MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 20/81

Gruppentheorie

3.5. bis 9.5.1981

Die Tagung wurde von den Herren Professoren Dr.W. Gaschütz (Kiel) und K.W. Gruenberg (London) geleitet. Im Mittelpunkt der Vorträge und Gespräche standen neue Entwicklungen der Theorie der endlichen und unendlichen Gruppen sowie der Darstellungstheorie.

Vortragsauszüge

R.B.J.T. ALLENBY:

Potency

In the first part of the lecture it was observed how the concept of 'potency' (a strong residual finiteness condition, first defined explicitly in connection with investigations concerning certain 1-relator groups) had quite quickly appear to become a property worthy of investigation in its own right in the areas of soluble groups and finite groups.
An attempt was then made to indicate the plan of a proof, not very difficult in concept but rather involved in detail, of the potency of cyclically pinched one relator groups, that is of groups which are generalized free products of free groups with an infinite cyclic subgroup amalgamated.

ZVI ARAD:

Powers and Products of conjugacy classes in simple groups

We consider problems of covering a group G by a power or product of

conjugacy classes of G , for example -- determining the least number m such that $C^m = G$ for all non-trivial conjugacy classes C of G (this number, called the covering number of G , exists if G is finite, simple and non-abelian).
Covering problems for special families of groups, especially the groups A_1 , were studied by j . Brenner et al. in a series of papers, with emphasis on classes C such that $C^2 = G$. Covering problems also arise from the direction of model theory and universal algebra. It is on this background that our interest in the subject began.

With Herzog and Stavi we study the following aspects and ramifications of covering problems: Bounds on the covering number and related invariants of finite simple groups, covering theorems for other types of finite groups and for some infinite groups with finitely many conjugacy classes, the situation for particular families of groups (we have recently proved a conjecture of Brenner that the covering number of A_1 , $n \geq 6$, is precisely $[\frac{n}{2}]$. We also proved that the covering number of Sz(q), is precisely 3), theorems on the product of two conjugacy classes, with applications to the theory of factorizations of finite groups, and the development of counterparts to the covering theorems in the framework of universal algebra.

A.0. ASAR:

On a problem of Kegel and Wehrfitz

O.H. Kegel and B.A.F. Wehrfitz, after solving the Cernikov Conjecture on locally finite groups which satisfy the minimum condition on subgroups, raised the following question in their book Locally Fini[te] Groups. Question (V.1). Let G be a locally finite group and i be an involution of G such that $C_G(i)$ is Cernikov. Does G then have a locally soluble subgroup of finite index? Since none of the known simple groups has an involution with this property the answer was anticipated to be positive. Indeed the answer in "YES" and the structure of G is contained in the following theorem.
THEOREM. Let G and i be given as in the above question. Then G has a series of normal subgroups

$$1 \leq OG \leq M \leq G$$

such that G/M is finite, M/OG is a divisible 2-group and OG is a soluble group such that $[OG,i]'$ is Cernikov.

H. BASS:

Automorphisms of groups and of algebraic varieties

We study the following properties of a group Γ :

(RF) Γ is residually finite

(VTF) Γ is virtually torsion free, ie. has a torsion free subgroup of finite index.

Note that (VTF) implies that the finite subgroups of Γ have bounded orders. Let F be a field, V a finite dimensional F-module, and Γ a finitely generated subgroup of $Aut_F(V)$. Then it is well known that satisfies (RF) and, if char(F) = 0 , also (VTF). We generalize this by allowing V to be any algebraic variety over F and $Aut_F(V)$ the group of automorphisms of the variety V .

Let Γ be a finitely generated group. Then $Hom(\Gamma, GL_n(F))$ is naturally an affine variety, which we denote $R_n(\Gamma)$. $GL_n(F)$ acts on it by conjugation; the quotient variety $S_n(\Gamma) = R_n(\Gamma)/GL_n(F)$ parametrizes isomorphism classes of semi-simple n-dimensional representations of Γ . The group $Aut(\Gamma)$ acts on $R_n(\Gamma)$, and induces an action of $Out(\Gamma)$ on $S_n(\Gamma)$. When these actions are faithful we can deduce (RF) and (if char(F) = 0) (VTF). This applies for example to free groups and to surface groups.

F.R. BEYL:

The Schur multiplicator of $SL(2,\mathbb{Z}/m)$.

Theorem 1. The Schur multiplicator of $SL(2,\mathbb{Z}/m)$ is $\mathbb{Z}/2$ if $4 \mid m$ and vanishes otherwise.

Corollary. $SL(2,\mathbb{Z}/4)$ is efficient in the sense of D.B.A. Epstein by virtue of the presentation $\langle A,B : A^4 = B^4 = 1 , (AB)^3 = B^2\rangle$.

Theorem 2. The Schur multiplicator of $PSL(2,\mathbb{Z}/m)$ = SL/Center is

$$\begin{cases} (\mathbb{Z}/2)^d & \text{for } 4 \nmid m \\ (\mathbb{Z}/2)^{d+1} & \text{for } 4 \mid m \end{cases}$$, where d is the No. of odd prime factors in m .

These results partly confirm and partly refute an assertion of Mennicke [Invent.Math.4 (1967), 202-228] according to which the multiplicator of $SL(2,\mathbb{Z}/m)$ vanishes for all m . We note that the original reasoning of Mennicke can be repaired as to give the same conclusion as before, viz. that $SL(2,\mathbb{Z}[\frac{1}{p}])$ has the Congruence Subgroup Property. The latter point was further clarified in discussions with Professor Mennicke at the conference.

F.B. CANNONITO:

Algorithms for Solvable Groups

This lecture reports an joint work with C. Boumslag and C.F. Miller, III.

The main Theorem proved is

Theorem 1. There is an algorithm which decides if an arbitrary finitely presented solvable group is polycyclic.

As an immediate consequence, we see we can effectively recognize when a finitely presented solvable group is a) polycyclic, b) supersolvable, c) nilpotent, d) abelian, e) cyclic, f) finite or g) trivial.

The proof of the Theorem depends on the following two propositions of independent interest.

Proposition 1. There is an algorithm which decides if an arbitrary finitely presented right $\mathbb{Z}P$-module M, where P is a polycyclic group, is finitely generated as an abelian group.

Proposition 2. The wordproblem is solvable for finitely generated nilpotent-by-polycyclic groups which satisfy Max-n.

Proposition 2 has the following corollories of note.

Corollary 1. The word problem is solvable for finitely presented groups in the $\mathfrak{N}_2\mathfrak{A}$ (nilpotent of class 2-by-abelian).

Corollary 2. Let G be a finitely generated locally finitely presented solvable group. Then G has solvable word problem.

K.W. GRUENBERG:

Decomposition of relation modules

A survey of the present state of knowledge concerning direct sum decompositions of relation modules was given and the following two new results stated and their proofs sketched:

(A) If S is a 2-generator simple group and S $G \trianglelefteq$ Aut S , if the prime graph of G/S in connected and G/S is not cyclic, then the relation cores of G are indecomposable.

(B) If G $\twoheadrightarrow$ H and the prime graph of H is connected and $d_G(\mathcal{G}) = d_H(\mathcal{H})$, then the relation cores of G are indecomposable.

F. GRUNEWALD:

Decideability in Algebra

The talk was a report on recent work of Dan Segal and myself on decideability questions in arithmetic and algebra. The aim of our efforts was an effective version of reduction-theory. Some of the resulting theorems are:

1) The isomorphism problem for finitely generated nilpotent groups is effectively solvable.
2) The isomorphism problem for finite dimensional R-algebras is effectively solvable.
3) The conjugacy problem for arithmetic groups is effectively solvable.

H. HEINEKEN:

Finite subnormal subgroups and their nilpotent residue

If A is a subnormal subgroup of a finite group G and B is its nilpotent residue, then there is a normal subgroup L of A such that L is also normal in L^G which is equal to B^G .

Generalization of this statement leads to statements on the structure of groups all of whose finite subsets of elements are contained in finite (locally) subnormal subgroups.

(joint work with P. Schwittek)

D.F. HOLT:

Ends of Locally Finite Groups

Let G, A be groups, let $\overline{AG}$ be the group of functions $G \longrightarrow A$ and AG the subgroup of functions having finite support G acts on $\overline{AG}$ (and AG) by the rule $f^g(h) = f(hg^{-1})$ $(g,h \in G,\ f \in \overline{AG})$. Observe that the semidirect product $G \rbrack \overline{AG}$ is the wreath product $AWrG$, and $G \rbrack AG$ is the restricted wreath product. We shall prove the following two theorems.

Theorem 1. If G is uncountable locally finite, then all derivations $d : G \longrightarrow AG$ are inner. In particular, if A is abelian, then $H'(G, AG) = 0$, and so G has one end.

[Remark. If G is countable and locally finite, then $|H'(G,AG)| = |A|^{\aleph_0}$.]

Theorem 2. If $H \subseteq G$ with H countably finite and G locally finite, and the derivation $d : G \longrightarrow AG$ satisfies $d(H) \subseteq AH$, then d is inner.

B. HUPPERT:

Some remarks on the Lorentz-group

Theorem 1. Let V be a K-Vector space (char. $k \neq 2$) of finite dimension with a symmetric, bilinear, regular scalar product $(,)$. Suppose ind $V > 0$, If G is a linear mapping of V into V preserving isotropie, then G is a similarity.

Theorem 2. Suppose V as above with $K = \mathbb{R}$.

a) If ind V is odd, G has a real eigen-value.

b) If dim V is even, ind V odd and det $G = 1$, G has a real eigen-value with isotropic eigenvector.

From Theorem 2 follows easily a complete classification of the Lorentztransformation.

A. KERBER:

Charaktere endlicher Gruppen und Teilbarkeitsprobleme

If $n \in \mathbb{N}$, S_n := symmetric group of degree n , G := a finite group, $D : G \longrightarrow GL(V)$ an ordinary representation of G on V , D_i an ordinary irreducible constituent of $\overset{n}{\otimes} D$ with character ς^i and dimension f^i , the following defines a character of S_n :

$$\chi^{D \Delta_n D_i}(\pi) := \frac{1}{|G|} \sum_g \varsigma^i(g^{-1}) \prod_{i=1}^{n} \chi^D(g^i)^{a_i(\pi)}$$

($c_i(\pi)$:= no. of i-cycles in $\pi \in S_n$). For D_1 := identity representation we put

$$c_{i,n} := \chi^{D_i \Delta_n D_1}((1\ldots n)) = \frac{1}{|G|} \sum_g \varsigma^i(g^n).$$

Theorem. Given $n_j \in N$ and a conjugary class $C \subseteq G$, then the number of solutions $(g_1,\ldots,g_k) \in G^k$ of

$$g_1^{n_1} \ldots g_k^{n_k} \in C$$

is equal to (if $k \geq 2$)

$$|G| \sum_{f_i} \left(\frac{|G|}{}\right)^{k-2} \left(\prod_j c_{i,n_j}\right) w^i(g^{-1}), \text{ any } g \in C ,$$

and hence divisible by $|G| \gcd \left(\frac{|G|}{f_i}\right)^{k-2}$.

(For further details and results see § 5.3 in the book by James/Kerber (in print) and the joint paper with B. Wagner in Archiv der Math.)

J.C. LENNOX:

Nearly maximal subgroups of finitely generated soluble groups

M is a nearly maximal subgroup if a group G if M is maximal with respect to having infinite index in G. Near splitting techniques are used to prove the

Theorem. A finitely generated soluble group G is finite by nilpotent if and only if every nearly maximal subgroup M of G has normalizer of finite index.

Related characterizations of nilpotent by finite and finite by supersoluble groups are discussed.

<u>Corollary.</u> A finitely generated soluble group G is finite by nilpotent if and only if

$$G/H^G \text{ finite} \Longrightarrow |G : H| \text{ finite.}$$

M a subgroup of G .

F. LEVIN:

<u>Automorphism groups of certain one-relator groups</u>

Common paper with Don Collins: We determine the automorphism groups in terms of generators and relations, of groups of the form $G = \langle x,y \mid x^{-1}y^r = y^{rs}\rangle$, $r,s \geq 2$. A side result is that where r and s have the same prime divisors, then the group is also hopfian. Further, we determine the autormophism group of the automorphism group in the latter case. In particular, Aut(Aut G) turns out to be an extension of Jnn(Aut G) by a cycle of order two.

P.A. LINNELL:

<u>Decomposition of augmentation ideals and relation modules</u>

Let G be a group and $\mathfrak{g}$ its augmentation ideal. I will be concerned with the question of characterising the groups G for which $\mathfrak{g}$ has a nontrivial decomposition; that is we may write $\mathfrak{g} = U \oplus V$ as $\mathbb{Z}G$-modules with $U \neq 0 \neq V$. The starting point for this research is the following result.

<u>Theorem 1.</u> If $H^0(G,\mathbb{Z}G) = H'(G,\mathbb{Z}G) = 0$, then $\mathfrak{g}$ is indedomposable.

It is well known that $H^0(G,\mathbb{Z}G) = 0$ if and only if G is infinite.

Now a celebrated result due to Stallings states that if G is finitely generated, then $H'(G,\mathbb{Z}G) \neq 0$ if and only if we can write $G = A*_F B$ or $G = A*_{F,0}$ with F finite and $A \neq F \neq B$. Thus it is particularly relevant to consider the problem for when G is the fundamental group of a graph of groups with each edge group finite and each vertex group satisfying $H'(G,\mathbb{Z}G) = 0$.

The same techniques can also be used to characterise groups for which the minimal relation modules decompose. The results here are not so good; one reason is that the groups for which $H^2(G,\mathbb{Z}G) \neq 0$ have not been classified. One result we do have is

Theorem 2. If G is a finitely generated nilpotent group, then its minimal relation modules are indecomposable.

J. MENNICKE:

Linear groups over complex quadratic number fields

Es wurde über Zusammenhänge zwischen linearen Gruppen vom Typ SL_2 über komplexquadratischen Zahlkörpern und Verteilungen von Klassenzahlen berichtet.

R.E. PHILLIPS:

Finitely generated subgroups of wreath products

Let G be any non-Abelian group and C an infinite cyclic group. The unrestricted wreath product W = G Wr C has $2^{\aleph_0}$ pairwise non-embeddable 2-generator subgroups. If B is any infinite k-generator group ($k \geq 2$) and G has a class 2-nilpotent subgroup, then W = G Wr B also has $2^{\aleph_0}$ pairwise non-embeddable 2-generator subgroups.

Using this result and other known constructions it is easy to produce large classes of pairwise non-embeddable, 2-generator groups G which have prescribed properties; a partial list of such properties is (1) p-groups, (2) groups of small exponent (3) residually finite.

The results pertaining to (3) make essential use of recent work of J.S. Wilson [Math. Z., 174, 149-157 (1980)].

J. POLAND:

On a class of residually finite groups

A group G is calles "potent" iff for every x in G and every positive integer n (dividing the order of x if this order is finite) there is a normal subgroup N of G of finite index in G such that xN has order precisely n . Every group which is residually-finite-p for all primes p is potent, and every potent group is residually finite. We outline what is presently known about how the class of potent groups sits between these two classes, comparing their properties. The topics discusses are: closure operations, the position of relatively free groups, of polycyclic groups, of linear groups, of abelian-by-nilpotent groups, and which finite groups lie in these classes.

K.W. ROGGENKAMP:

The isomorphism problem and units in group rings of finite groups

If RG is the group ring of the finite group G over the cummutative ring R with identity, we denote by U(RG) the units in RG , and for a two-sided ideal I the congruence subgroup of U(RG) with respect to I is $V_I(RG) = \{1 + x \in U(RG) , x \in I\}$.

In particular for g_R the augmentation ideal

$$V_g(RG) = V(RG)$$

are the normalized units. If there is a homomorphism $\varphi : V(RG) \longrightarrow G$, split by the natural injection $G \longrightarrow V(RG)$, we way that V(RG) is split Apart from interest eo ipso, a splitting of V(RG) has a severe impact on the famous isomorphism problem: Does RG = RH imply G = H . As one result, for example, if G is a p-group and R an integral domain of characteristic zero with $pR \neq R$, then if V(RG) is split and RG = RH then $G \cong H$. The situation is more difficult in characteristic p , but we are able to prove

Theorem I: Let $k = \mathbb{Z}/p\mathbb{Z}$ and $\mathfrak{n}$ a finite dimensional nilpotent k-algebra with

(i) $\mathfrak{n}^{(p-1)} = 0$,

(ii) for x,y in the centre of $\mathfrak{n}$, xy = 0 .

Then if $G = \{1+n,\ n \in \mathfrak{n}$, $kG = kH$ implies $G \cong H$.

This Theorem extends the only positive result previously known for modular group algebras: If G is p-group of exponent p and class of nilpotency 2 , then G is determined by $\mathbb{Z}/p\mathbb{Z}G$. (Passi-Sehgal 1972). For groups other than p-groups it is known (Cliff-Sehgal-Weiss 1981) that $V(\mathbb{Z}G)$ is split if G is metabelian and G/G' is odd (and counter-examples by us show the condition on G/G' is necessary). Though for metabelian groups a splitting of the units occurs rather frequently, the behaviour of simple groups is quite different. We have obtained the following result for groups with all Schur indices 1 (for example, the alternating groups) and suspect it is true in general.

Theorem II: If G is simple and not isomorphic to PSL(n,q) for some integers n > 0 and prime power q , then the map G => V(G) is not split.

The proof of this uses an extension of Bass-Milnor-Serre's congruence subgroup theorem by Prassad.

G. ROSENBERGER:

On certain discrete subgroups of $PSL(2,\mathbb{C})$

Eine Untergruppe der $PSL(2,\mathbb{C})$ heisst elementar, wenn je zwei Elemente unendlicher Ordnung mindestens einen gemeinsamen Fixpunkt haben. Die Klassifizierung der elementaren diskreten Untergruppen der $PSL(2,\mathbb{C})$ ist seit langem bekannt.

Sei nun $A,B \in PSL(2,\mathbb{C})$ und $G = \langle A,B \rangle$ nicht elementar.

Ist sogar $G \subseteq PSL(2,\mathbb{R})$, so lässt sich nach Ergebnissen von N. Purzitsky und G. Rosenberger algorithmisch entscheiden, ob G diskret ist oder nicht; als Folgerung ergeben sich einfache notwendige und hinreichende Bedingungen für die Diskretheit von G . R.J. Evans und andere haben kürzlich Bedingungen angegeben, unter denen G diskret ist, falls wenigstens Sp A, Sp B, Sp $AB \in \mathbb{R}$. Der folgende Satz zeigt nun, dass sie in Wirklichkeit keine neuen Ergebnisse erzielt haben:

Seien $A,B \in PSL(2,\mathbb{C})$ mit Sp A, Sp B, Sp $AB \in \mathbb{R}$; und sei $G = \langle A,B \rangle$ nicht elementar. Ist G diskret, so ist G in $PSL(2,\mathbb{C})$ konjugiert zu einer zweielementig erzeugten Fuchsschen Gruppe $\tilde{G} \subseteq PSL(2,\mathbb{R})$. Insbesondere ist G diskontinuierlich.

P. SCHMID:

Theorem B of Hall-Higman Revisited

Applying Green's theory of vertices and sources, J. Thomson described an elegant approach to the celebrated Theorem B of Hall-Higman. We extend and complete this from the point of block theory. The crucial result is the following, obtained in joint work with W. Knapp.

<u>Theorem $B^{\#}$.</u> Let G be a finite p-solvable group with $O_p(G) = 1$ and with a cyclic Sylow p-subgroup $\langle x \rangle$ of order $p^n > 1$. Clearly

$Q = [O_{p'}(G), x^{p^{n-1}}]$ then is a nontrivial normal subgroup of G .
Suppose G has a p-block B of defect a , say , containing a faithfull kG-module V over some field k of characteristic p . Then the degree of the minimal polynomial of x on V either is $d = p^n$ or $d = p^{n-a}(p^a-1)$.
The latter happens only if the following holds:
(a) In case p = 2 the integer $q = 2^a-1$ is a Mersenne prime, and Q is a nonabelian special q-group.
(b) If p is odd, either B is of defect a = 1 or p = 3 and a = 2 . Moreover, p is a Fermat prime and Q is a nonabelian special 2-group of order larger than $(p^a-1)^2$.
Of course, this applies to the original situation of Theorem B . (There x is assumed to be any p-element, say of order p^n , in some p-solvable group G_o with $O_p(G_o) = 1$. Let $H = O_{p'}(G_o)$ and $G = \langle x\rangle H$. If V_o is a faithful kG_o-module, consider the indecomposable summands V of $(V_o)_G$ for which $H/C_H(V)$ is not centralized by $x^{p^{n-1}}$. Observe that $[H, x^{p^{n-1}}]$ acts faithfully on the direct sum of these summands.)

R. SCHMIDT:

Affinitäten von Gruppen

Sei σ eine 1-Affinität der Gruppe G auf die Gruppe $\bar{G}$, d.h. eine bijektive Abbildung von G auf $\bar{G}$, die Untergruppen auf Untergruppen und Restklassen nach einer Untergruppe auf Restklassen nach deren Bildgruppe abbildet. Für $x,y \in G$ sei $a(x,y) = (y^\sigma)^{-}(x^\sigma)^{-}(xy)^\sigma$ und sei schliesslich $A = \langle a(x,y) \mid x,y \in G\rangle$. Dann gilt:
(1) $A \leq Z(\bar{G}) = Z(G)^\sigma$.
(2) A ist eine Torsionsgruppe mit p-Komponente $A_p \simeq Z_{p^n}$, $n \in N_o \cup \{\infty\}$.

Ist ferner $Reg(\sigma) = \{g \in G \mid (xg)^\sigma = x^\sigma g^\sigma$ und $(gx)^\sigma = g^\sigma x^\sigma$ für alle x G$\}$ und $G_o = \bigcap_{1\text{-Aff von } G} Reg(\sigma)$, so gilt:

(3) Jedes Element unendlicher Ordnung von G liegt in G_o .

(4) G_o ist eine charakteristische Untergruppe von G , und G/G_o ist π-abgeschlossen für jede Primzahlmenge π .

(5) Sind g,h p-Elemente aus $G \setminus G_o$, so ist $\langle g\rangle \cap \langle h\rangle \neq 1$.

Ist insbesondere Z(G) = 1 oder G eine endliche perfekte Gruppe, so ist jede 1-Affinität von G ein Isomorphismus.

D. SEGAL:

Nilpotent groups of Hirsch length 6

1. The equivalence of the classification of nilpotent torsion-free groups of class 2 with the classification of certain bilinear mappings.
2. In the case of Hirsch length 6, the equivalence of this classification with that of binary quadratic forms over $\mathbb{Z}$, using a sort of Pfaffian. Consequences for the theory of nilpotent groups.

U. STAMMBACH:

Some remarks on the cohomology of a finite groups with simple coefficents.

Let G be a finite group with $p \mid |G|$ and let A be a simple module in the principal block of kG where k is the field of p elements. Gruenberg has asked the question, whether it can happen that $H^i(G,A) = 0$ for all $i \geq 1$.
Even for p-solvable groups the answer to the question is not known. For groups G which are extensions by a p-group of a group of p-length one Thomas D. (Zürich) has recently shown, that the answer is negative. In his proof D. uses some remarkable results on the spectral sequence

of a group extension (*) $E \longrightarrow P \longrightarrow\!\!\!> P/E$ where P is a p-group and E is an elementary abelian central subgroup of P . (i) The cohomology ring $H^*(P,k)$ contains a copy of $k[y_1,\dots,y_n]$, where $y_1,\dots,y_n$ is a basis of E . (ii) If Q is a p-group acting on the extension (*) and if the simple Q-module A appears in $H^*(P/E,k)$ then it appears in $H^*(P,k)$, also.

S.E. STONEHEWER:

Projectivities of Groups

The following Theorem has been proved recently by Rips:

1. Let $H \lhd G$ with $|G:H| = p$(prime) and suppose that there is a projectivity (i.e. lattice isomorphism) from G to a group $\bar{G}$ (Denote the image of $U \leq G$ by $\bar{U}$). Then $|G : H| = q$ (prime). Moreover if $\bar{H} \ntriangleleft \bar{G}$, then there are subgroups $K \geq L$ of G with $H \cap K = L \lhd G = HK$, $|H : L| = q$, $p = q$ oder $p \mid (q-1)$, $\bar{K} \lhd \bar{G}$, $|\bar{G} : \bar{K}| = r$ (prime), $r \mid (q-1)$ and $\bar{L} \lhd \bar{G}$. (A second and shorter proof has been given independently by Zacher.) Many long-standing questions about projectivities can now be answered. For example in 1976 Stonehewer proved that if M is a modular subgroup of a soluble group G with $\mathcal{L}(G:M) \cong \mathcal{L}(C_\infty)$, then $M \lhd G$. As a corollary of this result and 1 it is easy to prove.

2. Let $N \lhd G$ with $G/N \cong C_\infty \neq$ and let $\bar{G}$ be lattice-isomorphic with G . Then $\bar{N} \lhd \bar{G}$. Using work by Gross on quasinormal subgroups, it is also possible to prove.

3. Let $N \lhd G$ with G/N non-periodic and let $\bar{G}$ be lattice-isomorphic with G . Then $\bar{N} \lhd^2 \bar{G}$ and $\bar{N}$ modulo its core in $\bar{G}$ is abelian. Another immediate corollary of 1 is

4. Suppose that there is an index-preserving projectivity from G

to $\bar{G}$ and let $H \triangleleft G$ with $|G:H|$ prime. Then $\bar{H} \triangleleft \bar{G}$. There are many other corollaries of this type.

B.A.F. WEHRFRITZ:

<u>Groups whose irreducible representations have finite degree</u>

For a given field F let $\mathfrak{X}_F$ denote the class of all groups G for which every irreduzible FG-module has finite dimension over F . This talk we describe for an arbitrary field F all the soluble $\mathfrak{X}_F$-groups. There are five very different cases a) char F = 0 and F algebraically closed or real closed, b) char F = 0 but otherwise F not as in (a), c) char F > 0 and F algebraically closed but not locally finite, d) char F > 0 and F neither algebraically closed nor locally finite, e) F locally finite. In the first four cases our descriptions is complete. In case e) there remains a gap between our necessary and sufficient conditions.

H. WILLEMS:

<u>Vertices of irreducible modules in a p-soluble group</u>

By well known results of Brauer and Green, each defect group D of a p-Block B of a finite group G satisfies the following two conditions.

(i) $D = P \cap P^x$ where P is a Sylow p-subgroup of G and $x \in C_G(D)$,

(ii) $D = O_p(N_G(D))$

The subject of this talk is concerned with an extension of these properties to vertices of irreducible FG-modules where G is a finite p-soluble group and F a field of characteristic $p > 0$. Examples show that in case of arbitrary finite groups no extension is available.

J.S. WILSON:

Abelian subgroups of polycyclic groups

Some theorems were discussed relating the structure of a polycyclic group G to the structure of its Abelian subgroups.

Theorem 1. If all subnormal Abelian subgroups of G have torsion-free rank at most n then so do all Abelian subgroups of G .

Equivalently, if $\mathbb{Z}^r$ embeds in a polycyclic group G , then it also embeds in the Fitting subgroup Fitt G of G .

Theorem 2. If all subnormal Abelian subgroups of G have torsion-free rank at most n , then all subnormal Abelian subgroup of G/Fitt G have torsion-free rank at most n - 1 .

This bound is attained, but only for groups with very restricted structure.

Corollary 1. Under the hypotheses of Theorem 2, the torsion-free rank of G is at most $\frac{1}{2}(n^3 + 3n - 2)$.

Corollary 2. Under the hypotheses of Theorem 2, G has a finite by torsion-free nilpotent by torsion-free Abelian subgroup of index bounded by a function of n .

An example was discussed which illustrated some ideas occurring in the proof and the precise degree to which Dirichlet's units theorem is involved.

H. ZIESCHANG:

On some subgroups of the modular group

The subgroup of $Z_{n_1} * \ldots * Z_{n_k}$ which are generated by elements of finite order are classified, and if is described a method to determine all those subgroups to a given index. To each subgroup a "reduced" system of generators is constructed by Nielsen's method. This system is bijectively determined by the subgroup.

St.J. PRIDE:

Epimorphisms of Groups

Let G be a homomorphic image of the free group F . Two maps f,f' lie in the same T-system if $f' = \varphi f \vartheta$ for some $\varphi \in \mathrm{Aut}\ F, \vartheta \in \mathrm{Aut}\ G$. Let $f_i (i \in I)$ include a set of representatives for the T-systems, and let $N_i = \mathrm{Ker} f_i$. Define a category $\bar{\Phi}$ by: the objects are the N_i , and $N_i \xrightarrow{\varphi} N_j$ is a map if $\varphi \in \mathrm{Aut}\ F$ and $N_i \varphi \subseteq N_j$. Then G is hopfian if and only if $\bar{\Phi}$ is a groupoid. To compute the objects of $\bar{\Phi}$ one must compute the T-systems of G . In recent years several techniques have been developed for doing this when G is on HNN group or an amalgamated product. Here I will discuss a new technique which applies when G is a two-generator small concellation group. The technique can be used to obtain a description of all the two-generator subgroups of any finitely presented group satisfying a suitable small cancellation condition. To help compute the maps in $\bar{\Phi}$ I will discuss a very useful functor from $\bar{\Phi}$ to a category associated with a certain lie algebra. Using this functor it is possible to establish the hopficity of many groups. Finally, I will show how this work is related to computing the automorphisms of a group, and in determining whether or not two groups are homomorphic images of eachother.

Berichterstatter: Karsten Johnsen

Liste der Tagungsteilnehmer

Allenby, R.B.J.T.
Department of pure Mathematics
University of Leeds
Leeds LS 2 9 JT (England)

Amberg, B.
Fachbereich 17, Universität Mainz
6500 M a i n z

Arad, Z.
Department of Mathematics
Bar-Ilan University
Ramat-Gan (Israel)

Asar, A.O.
Department of Mathematics
The University of Manchester
Manchester (England)

Bass, H.
Ecole Normale Superieure,
45 rue d'Ulm
Paris 5
France

Baumslag, B.
Maths. Department
Imperial College
London SW 7
England

Baumslag, G.
Maths. Department
City-College, 138. Street
N e w Y o r k
N.Y. 10031 USA

Bender, H.
Math. Seminar Universität Kiel
2300 Kiel

Beyl, R.
Math. Institut
Im Neuenheimer Feld 288
6900 Heidelberg 1

Blessenohl, D.
Math. Seminar Universität Kiel
2300 Kiel

Brandis, A.
Math. Inst. d. Univ. Heidelberg
Im Neuenheimer Feld 288
6900 Heidelberg

Cannonito, F.B.
Department of Mathematics
University of California
Irvine, CA 92717
USA

Collins, D.J.
Ruhr Universität Bochum
Institut für Mathematik
Postfach 102148
4630 Bochum 1

Dark, R.S.
Mathematisches Inst.d.Univ.
Am Hubland
8700 Würzburg

Doerk, K.
FB Mathematik
Universität Mainz
Saarstr.21
6500 Mainz

Gaschütz, W.
Math.Sem.d.Univ.Kiel
2300 Kiel

Green, J.A.
Mathematics Institute, University
of Warwick
Coventry CV 4 7AL
ENGLAND

Gruenberg, K.
Queen Mary College
London
England

Grunewald, F.
SFB Theoretische Mathematik
Universität Bonn
Beringstr.4
53 Bonn 1

Heineken, H.
Math.Institut
Am Hubland
87 Würzburg

Holt, D.F.
Mathematics Institute
University of Warwick
Coventry England

Huppert, B.
FB Mathematik Universität Mainz
Saarstr.21
6500 Mainz

Kegel, O.
Albertstraße 23b
7800 Freiburg i.Br.

Kerber, A.
Lehrstuhl II für Mathematik
Postfach 3008
8580 Bayreuth

Laue, R.
Lehrstuhl D für Mathematik
der Techn.Hochschule Aachen
Templergraben 55
5100 A a c h e n

Lennox, J.C.
Mathematics Institute
University College, Po Box 78
Cardiff CFI IXL (Great Britain)

Levin, F.
Ruhr Universität
4630 Bochum 1

Linnell, P.A.
Department of Mathematics
Umist
PO Box 88
Manchester M60 1QD (England)

Lyndon, R.
15 Place Dauphine
75001 Paris
FRANKREICH

Mennicke, J.
Fakultät für Mathematik
Universität Bielefeld
Universitätsstr.1
48 Bielefeld 1

Neubüser, J.
Lehrstuhl D f.Mathematik
RWTH Aachen
Templergraben 64
5100 Aachen

Pahlings, H.
Lehrstuhl D f. Mathematik RWTH
Aachen, Templergraben 64
5100 Aachen

Phillips, R.E.
Michigan State University,
Dept. of Mathematics
East Lansing, Michigan 48824
USA

Poland, J.
Mathematics Institute
University of Warwick
Coventry CV4 7Al
ENGLAND

Pride, S.J.
Dept.of Math.University Gardens
Glasgow G 12 8QW
Scotland

Roggenkamp, K.W.
Universität Stuttgart
Mathematisches Institut B
7000 Stuttgart 80
Pfaffenwaldring 57

Roseblade, J.E.
Jesus College
Cambridge
England

Rosenberger, G.
Abteilung Mathematik der
Univ.Dortmund, Postfach 500500
4600 Dortmund 50

Schaller, K.U.
Math.Sem.d.Universität Kiel
2300 Kiel

Schmid, P.
Mathematisches Inst. d. Univ.
Auf der Morgenstelle 10
D-7400 Tübingen

Schmidt, R.
Math.Sem.d.Univ.Kiel
2300 Kiel

Segal, D.
Dept. of Mathematics UMIST
Po. Box 88, Manchester
M60 1QD,England

Stammbach, U.
Departement für Mathematik
ETH, 8092 Zürich
8092 Zürich
SCHWEIZ

Stonehewer, S.E.
Mathematics Institute Warwick
University
Coventry CV4 7AL
ENGLAND

Wehrfritz, B.A.F.
Pure Mathematics Dept.
Queen Mary College, Mile End Rd.
London E1 4 NS
ENGLAND

Willems, W.
Fachbereich Mathematik
Universität Mainz
6500 Mainz

Wilson, J.S.
Department of Pure Mathematics
and Mathematical Statistics
16, Mill Lane
C a m b r i d g e CB2 1SB
ENGLAND

Zieschang, H.
Institut für Mathematik
Ruhr Universität, Postf.102148
4630 B o c h u m