MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

TAGUNGSBERICHT 18/81

Dynamische Systeme

21.4. bis 25.4.1981

Die Tagung fand unter der Leitung von Herrn J. Moser (ETH-Zürich) und von Herrn E. Zehnder (RUB-Bochum) statt. Die Tagung hat sich auf vier Schwerpunkte konzentriert:

1. Integrable Systeme:
   Beispiele klassischer integrabler Systeme, Zusammenhang mit algebraischer Geometrie; Partielle Differentialgleichungen wie KdV-Gleichungen und Zusammenhang mit einfachen Lie Algebren. Kriterien für die Integrabilität von Hamilton'schen Systemen, Spektraltheorie quasiperiodischer Potentiale.

2. Kleine Nenner Probleme:
   Störungstheorie integrabler Hamilton'scher Systeme; differenzierbare Foliierung in invariante Tori, optimale Glattheitsaussagen, Kreisabbildungen. Anwendungen der Existenzsätze quasiperiodischer Lösungen auf Bifurkationsprobleme gewöhnlicher und partieller Differentialgleichungen.

3. Unstabile Systeme:
   Ergodentheorie differenzierbarer dynamischer Systeme, Pesin's Theorie für Abbildungen mit Singularitäten. Stochastische Teilsysteme differenzierbarer Systeme, homoklinische Phänomene.

4. Spezielle Probleme der Himmelsmechanik:
   Kollisionssingularitäten in Newton'schen Mehrkörpersystemen, periodische Lösungen.

Weitere Resultate bezogen sich auf den Fuller Index, Gleichungen gekoppelter chemischer Reaktoren, Linearisierungssätze, Klassifikation des genus komplexer Differentialgleichungen. Es war nicht das Ziel dieser Tagung, sich auf ein spezielles Gebiet zu beschränken, sondern Mathematiker mit verschiedenen Interessensgebieten zusammenzubringen. Es ist unser Eindruck, daß dies auch gelungen ist. Die Tagung dauerte nur 4 statt wie üblich 5 Tage. Dadurch waren die Tagungsprogramme mit Vorträgen stärker belastet als wünschenswert wäre. Aus demselben Grund war die Tagungsleitung auch bemüht, die Teilnehmerzahl niedrig zu halten. Die eingeladenen Spezialisten aus Russland haben an der Tagung leider nicht teilgenommen.

Vortragsauszüge

H.W. BROER:

Some divergence free bifurcations involving quasi periodic flow

We consider local bifurcations in volume preserving (divergence free) vector fields. The local phase portraits in and near the bifurcations are to be classified modulo topological equivalences: homeomorphisms mapping integral curves to integral curves. These homeomorphisms are not required to be volume preserving. Arnol'd and others have developed a theory for 1-parameter families of not necessarily volume preserving vector fields and they have shown that, up to considerations involving centre manifolds, generically only two bifurcations appear which are topologically stable. These are the saddle node and the Hopf bifurcation. In our divergence free case the same result holds if the dimension is at least 5. For lower dimensions new facts show up. Especially the dimensions 3 and 4 are of interest, since here we find bifurcations with invariant tori and quasi periodic motions. This unstable situation therefore occurs openly.

M. CHAPERON:

On the local structure of smooth abelian Lie group actions

Generalizations of the Sternberg and Hartman Linearisation theorems of "bigger" abelian group actions were stated, as well as a theorem showing that these generalizations cannot be improved. The proofs were only sketched.

A. CHENCINER:

Non normally hyperbolic invariant curves in the neighborhood of degenerate Hopf bifurcations of diffeomorphisms of $\mathbb{R}^2$

Let $H_{\mu,a}$ be a "generic" two-parameter unfolding of a local diffeomorphism $H_{o,o}$ of $(\mathbb{R}^2,0)$ whose derivative $DH_{o,o}(0)$ has eigenvalues $e^{\pm 2\pi i\omega_o}$, ω_o irrational, and whose first non linear term in a normal form is zero. If ω_o is sufficiently irrational, there exists a Cantor set of values of (μ,a) near $(0,0)$ for which $H_{\mu,a}$ leaves invariant a "non-normally hyperbolic" smooth closed curve (formally there is a whole curve of such values). To prove this we use a version of Rüssmann's "translated curve" theorem. The intersection property is replaced by the presence of parameters and the Cantor set of closed invariant smooth curves found in K.A.M. theorems is "unfolded" in the parameter space.

C. CONLEY:

Another definition of the Fuller-index

Let M be a manifold and let $(x,t) \to x \cdot t$ be a flow on M. Let $P \subset M \times R^+$ be the set of pairs (x,t) such that $x \cdot t = x$. An isolated periodic set, C, is a compact, closed-open subset of P. The Fuller-index of C has been defined as the "tail" of the sequence, $\{\beta_k\}$, of homology classes in the braid spaces, $M_k \equiv (\times_k M \setminus \Delta) / Z_k$, where Δ is the generalized diagonal, Z_k acts by cyclic permutation of the coordinates, and k runs through the primes. For example, let π be a repelling periodic solution with minimal period T. Then β_1 is the homology class in $H_1(M)$ determind by π. The curve π also determines simple closed curves, π_k, in M_k : namely the curves $(x_1 \cdot t, x_2 \cdot ((T/k)+t)$, $\ldots, x_k \cdot ((k-1)T/k+t))$ where x_j is a point of π and t runs from o to T/k. (Note that the end points are identified in M_k). Then β_k is the homology class in $H_1(M_k)$ determind by π_k. [In general, the index of an isolated periodic solution π with Poincaré map P and multiplicity m is $\mathrm{ind}(\pi) = (\deg P/m)$ $\mathrm{ind}(\pi_R)$ where π_R is a repelling periodic solution with the same orbit (but of another flow of course)]. Question 1. For simple repellers, say, it is obvious that the index depends only on the isotopy class of π. Does it depend only on the homology class? In other words, does β_1 determine the remaining β_k? This seems unlikely since $\beta_1 = o$ does not imply $\beta_k = o$. Question 2. If $\mathrm{ind}(\pi) = -\mathrm{ind}(\pi')$, can the flow be continuously changed so that π and π' "cancel"? (This is not true if $\dim M \leq 3$: how abour higher dimensions 3).

R.L. DEVANEY:

Some area-preserving mappings exhibiting stochastic behavior

Numerical work in classical mechanics often indicates the existence of sets of positive measure on which a given system exhibits highly random behavior (the "ergodic sea"). We discuss several examples of area preserving mappings which arise in mechanics and for which some sort of stochasticity can be verified. One example is linked twist mappings: these may hold the key to understanding ergodic behavior within the zones of instability near an elliptic fixed point of generic type. Another is the nonlinear mapping of the plane: $x_1 = x_0 + 1/y_0$, $y_1 = y_0 - x_0 - 1/y_0$ recently introduced by Henon as an asymptotic form of the equations of motion of the restricted three body problem. One can prove that this mapping is topologically conjugate to the well known Baker transformation. Similar mappings also seem to arise in numerical studies of the anisotropic Kepler problem and the isosceles three body problem.

J. DUISTERMAAT:

Periodic solutions near equilibrium points of Hamiltonian systems

Let $\dot{z} = \mathrm{Ham}_{F_\mu}(z)$ be a Hamiltonian system with a smooth (C^k, k large, C^∞, or real analytic) Hamiltonfunction F_μ depending smoothly on finitely many parameters μ, with the origin o as an equilibrium point. Let $\dot{z} = A_\mu z$ be the linear approximate at the origin. Let $N = \mathrm{Ker}(e^{\omega_0 A_0} - I)$, the space of periodic solutions of $\dot{z} = A_0 z$, be 4-dimensional, equal to the nilpotency space of $e^{\omega_0 A_0} - I$, and such that the eigenvalues of $A_0 \mid N$ have a ratio $k : \ell$ with $k \neq \ell$, k,ℓ integers.

Theorem 1. There exists an embedding Φ_μ from a neighborhood of o in N into the phase space, depending smoothly on μ, and a family of smooth functions G_μ depending smoothly on μ, such that a) G_μ is invariant under the circle action $t \to e^{tA_0}$ on N b) The set of periodic solutions of $\dot{z} = \mathrm{Ham}_F(z)$ near o and with period near ω_0 is equal to the image under Φ_μ of the set $x \in N$ where dG_μ is a multiple of dF_0^2 (F_0^2 = second degree part of F_0)

2) Under genericity assumptions for F_μ (algebraic inequalities for the Taylor expansion of F_μ up to the order $\min(k+\ell,6)$) there is a smooth change of coordinates depending smoothly on μ bringing G_μ into standard form = a polynomical depending on $\nu = \nu(\mu) \in \mathbb{R}^k$, k at most 4, $\nu(\mu)$ depending smoothly on μ, and mapping F_0^2 to a function of F_0^2 and G_μ. This is proved using Wasserman's theory of group-invariant normal forms. This brings the set of periodic solutions into standard position.

F.D. EHLERS:

The Bäcklund-Transformation for the KdV-equation in Krichever's algebraic geometric picture

By the work of Krichever those solutions u of the KdV-equation, the Schrödinger operators $-d^2/dx^2 + u$ of which commute with some operator of odd order, belong in an essentially bijective way to line bundles of some fixed degree on hyperelliptic algebraic curves. It is shown, how these algebraic geometric data are transformed under a Bäcklund transformation of the solutions of the KdV-equation. "Generically", the new curve is obtained from the old one by identifying two points which are in involution under the hyperelliptic conjugation, giving rise to a double point singularity on the new curve. In the language of differential operators, this corresponds to a conjugation $Q \to AQA^{-1}$ with a first order operator A in a ring of formally pseudodifferential operators, applied to the

Schrödinger operator of the KdV-equation. (Joint work with H. Knörrer).

B. FUCHSSTEINER:

Compatible deformations of Lie-algebras and nonlinear dynamical systems with infinite dimensional abelian symmetry groups

It is investigated wether equations, such as

(1) $u_{xt} - u_t = u_x + u_{xx} + 17(uu_x)_x - 2\,uu_x$

(2) $u_{xt} = u_{xx} + \sin u + 2u_x \sin u + u_{xx} \int_{-\infty}^{x} \sin u(\xi)\, ds$

(3) $u_t - u_{xxt} = 3(uu_x + (uu_x)_{xx} - \alpha u_{xx}) - u_x - u_x u_{xx}$

are completely integrable. For this purpose special deformations of Lie-algebras, having a linear interpolation property, are introduced. It turns out that these deformations are generating infinite dimensional abelian Lie subalgebras. Furthermore, they characterize the soliton-structure and the conservation laws for those dynamical systems which have elements of this particular abelian Lie subalgebras as infinitesimal generators. Examples are given (among them, of course, the popular equations like, KdV, Sinh-Gordon, Burger's etc.).

M.R. HERMAN:

Proof of the $C^{3+\varepsilon}$ - twist theorem for invariant curves of constant type rotation number

We outline the proof of the twist theorem of translated curve theorem in class $C^{r+\varepsilon}$ $r \geq 1$, $r \in \mathbb{N}$, $o < \varepsilon < 1$. The proof only uses the Schauder Tychonoff fixed point theorem. The curves obtained are of class $C^{r-1+\varepsilon}$. This proof outlined works only if we assume the rotation number on the invariant curve to be of constant type.

A. KATOK:

A new proof of ergodicity of smooth dynamical systems with non-zero Lyapunov exponents

We prove local ergodicity of a $C^{1+\alpha}$ dynamical system with respect to an absolutely continuous invariant measure on a set of regular points with non-zero Lyapunov exponent not using the families of expanding and contracting manifolds. This proof saves one limit process in comparison with the original proof given by Pesin in 1975 and based on the absolute continuity of the families of invariant manifolds. Our approach allows to avoid some painful technicalities involved in the proof of absolute continuity. Our proof is based on estimates of volume and shape of particular tubes around regular points. These estimates are also useful in other asymptotic considerations, including the computation of the entropy, construction of hyperbolic sets etc.

U. KIRCHGRABER:

Coupled Chemical Reactors

In a recent Paper Marek and Stuckl performed experiments on two almost identical chemical reactors of the Belousov-Zhabotinskii type which were weakly coupled; they have shown that a periodic response may be observed depending on the ratio of the difference of the reactors to the strengh of the coupling. The problem has been treated from a mathematical point of view by Neu; using the two-variable expansion procedure he has given a formal treatment. The purpose of this talk is to rigorously justify Neu's results. Using transformation techniques and invariant manifold theory the problem is reduced to a one-dimensional periodic equation depending on two small parameters. The number of periodic solutions is studied in a full neighborhood of the origin of the parameter space.

H. KNÖRRER:

Geodesics on quadrics and a mechanical problem of C. Neumann

Let A be a non-degenerate symmetric $(n \times n)$-matrix; and let $x(t)$ be a geodesic on the quadric $Q : < x,Ax > = 1$, parametrized such that

$$\ddot{x} = - \varepsilon Ax + \rho \dot{x} \quad \text{where} \quad \varepsilon = \pm 1, \; \rho = 2 \frac{< A\dot{x}, Ax >}{< Ax, Ax >}$$

Let $\xi(t)$ be the unit normal vector of Q in the point $x(t)$. Then

$$(*) \quad \xi_1^2 + \ldots + \xi_n^2 = 1 \quad \text{and} \quad \ddot{\xi} = - \varepsilon A \xi + u\xi \quad \text{with} \quad u = \varepsilon < \xi, A\xi > - < \dot{\xi},\dot{\xi} >$$

The differential equation (*), which has already been studied by C. Neumann in 1859, is intimately related to the one-dimensional Schrödinger-operator with quasi-periodic potential. It can be shown that every solution of (*) can be obtained from a geodesic on a suitable quadric (possibly a paraboloid) in the way described above. The relations between the integrals for the diff. eq. (*) and for the geodesic flow on quadrics is be discussed. Furthermore some new quadratic relations between eigenfunctions for the one-dimensional Schrödinger-operators with quasi-periodic finite-gap-potential are established.

M. KUMMER:

The 3-dim. rest. 3-body problem in the limit of large values of the Jacobian constant

We show that a natural setting for studying perturbations of the 3-dim. Kepler problem is the "twistor-space" C(2,2) of Penrose (with its canonical action of U(2,2)) or alternatively the space $T^{+}S^{3}$ which is obtained from $C(2,2)\setminus\{0\}$ by reducing out the U(1)-action. We recover a result of Sternberg who exhibits $T^{+}S^{3}$ as symplectic homogeneous space of SU(2,2) (or (SO(4,2). We apply our constructions to the 3-dim. rest. 3-body problem for high values of the Jacobian constant and find an integrable approximation which besides the four well known periodic solutions also possesses 4 families of quasi-periodic solutions with 2 frequencies (family parameter = 3rd comp. of angular momentum) which are "surrounded" by families of quasi-periodic solutions with 3 frequencies. Finally we discuss the continuation of these solutions from the integrable approximation to the exact problem.

E. A. LACOMBA:

Singularities in the total collision manifold of the rhomboidal 4-body problem

Consider the rhomboidal 4-body problem where one is given two equal masses symmetrically situated with respect to a symmetry line, while the remaining masses move along this line. Symmetrical initial velocities are given to keep a rhomboidal configuration at all times. We blow up the origin by a Mc Gehee transformation to get the quadruple collision manifold. Then we regularize double collisions, and hopefully blow up triple collisions, which remained as singularities in the collision manifold. If either of the masses along the symmetry line goes to zero, we get a 3+1 problem consisting of an isosceles one plus a negligeable mass point.

J. LLIBRE:

Families of periodic orbits near a homoclinic orbit to a saddle center equilibrium point of a Hamiltonian system with 2 degrees of freedom

We prove that in any neighborhood of a homoclinic orbit of the mentioned type there exists a countable set of families of simple periodic orbits (S.P.O.). Furthermore a contiunuous family of S.P.O. can exist ending in the homoclinic orbit. The Hamiltonian near the equilibrium point can be put into the form $H = \alpha\, x_1\, y_1 + \frac{\beta}{2}\,(x_1^2 + y_2^2) + O_4(x_1,y_1,x_2,y_2)$. Two case are possible. In the first on each level of negative energy, $h < o$, there is a finite set of S.P.O. and for zero and positive values of h there is a countable family. In the second case, P.O. of the studied type only exist for positive values of h and on each level set there is a countable set of such orbits. Several examples are given. We show that the countable set of S.P.O. in each level set of positive energy tends to an orbit which is homoclinic to the P.O. of this energy level belonging to the Lyapunov family which emerges from the fixed point.

J. MARTINET:

Analytic classification of genus of resonant differential equations in C^2

This is joint work with J.P. Ramis. We classify, up to local analytic diffeomorphisms of $(C^2,0)$, the genus of holomorphic differential equations $\omega = A(x,y)dx + B(x,y)dy = 0$, the linear part of which is resonant, i.e. of one of the following types: (i) $ydx = 0$ (one eigenvalue is 0, the other $\neq 0$) (ii) $px\, dy + qy\, dz = 0$ (quotient of eigenvalues is a negative rational) . In each case, there is an infinite dimensional "moduli space". For instance, the form of type (i) which are formally equivalent to the normal form $\omega_o = x^2\, dy - ydx = 0$, are up to analytic equivalence, in canonial bijection with the space $C \times H$, where H is the group of local analytic diffeomorphism of $(C,0)$ whose linear part is the identity. This bijection is "biholomorphic". The forms of type (ii), with $p = q = 1$, which are formally equivalent to the normal form $\omega_o = du - uydx = 0$ ($u = xy$) are classified by the space $H \times H$.

R. Mc GEHEE

C^∞-Denjoy counterexample

The following recent result of G.R. Hall was discussed. Given any irrational number α . There exists a homeomorphism $f : S^1 \to S^1$ such that the rotation number of f is α, f is not conjugate to a rigid rotation, and f is C^{∞}. Furthermore, f has at most two points of zero derivative.

R. MOECKEL:

Orbits near triple collision in the three-body problem

Orbits near triple collision in the three-body problem with fixed energy and zero angular momentum can be studied using Mc Gehee's collision manifold construction. The collision manifold M_0 is four dimensional and forms a boundary to the usual phase space. The vectorfield of the 3 b.p. extends to M_0 and is ∇-like there. Orbits which ended in finite time due to triple collision tend asymptotically to restpoints in M_0. Orbits in M_0 which connect two restpoints lead to orbits of the three body problem which approach collision near one central configuration, narrowly avoid collision and emerge from a neighborhood of the singularity near a different central configuration.

P. VAN MOERBEKE:

A criterion for algebraic complete integrability

A Hamiltonian system having enough degrees of freedom is algebraically completely integrable if and only if the system admits Laurent expansions in time (t) (possibly complex). This is shown for two classes of Hamiltonian systems 1) a system coming from non nearest neighbor interaction exponential systems 2) the geodesic flow on SO(4) for a leftinvariant metric. Application of this criterion leads in both cases to infinite dimensional extensions of the classical Lie algebras (Kac-Moody Lie algebras). Put in that framework, the problem can then be linearized on an Abelian variety and integrated.

J. MOSER:

Spectral theory of quasiperiodic potentials

We study the spectrum of the selfadjoint Sturm Liouville operator $L = -(\frac{d}{dx})^2 + q(x)$ densely defined on $L^2(-\infty,+\infty)$ in case $q(x)$ is an almost periodic function. It is wellknown that for periodic potentials $q(x)$ the spectrum is continuous and consists of a sequence of intervals extending to $+\infty$, the so-called band spectrum. In particular, there exists no point eigenvalue. Little is known about the spectrum of almost periodic potentials, but one encounters new phenomena. We give examples of a quasi-periodic potential with two frequencis ω_1, ω_2 with $\omega_2/\omega_1 = e$ for which there exists a point eigenvalue. Another example shows that the spectrum can be a nowhere dense Cantor set,probably this is a typical phenomena. For a systematic study of the spectrum we introduce the functional.

$$w = w(\lambda,q) = \lim_{x\to\infty} \frac{-1}{x} \int_0^x \frac{dt}{2G(t,t,\lambda)} \ , \quad \operatorname{Im}\lambda \neq o$$

where $G(x,y,\lambda)$ is the Green's function, i.e. the kernel of $(L-\lambda)^{-1}$. As a function of λ it is holomophic in $\operatorname{Im}\lambda \neq o$ and satisfies

$$\frac{dw}{d\lambda} = \lim_{x\to\infty} \frac{1}{x} \int_0^x G(t,t,\lambda)\, dt \ ,$$

which is holomorphic on the resolvent set of L. However, w is not one-valued in the resolvent set but suffers a jump, is, over any gap in the spectrum, where s belongs to the frequency module of $q(x)$. As a functional, i.e. in dependence of q, the expression w is translation invariant, i.e. under $q(x) \to q(x+t)$, as well as under the flow of the Korteweg - de Vries equation $q_t - 6qq_x + q_{xxx} = o$. Moreover, with respect to a symplectic structure appropriate for that theory the functionals $w(\lambda_1,q)$, $w(\lambda_2,q)$ are "in involution" for arbitrary λ_1,λ_2 in $\operatorname{Im}\lambda_j \neq o$. The asymptotic expansion of $w(\lambda,q)$ for large negative λ yields as coefficients the usual conserved quantities (conservation laws) of the Korteweg-de Vries equations.

S. NEWHOUSE:

Continuity properties of entropy

Let M be a compact C^∞ 2-dimensional manifold. Let $D^\infty(M)$ be the space of C^∞ diffeomorphisms with the C^∞ topology. For $f \in D^\infty(M)$, let $h(f)$ denote the topological entropy of f. Let $m(f)$ denote the space of f-invariant probability measures on M. If $\mu \in m(f)$, let $h_\mu(f)$ denote its metric entropy. We discuss the theorem:

1. $h(\cdot) : D^\infty(M) \to \mathbb{R}_+$ is uppersemicontinuous.
2. For fixed $f \in D^\infty(M)$, $\mu \to h_\mu(f)$ is uppersemicontinuous.

By means of a result of Katok we deduce as a Corollary: The map $f \to h(f)$ from $D^\infty(M)$ to R_+ is continuous. By means of the ergodic decomposition theorem we get another Corollary: For $f \in D^\infty(M)$ there is an ergodic invariant measure μ with $h_\mu(f) = h(f)$. The above theorem is false for any finite class of differentiability. If $\dim M > 2$, then $f \to h(f)$ is not lowersemicontinuous even on $D^\infty(M)$.

J. PÖSCHEL:

Differentiable foliation of invariant tori in Hamiltonian systems

We study differentiable perturbations of analytic, nondegenerate integrable Hamiltonian systems of n degrees of freedom. We show that there exists a differentiable foliation of invariant, n-dimensional tori over a Cantorset using Whitney's notion of a differentiable function on a closed set. We assume the perturbation to be of class C^r, $r > 3n-1$. In fact this foliation is more often differentiable on the tori than transversal to them, leading to a new class of differentiable functions both on open and closed sets, for which a generalization of Whitney's extension theorem is given. As corollaries we obtain on a Cantorset n differentiable integrals in involution and a differentiable solution of the Hamilton-Jacobi-equation. We also get an easy estimate of the Lebesgue-measure of the set of all invariant tori, which also applies for general elliptic fixed points.

J. SCHEURLE:

Quasiperiodic solutions of reversible systems bifurcating from an equilibrium

A family of q-dimensional dynamical systems $\dot{x} = X(\lambda,x)$, $\lambda \in \mathbb{R}^p$, is called reversible, if there is a reflection operator R in $\mathbb{R}^q (R^2 = id)$ such that $X(\lambda,Rx) = - RX(\lambda,x)$ holds. Assume that $x = o$ is an equilibrium for all values of λ. For this case a result about the existence of quasiperiodic solutions bifurcating at a certain point $\lambda=\lambda_o$, $x = o$ is described. The Jacobian $D_xX(\lambda,o)$ is assumed to possess exactly ν pairs of simple, purely imagenary eigenvalues $\pm i\omega_k(\lambda)$, where $1 \leq \nu \leq q/2$ and $p \geq \nu - 1$. Moreover, a non-resonance condition is imposed on the numbers $\omega_k(\lambda_o)$, and the rank of the ν x(p+1)-matrix $(D_\lambda\omega(\lambda_o), \omega(\lambda_o))$, $\omega = (\omega_1,\ldots,\omega_\nu)^T$ is assumed to be maximal. Also a generalization of this result to a class of infinite dimensional systems is discussed and applied to a semilinear boundary value problem in the two-dimensional strip $[0,1] \times \mathbb{R}$.

C. SIMO:

Splitting of separatrices for Henon-Heiles type problems

The Melnikov integral $M(t_o)$ is introduced to measure the splitting of separatrices. The example $\ddot{x} + \sin x = \varepsilon f(t)$ with f a 2π-periodic analytic function and $\bar{f} = o$ is considered. For all nonzero functions f one finds nonintergrability if ε is small enough. For other perturbations like $\sin(x+\alpha t)$ there is a countable set of α's such that $M(t_0) = o$, but a second order variational approach destroyes the tangencies of the invariant manifolds. In the HH-problem numerical work showed that for small energy the system looked integrable. We compute numerically the angle γ of the invariant manifolds at some heteroclinic point. It is quickly decreasing with h but up to $h = o{,}5$ it is definitely nonzero. The analytical computation proceeds as follows: 1) Produce the Gustavson Normal Form Γ_6 up to degree 6 with a tail $\Delta\Gamma$ of order 8. 2) Obtain the analytical expression of the separatrix $z(t-t_0)$ 3) Compute the Melnikov integral $\int_R \{\Gamma_6(z(t-t_0)), \Delta\Gamma(t-t_0),t\}dt$. The process has been carried out for 1),2) and stopped due to length of computations. For a simpler problem $H = \frac{1}{2}(|x|^2 + |y|^2) - \frac{\varepsilon}{2}(x_2^2\ x_1^2)$ which looked integrable for small h the 3 steps are carried out and γ different from zero is obtained analytically (we need 2^{nd} order variational equations) which agrees with numerical results. In general the angle is $\gamma = A \exp(-c/\lambda)$ where A,c are constants and λ is the eigenvalue of the critical point.

J.-M. STRELCYN:

Pesin theory for mappings with singularities

The study of (plane) billiards leads in a natural way to the study of some differentiable maps with singularities. In the joint work with A. Katok, we give an axiomatic characterization of a large class of smooth mappings with singulartities, for which all the results of Pesin's theory remain true. This class contains in particular the mappings arising from the study of not too wild billiards. This gives the possibility of applying Pesin's theory to the study of billiards.

T. THIMM:

Examples of degenerate integrable Hamiltonian systems

A method is described which allows the construction of families of first integrals in involution for Hamiltonian systems which are invariant under the Hamiltonian action of a Lie group G . These first integrals are obtained by composing the moment map with invariant functions on subalgebras of the Lie algebra. For two such subalgebras there are simple conditions guaranteeing that the first integrals generated by both subalgebras are in involution. This method is applied to invariant Hamiltonian systems on tangent bundles of real Grassmannian manifolds. A proof of their integrability is sketched. The degeneracy of these integrable Hamiltonian systems follows from the fact that their integral curves are images of 1-parameter subgroups in G . The same method allows to prove the integrability of invariant Hamiltonian systems on tangent bundles of complex Grassmannians and of SU(n+1) / SO(n+1) and of the distance spheres in CP^{n+1} .

G. WILSON:

Equations of KdV equations and simple Lie algebras

There are the following relationships between the KdV equation $u_t = u_{xxx} + 6uu_x$ the modified KdV equation $v_t = v_{xxx} - 6v^2 v_x$ and the sinh-Gordon equation $R_{xt} = \sinh 2R$: if we set $v_x - v^2$, then if v satisfies the MKdV equation, u satisfies the KdV equation; furthermore, the Hamiltonian form $v_t = (-\frac{1}{2}\partial)\delta H/\delta v$, $H = u^2$ for the MKdV equation implies the "second" Hamiltonian form $u_t = (\frac{1}{2}\partial^3 + u\partial + \partial u)\, \delta H / \delta u$ for the KdV equation $(\partial \equiv \partial/\partial x)$. Via the shift $u \to u + \lambda$ this gives also the "first" Hamiltonian form $u_t = (2\partial)\delta H_3/\delta_u$, $H_3 = \frac{1}{2} u^3 - \frac{1}{4} u_x^2$ for the KdV equation, and, more generally, the recursion relation $(2\partial)\, \delta H_n/\delta u = (\frac{1}{2}\partial^3 + u\partial + \partial u)\, \delta H_{n-1}/\delta u$ for the conserved densities H_q of the KdV equations. If we set $\partial R = v$, these same H_q are conserved densities also for the MKdV and sinh-Gordon equations.
All this machinery generalizes if we replace the KdV equations by the isospectral flows $L_t = [P,L]$ based on an n^{th} order differential operators $L = \partial^u + u_{n-2}\, \partial^{n-2} + \dots + u_0$; the Miura transformation is replaced by the factorization relation $L = (\partial - v_n) \dots (\partial - v_1)$, and the sinh-Gordon equation by the 2-dimensional Toda lattice equation $R_{i,xt} = \exp(R_{i-1}-R_i) - \exp(R_i-R_{i+1})$ $(i = 1,\dots,n)$. Here the least equations are the special case of $s\ell(n)$ of a construction that can be carried out for any simple Lie algebra; it is very likely that all the machinery described above admits a similar generalization.

J.C. YOCCOZ:

C^1-conjugation of C^3 diffeomorphisms of the circle

Theorem: Let f be C^3, orientation-preserving, diffeomorphism of the circle $T^1 = \mathbb{R}/\mathbb{Z}$. Suppose that the rotation number of f is irrational, and satisfies the following "small divisor" condition:

$$\exists\, \varepsilon > 0,\ C > 0,\ \forall\, p/q \in \mathbb{Q},\ |\rho(f) - p/q| > C\, q^{\varepsilon-3}$$

Then, f is C^1 conjugate to $R_{\rho(f)}$.
Corollary. The Arnold conjecture is true for type Roth numbers (for example

e , or algebraic numbers): Any C^∞ diffeomorphism of T^1, s.t. its rotation number satisfies.

$$\forall\ \varepsilon > o,\ \exists\ C_\varepsilon > o,\ \forall\ p/q \in \mathbb{Q},\ |\rho(f) - p/q| > \frac{C_\varepsilon}{q^{2+\varepsilon}}$$

is C^∞ conjugate to $R_{\rho(f)}$. The crucial estimate in the proof of the theorem is the following:

$$|Df^q - 1|_o < M|f^q - id - p|_o^{1/2} ,$$

where f is a C^3 diffeomorphism, p/q a convergent of $\rho(f)$ and M is a constant which depends only on the norm of f in the C^3 topology.

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