrectly stated is: $\theta_{\epsilon_0} = \delta_H$ and $\lambda_{\epsilon_0} = \gamma_H$.

G.MITSCHKE: *Lambda-Kalkül mit unendlich langen Termen*

Zu gewissen beweistheoretischen Zusammenhängen wird ein λ -Kalkül benutzt, der die Möglichkeit der Bildung "unendlich langer" Terme

$\langle M_i \mid i < \omega \rangle$ zuläßt. Im klassischen, endlichen λ -Kalkül erhält man eine

wirkliche Einsicht in die Reduzierbarkeits-Relation \geq erst durch die Betrachtung von sog. Reduktionen. Im vorliegenden Vortrag wird eine Möglichkeit gezeigt, den Begriff der Reduktion auf unendliche Terme zu erweitern. Es zeigt sich, daß viele Methoden und Ergebnisse aus dem endlichen Fall mit leichten Modifikationen auch für den unendlichen Fall gelten.

J. MLČEK: *Representations of some Models in Alternative Set Theory*

Our aim is to represent isomorphically models of ZF or ZF_{fin} as substructures of $\langle V, \in \rangle$ and models of arithmetic, stronger than Presbouer's arithmetic, as some substructures of $N = \langle N, +, \cdot, 0, 1, \in \rangle$. We work in the Alternative Set Theory where both $\langle V, \in \rangle$ and N are saturated.

Representations of models of arithmetics. Let L be the language $\langle 0, 1, +, \cdot, \in \rangle$ and let A be an arithmetic stronger than Presbouer's arithmetic (e.g. PA). Let $M \models A$. We denote by M_+ the restriction of M to the language $\langle 0, 1, +, \in \rangle$.

THEOREM. Let $M \models A$.

- 1) There exists a substructure M^* of N_+ and a set function f (i.e. a function coded in N) such that M and $\langle M^*, f \cap (M^*)^3 \rangle$ are isomorphic.
- 2) There exist an elementary extension \hat{M} of M and a function $F : N^2 \rightarrow N$ such that \hat{M} and $\langle N_+, F \rangle$ are isomorphic.
- 3) M is isomorphic to a substructure of N iff for every sentence $(\exists \vec{x})\phi(\vec{x})$, where $\phi(\vec{x})$ is an open formula of L , we have

$$M \models (\exists \vec{x})\phi(\vec{x}) \Rightarrow N \models (\exists \vec{x})\phi(\vec{x}).$$

Remark. There exists a model of PA which is not elementary equivalent to any cut on N .

Representations of models of ZF, ZF_{fin} .

THEOREM. Every model of ZF or ZF_{fin} is isomorphic to a substructure of $\langle V, \in \rangle$.

Let $\langle M, \in \rangle \not\models ZF$ or ZF_{fin} . Then the following are equivalent:

- 1) $\langle M, \in \rangle \prec_{\Delta_0} \langle V, \in \rangle$, where Δ_0 is the class of all bounded formulas,
- 2) for every sentence $(\exists \vec{x})\phi(\vec{x})$, where $\phi(\vec{x})$ is a bounded formula, we have $M \models (\exists \vec{x})\phi(\vec{x}) \Rightarrow (\exists \vec{x})\phi(\vec{x})$,
- 3) all gödelian operations are absolute.

Note that the transitive closure of M is its elementary extension.

We say that a class A is b -absolute (ag-absolute resp.) iff

- (i) $0 \in A$, A is an extensional class and $P_{fin}(A) \subseteq A$ (the same resp.),
- (ii) all boolean operations are absolute (we mean the operations \wedge, \vee, \neg)
(all gödel.operations without dom are absolute resp.)
- (iii) the predicate " \in is trichotomic on x " is absolute (the same and, moreover, the predicate " x is transitive" is absolute resp.).

Theorem. Let $M \models ZF$ or ZF_{fin} .

1) There exists a b -absolute class M^* such that M and $\langle M^*, \in \rangle$ are isomorphic.

2) There exists an ag-absolute class M^* such that $\langle K^M, \in_M \upharpoonright K^M \rangle$ and $\langle M^*, \in \rangle$ are isomorphic, where

$$K^M = \bigcup \{ K_n^M : n \in \text{FN} \}, \quad K_0^M = \text{Ord}^M \quad \text{and} \quad K_{n+1}^M = \{ x \in M : \in_M^*(x) \subseteq K_n^M \}.$$

P.PÄPPINGHAUS: *Nichtdeterministische partielle Logik*

(Ergebnisse gemeinsam mit M.WIRSING)

In der Informatik werden Programmiersprachen betrachtet, die partielle Funktionen und nichtdeterministische Ausdrücke enthalten wie etwa $\epsilon_x A(x)$ ("ein x mit $A(x)$ ") oder $t_1 \sqcup t_2$ ("wahlweise t_1 oder t_2 "). Die Bedeutung eines solchen Ausdrucks ist gegeben durch die Menge seiner möglichen Werte, wobei U ("undefined") als uneigentlicher Wert dem jeweiligen Objektbereich hinzugefügt wird. Entsprechend wird den Formeln eine nichtleere Teilmenge von $\{T, F, U\}$ als Menge möglicher Wahrheitswerte zugeordnet. Wir untersuchen eine dieser Semantik entsprechende Aussagenlogik. Dabei legen wir ein informelles Konzept von Wahrheitsfunktionen zugrunde, die durch eine nicht-deterministische Prozedur berechnet werden können. Die Klasse dieser Wahrheitsfunktionen ist syntaktisch und semantisch charakterisierbar (i) als die Klasse der \subseteq -isotonen, \subseteq^- -isotonen, erblich bewachten, erblich wachsenden Funktionen; sowie (ii) als die Klasse der aus \sqcup , if...then...else... und Konstanten (in einer gewissen Normalform) explizit definierbaren Funktionen.

W.POHLERS: *Spectral Theory for Formal Systems*

We introduce and discuss two concepts

I The spectrum of a formal system

II The spectrum of an ordinal.

For the sake of simplicity we restrict ourselves to formal systems for iterated inductive definitions. The level $S\xi$ of an ordinal ξ is the greatest admissible which is $\leq \xi$, the level SA of a formula A is its status in the hyperjump hierarchy. Then

$$sp(T) := \{ |n|_A : T \vdash n \in I_A \wedge S(|n|_A) = S(A) \}.$$

For the proof theoretic ordinal $\|T\|$ of T it then follows $\|T\| = sp(T) \cap \Omega_1$.

Moreover one can prove for a formula A of level Ω_μ it holds

$$\tilde{JD}_v \vdash A \Leftrightarrow \tilde{JD}_\mu + TJ(sp(\tilde{JD}_\mu) \cap \Omega_{\mu+1}) \vdash A \text{ generalizing the older result } \quad JD_v \vdash A \Leftrightarrow PA + TJ(<\|JD_v\|) \vdash A \text{ for arithmetic } A.$$

To compute $sp(T)$ we have to introduce certain collapsing functions

$D_\mu : \text{On} \rightarrow \Omega_{\mu+1}$ and connected relations $\alpha << \beta \Leftrightarrow \alpha < \beta \wedge D_\rho \alpha < D_\rho \beta$ for all ρ . The definition of the functions D_μ uses heavily ordinal notations. Then one may define $sp(\alpha) = \{\beta : \beta << \alpha\}$. It is possible to compute $sp(\alpha)$ and we obtain $sp(\alpha) = C(\alpha, \alpha) \cap \alpha$.

We obtain the following

Theorem: For all suitable T there is an ordinal $\tau(T)$ such that

$$sp(T) = sp(\tau(T)).$$

This shows (1) that the spectrum of $\tau(T)$ does not depend upon the notation system, which is quite surprising, and that the provable sentences of T which belong to the hyperjump hierarchy are quite well characterized by a single ordinal.

A.PRESTEL: On the Model Companions of Some Classes of Fields

Let K be a field of characteristic zero. A *preordering* S is a subset of K such that $S + S \subseteq S$, $S \cdot S \subseteq S$, $K^2 \subseteq S$. It is called *proper* if $-1 \notin S$. In that case K is orderable. An *ordering* P is a proper preordering such that $P \cup -P = K$. The set $X_K(S) = \{P \mid P \text{ ordering s.t. } S \subseteq P\}$ can be given a Hausdorff topology generated by the sets $H(a) = \{P \mid a \in P\}$ where $a \in K \setminus \{0\}$. Let POF_n , for $0 \leq n \leq \omega$, be the theory of preordered fields such that $|K^\times / S^\times| = 2^n$ and $|X_K(S)| = n$ (this can be axiomatized in the 1.order language of preordered fields). Now Tarski's and van den Dries' results imply that

$$\overline{POF}_n := POF_n + "K \text{ is maximal PRC}" + "S = K^2"$$

is the model companion of POF_n . The author's new result is that

$$\overline{POF}_\omega := POF_\omega + "K \text{ is maximal PRC}" + "S = K" + "X_K \text{ has no isolated point}"$$

is the model companion of preordered fields.

Here PRC means that K is existentially closed in each extension field L of K in which K is algebraically closed and to which all orderings of K extend. "maximal PRC" means that, in addition, K has no algebraic extension to which all orderings of K extend. All these properties can be expressed in the 1.order language of fields (see the author's article "Pseudo real closed fields" in a forthcoming Springer Lecture Notes: "Set Theory and Model Theory", Nr.8.,).

M.v.RIMSCHA: *Kumulative Hierarchien in der nichtfundierten Mengenlehre*

Den Ausgangspunkt bildet das System $ZF^0 + \text{Sext}$ ($ZF^0 = ZF$ ohne Fundierungsaxiom; Sext ist das Axiom der starken Extensionalität, welches besagt, daß Mengen mit \in -isomorphen transitiven Hüllen gleich sind).

In $ZF^0 + \text{Sext}$ - Modellen läßt sich folgende Hierarchie transitiver Mengen definieren: $US(\emptyset) := \emptyset$

$$US(\alpha+1) := P(US(\alpha)) \cup \{v \mid \exists x \exists u \in US(\alpha) (\in \cap x^2 \text{ isou} \wedge \text{trans } x \wedge v \in x \})$$

$$US(\lambda) := \bigcup_{\alpha < \lambda} US(\alpha)$$

Setze $US := \bigcup_{\alpha \in \text{On}} US(\alpha)$

Charakterisierung von US : Der US -Anteil eines $ZF^0 + \text{Sext}$ - Modells ist ein maximales, inneres, transitives Modell von $ZF^0 + \text{Sext} + R$, wobei die Bedingung R besagt, daß jede Menge bijektiv abbildbar auf eine fundierte Menge sein soll.

In $ZF^0 + \text{Sext}$ - Modellen gilt: $R \Leftrightarrow US$ ist die Klasse aller Mengen ("V=US") $V=US$ ist unabhängig von $ZF^0 + \text{Sext}$. Es scheint daher vernünftig, das System $ZF^0 + \text{Sext}$ um die Bedingung $V=US$ zu erweitern, da dadurch keine Beschränkungen hinsichtlich der Freiheit der Mengenbildung eintreten, was die folgenden Sätze zeigen:

- $ZF^0 + \text{Sext} + V=US$ ist konsistent mit U_2 (U_2 ist das stärkste, mit $ZF^0 + \text{Sext}$ verträgliche Universalitätsaxiom).
- Jedes $ZF^0 + \text{Sext}$ - Modell, das auch im Metasinne das Axiom der starken Extensionalität erfüllt, kann durch Enderweiterung zu einem $ZF^0 + \text{Sext} + V=US$ - Modell gemacht werden.

Insgesamt erscheint das System $ZF^0 + \text{Sext} + V=US$ eine vernünftige Alternative zu ZF zu sein, dies auch deshalb, weil in diesem System die gleichen (soweit bekannten) AC - Äquivalenzen gelten wie in ZF .

U.SCHMERL: *Peano Arithmetic and the Bachmann/Howard Ordinal*
We prove a partial characterization of Peano arithmetic by a uniform
or pointwise version of transfinite induction up to the Bachmann/Howard
ordinal φ_{ϵ_0} . This answers a conjecture by J.-Y.Girard.

P.ŠTĚPÁNEK: *Automorphisms of Boolean Algebras and Embeddings*
Several important classes of Boolean algebras are defined by automor-
phism properties e.g. the classes of rigid Boolean algebras or homo-
geneous algebras. There are general embedding theorems for these classes
of algebras.

Theorem 1. (Kripke) Every Boolean algebra can be completely embedded
in a complete homogeneous Boolean algebra.

Theorem 2. (Balcar, Štěpánek) Every Boolean algebra can be completely
embedded in a rigid complete Boolean algebra.

If B is completely embedded in C one may ask whether every automor-
phism of B extends to an automorphism of C . Let $\text{Aut}(C)$ denote the
group of automorphisms of C .

The center of a Boolean algebra C is defined as follows

$$\text{center}(C) = \{u \in C : \varphi(u) = u \text{ for every } \varphi \in \text{Aut}(C)\}.$$

Trivial cases: if $B \subseteq \text{center}(C)$ then no nontrivial automorphism of
 B extends to an automorphism of C . As we have $C = \text{Center}(C)$ for
every rigid C , no nontrivial automorphism of any complete subalgebra
of C can be extended to an automorphism of C .

On the other hand, it can be shown that for the Kripke's embeddings
every automorphism of the smaller algebra extends to an automorphism
of the larger algebra.

Definition: If u is a nonzero element of a BA B , let $B \upharpoonright u$ denote
the principal ideal of all $v \in B, v \leq u$. Then $B \upharpoonright u$ with the re-
stricted operations is a Boolean algebra; we shall call it a factor
of B .

We say that B has no rigid or homogeneous factors if no factor of B
is rigid or homogeneous. We have the following results:

Theorem 3. Every Boolean algebra B can be completely embedded in a
complete Boolean algebra C with no rigid or homogeneous factors such
that every $\varphi \in \text{Aut}(B)$ extends to an automorphism of C .

Theorem 4. Every Boolean algebra B can be completely embedded in a
complete Boolean algebra C with no rigid or homogeneous factors such

that $B \subseteq \text{center}(C)$.

Problem. Is there a complete homogeneous Boolean algebra C with a complete subalgebra B such that no nontrivial automorphism of B extends to an automorphism of C ?

J.VAUZEILLES: *A Functorial Construction of the Veblen Hierarchy*

The author considers a functor V from $\text{ON} \times \text{ON}$ to ON commuting to direct limits and to pull-backs; in particular, the restriction of V to the subcategory of finite ordinals determines V completely (and effectively). V is connected to the usual Veblen hierarchy K by $K^\alpha(\kappa) = V(\omega^\alpha, \kappa)$. So, we have established that the Veblen hierarchy is a functor of the two variables.

A.VISSEER: *On the Principle $A \rightarrow \square A$ and the Provability Logic of HA and Extensions*

Let U be an RE extension of HA . T is a *selfcompletion* of U if $T = U + \{A \rightarrow \square_T A \mid A \in L_{\text{HA}}\}$. We study selfcompletions (i) for the intrinsic interest of the principle $A \rightarrow \square A$, (ii) as a tool to derive certain provability principles for HA , (iii) as a tool to get certain independence results. For a certain class B of RE extensions of HA (e.g. HA , $\text{HA} + \neg M_{\text{PR}}$, $\text{HA} + \text{DNS}$, $\text{HA} + \{\square_{\text{HA}} A \rightarrow A \mid A \in L_{\text{HA}}\} \in B$) we give a characterization of the selfcompletions of its elements, including a conservativity result. Some applications:

$$\vdash_{\text{HA}} \square_{\text{HA}} (\neg \square_{\text{HA}} A \rightarrow \square_{\text{HA}} A) \rightarrow \square_{\text{HA}} \square_{\text{HA}} A$$

and for any $U \in B$ satisfying the Π_2^0 Reflexion principle

($A \in \Pi_2^0$ and $\vdash_U A \Rightarrow A$) : $\vdash_U \text{KLS}$.

(The last mentioned result is an adaptation of Beeson's result that
 $\vdash_{\text{HA}} \text{KLS}$.)

M.WIRSING: *Abstrakte Datentypen: Anwendungen universeller Algebra in der Theorie der Programmierung*

Unter dem Schlagwort "Abstrakte Datentypen" wurden in den letzten Jahren algebraische Methoden zur Beschreibung von (Systemen von) Datenstrukturen und Programmiersprachen eingeführt. Verschiedene dieser Methoden werden vorgestellt und deren Mächtigkeit wird untersucht: Initiale und terminale Spezifikationen werden durch die Klassen der

"semicomputablen" und "cosemicomputablen" Algebren charakterisiert, während hierarchische Spezifikationen exakt die Klasse der hyperarithmetischen Algebren beschreiben. Diese Resultate lassen sich ohne große Schwierigkeiten auch auf Spezifikationen von partiellen Algebren verallgemeinern.

A.WRÓNSKI: *Strengthenings of the Intuitionistic Consequence Operation*

The lattice of all structural strengthenings of the intuitionistic consequence operation in the implicationless language is isomorphic to a chain of type $1 + \omega^*$. Every consequence operation of this lattice is finitistic, finitely axiomatizable and structurally complete. The smallest element of the lattice i.e. the implicationless reduct of the intuitionistic consequence operation can be determined by a family of finite matrices and all remainig elements have single finite matrices adequate.

Berichterstatter: J.Schulte Mönting , Tübingen

Tagungsteilnehmer

- 20 -

Prof.Dr. Z.Adamowicz
ul.Sniadeckich 8
IM PAN

PL - 00-950 Warszawa

Prof.Dr. S.Basarab
Institute of Mathematics
Str. Academici 14

Bucuresti Rumänien

Prof.Dr. M.J.Beeson
Mathematisch Instituut
Budapestlaan 6

TA 3508 Utrecht , Nederland

Prof.Dr. E.Börger
Lehrst.Informatik II , Univ.
Postf. 500 500

4600 Dortmund

Prof.Dr. W.Buchholz
Math.Inst.d.Univ.
Theresienstr.39

8000 München 2

Prof.Dr. A.Cantini
Via Adriano Mari 5

I - 50136 Firenze

Prof.Dr. H.G.Carstens
Fakultät für Mathematik
Universität Bielefeld

4800 Bielefeld 1

Prof.Dr. E.Casari
Istituto di Filosofia
via Bolognese 52

Firenze Italien

Prof.Dr. G.Cherlin
Inst.Advanced Study
Mt.Scopus
Hebrew University
Jerusalem Israel

Prof.Dr. M.A.Dickmann
UER de Mathématique
Université Paris VII
Tour 45-55 , 2,Place Jussieu
F - 75005 Paris

Prof.Dr. K.H.Diener
Math.Inst.d.Univ.Köln
Weyertal 86-90

5000 Köln 41

Prof.Dr. J.Diller
Inst.f.mathem.Logik
Einsteinstr.64

4400 Münster

J.Dvorák
Im Acker 37

5300 Bonn

Prof.Dr. U.Felgner
Math.Inst.d.Univ.Tübingen
Auf der Morgenstelle 10

7400 Tübingen

Prof.Dr. W.Felscher
Math.Inst.d.Univ.Tübingen
Auf der Morgenstelle 10

7400 Tübingen

W.Friedrich
Heinrich-Hofmann-Str.17 1/2

6467 Hasselroth

Dr.D.Giorgetta
FU Berlin,II.Math.Inst.
Königin-Luise-Str.24/26

1000 Berlin 33

Prof.Dr. J.Y.Girard
Dept.de Mathematiques
Université Paris VII
12 Place Jussieu
F - 75005 Paris

Dr. L.Gordeev
Mathematisches Institut
Theresienstr.39

8000 München 2

Prof.Dr. R.J.Grayson
Institut f. Mathem.Logik
Einsteinstr. 64

4400 Münster

Dr. W.Guzicki
Instytut Matematyki UW
PKiN IX_p

PL - 00-901 Warszawa

Prof.Dr. A.Hajnal
Math.Inst.Hung.Acad.Sci.
Realtanoda U 13-15

H - 1053 Budapest

Dr. G.Jäger
Mathem.Institut
Theresienstr.39

8000 München 2

Prof.Dr. M.Karpinski
Math.Inst.Polish Acad.Sci.

PL - 61-725 Poznań

P.Koepke
Abt.f.Math.Logik
Albertstr. 23b

7800 Freiburg

Prof.Dr. S.Koppelberg
FU Berlin, II.Math.Inst.
Königin-Luise-Str. 24-26

1000 Berlin 33

Dr. R.Kossak
Inst.Zast.Mat.i Statyst.SGGW-
AR
Nowoursynowska 166

PL - 02-766 Warszawa

Dr.H.Kotlarski
Inst.Zast.Mat.i Stat. SGGW-AR
ul. Nowoursynowska 166

PL - 02-766 Warszawa



Prof.Dr. P.Krauss GhK - Fb 17 Heinrich - Plett - Str. 40	Dr. P.Päppinghaus Inst.f.Math., Univ.Hannover Welfengarten 1
3500 Kassel	3000 Hannover 1
Prof.Dr. H.Läuchli Mathematik ETH	Prof.Dr. W.Pohlers Math.Inst.d.Univ.München Theresienstr.39
CH - 8092 Zürich	8000 München 2
Prof.Dr. H.Luckhardt Math.Seminar Robert-Meyer-Str. 6-16	Prof.Dr. K.Potthoff Philos.Seminar d.Univ. Ohlshausenstr. 40 - 60
6000 Frankfurt	2300 Kiel
Prof.Dr. M.Masseron Université Paris - Nord	Prof.Dr. A.Prestel Fakultät für Mathematik Universität Konstanz
F - 75 Paris	7750 Konstanz
Dr. G.Mitschke Paosostr. 105	Prof.Dr. Z.Ratjczyk Instytut Matematyki UW PKiN IX _p
8000 München 60	PL - 00-901 Warszawa
Prof.Dr. J.Mlcek Math.Inst. Charles Univ. Sokolovská 83	Prof.Dr. J.Reineke Inst.f.Math.,Univ.Hannover Welfengarten 1
Praha 8 CCSR	3000 Hannover
Prof.Dr. G.H.Müller Math.Inst.d. Univ.Heidelberg Im Neuenheimer Feld 288	Dr. M.v.Rimscha Inst.f.Informatik Univ.Stgt. Azenbergstr.12
6900 Heidelberg	7000 Stuttgart 1

Dr. U.Schmerl
Math.Inst.d. Univ.
Theresienstr. 39

8000 München 2

Dr. J.Schulte Mönting
Math.Inst.d.Univ.Tübingen
Auf der Morgenstelle 10

7400 Tübingen

Prof.Dr. W.Schwabhäuser
Inst.f.Informatik Univ.Stgt.
Azenbergstr. 12

7000 Stuttgart

Prof.Dr. H.Schwichtenberg
Math.Inst.d. Univ.
Theresienstr. 39

8000 München 2

Prof.Dr. E.Specker
Mathem.Seminar ETH

CH - 8092 Zürich

Dr. P.Stepanek
Dept.of Cybernetics and OR
Charles University
MalostranskéNaýiesti 25
CSSR - 11800 Praha 1

Dr. M.Stein
WWU Münster FB 24
Fliednerstr.

4400 Münster

Prof.Dr. E.J.Thiele
Breisgauer Str. 30

1000 Berlin 38

Prof.Dr. W.Thomas
Math.Inst.d.Univ.
Albertstr.23 b

7800 Freiburg

Prof.Dr. J.Vauzeilles
44 rue de la Pépinière

F - 92 160 Antony

Prof.Dr. A.Visser
Mathematisch Instituut
Budapestlaan 6

3508 TA Utrecht , Niederlande

Prof.Dr. H.Volger
Math.Inst.d.Univ.Tübingen
Auf der Morgenstelle 10

7400 Tübingen

Dr. M.Wirsing
Dept.of Comp.Sci.,Univ.Edinb.
Mayfield Road

Edinburgh EH9 3JZ England

Prof.Dr. A.Wróński
Dept.of Logic, Jagellon.Univ.
str.Grodzka 52

Kraków

Polska



