MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 22/1981

Quadratische Formen

18.5. bis 23.5.1981

Die dritte Tagung über quadratische Formen fand wie die früheren unter der Leitung der Herren Knebusch (Regensburg), Pfister (Mainz) und Scharlau (Münster) statt.

Die Schwerpunkte der Tagung lagen auf der arithmetischen und der algebraischen Theorie der quadratischen Formen, der Theorie reeller Körper und auf Aspekte der algebraischen Geometrie. Die starke internationale Beteiligung ermöglichte eine umfassende Darstellung aktueller Ergebnisse.

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Vortragsauszüge

J.K. ARASON: Extensions of symmetric bilinear spaces

An extension of a symmetric bilinear space $(\mathcal{F},\phi)$ over a scheme X by a vector bundle U over X is a symmetric bilinear space $(\mathcal{E},\psi)$ together with an embedding of U in $\mathcal{E}$ as a totally isotropic subbundle and an isomorphism between the induced symmetric bilinear space $(U^{\perp}/U,\overline{\psi})$ and $(\mathcal{F},\phi)$. We study the functorial properties of the set $\mathrm{Ext}((\mathcal{F},\phi),U)$ of equivalence classes of extensions of $(\mathcal{F},\phi)$ by U and the connections with a forgetful map $\mathrm{Ext}((\mathcal{F},\phi),U) \quad \mathrm{Ext}(\mathcal{F},U)$. In the case $\mathcal{F} = 0$ we obtain the set Met(U) of equivalence classes of metabolic spaces with U as a lagrangian. It turns out that Met(U) carries a natural group structure and that this group operates faithfully and transitively on the fibres of the forgetful map. If 2 is invertible, one can describe Met(U) and the image of the forgetful map in terms of the usual (linear) theory of extensions.

E. BAYER: Unimodular hermitian forms

Let K be a number field with a non-trivial $\mathbb{Q}$-involution $x \to \bar{x}$, and let F be the fixed field of this involution. Let A be the ring of integers of K. Assume that there exists an a in A such that $a + \bar{a} = 1$. Let L be a torsion free A-module of finite rank and let $h : L \times L \to A$ be a unimodular hermitian form. Let us consider the determinant of (L,h), i.e., the unimodular rank one form which is obtained by taking the nth exterior power of (L,h), where $n = \mathrm{rank}(L)$. Then we have:

Two indefinite unimodular hermitian forms are isometric if and only if they have the same rank, signatures and isometric determinants.

The proof uses the strong approximation theorem of G. Shimura.

E. BECKER: Summen n-ter Potenzen in Körpern und der reelle Holomorphiering

Sei K ein formal-reeller Körper, sei $n \in \mathbb{N}$ und sei

$P_{2n} := \min\{ k \mid$ jede Summe 2n-ter Potenzen in K ist bereits Summe von k 2n-ten Potenzen$\}$ oder ∞.

Sei H der reelle Holomorphiering von K, definiert als Durchschnitt aller Bewertungsringe von K mit formal-reellem Restklassenkörper. Es werden folgende Aussagen bewiesen:

Theorem 1: i) $H^{\cdot} \cap \Sigma K^2 = \{r \frac{s+q}{t+q} \mid r,s,t \in \mathbb{Q}_+^{\cdot},\ q \in \Sigma K^2\} \bigcap_n \Sigma K^{2n}$.

ii) $H^{\cdot} \cap \Sigma K^2 = \bigcap_n \Sigma K^{2n}$, wenn z.B. K ein Funktionenkörper über $\mathbb{Q}$, $\mathbb{R}$ ist.

iii) $\Sigma K^{2n} = (H^{\cdot} \cap \Sigma K^2)(\Sigma K^2)^n$.

Theorem 2: $P_2(K) = k < \infty \Rightarrow P_{2n}(K) \leq G(2n)k\binom{2n+k-1}{k-1}\binom{2n+2+k}{k}$

Hierbei ist $G(2n)$ die Waringkonstante für die Darstellung genügend großer Zahlen als Summen 2n-ter Potenzen natürlicher Zahlen. Für den Beweis werden alte Ideen von Siegel und Kamke mit Hilfe von Struktursätzen über H und der Theorie der Ordnungen höherer Stufe auf beliebige formal-reelle Körper übertragen.

J. BIERMANN: Gitter mit kleiner Automorphismengruppe in Geschlechtern von $\mathbb{Z}$-Gittern mit positiv-definiter quadratischer Form

M sei stets ein $\mathbb{Z}$-Gitter mit positiv-definiter quadratischer Form. Sei weiter $h(M)$ die Klassenzahl von M und $h_O(M)$ die Anzahl der Klassen der Gitter mit nichttrivialer Automorphismengruppe im Geschlecht von M. Dann gilt folgender Satz:

Zu $m \in \mathbb{N}$ gibt es ein $d(m) \in \mathbb{N}$ und $\ell(m) > 0$, so daß für $\operatorname{rg} M = m$, $(b(M,M)) = \mathbb{Z}$ und $dM > d(m)$ gilt:

$$\frac{h_O(M)}{h(M)} \leq (dM)^{-\ell(m)}$$

R. BOS: Value sets of binary forms under finite field extensions

Let $K = F(\sqrt{\ })$ be a quadratic extension, $d \in F \setminus F^2$. It is proved that the following sequence is exact:

$$1 \to \{1,d\}\dot{F}^2 \to D_F(<1,x>) \cdot D_F(<1,dx>)/\dot{F}^2 \to D_K(<1,x>)/\dot{K}^2 \xrightarrow{N}$$

$$\to (D_F(<1,x>) \cap D_F(<1,dx>))/\dot{F}^2 \to 1 .$$

Using this, results on the size of $D_F(<1,x>)/\dot{F}^2$ and

$D_L(\langle 1,x\rangle)/\dot{L}^2$, where L is a finite field extension of F , is deduced. If L/F is normal and $|\dot{F}/\dot{F}^2| = |\dot{L}/\dot{L}^2| < \infty$, then under certain additional assumptions it is proved that $W(F) \cong W(L)$.

J. BRZEZINSKI: Spinor genera of orders and quadratic forms

If (V,q) is a quadratic space over the field of fractions F of a Dedekind ring A , then each A-lattice L on V defines an A-order O(L) in the (even) Clifford algebra $C_0(V,q)$. It is proven that in the case of ternary quadratic spaces, the orders O(L) are precisely the Gorenstein orders in $C_0(V,q)$. We consider spinor genera of orders and for an A-order Λ , a group $SG(\Lambda)$, whose elements are in a 1-1 correspondence with the spinor genera of orders in the genus of Λ . In the case of global fields and algebras satisfying the Eichler condition, $SG(\Lambda) = (1)$ if and only if there is only one isomorphism class of orders in the genus of Λ . In the particular case of Gorenstein orders in quaternion algebras over the rational numbers, we prove the following result:

Theorem: Let Λ be a Gorenstein Z-order in a quaternion Q-algebra. Assume that for each prime number $p \neq 2$, $e_p(\Lambda) \neq 0$ or $e_p(\Lambda^d) \neq 0$ and if $e_2(\Lambda) = 0$ and $e_2(\Lambda^d) = 0$, then $2^6 \nmid d(\Lambda)$ or $2^6 \nmid d(\Lambda^d)$. Then $SG(\Lambda) = (1)$. (Λ^d is the dual order to Λ , $d(\Lambda)$ the discriminant, $e_p(\Lambda)$ a local invariant.)

As a consequence of this theorem, we get a slight generalization and an easy proof of a well-known theorem of A. Meyer.

L. BRÖCKER: Distribution of signatures of quadratic forms on real algebraic varieties

Let R be a real closed field and V an R-variety (complete integral in general). Divide $V(R)$ into semi-algebraic subsets (up to closed sets of lower dimension) and provide each subset with an integer to get a map $\{\text{subsets}\} \to \mathbb{Z}$, called a distribution of signatures. The problem was discussed under which conditions such a distribution can be solved. A solution means that there is a $p \in W(R(V))$ such that sign $p(x)$, $x \in V(R)$, is the given element of $\mathbb{Z}$, whenever this is defined. A complete solution is given in the case where $R = \mathbb{R}$ and V is smooth of dimension 2 .

J.-L. COLLIOT-THELENE: Quadratische Formen auf $k(t)$ und Kegelschnitt - Familien auf der projektiven Geraden

(gemeinsame Arbeit mit J.-J. Sansuc)

Sei k ein vollkommener Körper, sei $K = K(t)$, seien $a(t)$ und $b(t)$ nicht null in $k[t]$, sei $q = \langle\langle -a,-b \rangle\rangle$ die quadratische Form, die der reduzierten Norm $N : A \to K$ der Quaternionalgebra $A = \left(\frac{a\ b}{K}\right)$ entspricht. Sei

$K_q^* = \{f \in K^* \mid fq \perp -q \in \text{Bild}\,(W(k) \to W(K))\}$ und sei

$k_q^* = \{\alpha \in k^* \mid \alpha q \perp -q \in \text{Bild}\,(W(k) \to W(K))\}$. Sei schließlich L der Funktionenkörper der Fläche $y^2-a(t)z^2-b(t) = 0$, und sei X ein glattes projektives Modell dieser Fläche. Beim Studium dieser Fläche treten die Gruppen K_q^*/NA^* und $K_q^*/k_q^*NA^*$ auf.

Satz: a) Wenn $X(k) \neq \emptyset$ ($\Longleftrightarrow$ L hat eine k-Stelle $L \to k$) , dann ist $K_q^*/NA^* = 0$.

b) Wenn $k = \mathbb{R}$, ein $\mathfrak{p}$ -adischer Körper oder ein Zahlkörper ist, dann gilt $K_q^*/k_q^*NA^* = 0$.

Kommentare:

1) Bei $k = \mathbb{C}((u))((v))((w))$ kann man ein Beispiel angeben, mit $K_q^*/k_q^*NA^* \neq 0$.

2) Aus dem Satz folgt, daß die reduzierte Chow-Gruppe $A_0(X)$ endlich ist, falls k eine endlich erzeugte Erweiterung von $\mathbb{Q}$ ist, und $X(k) \neq \emptyset$ ist: Dies hat Bloch bewiesen, wenn k ein Zahlkörper ist.

3) Man kann ein ähnliches Problem wie oben bei $q = \langle\langle -a(t)\rangle\rangle$ (binär) studieren. Hier hängt das Problem mit der Arithmetik der Kurve $y^2 = a(t)$ zusammen. Es gilt das Analoge zu a) . Man kann Gegenbeispiele zu b) geben, falls $k = \mathbb{Q}_p$.

4) Anschließend wurde eine Frage bezüglich quadratischer Formen auf $k(t)$, k ein Zahlkörper, gestellt. Eine positive Antwort zu dieser Frage hätte eine direkte Anwendung auf die Berechnung von $A_0(X)$, im Falle daß k ein Zahlkörper ist.

D. CORAY: A remark on systems of quadratic forms

The following three quadratic forms have no non-trivial common zero in the finite field $\mathbb{F}_3$:

$$f_1 = X_1X_5+X_2X_3-X_2X_4-X_3^2$$

$$f_2 = X_1X_4+X_2^2-X_3X_5+X_6^2$$

$$f_3 = X_1^2-X_1X_3+X_1X_4+X_1X_5-X_2X_3+X_2X_5+X_3X_4-X_4^2+X_4X_5+X_5^2$$

However, they have a non-trivial common zero in every proper algebraic extension of $\mathbb{F}_3$. Indeed, the variety defined by these equations has the exceedingly nice property that it is a smooth K3 surface in $\mathbb{P}^5_{\mathbb{F}_3}$! This example has been constructed with the method of a recent paper (Archiv der Mathematik, 34 (1980), 403-411). From it we derive examples in 12 variables over $\mathbb{Q}_3$ and over $\mathbb{Q}$ with non-trivial common zeros in some odd-degree extensions, but none in the ground field, thereby answering a question of Pfister.

T.C. CRAVEN: Higher level orderings and semilocal rings

We define signatures of higher level for semilocal rings. This extends both the usual definition of signatures on semilocal rings defined by Knebusch, Rosenberg and Ware and the concept of ordering of higher level in a field developed by Becker. These signatures take the form of homomorphisms from a Witt ring of higher level to the ring of integers in a cyclotomic field of 2^nth roots of unity. Given a ring A and a signature, one obtains a corresponding subgroup P of A^* , the group of units of A , such that A^*/P is cyclic of order 2^n . At least for local rings with 2 a unit, there always exists a prime ideal p of A such that the signature extends uniquely to A(p) , the field of fractions of A/p . Using this prime ideal to lift results concerning fields to semilocal rings, we are able to define and prove the existence of chains of orderings (defined similarly to those of Harman for fields) and chain closures (making use of the notion of real closures for rings developed by Knebusch).

A.G. EARNEST: Representation Properties of Integral Ternary Quadratic Forms

An integer c is (primitively) exceptional for a genus G of indefinite integral ternary quadratic forms if c is (primitively) represented by some, but not all (and consequently, by exactly half) of the inequivalent classes in G. From the work of Jones-Watson, Kneser, Hsia and Schulze-Pillot has emerged a theoretical basis for the identification of these integers by essentially local criteria. If c is primitively exceptional for G and t is a positive integer relatively prime to twice the discriminant d of G, then ct^2 is also primitively exceptional for G. It is proven that for a form f in G, f itself primitively represents c if and only if f primitively represents all such integers ct^2 for which the Jacobi symbol $(-cd/t)$ equals $+1$. Several ramifications of this result, which is a special case of a theorem on "spinor exceptional integers" of a general ternary genus, are discussed.

R. ELMAN: Galois cohomology

It is shown how parts of Arason's thesis easily generalize and thereby lead to an easy proof of the theorem of S. Bloch which asserts the natural maps $K_2F/(K_2F)^n \to H^2(\mathrm{Gal}(F_s/F),\mu_n)$ and $K_2F(t) / (K_2F(t))^n \to H^2(\mathrm{Gal}(F(t)_s/F(t),\mu_n)$ have isomorphic kernels and cokernels whenever char $F \nmid n$ and $\mu_n \subset F$. Joint work with Bill Jacob was also discussed. In particular it was announced that $GWF \tilde{\to} H^*(\mathrm{Gal}(F_s/F),\mu_2)$ if char $F \neq 2$ and if L is linked for every $F_s|L|F(\sqrt{-1})$ (e.g. F local, global, or $\mathrm{trdeg}_{\mathbb{R}}(F) \leq 2$.

L. GERSTEIN: Orthogonal splitting of quadratic forms

Let $\Theta = \Theta(s)$ be a Hasse domain of a global field F, with char $F \neq 2$. In earlier work it was shown that there exists a natural number $n_0(\Theta)$ such that the genus of every quadratic Θ-lattice L of rank $\geq n_0$ contains a split lattice; and in the

indefinite case there is in fact a splitting $L = L_1 \perp L_2$.
The choice $n_0 = 7$ always works when F is a (global) function field. In the lecture, conditions are given under which $n_0 = 5$ suffices; in particular, this is so when the ideal class group $C = C(\Theta)$ has odd order. Finally, an example of a genus of indecomposable lattices is exhibited in the following setting:
$F = k(x)$, where k is a finite field in which -1 is not a square; $\mathfrak{q}$ is the $\mathfrak{p}$-adic prime, where $\mathfrak{p} = x^2+1$; and $\Theta = \bigcap_{\mathfrak{p} \neq \mathfrak{q}} \Theta(\mathfrak{p})$.
(Here $\# C = 2$.) This shows the number $n_0 = 7$ to be best possible for global function fields in general.

H. GROSS: The lattice method in the theory of quadratic spaces of non-denumerable dimensions

There is exactly one isometry class of regular symmetric bilinear forms of dim n over $\mathbb{C}$ if n is finite or $n = \aleph_0$ but there are $2^{\aleph_1}$ classes if $n = \aleph_1$. Thus in non-denumerable dimensions there are other features besides the arithmetic of the base field that are relevant. Among the $2^{\aleph_1}$ classes there is 1 class of diagonal forms, $\aleph_0$ classes of prediagonal forms (i.e. subforms of diagonal forms) and hence $2^{\aleph_1}$ classes of indecomposable forms (i.e. not prediagonal). The classes of prediagonal forms are completely characterized by one single cardinal number d with $0 \leq d \leq \aleph_1$; the diagonal class has $d = 0$. Classification of prediagonal spaces is accomplished by theorems of the following type:

Theorem: Let $\eta : V \to V'$ be a lattice isomorphism between lattices of subspaces in (E,φ) $(\dim E = \aleph_\alpha , \alpha > 0)$ preserving dimensions. Assume that V,V' are stable under $\perp$ and the closure operators $x \to \bar{x}^{(\gamma)}$ for Ogg's topologies $(\gamma \in [0,\omega_\infty[)$ and that η commutes with these operations in V,V' . Then in order that η be induced by an element of the orthogonal group of E, it suffices that:

i) V is finite and distributive,

ii) $\dim E < \aleph_{\omega_1}$.

H.A. KELLER: Nicht-klassische, hilbertsche Räume

Ein nicht-ausgearteter, symmetrischer Bilinearraum (E,ϕ) (über beliebigem Körper K mit char $(K) \neq 2$) heißt hilbertsch, falls gilt: (*) $F \subseteq E$, $F = F^{\perp\perp} \Rightarrow E = F \oplus F^{\perp}$. Im Fall unendlicher Dimension ist (*) eine äußerst starke Forderung, lange Zeit war außer den klassischen Hilberträumen kein anderes Beispiel bekannt. Kürzlich sind neuartige Räume (E,ϕ) mit (*) entdeckt worden. Solche Räume werden dargestellt, zunächst solche über (nicht-archimedisch) angeordneten Körpern, dann über bewerteten Körpern. Es werden die zentralen Punkte der Konstruktion (Begriff der Typen, Konvergenzverhalten der Reihen, welche Vektorlängen geben) besprochen. Schließlich wird ein starkes Charakterisierungstheorem für hilbertsche Räume aufgestellt.

M. KNESER: Komposition binärer quadratischer Formen

Sei q eine quadratische Form auf einem projektiven R-Modul vom Rang 2, $\ker q = \{x \in M^{\perp} \mid q(x) = 0\}$. M heißt pbq-Modul, falls $Rq(M) = R$ ist. Eine Kompositionsabbildung $\mu : M_1 \times M_2 \to M$ ist bilinear und multiplikativ: $q(\mu(x_1,x_2)) = q_1(x_1)q_2(x_2)$.

Satz 1: Ist μ eine Kompositionsabbildung zwischen pbq-Moduln M_1, M_2, M und $\ker M_1 = \{0\}$, so gibt es eindeutig bestimmte Homomorphismen $\gamma_i : C^+(M_i) \to C(M)$ derart daß: (*) $\mu(c_1x_1, c_2x_2) = \gamma_1(c_1)\gamma_2(c_2)\,\mu(x_1,x_2)$ gilt für $x_i \in M_i$, $c_i \in C^+(M_i)$.

Ein pbq-Modul M heißt vom Typ C , wenn ein Isomorphismus zwischen $C^+(M)$ und einer quadratischen Algebra C gegeben ist. Sind M_i vom Typ C_i und $\gamma_i : C_i \to C$ Algebrenhomomorphismen, so heißt μ vom Typ (γ_1, γ_2) wenn (*) für $x_i \in M_i$, $c_i \in C_i$ gilt.

Satz 2: In gegebenen pbq-Moduln M_i vom Typ C_i und Homomorphismen $\gamma_i : C_i \to C$ gibt es einen (bis auf Isomorphie) eindeutig bestimmten pbq-Modul M vom Typ C und Komposition μ vom Typ (γ_1, γ_2) .

Satz 3: Die Isomorphieklassen der pbq-Moduln vom Typ C bilden eine abelsche Gruppe G(C) , und es gibt eine exakte Sequenz $C^* \to R^* \to G(C) \to \text{Pic}\ (C) \to \text{Pic}\ (R)$.

M.A. KNUS: Quadratic spaces over the affine plane and the projective plane
(joint work with R. Parimala, R. Sridhoran)

If K is a field of char $\neq 2$, then any quadratic space over $\mathbb{A}_2(K)$ admits an extension to $\mathbb{P}_2(K)$ and the extension is unique (up to isometries) if the given space is anisotropic. The analogue result is valid for hermitian forms (with respect to an involution of K). It follows that Krull-Schmidt is valid for positive definite spaces (resp. hermitian forms) over $\mathbb{R}[X,Y]$ (resp. over $\mathbb{C}[X,Y]$). One can associate with each non-free projective ideal a 2-bundle over $\mathbb{P}_2(\mathbb{C})$, called $\mathfrak{p}$-bundle. They are stable and of type (0,2m) (i.e., first Chern number $c_1 = 0$, the second $\equiv 0 \pmod 2$). For any even integer m , there exist $\mathfrak{p}$-bundles with $c_2 = 2m$. Using Barth's classification of stable bundles of type (0,2) , it is shown that for $\mathfrak{p}$-bundles with $c_2 = 2$ the construction is independent of the chosen embedding $\mathbb{A}_2 \to \mathbb{P}_2$ and a complete classification of these bundles is given. Finally, it is shown that the extension of quadratic bundles from $\mathbb{A}_2$ to $\mathbb{P}_n$ is not possible for every n .

O. KÖRNER: <u>On the arithmetic of quaternion algebras</u>

Let A be a quaternion algebra over a local field F and M be an order of A . We show M to be isomorphic to a subring M' of M(2,K) , where K is a suitable 2-dimensional semi simple F-algebra, and M' is essentially constructed of a pair of orders of K in an explicit manner. Among the orders M' a complete system of representatives for the classes of isomorphy of all orders of A is found by means of the results of the theory of quadratic forms using the fact that A is a quadratic F-space relative to the norm and that two orders of A are isomorphic if and only if they are isometric as lattices on A . These local results can be applied to the computation of class numbers in global situations, e.g., the class number of all left ideals of M is obtained if A is a definite quaternion algebra over the field of rational numbers.

M. KULA: <u>Axiomatic theory of quadratic forms</u>

A quadratic form scheme /q.f. scheme/ is a triple (g,-1,d) , where g is a group of exponent 2 with distinguished element -1 and d is a function from g into the family of all subgroups of g , satisfying some natural axioms. A scheme homomorphism from (g,-1,d) to (g',-1',d') is a group homomorphism f: g → g' such that f(-1) = -1' and $f(d(a)) \subset d'(f(a))$ for all $a \in g$. The class of q.f. schemes and scheme homomorphisms forms a category. This talk will be a survey of various constructions in this category. In particular, it will be a characterization of those pythagorean q.f. schemes. The last result is due to A. Sladek and it is still unpublished.

T.-Y. LAM: Quadratic forms via Stiefel manifolds

(joint work with Z.D. Dai)

Topological methods can be used effectively to solve certain problems in the theory of quadratic forms. For example, a trivial proof using the Borsuk-Ulam Theorem shows immediately that $A = \{f : S^{n-1} \to \mathbb{C} \mid f \text{ continuous and } f(-x) = \overline{f(x)}\}$ has level n. In the talk it is shown that the above idea leads to a systematic way of studying decompositions $n\langle 1\rangle \cong r\langle 1\rangle \perp t\langle -1\rangle \perp \ldots$ over affine algebras by exploiting equivariant maps between spaces with involutions and Stiefel manifolds. In particular, various well-known properties of Stiefel manifolds may be used to obtain remarkable results on certain "generic" affine algebras arising naturally in quadratic form theory.

T.-Y. LAM: Pythagorean numbers

The pythagorean number $p(A)$ of a ring A is defined as $p(A) = \sup\{n \mid a \in \Sigma A^2 \Rightarrow a = \sum_{i=1}^{n} a_i^2\}$. A is formally real if $\sum_{i=1}^{n} a_i^2 = 0 \Rightarrow a_1 = \ldots = a_n = 0$. If A is an affine algebra over $\mathbb{R}$ with $\mathrm{trd}_{\mathbb{R}} \le 1$, it is shown that $p(A) < \infty$. If $A = k[x,y]$ has $\mathrm{trd} = 2$ over the formally real field k, then $p(A) = \infty$.

Main Theorem: Let k be a field, A a formally real affine k-algebra of $\mathrm{trd}_k A \ge 3$, then $p(A) = \infty$.

The pythagorean numbers of power series rings over $\mathbb{R}$ and $\mathbb{Z}$ are calculated, obtaining the following table of pythagorean numbers:

n	1	2	3	4
$\mathbb{R}[X_1,\ldots,X_n]$	2	∞	∞	∞
$\mathbb{R}(X_1,\ldots,X_n)$	2	4	≤ 8	≤ 16
$\mathbb{R}[[X_1,\ldots,X_n]]$	1	2	∞	∞
$\mathbb{Z}[X_1,\ldots,X_n]$	∞	∞	∞	∞
$\mathbb{Q}(X_1,\ldots,X_n)$	5	??	??	??
$\mathbb{Z}[[X_1,\ldots,X_n]]$	5	∞	∞	∞

D. LEEP: Systems of quadratic forms over non-real fields

Assume char $F \neq 2$, $u(F) < \infty$. Say that a system of r quadratic forms $q_1,\ldots,q_r$ is an anisotropic system if $q_1,\ldots,q_r$ have no nontrivial common zero. Let $U_r(F) = \max\{m \mid$ there exists an anisotropic system of r quadratic forms in m variables$\}$. Then Theorem 1 says $U_r(F) \leq U_{r-1}(F) + rU_1(F)$. It follows that a system of r quadratic forms in more than $\frac{r(r+1)}{2} U_1(F)$ variables has a nontrivial common zero. If F is a $\mathfrak{p}$-adic field, then $U_1(F) = 4$ so r forms in more then $2r^2 + 2r$ variables have a nontrivial common zero. Theorem 2 states if $[K:F] = r$, then $U_1(K) \leq \frac{rH}{2} U_1(F)$. Theorem 1 is best possible when $r = 1,2,3$. Theorem 2 is best possible for $r = 1,3$. The case $r = 2$ in Theorem 2 leads to a very hard open problem.

M. MARSHALL: Sheaves of reduced Witt rings

Suppose I is a Boolean space and for each $i \in I$ that (X_i,G_i)

is a space of orderings. Then there exists a space of orderings (X,G) such that each (X_i,G_i) is a subspace of (X,G), $X = \bigcup_{i\in I} X_i$, $\pi: X \to I$ is continuous, and each non-trivial fan in X lies wholly in some X_i. Note, by the representation theorem, this implies that G consists of all continuous $g: X \to \pm 1$ satisfying $g|X_i \in G_i \ \forall\, i \in I$, and the Witt ring $R(X,G)$ consists of all continuous $g: X \to \mathbb{Z}$ satisfying $g|X_i \in R(X_i,G_i) \ \forall\, i \in I$ and $g(\sigma) \equiv g(\tau) \bmod 2 \ \forall\, \sigma, \tau \in X$. (X,G) is not, in general, unique although this will be the case if $|I| < \infty$; then (X,G) is just the direct sum. For example, if $I = \mathbb{N} \cup \{\infty\}$, the one-point compactification of the natural numbers, (X_i,G_i) is a 4-element fan for $i \neq \infty$, and (X_∞,G_∞) is a 2-element fan, then there are five choices for (X,G). It would be nice to know that if each (X_i,G_i) is realized as the space of orderings of a field, then (X,G) is too. So far this has only been shown in the SAP case, and in the case $|I| < \infty$, both results of T. Craven.

J. MERZEL: <u>The square-class graph of a preordered field</u>

For a field F with preordering T, a partial ordering is defined on $F^\bullet/T^\bullet$. In the case $|F^\bullet/T^\bullet| < \infty$, this is shown to impart a lattice structure to $F^\bullet/T^\bullet$; the resulting family of lattices is characterized inductively.

A. MICALI: <u>Algèbres quadratiques et formes quadratiques</u>

(joint work with A. Paques)

Soit K un anneau commutatif à élément unité et soient (A,f,e) et (A',f',e') deux K-algèbre quadratiques libres sur K (Nous nous bornerons ici au cas où $e = f(e) = 1$,

$e' = f(e') = 1')$ de bases $\{1, e_1, \ldots, e_m\}$ et $\{1', e'_1, \ldots, e'_n\}$ respectivement et φ (resp. φ') la forme K-bilinéaire symétrique associée à f (resp. f'). Il existe sur A (resp. A') un K-automorphisme linéaire $\sigma: A \to A$ (resp. $\sigma': A' \to A'$) donné par $\sigma(e_i) = \alpha_i - e_i$ (resp. $\sigma'(e'_i) = \alpha'_i - e'_i$) où $\alpha_i = \varphi(e_i, 1)$ (resp. $\alpha'_i = \varphi'(e'_i, 1')$. On considère le sous-K-module libre de $A \otimes_K A'$ défini par $A * A' = (A \otimes_K A')^{\sigma \otimes \sigma'}$ dont une base est donnée par $\{1 \otimes 1', e_{ij}, 1 \le i \le m, 1 \le j \le n\}$ avec $e_{ij} =$ $= \alpha_i \otimes e'_j + e_i \otimes \alpha'_j - 2e_i \otimes e'_j$. Sur le K-module libre $A * A'$, il existe une forme quadratique F telle que si A et A' sont des K-algèbres commutatives, alors $A * A'$ est muni d'une structure naturelle d'algèbre quadratique pour la forme F. On fait une étude exhaustive de l'operation $*$.

M. OJANGUREN: Die Wittgruppe eines regulären Ringes

Satz 1: Sei A ein regulärer Ring mit $2 \in A$ und $q(A)$ sein Quotientenkörper. Ist $\dim(A) < 4$, so ist die kanonische Abbildung $W(A) \to W(q(A))$ ein Monomorphismus.

Ein Gegenbeispiel für $\dim(A) = 4$ wird angegeben, jedoch gilt allgemein folgender Satz:

Satz 2: Ist K ein Körper der Charakteristik $\neq 2$, B eine affine K-Algebra, m ein reguläres maximales Ideal von B und $A = B_m$, so ist $W(A) \to W(q(A))$ injektiv.

Der Beweis benutzt eine Methode von H. Lindel.

P. REVOY: Forme quadratique trace dans le extensions separables

One can associate with an extension of fields, finite and separable, a quadratic form via the trace map. In the characteristic 2 case, one takes the trace of the second exterior power of the multiplication so that one has the alternate form $(x,y) \mapsto \mathrm{tr}_{L|K}(x)\, \mathrm{tr}_{L|K}(y) - \mathrm{tr}_{L|K}(xy)$. In the odd dimensional case, one restricts to the kernel of the trace map. So in all cases an Arf invariant can be defined, which appears to be quite related with Berlekamp's analog of the discriminant. In many cases it is proven that these invariants are either equal or differ from 1, depending on the dimension of L over K modulo 8. Also the quadratic form which is defined via the reduced trace for central simple algebras is studied. Simple results can be obtained for the odd dimensions. In the even case, the signature determines the 2-component of the class of the algebra in $Br(K)$ if K is a number field.

J.-J. SANSUC: On Gersten's spectral sequence in L-theory
(joint work with J. Barge and P. Vogel)

Theorem. Let A be a commutative noetherian domain of finite dimension d. Assume A is regular and 2 is a unit. There exists a cohomological spectral sequence

$$E_1^{p,q} = \bigoplus_{ht\mathfrak{p}=p} L_{-q}(\mathbb{K}(\mathfrak{p})) \Rightarrow L_{-p-q}(A)$$

where $\mathfrak{p}$ runs through the prime ideals of A and $\mathbb{K}(\mathfrak{p})$ is the residue class field at $\mathfrak{p}$.

Corollary. Moreover, if $\dim A \leq 4$ and A is local with maximal ideal $\underline{m}$, there exists an exact sequence:

$$W(A) \xrightarrow{\alpha} W(K) \to \bigoplus_{ht\mathfrak{p}=1} W(\mathbb{K}(\mathfrak{p})) \to \bigoplus_{ht\mathfrak{p}=2} W(\mathbb{K}(\mathfrak{p})) \to \sum_{ht\mathfrak{p}=3} W(\mathbb{K}(\mathfrak{p})) \xrightarrow{\beta} W(A/\underline{m})$$

where ker α = coker β .

If A is Dedekind, there exists an exact sequence:

$$0 \to W(A) \to W(K) \to \bigoplus_{\underline{m}\ \max} W(a/\underline{m}) \to L_3(A) \to 0 .$$

If dim A $\leq$ 3 and A is global regular, there is an exact sequence

$$0 \to W(A) \to W(K) \to \bigoplus_{ht\mathfrak{p}=1} W(\mathbb{K}(\mathfrak{p})) .$$

Let $\underline{C}$ be the category of bounded complexes $C = (C_n)_{n\in\mathbb{Z}}$ of A-projective modules of finite type. The proof uses groups $L_n(\underline{A},\underline{B})$ analogous to those of Ranicki, defined for subcategories $\underline{C} \supset \underline{A} \supset \underline{B} \supset$ (acyclic complexes). It uses a filtration $\underline{C} = \underline{C}^0 \supset \underline{C}^1 \supset \ldots \supset \underline{C}^d \supset \underline{C}^{d+1}$ = {acyclics} given by the co-dimension of the support (here supp (C) = = $\{\mathfrak{p} \mid C_{\mathfrak{p}}$ not acyclic$\}$).

R. SCHARLAU: Quadratic matrix problems

Es wurden zwei Beispiele der "Matrizen-Theorie" behandelt, die nicht als Klassifikationsprobleme in additiven Kathegorien, sondern als Klassifikationsprobleme für "hermitesche Formen in abelschen Kategorien" im Sinn von [QSSS] anzusehen sind, nämlich beliebige Bilinearformen (V,b) ohne Symmetrie-Bedingung und Isometrien von symmetrischen Formen (V,b,σ) über einem festen Körper K . Im 1. Schritt wird reduziert auf den "isotypischen" Fall, d.h. V als K[X]-Modul bzgl. σ bzw. der "Asymmetrie" σ_b von b ist von der Form A^n , $A = K[X]/(p^r)$ für "selbstduales" $p \in K[X]$.

Im 2. Schritt wird mittels der Verlagerung bezüglich einer "regulären Spur" $s: A \to K$ das Problem reduziert auf die Klassifikation der "klassischen Typen" von Formen über K und endlichen Erweiterungen.

[QSSS]: Quebbemann, Scharlau, Scharlau, Schulte in Bull. Soc. Math. France Mem. 48 (1976).

J.P. TIGNOL: Non 1-amenable fields and indecomposable division algebras with involution

(joint work with R. Elman, T.-Y. Lam and A.R. Wadsworth)

Let F be a field of char $\neq 2$. For any $u \in F^{\bullet}$, let $N(u)$ be the group of elements of $F^{\bullet}$ represented by the Pfister form $\langle\langle -u \rangle\rangle$. It is shown that if a,b,c are elements of $F^{\bullet}$ and if there exists $t \in [N(a) . N(c)] \cap [N(b) . N(c)] \cap [N(ab) . N(c)]$ such that the system $U^2 - aV^2 = W^2 - bX^2 = t(Y^2 - cZ^2)$ only has the trivial solution, then the kernel of the map from the Witt ring of F to the Witt ring of $F(\sqrt{a},\sqrt{b},\sqrt{c})$ is not generated by the Pfister forms $\langle\langle -a \rangle\rangle$, $\langle\langle -b \rangle\rangle$ and $\langle\langle -c \rangle\rangle$, thus, F is not 1-amenable.

The method of proof is to compare a complex of Witt rings with a complex of Brauer groups which was first used by Amitsur et al to construct an indecomposable division algebra of degree 8 with involution. This result is then used to show that the field of rational fractions in two indeterminates over any field of char 0 is not 1-amenable.

T.M. VISWANATHAN: The u-invariant and Galois theory

Let Ω be an algebraically closed field, $x_1,\dots,x_n \in \Omega$ and k a subfield of Ω maximal with respect to the exclusion of these n elements. In the case $n = 3$ the following theorem is proven:

Theorem: i) Every finite extension of k is normal with cyclic Galois group; or

ii) k is perfect, the algebraic closure $\bar{k}$ of k coincides with the quadratic closure and $|k^{\cdot} : k^{\cdot 2}| = 4$.

Further, if k is non-real, one has one of the following situations:

i) $\mathrm{Gal}\,(\bar{k}/k)$ is the free pro-2-group on 2 free generators.

ii) $\mathrm{Gal}\,(\bar{k}/k) = \langle s,t\rangle$, s,t generate $\mathbb{Z}_2$ and commute.

iii) $\mathrm{Gal}\,(\bar{k}/k) = \mathbb{Z} \rtimes \mathbb{Z}_2$, generated by y and t respectively, $sts^{-1} = t^{-1}$.

iv) $\mathrm{Gal}\,(\bar{k}/k) = \mathbb{Z}_2 \rtimes \mathbb{Z}_2$, $sts^{-1} = t^{2^m+1}$, for some $m \geq 2$.

v) $\mathrm{Gal}\,(\bar{k}/k) = \mathbb{Z}_2 \rtimes \mathbb{Z}_2$, $sts^{-1} = t^{2^m-1}$, for some $m \geq 2$.

J. YUCAS: Quaternionic structures, rigid elements and non-real preorders

Let $q: G \times G \to B$ be a quaternionic structure. A subgroup T of G containing -1 is called a non-real preorder if for every $-1 \neq t \in T$, $D\langle 1,t\rangle \leq T$. In the note the classification of non-real preorders is studied, i.e., the conjecture is discussed that there are only two types of non-real preorders:

i) subgroups T of G containing -1 such that a is rigid $\forall -1 \neq a \in T$ and

ii) subgroups T of G containing the basic part of G.

Berichterstatter: Heinz-Werner Schülting

Liste der Tagungsteilnehmer

Prof. Dr. J. K. Arason
University of Iceland/Science Inst
Dunhaga 3
Reykjavik
Island

Prof. Dr. R. P. Brown
University of Hawaii
Dep. of Mathematics
Honolulu Hi 96822
USA

Prof. Dr. A. Bak
Math. Institut der
Universität
48oo Bielefeld

Prof. Dr. D. Coray
Math. Inst.
Universite, CP 124
CH-1211 Genf 24
Schweiz

Prof. Dr. H. J. Bartels
Mathematisches Institut der
Universität
34oo Göttingen

Dr. H. Delfs
Fakultät für Mathematik der
Universität
Universitätsstr. 31
84oo Regensburg

Dr. Eva Bayer
Mathematisches Institut der
Universität , CP 124
CH-1211 Genf 24
Schweiz

Prof. Dr. R. Bos
Math. Instituut
Rijksuniversiteit
Budapestlaan 6
35o8 TA Utrecht, Niederlande

Prof. Dr. E. Becker
Abt. Mathematik der
Universität
Postfach 5oo5oo
46oo Dortmund

Prof. Dr. L. Bröcker
Math. Institut der Universität
Einsteinstr. 64
44oo Münster

Dr. J. Biermann
Math. Institut der
Universität
Bunsenstr. 3-5
34oo Göttingen

Prof. Dr. J. L. Colliot-Thelene
C.N.R.S. Mathematique
Batiment 425
Universite de Paris-Sud
F-914o5 Orsay
France

Prof. Dr. J. Brzezinski
Math. Inst.
Chalmers University
Goteborg
Schweden

Prof. Dr. T. Craven
University of Hawaii
Dep. of Mathematics
Honolulu Hi 96822
USA

Prof. Dr. A. G. Earnest
University of Southern California
Dep. of Mathematics
Los Angeles CA 9ooo7
USA

Prof. Dr. R. Elman
University of California
Dep. of Mathematics
Los Angeles CA 9oo24
USA

Prof. Dr. L.J. Gerstein
University of California
Dep. of Mathematics
Santa Barbara, Ca 931o6
Usa

Prof. Dr. H. Gross
In der Halden 2
CH-86o3 Schwerzenbach
Schweiz

Prof. Dr. M. Kassner
Fachbereich Mathematik der
Universität
Saarstr. 21
65oo Mainz

Prof. Dr. H. A. Keller
Math. Institut der
Universität
Freiestr. 36
CH-8o32 Zurich

Prof. Dr. M. Knebusch
Fakultät für Mathematik der
Universität
Universitätsstr. 31
84oo Regensburg

Prof. Dr. M. Kneser
Math. Institut der
Universität
Bunsenstr. 3-5
34oo Göttingen

Prof. Dr. M. A. Knus
Math. Seminar der
ETH Zürich
CH-8o92 Zürich

Dr. M. Kolster
Math. Institut der
Universität
Einsteinstr. 64
44oo Münster

Prof. Dr. O. Körner
Abt. für Mathematik IV der
Universität
Oberer Eselsberg
79oo Ulm

Prof. Dr. M. Kula
Math. Institut
Universität Katowice
Katowice
Polen

Prof. Dr. T. Y. Lam
University of California
Dep. of Mathematics
Berkeley CA 9472o
USA

Dr. D. Lee
University of Chicago
Dep. of Mathematics
Chicago, Illinois [illegible]
USA

Prof. Dr. F. Lorenz
Math. Institut der Universität
Roxelerstr. 64
4400 Münster

Prof. Dr. J. Merzel
5645 Miles Ave. Apt. 2
Oakland, CA 94618
USA

Prof. Dr. M. Marshall
University of Saskatchewan
Saskatoon
Saskatchewan S7N0W0
Canada

Dr. W. Meißner
Abt. Mathematik der
Universität
Postf. 500 500
4600 Dortmund

Prof. Dr. A. Micali
Math. Institut
Universite
Monpellier
France

Prof. Dr. M. Ojanguren
Route de la Plaine 5
CH-1022 Chavannes
Schweiz

Prof. Dr. A. Paques
Math. Institut
Universite
Montpellier
France

Prof. Dr. R. Parimala
University
Dep. of Mathematics
Bombay
India

Prof. Dr. M. Peters
Math. Institut der
Universität
Einsteinstr. 64
4400 Münster

Prof. Dr. A. Pfister
Fachbereich Mathematik der
Universität
6500 Mainz

Prof. Dr. A. Prestel
Hoheneggstr. 13
7750 Konstanz

Prof. A. Ranicki
Princeton University
Dep. of Mathematics
Princeton NJ 08544
USA

Prof. Dr. P. Revoy
Universität, Math. Institut
Montpellier
France

Prof. Dr. C. Riehm
Mc Master University
Dep. of Mathematics
Hamilton, Ontario
Canada

Prof. Dr. J. J. Sansuc
E.N.S.
45 rue d'Ulm
F-7523o Paris Cedex o5
France

Dr. R. Scharlau
Fakultät f. Mathematik der
Universität
Universitätsstr. 25
48oo Bielefeld 1

Prof. Dr. W. Scharlau
Math. Institut der
Universität
Einsteinstr. 64
44oo Münster

Dr. H. W. Schülting
Abt. Mathematik der
Universität
46oo Dortmund
Postfach 5oo5oo

Prof. Dr. D. B. Shapiro
Ohio State University
Dep. of Mathematics
Columbus, OH 4321o
USA

Prof. Dr. J. P. Tignol
Universite Catholique
B-1348 Louvain-la-Neuve
Belgien

Dr. A. Tsch[illegible]
Math. Institut der
Universität
Einsteinstr. 6[illegible]
44[illegible] Münster

Prof. Dr. T. M. Viswanathan
Inst. for pure and appl. math
Rua Luis de Camoes 68
Rio de Janeiro
Brazil

J. L. Yucas
3o9 Mc AlluterBlvd.
University Park, PA 168o2
USA

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